Characterization of Higher Order Modes in Optical Fibers

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Abstract

Higher-order-mode fibers generate today a great interest and find various applications in new photonic devices. The knowledge of the intermodal dispersion characteristics of higher-order-mode fibers is of interest for dispersion compensation, fiber refractive-index profile characterization, and a variety of in-fiber devices. In this context it is necessary to identify the individual transverse modes and to precisely characterize the dispersion properties of these modes.

On the other hand, light beams, possessing special polarization structure, have recently attracted a great deal of interest. Several fiber techniques have been developed to generate such beams while employing higher-order modes of respective shapes. For example, radially or azimuthally polarized doughnut beams have found applications in various fields. They have a field similar to $TM_{01}$ or $TE_{01}$ vector mode in the $LP_{11}$ mode group of a circular fiber. Therefore, these doughnut beams can be considered to be generated by an optical fiber. It is shown in the present work that in practice a $TM_{01}$ or $TE_{01}$ mode can not propagate in step-index fibers which support $LP_{01}$ and $LP_{11}$ modes. The reason for this is the anisotropy of the fiber and the degeneracy of the modes in the $LP_{11}$ mode group. However, these doughnut beams can be generated at the end facet of the fiber by selective excitation of $LP_{11}$ modes in the fiber under adequate pressure and twist. Unfortunately, since this mode selection process involves several fiber modes propagating at different phase velocities, the transverse intensity distribution will vary strongly along the fiber. However, in many applications, a stable transverse beam pattern is required. In order to excite a single higher-order mode in optical fibers, a full characterization of the intermodal dispersion and mode profiles as well as the polarization state of these modes is essential.

This thesis focuses on a novel time-domain low-coherence interferometry technique for characterizing the intermodal dispersion and mode profiles of any type of higher-order-mode optical
fibers. The measurement can be done without any prior knowledge of the fiber properties. The interferometry uses an erbium-doped fiber amplifier (EDFA) as a low-coherence broadband light source. In the fiber arm the light is coupled into the tested few-mode fiber. In the reference arm, a mirror is controlled to obtain the same optical path as each of fiber modes. A phase-shifting algorithm is used to interpret the interference fringes obtained on a camera at the interferometer’s output. The mode intensity profile and the amplitude of the mode are obtained in real time. Depending on the features of the tested fiber, an interferometer of Michelson or Mach-Zehnder type is used.

One of the fibers under test is a few-mode all-glass fiber with special refractive-index profile. The Michelson low-coherence interferometer is used to characterize the intermodal dispersion and the mode profiles of the fiber. The measured intermodal dispersion of the LP_{01}, LP_{11}, LP_{21} and LP_{02} modes agrees very well with the results of our numerical simulations. It is shown that the four modes in the LP_{11} mode group of the tested fiber have a larger group-index difference than those of step-index fibers. This is indeed the advantage of the fiber design. The group-index difference of the degenerate HE_{21} modes in the LP_{11} mode group is used to determine the ellipticity of the fiber core. The estimated ellipticity is experimentally confirmed by the measurement of the group-index difference of the fundamental LP_{01} modes using a frequency-domain low-coherence interferometry technique.

Additionally, the Mach-Zehnder low-coherence interferometer is used to characterize the chromatic dispersion of the fundamental modes and the intermodal dispersion of a few-mode solid-core photonic crystal fiber. The group-index difference between the LP_{01} and LP_{11} modes agrees very well with the results of the numerical simulations. The absolute chromatic dispersion of the LP_{01} mode is estimated without sweeping the wavelength. The measurement resolution is discussed.
Zusammenfassung


Eine der untersuchten Fasern leitet wenige Moden und besteht vollständig aus Glas mit einem speziellen Brechungsindexprofil. Das Michelson-Interferometer mit geringer Kohärenz wird benutzt, um die intermodale Dispersion und die Modenprofile der Faser zu charakterisieren. Die gemessene intermodale Dispersion der $LP_{01}$-, $LP_{11}$-, $LP_{21}$- und $LP_{02}$-Moden stimmt sehr gut mit den Ergebnissen der im Rahmen dieser Arbeit durchgeführten numerischen Simulationen überein. Es wird gezeigt, dass die vier Moden der $LP_{11}$-Modengruppe der untersuchten Faser einen größeren Gruppenindex-Unterschied aufweisen als die in Standardfasern. Dies ist der Vorteil dieser Faserart. Der Gruppenindex-Unterschied der entarteten $HE_{21}$-Moden in der $LP_{11}$-Modengruppe wird genutzt, um die Elliptizität des Faserkerns zu bestimmen. Die Abschätzung der Elliptizität wird durch die Messung des Gruppenindex-Unterschieds der $LP_{01}$-Grundmoden experimentell bestätigt, wobei eine Interferometrie-Technik im Frequenzbereich mit geringer Kohärenz benutzt wird.

