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# Fatigue assessment of amplitude-modulated non-stationary random vibration loading

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## Abstract

Designing mechanical structures exposed to random vibration loading compromises the central challenges of defining comprehensive load assumptions and of processing these efficiently in a fatigue assessment. For this matter of statistical load description, frequency-domain methods withhold major advantages. They describe random vibration loading by its power spectral density, which allows drastic data reduction, to conduct efficient response analyses and to derive a statistical description of resulting load spectra. Nevertheless, this procedure is limited to stationary Gaussian loading. Thus, this paper proposes an extension of the frequency-domain approach to a special class of non-stationary loading – amplitude-modulated processes. These consist of a unique vibration state that varies in intensity, which is represented by a modulating signal. This paper develops a methodology to test for amplitude-modulated processes, to derive efficient measures for the intensity variation and to include this behavior in a fatigue assessment carried out in frequency-domain. The full methodology is presented via a set of simulated data.

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*Keywords:* fatigue assessment, random loading, non-stationary loading, frequency-domain methods, spectrogram

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## 1. Introduction

In numerous technical fields, mechanical structures are subjected to random stresses due to vibration loading. Random vibration loading is common in the automotive, railway and aerospace sector. It must be analyzed within a fatigue strength assessment whose purpose is to ensure the structural integrity and operation for the requested lifetime. This makes it an essential element throughout the design and test stages. Carrying out a fatigue assessment for

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structures exposed to random vibration loading compromises two central challenges. The first is to abstract or to define representative load assumptions and the second is to efficiently process these load assumptions to identify the central failure modes. Both are primarily a matter of statistical load description.

Defining random vibration loading via the power spectral density (PSD) combines an effective statistical description with the ability to efficiently perform fatigue analyses for linear structures using frequency-domain methods such as the Dirlik method. The PSD is a fundamental tool for vibration fatigue. For stationary Gaussian loading the PSD offers a full stochastic description. Nevertheless, most real-world phenomena, e.g. measured field data depicting vehicle vibration, usually differ significantly from this assumption. This is due to changing operational, environmental or excitational conditions which result in diverse vibration with varying intensity [1,2]. An exemplary load is shown in Fig. 1, which was measured on the axle box of a locomotive. The moving root mean square (RMS) value is used to determine the evolution of the signal's intensity. Implementing the PSD for these conditions results in the averaged PSD, which produces lower stress amplitudes than the referencing measured load. Hence processing the PSD for random loading leads to non-conservative deviations for a structure's lifetime estimation [3].

Since the PSD is not a sufficient descriptor for such loading, it must be complemented by stochastic parameters that reflect the deviations from a stationary Gaussian assumption. In recent research, higher-order statistical moments such as skewness and kurtosis have been used to describe how the probability density function (PDF) of realistic loading differs from stationary Gaussian vibration loading [4]. These scalar measures can be very efficient descriptors, however their use requires some knowledge about the nature of a load [5]. This is due to the fact that higher-order moments are sensitive to different effects that lead to a significant deviation from the stationary Gaussian hypothesis. These may be dominant periodical components, nonlinearities or non-stationary behavior. All of these can even occur simultaneously, therefore the description of general vibration loading via higher-order moments may be ambiguous. Such instances were shown in [6] that loads of same PSD and kurtosis value can distinctly deviate in their fatigue potential.

Specifying the nature of a loading can ensure that the use of kurtosis and other higher-order moments is not misleading. Therefore, this paper focuses on a special class of non-stationary loading – amplitude-modulated (AM) processes. They are composed of a single PSD, representing the average vibrational loading, and its varying intensity. Considering vehicle excitations, this non-stationary behavior may be caused by varying surface conditions, roughness, curves or gradient. The varying intensity is expressed by a low-frequency function that modulates the average vibration process. The latter can be represented by a stationary Gaussian process.

