

# Comparison of basic ferromagnetic coil hysteresis models

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## Abstract

In this paper scalar hysteresis models are discussed such as Jiles-Atherton, Preisach, Chua, Choleman-Hogdon and Bouc-Wen model. Hysteresis modeling problem will be discussed on example of ferromagnetic coil. Parameter estimation procedure for each model is presented briefly. Hysteresis models are compared by number of parameters, complexity of parameter estimation procedure and accuracy.

## 1. Introduction

The phenomenon of hysteresis is encountered in many different areas of science and technology. The hysteresis is a special type of dynamic nonlinearity, because it is multi-branching nonlinearity. The best known examples can be found in ferromagnetism, piezoelectricity, spin glasses, semiconductors, economics and physiology.[1]

The hysteresis phenomenon can be useful for example in magnetic recording for the storage of information (hard disk). While in most of other applications hysteresis effects have harmful impact on device performance thus they should be appropriately reduced or neutralized. [1]

For example, in modeling transformer transients, it is necessary to reproduce hysteretic properties of the core material [2], [3]. The need to account for these features arises when modeling transformer energizing and reenergizing, short-circuit events and operation of generator transformers under synchronization. The ability of the transformer model to simulate hysteresis and core dynamics is also required in describing voltage switching and ferroresonance phenomena [4] and to distinguish between frequency contents of inrush and short-circuit currents [5]. This list can be extended with the analysis of geomagnetically induced currents [6] as well as with modeling current, pulse, and transducer transformers [7], [8], for which hysteretic and eddy-current effects are closely linked. [9]

In this paper five different hysteresis models are considered. Four of them originate from magnetic hysteresis modeling problem and one (Bouc-Wen) model is originally used in vibration mechanics.

## 2. Jiles-Atherton model

In the 1980's magnetic hysteresis model was proposed by D. Jiles and D. Atherton [10]. Since then model is often used due to physical meaning of parameters, but main drawback of JA model is difficult parameters estimation procedure.

### 2.1. Model equations

Mathematical model is given by equations (2.1). It consists of two parts: 1) anhysteretic curve that relates magnetization to magnetic field intensity; 2) a differential equation whose solution

is hysteresis curve wrapped around anhysteretic curve. There are five parameters ( $\alpha$ ,  $k$ ,  $a$ ,  $M_s$ ,  $c$ ) for estimation procedure.

$$M_{an} = M_s \left[ \coth \frac{H + \alpha M}{a} - \frac{a}{H + \alpha M} \right] \quad (2.1)$$

$$\frac{dM}{dH} = \frac{1}{1+c} * \frac{M_{an} - M}{k * \operatorname{sgn}\left(\frac{dH}{dt}\right) - \alpha(M_{an} - M)} + \frac{c}{1+c} \frac{dM_{an}}{dH} \quad (2.2)$$

Where  $M_{an}$  is anhysteretic magnetization of magnetic core,  $M_s$  is saturated magnetization occurred during one cycle of excitation period,  $\alpha$  is average coefficient describing the coupling between magnetic material domains,  $a$  is describing shape of anhysteretic curve,  $c$  is coefficient related with reversible and irreversible domain wall movement.

Since hysteretic behavior is very often expressed in terms of B-H characteristics rather than M-H characteristics, equation (2.3) gives proper transition.

$$\frac{dB}{dH} = \mu_0 \left( 1 + \frac{dM}{dH} \right) \quad (2.3)$$

Where  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$  is magnetic permeability of vacuum.

## 2.2. Estimation methods for JA model

For estimation procedure measurements of B-H pairs of hysteresis curve in deep saturation is needed. From equations (2.1, 2.2) and measured hysteresis pairs model parameters can be estimated. Model parameters are estimated in terms of least squares, thus cost function is:

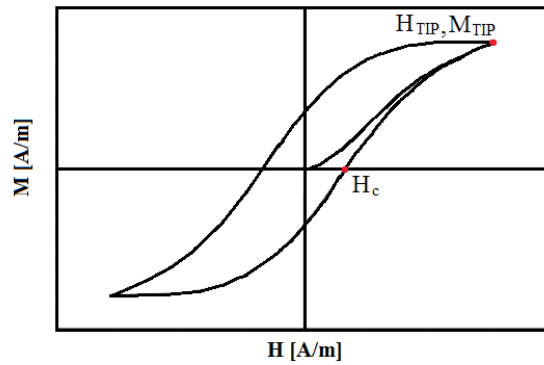
$$S = \sqrt{\frac{\sum_{i=1}^N (B_{i,estimated} - B_{i,measured})^2}{N}} \quad (2.4)$$

Where N is number of samples.