Außerdem wird das Mach-Zehnder-Interferometer mit geringer Kohärenz eingesetzt, um die
chromatische Dispersion der Grundmoden und die intermodale Dispersion einer HOM photonischen Kristallfaser mit Glaskern zu bestimmen. Der Gruppenindex-Unterschied zwischen den \( \text{LP}_{01} \)- und den \( \text{LP}_{11} \)-Moden stimmt sehr gut mit den Ergebnissen numerischer Simulationen überein. Die absolute chromatische Dispersion der \( \text{LP}_{01} \)-Mode wird ohne Durchstimmen der Wellenlänge abgeschätzt. Abschließend wird die Auflösung der Messung diskutiert.
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List of Abbreviations

CCD ............. Charge-coupled device
CVD ............. Chemical vapor deposition
DGD ............. Differential group delay
DWDM ........... Dense wavelength division multiplexing
EDFA ........... Erbium-doped fiber amplifier
FWHM ........... Full width at half maximum
LP .............. Linearly polarized
MCVD ........... Modified chemical vapor deposition
MFD ............ Mode field diameter
OVD ............ Outside vapor deposition
PCF ............ Photonic crystal fiber
PCVD ........... Plasma-assisted chemical vapor deposition
PMD ............ Polarization mode dispersion
VAD ............ Vapor axial deposition
WDM ............ Wavelength division multiplexing
Chapter 1

Introduction

The objective of this work

Optical waveguide theory tells that when the refractive-index profile of the fiber and the wavelength of transmission light satisfy the certain condition an optical fiber may support multiple transverse modes simultaneously. In these fibers different transverse modes possess different characteristic group velocities. Therefore, since long time the higher-order modes of an optical fiber are regarded as the drawback of the system because the intermodal dispersion limits the capacity of the fiber transmission system.

However, the knowledge of the intermodal dispersion characteristics of multimode fibers is of great interest for design of optical transmission system, as well as for other applications such as dispersion compensation [1–3] and fiber refractive-index-profile characterization [4, 5]. In order to precisely characterize the dispersion properties of all guided fiber modes, in many cases it is necessary to identify the individual transverse modes.

Higher-order modes of fibers are important not only in the transmission link, but also for design of novel photonic devices such as optical mode converter, optical switch, polarization controller, optical sensor and optical attenuator. Interesting results have been achieved by Prof. Ramachandran and his colleagues in OFS lab in America [6–17]. In these all-fiber devices, the optical properties of the fiber modes play a dominant role on the behavior of the whole system.

On the other hand, light beams, possessing special polarization, for instance, radially or az-
imuthally polarized beams [18–26], have recently attracted a great deal of interest. Several fiber techniques have been developed to generate such beams while employing higher-order modes of respective shapes [15, 27–31]. A full characterization of the mode profiles as well as the polarization state of these modes is essential to successfully apply these novel fiber techniques.

The aim of this thesis is to study the higher-order modes of an optical fiber. Stimulated by this motivation theoretical investigations on optical waveguide theory, experimental work done on different kinds of optical fibers, and relevant numerical simulations are accomplished. The experimental work, which is the main part of the thesis, includes the investigation of the modes in the lowest higher-order linearly polarized mode group ($\text{LP}_{11}$) in step-index fibers and characterization of mode properties of a special few-mode glass fiber as well as a few-mode solid-core photonic crystal fiber (PCF) using an optical low-coherence interferometry.

In this dissertation a novel time-domain optical low-coherence interferometry [5, 32–34] is proposed to measure the intermodal dispersion, mode profiles and mode weights of any type of optical fibers without any prior knowledge of the fiber physical properties [35]. The chromatic dispersion of the fiber mode can be estimated without sweeping the wavelength. For the measurements, a low-coherence light source and the phase-shifting algorithm [36–39] are used.

**Organization of the thesis**

The general optical waveguide theory and scalar approximation for weakly-guiding fibers are introduced in chapter 2 as the theoretical background of the thesis. Under scalar approximation, linearly polarized (LP) modes are obtained [40, 41]. The dispersion properties of modes in step-index fibers are shown. Beyond the conventional step-index fibers and several well-known specialty fibers scientists have recently designed many novel specialty higher-order-mode fibers for different applications [6–15, 17]. In these fibers the refractive-index profile of the fiber is tailored in order to obtain peculiar dispersion properties of certain higher-order modes. Some areas where higher-order modes of an optical fiber have found applications are reviewed.
In chapter 3 as an example of applications of higher-order modes in an optical fiber, it is shown that radially and azimuthally polarized beams can be obtained experimentally from a conventional step-index fiber which supports LP_{11} modes [27–31]. However, the anisotropy of the fibers and the degeneracy of modes in the LP_{11} mode group are clearly seen in experiments, as expected by theory. This leads to the instability of the produced radially and azimuthally polarized beams and degrades the purity of the beams because the generated beams are the coherent suppositions of several modes with almost identical propagation constants. The polarization properties of the generated radially and azimuthally polarized beams are evaluated using polarization optics.

In chapter 4 the measurement technique of the proposed time-domain optical low-coherence interferometry is presented. The fiber under test is a few-mode all-glass fiber with special refractive-index profile. This fiber is designed to have a large splitting of propagation constants for the four vector modes in the LP_{11} mode group. The fiber design has found applications such as polarization-insensitive microbend fiber gratings [12, 42]. Two-dimensional phase-shifting algorithm [36–39] is used to reconstruct the mode profiles and can be also used to determine the mode weights of the fiber. The measured mode dispersion and mode profiles are compared to the results of the numerical simulations. A very good agreement is obtained. The mode dispersion properties provide valuable information not only on the fiber refractive-index profile, but also on the birefringence of the fiber. The group-index difference of the degenerate HE_{21} modes in the LP_{11} mode group obtained in the measurement is used to characterize the ellipticity of the fiber core. The estimated ellipticity of the fiber core is confirmed by measuring the group-index difference of the fundamental LP_{01} modes using an independent measurement—a frequency-domain low-coherence interferometry [43–45]. The ellipticity of the fiber is very important for the polarization-insensitive microbend fiber grating where the tested fiber is a crucial element.