This paper gives a comprehensive framework for non-stationary loading under the assumption of an AM origin. It begins with an introduction to statistical load analysis. The second section covers non-stationary loading and reviews the carrier-noise model. An efficient description of these processes is derived on the basis of the abovementioned quantities and how this description can be estimated from real load series. Furthermore, it is demonstrated how the proposed measures are extended to frequency-domain, which enables testing procedures for this model. The effects of AM excitation on fatigue damage is investigated and a correction strategy is proposed.

## Nomenclature

$\alpha_2$	bandwidth parameter	$h(\tau)$	transfer function	$p(x)$	probability density function
$\beta$	kurtosis	$H(f)$	transfer function	$\sigma/\sigma^2$	standard deviation/variance
$D_i$	Dirlik parameter	$\lambda_n$	$n$ -th spectral moment	$\varrho_n$	$n$ -th order irregularity factor
$E[\cdot]$	expected value	$m(t)$	modulating signal	$s$	stress amplitude
$[\cdot]!!$	double factorial	$m_n$	$n^{th}$ -order moment	$x(t)$	excitation signal
$F(s)$	load spectrum	$\mu_x$	mean	$X(t)$	stochastic process
$g(\cdot)$	transformation function	$\mu_n$	$n^{th}$ -order central moment	$X(f)$	Fourier coefficients
$G_{xx}(f)$	power spectral density	$\nu_0$	zero-crossing-rate	$y(t)$	response signal
$\gamma$	skewness	$\nu_p$	peak-rate	$Z$	normalized stress amplitude
AM	Amplitude-Modulation	CN	Carrier-Noise	PDF	Probability Density Function
PSD	Power Spectral Density	RFC	RainFlow Counting	SDOF	Single-Degree-Of-Freedom

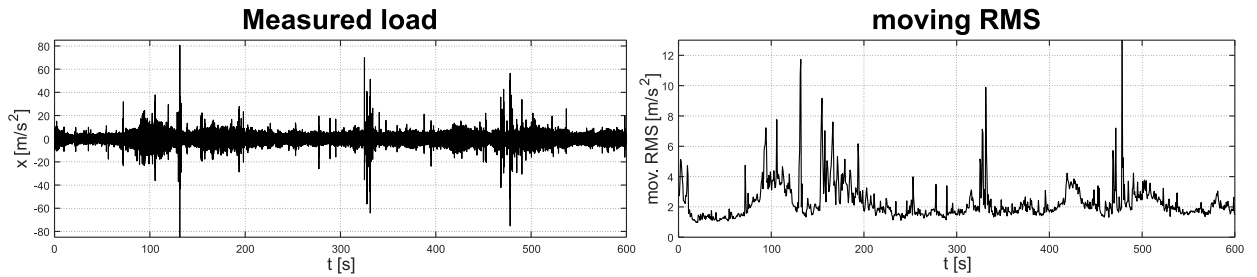


Figure 1. Measured load that varies in intensity

## 2. Statistical load description

Stationary random vibration loading conforming the central limit theorem follows the Gaussian PDF  $p_g(x)$ , which is fully defined by its mean  $\mu_x$  and variance  $\sigma^2$ .

$$p_g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x-\mu_x)^2}{2\sigma^2}\right)} \quad (1)$$

Both quantities belong to a wider range of statistical measures termed statistical moments  $m_n$ , including the mean  $\mu_x = m_1$ , and central moments  $\mu_n$  ( $\sigma^2 = \mu_2$ ) which measure the spread about the mean.

$$m_n = E[X^n(t)] = \int_{-\infty}^{\infty} x^n p(x) dx; \quad \mu_n = E[(X(t) - E[X(t)])^n] = \int_{-\infty}^{\infty} (x - \mu_x)^n p(x) dx \quad (2)$$

The generalization from  $2^{nd}$  to higher-order moments is required for a more thorough description of random loading and to determine the deviation of its PDF  $p(x)$  from the stationary Gaussian assumption. Higher-order moments of a Gaussian distribution  $\mu_{n,g}$  can be stated in terms of the second central moment, with  $[\cdot]!!$  denoting the double factorial.