Since equations (2.1) and (2.2) form nonlinear least squares problem, estimation process is performed in iterative manner. In iterative calculation good initial guess of unknown parameters is very important because it can reduce computing time. Classical iterative approach such as Lavenberg-Marquardt algorithm can be used [11]. Main drawback of classical methods is local/global optimum convergence problem if poor initial guess is chosen.

In literature [12] are given intervals for choosing good initial guess based on characteristic points in M-H plane. Knowing values of coercive field  $H_c$ , maximum field  $H_{TIP}$  and maximum magnetization  $M_{TIP}$  (figure 2.1).

$$\begin{aligned} \alpha &\in \langle 0, 5H_c/M_{TIP}; 0, 7H_{TIP}/M_{TIP} \rangle \\ a &\in \langle 0, 5H_c; 5H_c \rangle \\ c &\in \langle 0; 1 \rangle \\ k &\in \langle 0, 5H_c; 5H_c \rangle \\ M_s &\in \langle M_{TIP}; 1, 5M_{TIP} \rangle \end{aligned}$$



**Figure.2.1:** Characteristic points on M-H plane used for choosing good initial guess.

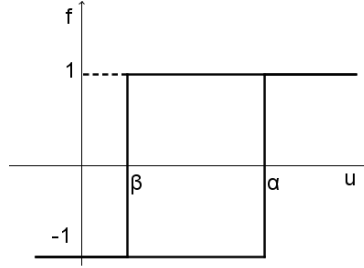
Literature [10] reveals better methods for nonlinear least squares problem like artificial intelligence algorithms: genetic algorithms, particle swarm optimization and neural networks. These algorithms can avoid local/global optimum problem due to exploration of parameter space inside defined boundaries. So good initial guess is not necessary, but can reduce computation time. Those algorithms still demands boundaries of unknown parameters, better (narrower) boundaries also reduces computation time.

### 3. Preisach model

German physicist F. Preisach proposed a scalar model of magnetic hysteresis based on some physical mechanisms of magnetization [13]. Model is further developed by Russian mathematician Krasnoselskii which made model purely mathematical i.e. applicable to any hysteretic phenomenon. [14]

#### 3.1. Mathematical model

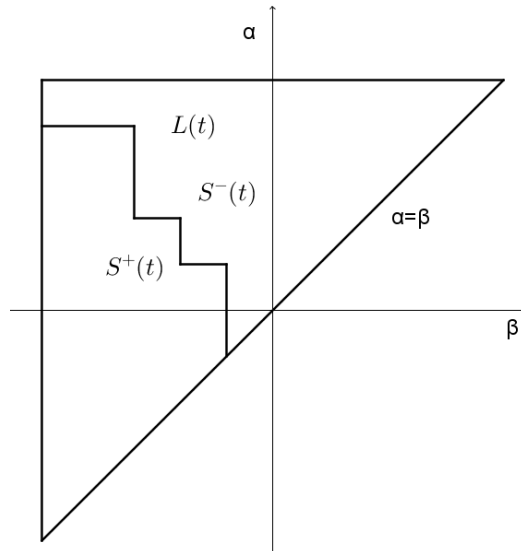
Elementary part of Preisach model is hysteresis operator (Fig. 3.1).