The similar measurement has been done on a few-mode solid-core PCF in chapter 5. The measured group-index difference between the LP_{01} mode and the LP_{11} mode as well as the chromatic dispersion of the LP_{01} mode agree very well with the results of the numerical simulations. The measurement resolution of the system is discussed.
Chapter 2

Theoretical background and review of optical fibers

2.1 Introduction

Optical waveguides are dielectric structures which are ideally uniform along the axis of propagation. It is therefore their transverse characteristics that basically prescribe their optical properties. Optical fibers can have different transverse structures and consequently different guiding regimes. The basic properties of an optical fiber are loss, dispersion and nonlinear optical effects. The arbitrary refractive-index profile of an optical fiber $n(r)$ is defined as [46]

$$n(r) = n_{co} [1 - 2\Delta f(r)]^{\frac{1}{2}}$$  

(2.1)

where $r$ is the radial coordinate; 

$$\Delta = (n_{co}^2 - n_{cl}^2) / 2n_{co}^2$$  

(2.2)

is called relative refractive index or refractive-index height of the fiber; $f(r)$ describes the shape of the profile; $n_{co}$ is defined as the peak index of refraction in the core; $n_{cl}$ is the refractive index of the cladding. $f(r)$ is a heaviside-step function for a step-index fiber. When $0 \leq r \leq a$, where $a$ is the radius of the core, $f(r) = 0$ and $n(r) = n_{co}$. When $r > a$, $f(r) = 1$ and $n(r) = n_{cl}$.

A step-index fiber has the simplest index profile and since many years has been widely used in optical communication as single-mode fiber or multimode fiber. In a step-index fiber, a circular
core with constant refractive index is surrounded by the cladding with slightly lower refractive index [47]. Figure 2.1 shows the cross section and refractive-index profile for a step-index fiber. The step-index profile is interesting from the theoretical point of view because it allows for a simpler mathematical treatment. Also, the step-index geometry has become the preferred choice for many applications in telecommunications and sensing.

In a step-index fiber, the guidance takes place within the core as a consequence of total internal reflection at the core-cladding interface. This guiding regime is shown in Fig. 2.2. From Snell’s

Figure 2.1: (a) Cross-section image for step-index fibers [47]; (b) refractive-index profile for step-index fibers [47].

Figure 2.2: Light confinement through total internal reflection in step-index fibers [47]. Rays for which $\theta > \theta_0$ are refracted out of the core.
law one obtains the critical angle for the total internal reflection at the core-cladding interface [48]

$$\theta_c = \sin^{-1}(n_{cl}/n_{co})$$ (2.3)

Following the constraint imposed by total internal reflection one can easily deduce that light rays incident at the fiber end within the cone of acceptance defined by the maximum angle

$$\theta_0 = \sin^{-1}(n_{co}^2 - n_{cl}^2)^{1/2}$$ (2.4)

will be trapped within the core. Consequently, the numerical aperture of the step-index fiber is defined as [46, 47, 49]

$$NA = \sin \theta_0 = (n_{co}^2 - n_{cl}^2)^{1/2}$$ (2.5)

NA provides an immediate indication of the characteristics of the light injection into a fiber.

When describing the optical properties of a step-index fiber, the parameter $V$ is often used. It is defined as [46, 47, 49]

$$V = ak(n_{co}^2 - n_{cl}^2)^{1/2} = akNA,$$ (2.6)

where $k = 2\pi/\lambda$ is the wavenumber in vacuum. This V-parameter, which is also called the normalized frequency, is a key parameter in characterizing optical fibers. It will be shown in section 2.2 that the parameter $V$ actually quantifies the fibers’ ability to support transverse modes. In fact, the number of transverse modes supported by a fiber is an increasing function of $V$. The total number of modes that can propagate in a multimode step-index fiber is given by [49]

$$N \approx V^2/2$$ (2.7)

This equation is only valid for large $V$ numbers. It has been shown that in photonic crystal fibers where different guiding mechanism was used, the second-mode cutoff wavelength and spot size of the fundamental Gaussian mode can be expressed using the effective $V$ value [50, 51], similar to
CHAPTER 2. THEORETICAL BACKGROUND AND REVIEW OF OPTICAL FIBERS

In section 2.2 the theoretical background of optical fibers and fiber modes are introduced. For weakly-guiding step-index fibers the LP designation is used. In section 2.3 several types of specialty fibers, especially, dispersion tailored higher-order-mode fibers are reviewed. The conventional fiber fabrication methods are introduced at the end of the chapter.