$$\mu_{n,g} = \begin{cases} (n-1)!! \sigma^n = (n-1)!! \mu_2^{(n/2)} & n \text{ even} \\ 0 & n \text{ uneven} \end{cases} \quad (3)$$

To compare loading of different variance, higher-order moments are normalized by the standard deviation  $\sigma = \sqrt{\sigma^2}$ . This introduces the well-known normalized moments skewness  $\gamma = \mu_3/\sigma^3$ , describing asymmetric properties of the spread, and kurtosis  $\beta = \mu_4/\sigma^4$ , specifying the tails of the PDF. Studying structures subjected to random vibration loading inevitably leads to the PSD  $G_{xx}(f)$ . The PSD depicts the squared amplitudes of all harmonics that compose a load series. The integral of these contributions also results in the second central moment  $\mu_2$ , suggesting the interpretation of the PSD as the decomposition of variance over frequency.

$$\mu_2 = \sigma^2 = \int_0^{\infty} G_{xx}(f) df \quad (4)$$

The frequency-domain description via PSD allows an efficient definition of a random loading as a continuous function, which enables them to be processed in stress response analyses of linear structures or to analyze the response of unknown structures. For stationary Gaussian processes, the PSD carries further information that is accessible via spectral moments  $\lambda_n$ , which are defined by

$$\lambda_n = \int_0^{\infty} f^n G_{xx}(f) df \quad (5)$$

This definition can be related to derivatives in frequency-domain where the PSD of a derivative process  $\dot{x}(t)$  can simply be determined by means of  $G_{\dot{x}\dot{x}}(f) = (2\pi f)^2 G_{xx}(f)$ . On this basis, statistical measures of time-domain quantities, such as zero-crossing-  $v_0$  and peak-rate  $v_p$ , can be stated by spectral moments. Their ratio further indicates the bandwidth  $\alpha_2$  of a vibration process.

$$v_0 = \sqrt{\frac{\lambda_2}{\lambda_0}}; \quad v_p = \sqrt{\frac{\lambda_4}{\lambda_2}}; \quad \alpha_2 = \frac{v_0}{v_p} = \frac{\lambda_2}{\sqrt{\lambda_0\lambda_4}} \quad (6)$$

This establishes a framework that enables a fatigue analysis in the frequency-domain. For narrow-band stationary processes ( $\alpha_2 = 1$ ), the distribution of stress cycles in a structural response can be expressed by an analytical solution based on the Rayleigh distribution [7]. For common broadband processes, empirical solutions such as the Dirlik method have been derived [8]. They approximate the stress cycle distribution by superimposing parametrical distributions. Eq. 8 estimates the load spectrum  $F^{DK}(s)$  via the Dirlik method which requires determining the Dirlik-parameters ( $D_1, D_2, D_3, x_m, Q, R$ ).

$$D_1 = \frac{2(x_m - \alpha_2^2)}{1 + \alpha_2^2}; D_2 = \frac{1 - \alpha_2 - D_1 + D_1^2}{1 - R}; D_3 = 1 - D_1 - D_2; x_m = \frac{\lambda_1}{\lambda_0} \sqrt{\frac{\lambda_2}{\lambda_4}}; Q = \frac{1.25(\alpha_2 - D_3 - D_2 R)}{D_1}; R = \frac{\alpha_2 - x_m - D_1^2}{1 - \alpha_2 - D_1 + D_1^2} \quad (7)$$

$$F^{DK}(s) = D_1 e^{-Z/Q} + D_2 e^{-Z^2/2R^2} + D_3 e^{-Z^2/2}; \quad Z = \frac{s}{\sqrt{\lambda_0}} \quad (8)$$

As Eq. 4 suggests, all this information is limited to processes that are fully defined by the second central moment – stationary Gaussian processes. The following section considers non-stationary loading.

### 3. Non-Stationary loading

Unlike a stationary Gaussian loading which is stochastically fully defined by the PSD, non-stationary resp. non-Gaussian loading, i.e. realistic vibration, must cover additional stochastic parameters. Most of the commonly used parameters, such as the kurtosis are derived from higher-order statistical moments. They describe a non-Gaussian PDF relative to a Gaussian PDF.