**Figure 3.1:** An elementary hysteresis operator  $\gamma_{\alpha\beta}$

$$f(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma(\alpha, \beta, u(t)) d\alpha d\beta \quad (3.1)$$

Output  $f(t)$  is calculated using equation (3.1), where  $\mu(\alpha, \beta)$  represents continuous weight function, that needs to be estimated, and  $u(t)$  is input signal. Preisach model is often represented in  $\alpha\beta$  plane (Fig.3.2) because there is one to one relation between each elementary hysteresis operator and point in  $\alpha\beta$  plane. Only upper half of  $\alpha\beta$  plane is considered due to elementary hysteresis operator demands  $\alpha \geq \beta$ .



**Figure 3.2:** Preisach  $\alpha\beta$  plane

In  $\alpha\beta$  plane staircase line  $L(t)$  can be constructed which represents boundary between hysteresis operators:  $S^+(t)$  that represents hysteresis operators with output value 1 and  $S^-(t)$  that represents hysteresis operators with output value -1. Line  $L(t)$  is constructed using two simple rules: line is a horizontal one and moving up when the input  $u(t)$  increases, and it is a vertical one and moves from right to left when input decreases. Using description of Preisach model in  $\alpha\beta$  plane output value can be represented in following form:

$$f(t) = \iint_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta - \iint_{S^-(t)} \mu(\alpha, \beta) d\alpha d\beta \quad (3.2)$$

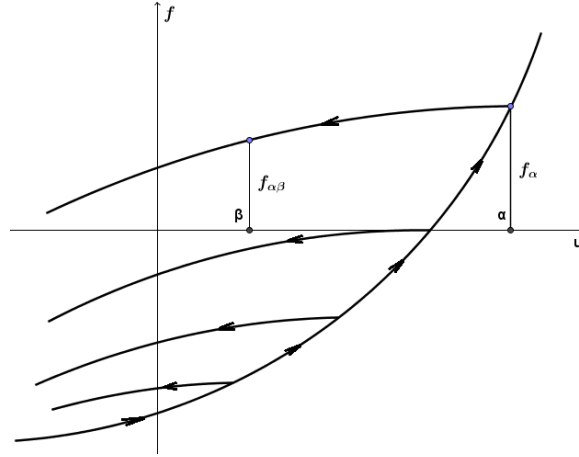
Output  $f(t)$  function can also be expressed, in simpler way, using Everett function  $E(\alpha, \beta)$  that represents double integral of weight function  $\mu(\alpha, \beta)$ .

$$f(t) = -E(\alpha_0, \beta_0) + 2 \sum_{k=1}^n [E(\alpha_k, \beta_{k-1}) - E(\alpha_k, \beta_k)] \quad (3.3)$$

Where  $\alpha_k$  and  $\beta_k$  represents decreasing and increasing sequences of  $\alpha$  and  $\beta$  coordinates of breaking points on staircase line  $L(t)$ , and  $n$  is number of horizontal lines.

### 3.2. Estimation methods for Preisach model

Measurements required for estimation, in classical sense, of  $\mu(\alpha, \beta)$  are very extensive because first order reversal curves are necessary.



**Figure 3.3:** First order reversal curves and limiting branch

In figure 3.3. first order reversal curves are shown along with a limiting ascending branch. Lowest branch is called limiting because there is no branch below it. Knowing previously described branches Everett function can be calculated using equation (3.4). In order to get better results first order reversal curves should be placed as close as possible.

$$E(\alpha, \beta) = (f_\alpha - f_{\alpha\beta})/2 \quad (3.4)$$

For calculation of output function from known input function only Everett function is sufficient, there is no need to find weight function  $\mu(\alpha, \beta)$ , but relation between Everett and weight function  $\mu(\alpha, \beta)$  is shown for sake of completeness.

$$\mu(\alpha, \beta) = -\frac{\partial^2 F(\alpha, \beta)}{\partial \beta \partial \alpha} \quad (3.5)$$

## 4. CHUA MODEL

In 1970 L.O. Chua presented nonlinear coil model based on magnetic materials hysteretic behavior [15]. Model parameters are relatively easy to estimate and measurements are

relatively simple. Chua model have also lumped circuit elements representation that is practical for software implementation such as Pspice.