2.2 Theoretical background of optical fibers

This section highlights some theoretical representations on the properties of different fiber modes. In section 2.2.1 the procedure of getting exact solutions of the Maxwell’s equations in an optical waveguide is introduced. Weakly-guiding approximation and LP modes [40] are discussed in section 2.2.2. In section 2.2.3 fiber modes are presented. They correspond to the exact or approximate solutions of the Maxwell’s equations. Deriving from the propagation constant, the normalized propagation constant and the normalized group delay are used to describe the fundamental properties of modes in section 2.2.4. The dispersion parameter is also derived. The calculated results of the normalized propagation constant and the normalized group delay for the lowest four LP modes in step-index fibers are obtained by Gloge using weakly-guiding approximation [40]. The results for the corresponding 12 exact modes are obtained by using COMSOL Multiphysics [52, 53]. COMSOL Multiphysics is a commercial software based on the finite-element method. The software is also used for numerical simulations of mode properties of the tested few-mode fiber in chapter 4.

2.2.1 Exact solutions for step-index fibers

A fiber mode refers to a specific solution of the wave equation that satisfies the appropriate boundary conditions and has the property that its spatial distribution ideally does not change with propagation as long as the refractive-index profile of the fiber remains constant. The fiber modes can be classified as guided modes (bound modes), leaky modes and radiation modes. The last two classes
of modes are not the topic of this thesis. Here the derivation of the propagation constant and the field distribution of the fiber modes is introduced according to [46] by Snyder. The electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ of a guided mode are solutions of source-free Maxwell’s equations with $J = 0$ and $\sigma = 0$, where $J$ is the current density and $\sigma$ is the charge density. We assume $\mathbf{E}$ and $\mathbf{H}$ are time harmonic, i.e. $\mathbf{E} = E_0 \exp(-i\omega t)$ and $\mathbf{H} = H_0 \exp(-i\omega t)$.

\[
\begin{align*}
\nabla \times \mathbf{E} &= i(\mu_0/\epsilon_0)^{1/2}k\mathbf{H} \\
\nabla \times \mathbf{H} &= -i(\epsilon_0/\mu_0)^{1/2}k n^2 \mathbf{E} \\
\nabla \cdot (n^2 \mathbf{E}) &= 0 \\
\nabla \cdot \mathbf{H} &= 0
\end{align*}
\] (2.8)

where symbols in bold represent vector quantities; $\nabla \times$ is the curl operator; $\nabla \cdot$ is the divergence operator; $\epsilon_0$ is the dielectric constant of free space; $\mu_0$ is the magnetic permeability of free space; $n$ is the refractive index; $k = 2\pi/\lambda$ is the free-space wavenumber, and $\lambda$ is the wavelength of light in free space.

Assuming the waveguide is translationally invariant, the electric and magnetic fields of of the waveguide are expressible as a superposition of fields with the separable form

\[
\begin{align*}
\mathbf{E}(x, y, z) &= e(x, y) \exp(i\beta z) \\
\mathbf{H}(x, y, z) &= h(x, y) \exp(i\beta z)
\end{align*}
\] (2.9)

where $\beta$ is the propagation constant. The corresponding forms for the cylindrical polar coordinates are

\[
\begin{align*}
\mathbf{E}(r, \phi, z) &= e(r, \phi) \exp(i\beta z) \\
\mathbf{H}(r, \phi, z) &= h(r, \phi) \exp(i\beta z)
\end{align*}
\] (2.10)
We decompose these fields into longitudinal and transverse components, parallel to and orthogonal to the waveguide axis, respectively, and denoted by subscripts $z$ and $t$, where

$$
\mathbf{E} = (\mathbf{e}_t + e_z \mathbf{z}) \exp(i \beta z)
$$

$$
\mathbf{H} = (\mathbf{h}_t + h_z \mathbf{z}) \exp(i \beta z)
$$

(2.11)

and $\mathbf{z}$ is the unit vector parallel to the waveguide axis. The fields with the separable form of Eq. 2.11 satisfy the source-free homogeneous vector wave equations

$$
(\nabla^2_t + n^2 k^2 - \beta^2) \mathbf{e} = - (\nabla_t + i \beta \mathbf{z}) (\mathbf{e}_t \cdot \nabla_t \ln n^2)
$$

$$
(\nabla^2_t + n^2 k^2 - \beta^2) \mathbf{h} = [(\nabla_t + i \beta \mathbf{z}) \times \mathbf{h}] \times \nabla_t \ln n^2
$$

(2.12)

where $k = 2\pi/\lambda$, the vector operator $\nabla_t$ and $\nabla^2_t$ are defined in Appendix A.

To facilitate description of the mode fields, dimensionless modal parameters $U_j$ and $W_j$ for the core and the cladding are introduced.

$$
U_j = a (k^2 n_{co}^2 - \beta_j^2)^{1/2}
$$

$$
W_j = a (\beta_j^2 - k^2 n_{cl}^2)^{1/2}
$$

(2.13)

where $n_{co}$ is the maximum core index (For step-index fibers $n_{co}$ is constant); $n_{cl}$ is the uniform cladding index; $U_j, W_j$ and the normalized frequency $V$ have the following relation.

$$
V^2 = U_j^2 + W_j^2
$$

(2.14)

where $V$ is defined in Eq. 2.6; the subscript $j$ in Eq. 2.13 and Eq. 2.14 stands for the different modes. When solving the above vector wave equations in the core and the cladding of the step-index fiber, where the $\nabla_t \ln n^2$ terms are zero, we need solve only for the longitudinal components $e_z$ and $h_z$ since the transverse components could be determined by the longitudinal components.
2.2. THEORETICAL BACKGROUND OF OPTICAL FIBERS