This paper covers a concrete model for non-stationary random loading, which consist of a unique vibrational state derived from a PSD, that varies in intensity. The varying intensity can be expressed by a modulating function that describes the evolution of the signal's intensity (magnitudes). As this elementary vibration process is generally a broadband process, the model is resumed as an AM 'carrier-noise' process in reference to the general modulation techniques.

#### 3.1. Carrier-noise model for non-stationary loading

Realistic loading often varies in intensity caused by diverse operational, environmental or excitational conditions, which motivates the definition of the carrier-noise (CN) model. It is based on amplitude-modulation (AM), where a low-frequency information signal generally modulates a high-frequency wave (carrier). To align this simple modulation technique to broadband random vibration, the CN model describes a broadband noise that is modulated by a varying low-frequency intensity. A realization  $x_{am}(t)$  for such a loading must conform to the model

$$x_{am}(t) = x_g(t) m(t) \quad (9)$$

whereby  $x_g(t) \sim \mathcal{N}(\mu = 0, \sigma = \sigma_{am})$  is a stationary zero-mean Gaussian realization that is linearly modulated by the modulating signal  $m(t)$ . The stochastic characteristics of the Gaussian realization are fully defined by the PSD  $G_{xx,g}(f)$ . The modulating function  $m(t)$  is a low-frequency signal which is normalized so that the standard deviation  $\sigma_{am} = \sigma_g$  remains equal.

To take a more detailed view at the latter condition and to establish a concise description for the non-stationary behavior, the  $n^{th}$ -order irregularity factor  $\varrho_n$  is introduced. This measure is motivated by the assumption that an in-service modulating function  $m(t)$  is of random nature and therefore suggests a description analogous to statistical moments. It is derived by inserting Eq. 9 in 2. The mean  $\mu_{x,am} = 0$  is set to zero and since a Gaussian  $X_g(t)$  and a modulating process  $M(t)$  are statistically independent, the expected value operator can be separated. This defines the  $n^{th}$ -order irregularity factor by  $\varrho_n = E[M^n(t)]$ .

$$\mu_{n,am} = E[(X_{am}(t) - \mu_{x,am})^n] = E[(X_g(t)M(t))^n] = E[X_g^n(t)]E[M^n(t)] = \mu_{n,g} E[M^n(t)] = \mu_{n,g} \varrho_n \quad (10)$$

Since the second moment  $\mu_{2,g} = \mu_{2,am}$  defines the condition of equal standard deviation, a normalized modulating function can now be stated by  $\varrho_{2,m} = 1$ . In case of a non-normalized realization  $\tilde{m}(t)$ , this condition can be executed via  $m(t) = \tilde{m}(t)/\sqrt{\varrho_{2,\tilde{m}}}$ . Generally, irregularity factors  $\varrho_n$  can only be calculated for even moments for the reason

that uneven Gaussian moments are zero (Eq. 3). Nevertheless, due to the symmetric Gaussian distribution, absolute moments may be used to determine irregularity factors  $\varrho_n$  of any arbitrary order. In order to derive the  $n^{th}$ -order irregularity factor  $\varrho_n$  from a given load  $x_{am}(t)$ , the ratio of the even higher-order moments  $\mu_{n,am}$  to its corresponding Gaussian moments  $\mu_{n,g}$  is determined (compare Eq. 3).

$$\varrho_n = \frac{\mu_{n,am}}{\mu_{n,g}} = \frac{1}{(n-1)!!} \frac{\mu_{n,am}}{\mu_{2,g}^{(n/2)}} \tag{11}$$

The proposed description permits an efficient definition for a non-stationary random loading of AM origin by its PSD  $G_{xx,g}(f) = G_{xx,am}(f)$  and a set of irregularity factors, e.g.  $\varrho_4, \varrho_6, \dots$ , that can be directly related to the statistical moments and the PDF of a referencing load. They indicate the deviation from a stationary Gaussian loading ( $\beta_g = 3$ ), here illustrated by the kurtosis  $\beta_{am} = \beta_g \cdot \varrho_4 = 3\varrho_4$ .