#### 4.1. Mathematical model

The model is completely specified by two strictly monotonically increasing functions: a restoring function:  $f(\cdot)$  and a dissipation function  $g(\cdot)$ .

$$\frac{dy}{dt} = g[x(t) - f(y(t))] \quad (4.1)$$

If  $x(t)$  represents inductor current  $i(t)$  and  $y(t)$  represents flux linkage  $\varphi(t)$  equation (4.1) becomes

$$\frac{d\varphi(t)}{dt} = g[i(t) - f(\varphi(t))] \quad (4.2)$$

This equation can be expressed in next form:

$$i(t) = g^{-1}(u(t) + f(\varphi(t))) \quad (4.3)$$

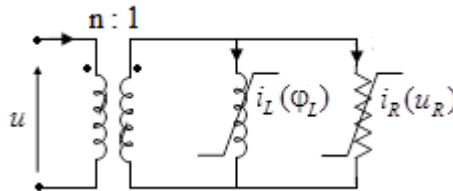
Where  $u(t) = d\varphi(t)/dt$  is voltage across inductor. Equation (4.3.) represents parallel combination of a nonlinear resistor and nonlinear inductor, where the  $v_R - i_R$  relationship of the resistor is given by

$$v_R = g(i_R) \quad (4.4)$$

And the  $i_L - \varphi_L$  relationship of the inductor is given by

$$i_L = f(\varphi_L) \quad (4.5)$$

The circuit model for a ferromagnetic core inductor of  $n$  turns is shown in figure 4.1, where the transformer shown is an ideal transformer. The characteristics of the nonlinear resistor are based on the terminal properties of a winding of one turn, thus characteristics of lumped elements is unchanged if different numbers of turns are applied to iron core.



**Figure 4.1:** Lumped circuit model of nonlinear inductor

## 4.2. Estimation of nonlinear coil characteristics

In order to identify these functions a pair of waveforms  $\{x(t), y(t)\}$  must be measured. For symmetric hysteresis loops restoring function:  $f(\cdot)$  and dissipation function  $g(\cdot)$  can be simply reconstructed. Restoring function:  $f^{-1}(\cdot)$  is defined as average values of two different input values  $x(t_1), x(t_2)$  that have same ordinate  $y(t_1) = y(t_2)$ , see fig.4.2. Dissipation function  $g(\cdot)$  is defined by distance  $d$  between midpoint and edge of hysteresis. Functions  $f(\cdot)$  and  $g(\cdot)$  are odd symmetric if hysteresis loop is odd symmetric.

If the given device is not characterized by symmetric hysteresis loops, then  $f(\cdot)$  and  $g(\cdot)$  need not be odd functions. In this case, appropriate optimization techniques may be used to determine the two functions  $f(\cdot)$  and  $g(\cdot)$ , which best approximate a given hysteresis loop.

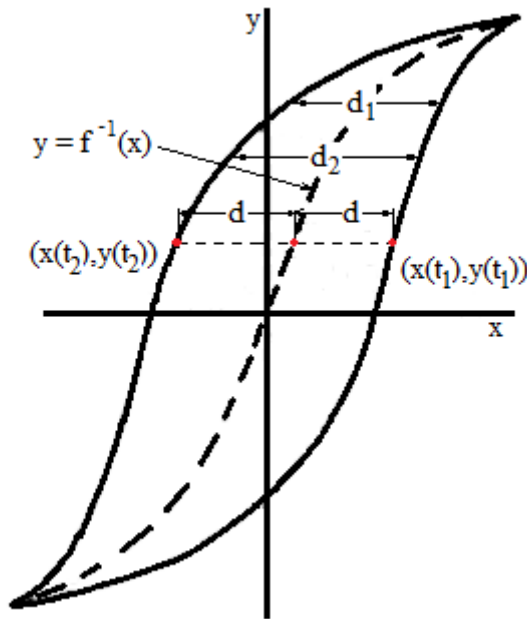


Figure 4.2: Geometrical representation of estimation procedure

## 5. Coleman-Hodgdon model

The Coleman-Hodgdon model was formulated in 1986 to describe rate-independent hysteresis in soft ferromagnetic materials [16]. For a class of hysteresis systems a first order scalar time-domain differential equation can be used to describe hysteresis behavior. A relatively simple differential model of hysteresis, which is appropriate for the representation of rate independent hysteretic systems, is the so-called Coleman-Hodgdon model proposed in [17, 18]. This model (a first-order nonlinear differential equation) is able to capture, in an analytical form, a range of shapes of hysteretic loops, which match the behavior of a wide class of hysteretic systems.