The relationships between the transverse components and the longitudinal components are obtained by substituting the field representations of Eq. 2.11 into the source-free Maxwell’s equations. Furthermore, the step-index fiber has a circularly symmetric cross section and its refractive-index profile is described as follows

\[
n(r) = \begin{cases} 
  n_{co} & 0 \leq r < a \\
  n_{cl} & a < r < \infty 
\end{cases}
\]  

(2.15)

Substitute the index profile of the fiber into the wave equation Eq. 2.12, it is deduced that \( \psi \) satisfies

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{U^2}{a^2} \psi = 0, \quad 0 \leq r < a \\
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} - \frac{W^2}{a^2} \psi = 0, \quad a < r < \infty
\]  

(2.16)

where \( \psi \) denotes \( e_z \) or \( h_z \). Then the mode field can be expressed by the Bessel function of the first kind \( J(Ur/a) \) inside the core and modified Bessel function of the second kind \( K(Wr/a) \) outside the core. By imposing boundary conditions, the exact expressions of all components of the mode fields and the eigenvalue equation in terms of \( U, W \) and \( \beta \) are determined. The boundary conditions satisfied by the field across an interface between media of differing refractive index, i.e. between the core and the cladding of the fiber, are: (1) continuity of the magnetic field and the component of the electric field tangential to the interface; (2) continuity of the normal component. In [46] Allan W. Snyder and John D. Love have listed the eigenvalue equations and the exact expressions for the mode fields of HE, EH, TE, and TM modes in a circular step-index fiber. The exact solutions contain the polarization properties of the modes.

Consequently, the following procedure can be used to solve a step-index fiber model. Knowing all of the physical quantities which define the waveguide, i.e. the refractive indices \( n_{co} \) and \( n_{cl} \), the core radius \( a \), and the wavelength of the light source \( \lambda \), one could solve the eigenvalue equation...
for modes in a step-index fiber [46] and get the values of $\beta$ numerically. The number of $\beta$ obtained indicates the number of transverse modes the fiber can support. All of the dispersion parameters of the fiber can be derived from $\beta$. This will be discussed later. Furthermore, $U$ and $W$ for each $\beta$ can be obtained according to Eq. 2.13. All of the field components can be determined by the Bessel function of the first kind $J$ function and the modified Bessel function of the second kind $K$ function in terms of $U$ and $W$ and a constant coefficient [46]. The coefficient can be easily obtained through normalizing the total intensity inside the core and the cladding. This procedure sounds easy. However, solving the eigenvalue equation in order to get $\beta$ is in general too complicated to accomplish without computer although the eigenvalue equation sometimes seems to have a simple form.

### 2.2.2 Weakly-guiding fiber

For weak guidance, the relative refractive index of the fiber $\Delta << 1$. Typical $\Delta$ values of a conventional step-index fiber are of the order of $10^{-3}$. From the fabrication point of view, a small index difference is advantageous since the core and the cladding are generally made of the same basic material to which is locally added one or a few dopants in order to tailor the refractive-index profile. Refractive-index difference of this order can be readily obtained without the need for adding high concentrations of dopants, thus preventing significant modification of the optical properties and of the stability of the glass. From the theoretical point of view, the great benefit of such a configuration is that it allows for a significant simplification of the mode description, which is, in general, rather involved [49].

The term “weakly-guiding fibers” is first introduced by Gloge [40]. Under weakly-guiding approximation the transverse fields of the fiber modes are assumed to be polarized in one direction. This is why the resultant modes are called LP modes (LP stands for linearly polarized modes.). Here we briefly introduce the approximate eigenvalue equation and the approximate expression for $U$ in a weakly-guiding fiber according to [40]
2.2. THEORETICAL BACKGROUND OF OPTICAL FIBERS

By differentiating the equations for longitudinal components expressed by the transverse components, the approximate expressions of the transverse components are obtained. By matching the boundary conditions and assuming \( n_{co} = n_{cl} \) and \( \epsilon_{co} = \epsilon_{cl} \), the eigenvalue equation in weakly-guiding step-index fibers reduces to [40]

\[
\frac{U J_{l-1}(U)}{J_l(U)} = -\frac{W K_{l-1}(W)}{K_l(W)}
\] (2.17)

which is expressed in terms of the azimuthal number \( l \). The equation is valid for both HE and EH modes. Therefore, in the weakly-guiding approximation the whole set of solutions can be obtained from the above eigenvalue equation. HE and EH modes share the same set of propagation constants. Accordingly, a new set of modes can be described on the basis of the azimuthal mode order \( l \) together with the radial mode order, \( m \), corresponding to the \( m \)th root of the eigenvalue equation. In fact the crucial point to recall is that under the weakly-guiding conditions, subgroups of modes possess essentially the same propagation constants (i.e. they are degenerate) and are actually superimposed on each other to form the so-called approximate LP modes. For most applications, indeed, the description in terms of LP modes is not only sufficient but more practical.

It is noted that the above eigenvalue equation for LP modes in weakly-guiding fibers is exactly the eigenvalue equation for \( TE_{0m} \) modes when \( l = 1 \) according to [40, 46]. This will be further discussed in section 4.2.1 where the theoretical background of the tested few-mode fiber is introduced.