### 3.2. Generating a carrier-noise load

In order to synthetically generate a varying load signal  $x_{am}(t)$ , a low-frequency signal  $m(t)$  is required that represents the variation of intensity whose magnitudes modulate the high-frequency broadband 'carrier-noise'  $G_{xx,g}(f)$  of the actual vibration process within the time-domain. This obligates the modulating signal  $m(t)$  to have the same length as the stationary Gaussian realization  $x_g(t) \sim G_{xx,g}(f)$ . It is created in a way, that the resulting load series  $x_{am}(t)$  has a specified kurtosis  $\beta_{am}$ .

Here it is generated as a random realization through the following steps:

- (i) Generating a stationary Gaussian realization  $x_g(t)$  from the PSD  $G_{xx,g}(f) = G_{xx,am}(f)$  which functions as the 'carrier-noise'.
- (ii) Defining a PSD shape  $G_{xx,m}(f)$  for the modulating signal  $m(t)$  that has a low-frequency content. Thereafter a stationary Gaussian realization  $m(t) \sim \mathcal{N}(\mu = 0, \sigma = 1)$  is generated with the same sampling as the 'carrier-noise'  $x_g(t)$ .
- (iii) Using the absolute values  $|m(t)|$  which ensures positive values. This prevents complications with the operations applied in the following step and produces an initial kurtosis  $\beta_{am} > 3$ .
- (iv) Iterating the exponent  $p$  (stretching the PDF) and adding a tolerance  $\Delta m$  (threshold to zero) until a desired kurtosis value is generated.

$$x_{am}(t) = x_g(t) (|\tilde{m}(t)|^p + \Delta m) \tag{12}$$

### 3.3. Testing procedure for carrier-noise model

While the former section presents a way of synthetically generating random AM loads, real applications raise the question of attributing loading to this model. A subjective approach would be to estimate the time-frequency evolution of a reference load via a spectrogram, which contains  $k = 1, 2, \dots, K$  frequency intervals and  $l = 1, 2, \dots, L$  time segments (compare Fig. 2). Visually examining the spectrogram can lead to the conclusion that the load has a unique PSD shape that varies in intensity and thus appears to be of AM origin. However, this subjective approach has two flaws, namely the spectrogram depends heavily on its parameters (window-length, -function, -overlap, etc.) and its results are based on an exclusively visual observation. Thus, a new measure is proposed that verifies the model quantitatively from the spectrogram. It is based on the assumption that the variation in intensity can also be found in the frequency-selective evolution of the spectrogram. Thus, the proposed irregularity factors  $\varrho_n \rightarrow \varrho_n(f)$  are extended to frequency. Its derivation is shown in Fig. 2. The

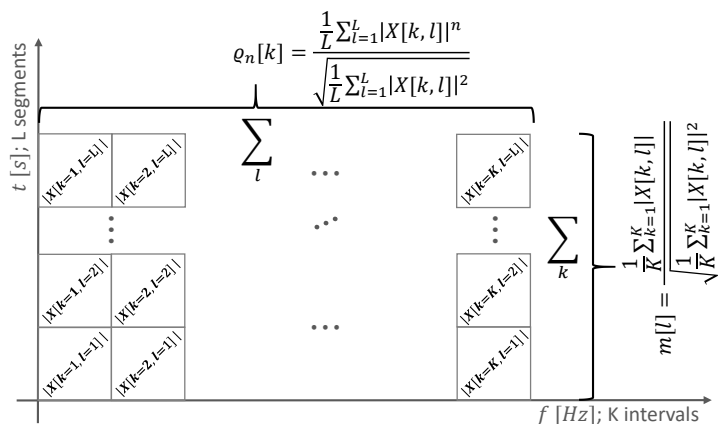


Figure 2. Schematic visualization of spectrogram and derivable quantities.

hypothesis is that if  $\varrho_n(f)$  is constant along the frequency axis and equals the global value  $\varrho_n$ , then the loading is of an AM origin. In practical application (compare Fig. 3), due to the random nature and the data split, the function  $\varrho_n(f)$  fluctuates around the global value  $\varrho_n$ . Thus, it should be pointed out that the conformity of  $\varrho_n(f)$  with  $\varrho_n$  can strongly depend on a suitable choice of the spectrogram's parameters. Fig. 2 further shows how a modulating signal can be recovered from an AM loading by summation and normalization along frequency.