### 5.1. Mathematical model

Mathematical Coleman-Hodgdon model can be expressed by equation:

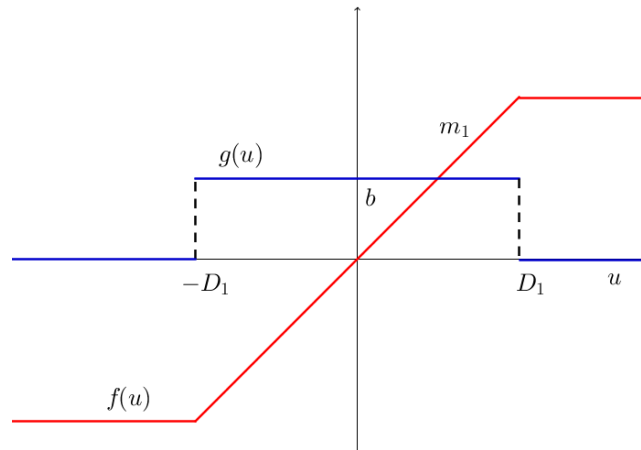
$$\frac{dy}{dt} = f(u, y) \left| \frac{du}{dt} \right| + g(u, y) \frac{du}{dt} \quad (5.1)$$

In literature [1] is given simple piecewise linear representation of functions  $f(u, y)$  and  $g(u, y)$  by expressions:

$$f[u(t)] = \begin{cases} m_1 D_1, & D_1 < u(t) \\ m_1 u(t), & -D_1 \leq u(t) \leq D_1 \\ -m_1 D_1, & u(t) < -D_1 \end{cases} \quad (5.2)$$

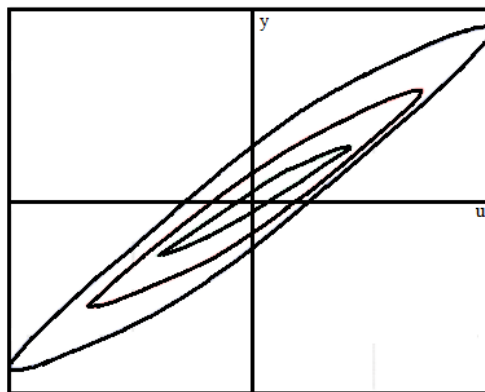
$$g[u(t)] = \begin{cases} 0, & D_1 < u(t) \\ b, & -D_1 \leq u(t) \leq D_1 \\ 0, & u(t) < -D_1 \end{cases} \quad (5.3)$$

Where  $m_1$  is slope of central segment of function  $f(\cdot)$ , the constant  $D_1 > 0$  determines range of central segment as well as the range of  $g(\cdot)$  with constant value  $b$  — see figure 5.1. Number of linear segments can be increased if better accuracy is needed. Also it is possible to model asymmetrical hysteresis loops, then function  $f(\cdot)$  is not odd symmetric while function  $g(\cdot)$  remains even symmetric.



**Figure 5.1:** Geometrical representation of piecewise linear functions  $f(\cdot)$  and  $g(\cdot)$

In figure 5.2. symmetric hysteresis loops, obtained by Coleman-Hodgdon model, are shown.



**Figure 5.2.:** Example of hysteresis curves



## 5.2. Estimation procedure

For estimation procedure pairs of input/output variables are required. In literature [1] is given iterative procedure for parameter estimation in sense of least squares problem. Parameter estimation problem is pseudo linear because functions  $f(\cdot)$  and  $g(\cdot)$  are piecewise linear. Number of parameters to be estimated is dependent on number of required piecewise sections of functions  $f(\cdot)$  and  $g(\cdot)$ , thus parameter estimation for asymmetrical hysteresis loops is more complicated because symmetry is broken and more piecewise lines are required to describe hysteretic behavior.