Furthermore, by differentiating the eigenvalue equation 2.17 and further mathematic approximations the expression for \( U \) of all modes except the fundamental \( HE_{11} \) is given out by

\[
U(V) = U_c \exp \left\{ \sin^{-1}\left(\frac{S}{U_c}\right) - \sin^{-1}\left(\frac{S}{V}\right)\right\} / S
\] (2.18)

with

\[
S = (U_c^2 - l^2 - 1)^{1/2}
\] (2.19)
where \( U_c \) is the value of \( U \) at the cutoff, corresponding to the \( m \)th root of \( J_{l-1}(U) \) for \( LP_{lm} \) modes.  

For \( HE_{11} \) mode,

\[
U(V) = \frac{(1 + \sqrt{2})V}{[1 + (4 + V^4)^{1/4}]}
\]  

(2.20)

For \( V \gg S \) (far enough from cutoff), the Eqs. 2.18 and 2.20 reduce to

\[
U(V) = U_\infty (1 - 1/V)
\]  

(2.21)

for all modes, using the \( m \)th root of \( J_l(U) \) for \( U_\infty \). Other parameters such as \( W \) and \( \beta \) then can be easily derived from \( U \).

### 2.2.3 Fiber modes

Following the exact eigenvalue equations of an optical fiber [46], accurate mode fields as well as the propagation constants of the modes can be derived, while optical properties of \( LP \) modes can be easily obtained using the simpler eigenvalue equation proposed by Gloge [40]. Figure 2.3 shows the lowest 12 modes of a circular step-index fiber [1, 35, 49, 54]. The upper part gives the notations and electric field distributions of the true eigenmodes. In principle each eigenmode has a unique propagation constant unless even modes and odd modes within the hybrid \( HE \) or \( EH \) modes are always degenerate for a circular fiber profile. The modes in the lower part are the corresponding four \( LP \) modes. Each \( LP \) mode has two polarizations \( E_x \) and \( E_y \) and two orthogonal intensity patterns, whose azimuthal dependences are indicated by \( \cos l\varphi \) and \( \sin l\varphi \) in the figure. For \( LP \) modes the propagation constants of the modes depend only on the mode intensity pattern and are independent of the polarization direction of the electric field. Thus, in the scalar approximation, the \( LP_{11} \) mode, for example, is fourfold degenerate since there are four possible combinations of vertically and horizontally oriented intensity patterns and polarizations. The degeneracy of modes in a step-index fiber are clearly shown in the results of the numerical simulations, which will be presented in the next section. Furthermore, because in the weakly-guiding approximation
2.2. **THEORETICAL BACKGROUND OF OPTICAL FIBERS**

The longitudinal fields are very small relative to the transverse fields (The ratio is proportional to $\sqrt{\Delta}$. $\Delta$ has the order of $10^{-3}$.), the field propagating in a weakly-guiding optical fiber is nearly transverse and linearly polarized wave.

<table>
<thead>
<tr>
<th>Exact modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE$_{11}^{\text{even}}$</td>
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</table>

<table>
<thead>
<tr>
<th>LP$_{01}$</th>
<th>LP$_{11}$</th>
<th>LP$_{21}$</th>
<th>LP$_{02}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>$E_y$</td>
<td>$E_x$</td>
<td>$E_y$</td>
</tr>
</tbody>
</table>

**Figure 2.3**: The lowest 12 exact modes of a circular step-index fiber and the corresponding four LP modes [1, 35, 49, 54]. Arrows indicate the electric field. Even and odd modes within the hybrid HE or EH modes are always degenerate in a perfectly circular fiber. Each LP mode has two polarizations $E_x$ and $E_y$. $\cos l\phi$ and $\sin l\phi$ indicate the azimuthal dependences of the intensity distributions of the LP$_{11}$ and LP$_{21}$ modes.

An optical fiber supports only a finite number of modes at a given wavelength. The number of the supported modes depends on not only the ratio of the fiber transverse dimension to the wavelength (i.e. $a/\lambda$), but also the numerical aperture of the fiber NA. Therefore the $V$ parameter which has been defined in Eq. 2.6 is actually the best suited for describing the evolution of the fiber mode content. $V$ is related to $U$ and $W$, as shown in Eq. 2.14. $U$ and $W$ appear as arguments of the Bessel functions of the model solutions. In particular $W = 0$ is the point where the mode is not guided any more (cutoff), $U_c = V_c$. At the opposite extreme, in the limit where $W \to V$, the
mode is well bounded.

The mode number that a step-index fiber can support has been given in Eq. 2.7. For example, for a $V$ value of about 20, corresponding to a core diameter of about 50 $\mu$m, there are approximately 200 modes propagating within the fiber. The propagation constants corresponding to such a large number of modes will fill the interval defined by $kn_{cl} < \beta < kn_{co}$. Each of the fiber modes will carry a portion of the light energy and propagate along the fiber with its own speed. This leads to the named “intermodal dispersion” [47, 49], which is the main factor limiting the fiber bandwidth in multimode step-index fibers. For this reason, multimode step-index fibers have a very limited range of applications in telecommunications and now are mainly used for short-distance light transport applications. Instead of the large-core step-index fibers, graded-index fibers were invented as multimode fibers to reduce the intermodal dispersion.