### 3.4. Adjustment procedure on response load spectra

The following section proposes an approach to reflect non-stationary behavior caused by varying intensity within a fatigue strength assessment and to implement load spectra estimators in frequency-domain. This involves the response behavior of structures subjected to an excitation. General mechanical structures are modelled as linear time-invariant systems, allowing the use of linear operator theory. This implicates that a linear time-invariant system responds proportionally to a varying intensity. Thus, the following approach takes on the separation of the AM non-stationary excitation  $x_{am}(t) = x_g(t) m(t)$  to assemble the response  $y_{am}(t)$  in the same manner, which results in a stationary Gaussian response process  $y_g(t)$  that is modulated by  $m(t)$ . The calculation can be carried out in time- (homogenous and particularly solution  $y_g(t)$ ) or frequency-domain (particularly solution  $\tilde{y}_g(t)$ ):

$$y_{am}(t) = y_g(t) m(t) = \int_0^\infty h(\tau) x_g(t - \tau) d\tau m(t); \tilde{y}_{am}(t) = \tilde{y}_g(t) m(t) = iFFT\{X_g(f)H(f)\} m(t) \quad (13)$$

In contrast to a direct solution of the non-stationary excitation ( $h(\tau)x_{am}(t - \tau)$ ), this procedure provides the opportunity to describe loading and response via statistical quantities. Therefore, the response analysis may also be carried out via PSD  $G_{yy,g} = G_{xx,g}(f)|H(f)|^2$ . As a consequence, frequency-domain methods can be adapted into the fatigue assessment. This allows to estimate the load spectrum  $F_{y,g}(s)$  using frequency-domain estimators such as the Dirlik-method (Eq. 8).

To tie in the non-stationary behavior, its influence is now be considered in terms of effects on the resulting load spectrum  $F_{y,am}(s)$ . While the normalized modulating signal  $m(t)$  does not affect the PSD, it does deform the PDF as well as the load spectrum compared to a stationary Gaussian process. This deformation can already be assessed from the excitation, comparing the excitational AM load spectrum  $F_{x,am}(s)$  to its corresponding stationary Gaussian spectrum  $F_{x,g}(s)$ . This defines a transformation  $g(\cdot)$  that corrects the load spectra of a linear time-invariant system under varying intensity.

$$g(\cdot) = \frac{F_{x,am}^{-1}(s)}{F_{x,g}^{-1}(s)} \quad (14)$$

Resulting load spectra  $F_{y,am}(s)$  can thereupon be derived by:

$$F_{y,am}(s) = g(F_{y,g}(s)) \quad (15)$$

The transformation  $g(\cdot)$  requires a single evaluation for the excitation and is then valid for all response load spectra. This procedure barely extends the expense of the efficient frequency-domain fatigue estimation and will reduce the duration of a fatigue assessment for random vibration loading by several orders compared to time-domain.

The presented approach is based on the prerequisite that a vibration excitation can be identified as having an AM origin and thus can be decomposed into a stationary Gaussian process and its varying intensity. As a consequence, the non-stationary and stationary response have the same PSD. Therefore, both responses share many fundamental characteristics. For example, the non-stationary and its corresponding stationary response have the same zero-crossing- and peak-rate and thus result in the same amount of damaging cycles (compare Eq. 5). The isolated non-stationary properties described by  $m(t)$  merely modifies the PDF and load spectra. In case that an excitation is non-stationary because of a composition of various vibrations states this conclusion does not apply. These different vibration states affect the response PSD in a way that it cannot be predicted on the basis of the averaged PSD  $G_{yy,am} \neq G_{yy,g}(f)$ .