## 6. Bouc-Wen model

The Bouc model of hysteresis —further modified by Wen —was initially used for nonlinear vibrational mechanics. It is important because it can model various shapes of hysteretic behavior booth static and dynamic. [19]

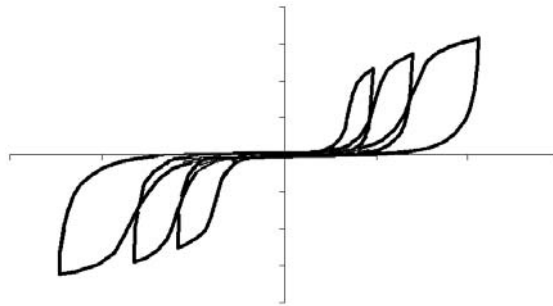


Fig.6.1.: Exotic hysteresis shapes

### 6.1. Mathematical model

The hysteretic BW model [20] consists of a nonlinear system described by differential equations.

$$\begin{cases} \frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \alpha\omega_n^2x + (1 - \alpha)\omega_n^2z = u(t) \\ \frac{dz}{dt} = -\gamma \left| \frac{dx}{dt} \right| |z|^{n-1}z - \beta \frac{dx}{dt} |z|^n + A \frac{dx}{dt} \end{cases} \quad (6.1)$$

Model is described with seven parameters.

$\alpha$  – rigidity ratio ( $0 \leq \alpha \leq 1$ )

$\xi$  - linear elastic viscous damping ratio ( $0 \leq \xi \leq 1$ )

$\omega_n$  - pseudonatural frequency of the system (rad/s)

$A$  - parameter controlling hysteresis amplitude

$\beta, \gamma, n$  – parameters controlling hysteresis shape ( $n \geq 1$ )

Very often equation is described in space state notation.

$$\begin{cases} \frac{dY_1}{dt} = Y_2 \\ \frac{dY_2}{dt} = -2\xi\omega_n Y_2 - \alpha\omega_n^2 Y_1 - (1 - \alpha)\omega_n^2 Y_3 + u(t) \\ \frac{dY_3}{dt} = -\gamma|Y_2||Y_3|^{n-1}Y_3 - \beta Y_2|Y_3|^n + AY_2 \end{cases} \quad (6.2)$$

$$[Y_1 \quad Y_2 \quad Y_3] = \left[ x \quad \frac{dx}{dt} \quad z \right]$$

## 6.2. Parameter estimation procedure

In literature [20] proposed method for parameter estimation is evolutionary computation. Measurement required for parameter estimation is set of input/output variables. It is also stated that BW model is rarely used in magnetism due to difficult estimation procedure and because it is necessary to detect the values of seven (or even in some case nine) parameters. In comparison with Jiles-Atherton model it is clear that BW model is not competitive because Jiles-Atherton model have only five parameters to estimate.

## 7. Conclusion

Since hysteretic behavior can be found in different scientific areas approaches in modeling are also wide and multidisciplinary. Quality of each approach is measured by several criterions: accuracy, difficulty of parameter estimation procedure and required measurements (due to physical limitations), Table 1.

Table 1. Comparison of modeling methods

	Error	Complexity of parameter estimation procedure	Complexity of required measurements
Jiles - Atherton	Medium	High	Low
Preisach	Low	Low	High
Chua	Medium	Medium	Low
Coleman - Hogdon	Medium	Medium	Low
Bouc - Wen	Medium	High	Low

Since parameter estimation procedure is often presented by nonlinear estimation problem, strong impact on applicability of each hysteresis model have methods used for solving such optimization problems. Modern optimization methods such as artificial intelligence and heuristic algorithms have bright future in hysteresis modeling problem. Modern computers made possible to automatize measurement procedure so special types of input functions can be generated to make estimation procedure simpler.

Future research will address the model ability to present hysteresis peculiarities like saturation, minor and major loops and impact of excitation frequency.

## 8. Literature

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