However, recently fibers that transmit only a limited number of modes (few-mode fibers) are of great interest. The characterization of the mode properties of the few-mode fibers in experiments is the focus of this thesis. It is presented in the chapter 4 and the chapter 5.

### 2.2.4 Dispersion properties of fiber modes

A very important parameter of the fiber modes is the effective index $n_{eff}$, defined as

$$n_{eff} = \beta/k$$

(2.22)

It is also called phase index, since that the propagation constant $\beta$ determines how fast the electric vectors are oscillating during the propagation through the fiber [46]. Following the derivation in section 2.2.2, $U(V)$ for weakly-guiding fibers can be easily obtained and $\beta$ can be calculated accordingly. In order to make the results independent of the particular fiber configurations, however,
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the ratio

\[ b_{\text{eff}} = \frac{(n_{\text{eff}}^2 - n_{\text{cl}}^2)}{(n_{\text{co}}^2 - n_{\text{cl}}^2)} \quad (2.23) \]

\[ = \frac{(\beta^2/k^2 - n_{\text{cl}}^2)}{(n_{\text{co}}^2 - n_{\text{cl}}^2)} \quad (2.24) \]

is more interesting than \( n_{\text{eff}} \) itself [40]. For small index difference \( \Delta << 1 \)

\[ \Delta \approx \frac{(n_{\text{co}} - n_{\text{cl}})}{n_{\text{co}}} \quad (2.25) \]

Therefore,

\[ b_{\text{eff}} \approx \frac{(n_{\text{eff}} - n_{\text{cl}})}{(n_{\text{co}} \Delta)} = \frac{(\beta/k - n_{\text{cl}})}{n_{\text{co}} \Delta} \quad (2.26) \]

Then the propagation constant \( \beta \) can be expressed by \( n_{\text{co}}, \Delta, k, \) and \( b_{\text{eff}} \) [40]

\[ \beta = n_{\text{co}}k(b_{\text{eff}} \Delta + 1) \quad (2.27) \]

\[ = n_{\text{co}}k[\Delta - \Delta(U^2/V^2) + 1] \quad (2.28) \]

Since \( \beta \) and \( b_{\text{eff}} \) are proportional, the quantity \( b_{\text{eff}} \) can be understood as a normalized propagation constant [40].

Furthermore, according to the Eq. 2.13, Eq. 2.18 and Eq. 2.20, the following formulas can be used to calculate \( b_{\text{eff}} \) for different \( \text{LP}_{lm} \) modes with the azimuthal order \( l \) and the radial order \( m \):

\[ \text{LP}_{01} : b_{\text{eff}} = 1 - U^2/V^2 = 1 - \left[ \frac{1 + \sqrt{2}}{1 + (4 + V^4)^{1/4}} \right]^2 \]

\[ \text{LP}_{lm} : b_{\text{eff}} = 1 - U^2/V^2 = 1 - \frac{U_e^2}{V^2} \exp\left[ \frac{2}{S} (\sin^{-1}(\frac{S}{U_e}) - \sin^{-1}(\frac{S}{V})) \right] \quad (2.29) \]

where

\[ S = (U_e^2 - l^2 - 1)^{1/2} \]
Figure 2.4: Calculated results for the lowest four LP modes under weakly-guiding approximation [40]: (a) Dependence of normalized propagation constant $b_{eff}$ on normalized frequency $V$; (b) dependence of the normalized group delay $b_g$ on $V$. 
and \( U_c \) is the value of \( U \) at the cutoff as before. Curves of “\( b_{eff} \) vs \( V \)” determine the dispersion properties of different fiber modes. The calculated results of “\( b_{eff} \) vs \( V \)” for the lowest four LP modes under weakly guiding approximation by Gloge are plotted in Fig. 2.4(a) [40]. Figure 2.4(b) [40] shows the corresponding normalized group delay of the modes \( b_g \) obtained from \( b_{eff} \). \( b_g \) will be introduced later. Note that several approximate formulas of \( b_{eff} \) with various ranges of validity were derived by various authors [47, 49, 55].

Dispersion inside an optical fiber includes intramodal dispersion and intermodal dispersion. They exist in all fibers. Intermodal dispersion is caused by the propagation of the different transverse modes. It exists even in a single-mode fiber, where the two fundamental Gaussian modes with orthogonal polarizations propagate simultaneously. Intramodal dispersion is introduced by the waveguide dispersion and material dispersion [40, 47, 56–58]. Material dispersion means the refractive indices of the core and the cladding \( n_{co} \) and \( n_{cl} \) depend on wavelength. It is an effect for any dielectric medium. The confinement of the mode in a waveguide causes its propagation constant \( \beta \) and thus the speed to depend on wavelength. This is referred to as waveguide dispersion. Material dispersion and waveguide dispersion are often referred to as chromatic dispersion or group velocity dispersion, measured in units \( ps/(km \cdot nm) \).