## 4. Sample Data

A simple single-degree-of-freedom system (SDOF) is used to determine and compare the fatigue damage caused by a stationary Gaussian and an AM loading. Fig. 3 covers the simulation of a SDOF with a damped eigenfrequency

$f_D = 60$  Hz. The left-hand side of Fig. 3 assembles the information of the excitation. It shows a Gaussian realization  $x_g(t) \sim G_{xx,g}(f)$  that is derived from the PSD. The Gaussian realization is modulated by the synthetically generated modulating signal representing the non-stationary character. Its effects can be summarized by the irregularity factors  $\varrho_2, \varrho_4$  that preserve the variance but scale the kurtosis. The representation of the latter over frequency  $\varrho_4(f)$  suggests a uniform modulation of the frequency content. Applied to measured data, this would mark the starting point of the method. The bottom shows the load spectra (inverted) that are derived from the Gaussian and the AM excitation. Their ratio defines the transformation  $g(\cdot)$  due to the non-stationary behavior which is to be retrieved in a response analysis.

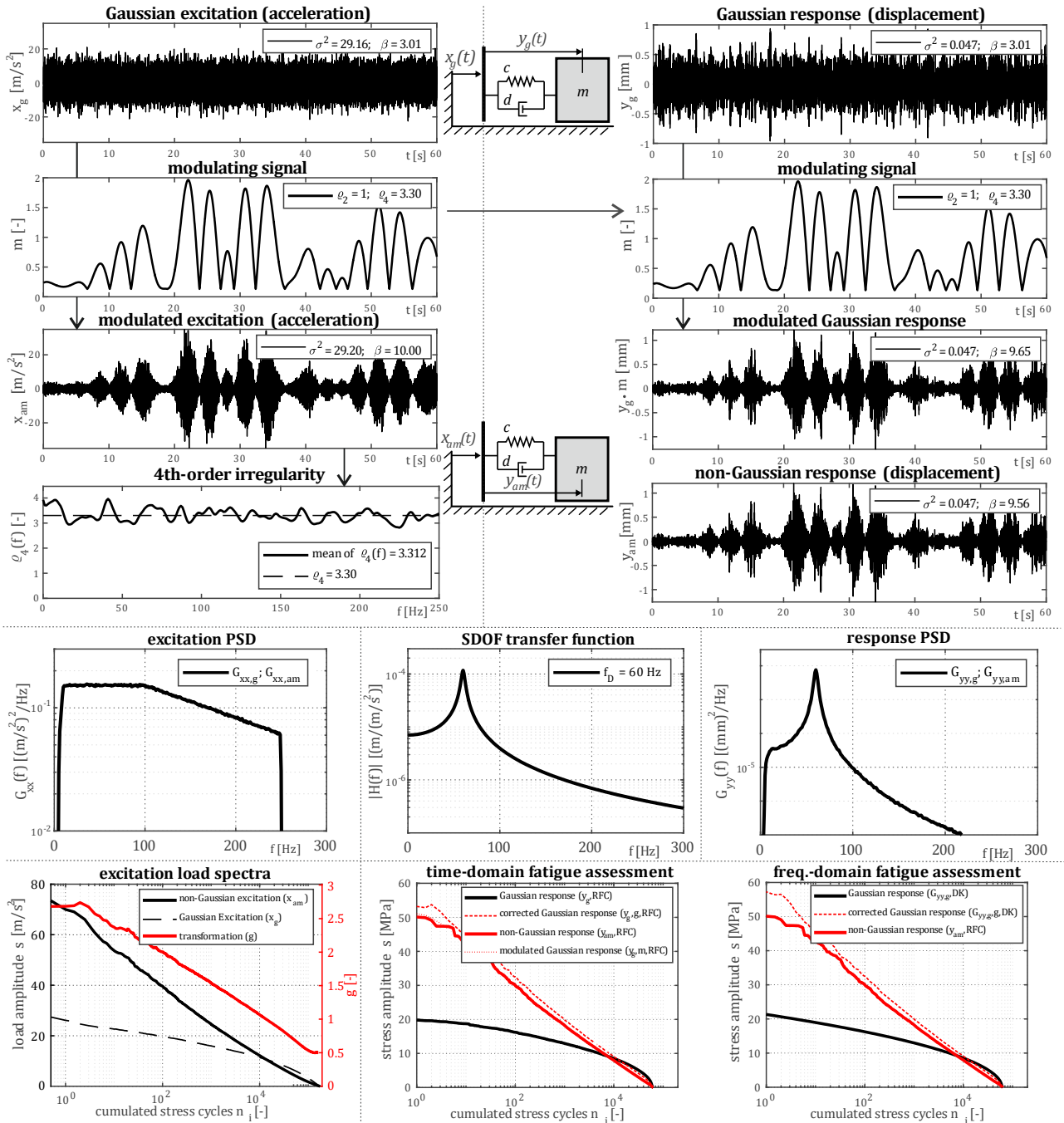


Figure 3. Simulation of SDOF system subjected to AM non-stationary loading.

The right-hand side depicts the evaluation of the SDOF. On the top it shows the response  $y_g(t)$  caused by the Gaussian excitation which is expectedly Gaussian. The modulating signal is used to modulate the response  $y_g(t) m(t)$ . This serves as a visual verification to the following time series, showing the non-Gaussian response  $y_{am}(t)$  due to the AM excitation  $x_{am}(t)$ . This confirms the equivalence of the non-Gaussian  $y_{am}(t)$  and the decomposed Gaussian  $y_g(t) m(t)$  responses, which emphasizes the assumption that the non-stationary behavior is transferred through the time-invariant system. Both responses result in the same PSD. The lower plots show the load-spectral evaluation of the responses. The discrepancies in the Gaussian and non-Gaussian load spectra remain unchanged. By applying the transformation  $g(\cdot)$  derived from the excitation load spectra, which serves as a correction function, the Gaussian load spectra can be adjusted to the non-stationary behavior. The lower right plot illustrates the central advantage of this procedure – which is to carry out the fatigue assessment in frequency-domain by using load spectra estimators. Extending this simulation to further SDOF systems (e.g. Fatigue Damage Spectra, finite-element mesh) would show that non-stationary excitation scales the damage of the corresponding stationary excitation proportionally upwards. In case of a stationary excitation, the transformation  $g(\cdot) = 1$  has no effect and  $F_{y,am}(s) = F_{y,g}(s)$ .

## 5. Conclusion

This paper discusses non-stationary amplitude-modulated (AM) vibration loading. Through the carrier-noise (CN) model, it is possible to generate synthetic loads that approximate realistic loading. While this portrays a limited class of non-stationary loading, it is worthwhile to be analyzed in detail. The CN model, in its simple composition, clearly addresses the disadvantages of a frequency-domain approach for non-stationary processes. It is that the power spectral density (PSD) averages non-stationary behavior but it does not cover any information of its evolution. Loading that follows a non-stationary characteristic generally causes greater damage than its averaged counterpart. The CN model provides a corrective approach to handle these flaws. Based on the assumption that a vibration process consists of a unique PSD shape, the non-stationary evolution can fully be expressed by a modulating function. To avoid the effort related with the modulating function's estimation, it is proposed to use irregularity factors. The concept is well known from higher-order moments to which it integrates seamlessly. By expanding these irregularity factors to frequency, it is further possible to test for the amplitude-modulated origin. It is shown how this can be done on the basis of the well-known spectrogram. Further, this paper addresses the desire to reflect the non-stationary behavior in an efficient fatigue assessment. Thus, the existing theory of frequency-domain fatigue assessment is extended to AM non-Gaussian vibration loading. A correction strategy is proposed on the basis of the excitational load spectra. Sample data shows how this extends the applicability of a frequency-domain fatigue assessment.

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