The dispersion of an optical fiber can be derived from the group delay \( \tau \) or group velocity \( v \) [40, 48, 56].

\[
\begin{align*}
\nu &= (d\beta/d\omega)^{-1} = c/n_g \\
\tau &= 1/\nu = d\beta/d\omega
\end{align*}
\tag{2.30}
\]

where \( \omega \) is the angular frequency; \( n_g \) is the group index; \( c \) is the speed of the light. Using

\[
\beta = n_{eff}k = n_{eff}\omega/c
\tag{2.31}
\]
one gets the group index \([40, 47, 48, 56]\)

\[
    n_g = c\left(\frac{d\beta}{d\omega}\right) = n_{\text{eff}} + \omega(\frac{dn_{\text{eff}}}{d\omega})
    = n_{\text{eff}} + k(\frac{dn_{\text{eff}}}{dk})
    = n_{\text{eff}} - \lambda(\frac{dn_{\text{eff}}}{d\lambda})
\] (2.32)

In the above equations, different expressions have been used to define the group index \(n_g\). They are equivalent. In order to precisely calculate \(n_g\) one should shift the wavelength and calculate the corresponding \(n_{\text{eff}}\) taking into account the material dispersion of the waveguide, i.e. the wavelength dependence of \(n_{\text{co}}, n_{\text{cl}}\) and \(\Delta\) \([58]\). It has been shown in \([40]\) and \([56]\) that in weakly-guiding fibers unless high precision is desired, a good estimation of the dispersion can be obtained under the assumption of additivity of material and waveguide dispersion. The material dispersion is the same for all fiber modes, and therefore, it could be neglected when the group-index difference between the modes is to be studied. Without taking into account the material dispersion, \(V\) changes monotonously by \(\lambda\) and \(dV/d\lambda = -V/\lambda\), hence

\[
    n_g = n_{\text{eff}} - \lambda(\frac{dn_{\text{eff}}}{dV})(\frac{dV}{d\lambda}) = n_{\text{eff}} + V(\frac{dn_{\text{eff}}}{dV})
\] (2.33)

Therefore, the group index can be basically deduced from the derivative \(dn_{\text{eff}}/dV\). Similar to normalized propagation constant \(b_{\text{eff}}\) we define normalized group delay \(b_g\) \([40]\)

\[
    b_g = (n_g^2 - n_{\text{cl}}^2)/(n_{\text{co}}^2 - n_{\text{cl}}^2) \approx (n_g - n_{\text{cl}})/(n_{\text{co}}\Delta)
\] (2.34)

The approximate expression is valid for \(\Delta \ll 1\). Substituting Eq. 2.33 into Eq. 2.34, one gets

\[
    b_g = d(Vb_{\text{eff}})/dV = V(d(b_{\text{eff}})/dV) + b_{\text{eff}}
\] (2.35)

which means the normalized group delay \(b_g\) can be obtained by using the derivative \(db_{\text{eff}}/dV\).
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Figure 2.5: Results of numerical simulations for the lowest 12 modes of a circular step-index fiber: (a) dependence of normalized propagation constant $b_{\text{eff}}$ on normalized frequency $V$; (b) dependence of the normalized group delay $b_g$ on $V$. 
The calculated results of “$b_{\text{eff}}$ vs $V$” and “$b_g$ vs $V$” for the lowest four LP modes under weakly-guiding approximation by Gloge have been plotted in Fig. 2.4 [40].

The calculated results of “$b_{\text{eff}}$ vs $V$” and “$b_g$ vs $V$” for the lowest 12 eigenmodes of a perfectly circular step-index fiber are plotted in Fig. 2.5. The commercial software “COMSOL Multiphysics” [52, 53] and MATLAB were used for the numerical simulations. The fiber model was firstly created in the graphic environment of COMSOL Multiphysics using the “Eigenfrequency analysis” of the RF application module. The model was then exported as a m-file, which meant the entire modeling session was saved as a sequence of commands. Then the m-file was run in “COMSOL with MATLAB”. The codes were modified to study the parameter dependency. The use of “COMSOL Multiphysics” for the numerical simulations of mode properties of an optical fiber is detailed in Appendix B. In the numerical simulations $\lambda = 1550 \text{ nm}$, the core radius $a = 5 \mu m$, the refractive index of the cladding $n_{cl} = 1.45$, which is the typical refractive-index value for pure silica at $\lambda = 1550 \text{ nm}$ wavelength and room temperature, were kept constant and $V$ was changed by varying the refractive index of the fiber core $n_{co}$.

As one can see in Fig 2.5(a), the effective index for all modes ranges between the index of the cladding at cutoff ($b_{\text{eff}} = 0$) and the index of the core as $V \rightarrow \infty$ ($b_{\text{eff}} = 1$), i.e. $n_{cl} \leq n_{\text{eff}} \leq n_{co}$. The normalized group delay $b_g$ shown in Fig 2.5(b) was obtained from the data shown in Fig. 2.5(a) using Eq. 2.35. It is seen in Fig. 2.5 for step-index fibers the modes in LP$_{11}$ and LP$_{21}$ groups have almost identical effective indices and group indices. The effective indices via $V$ for different modes may not cross, while group indices may cross. The relative relation in group index for different modes does not necessarily follow that in effective index. It is shown in Fig. 2.4 and Fig. 2.5 that for step-index fibers the solutions under scalar approximation have very high accuracy.

In the above numerical simulations the material dispersion was ignored for simplification. However, in practice both material dispersion and waveguide dispersion are present in any waveguide [40, 47, 56–58]. Next the expressions for the material dispersion and the waveguide dispersion will be derived from the group delay $\tau$ [47, 49]. Using the definition of the group delay Eq.
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2.30, the group delay $\tau$ can be expressed by

$$
\tau = \frac{d\beta}{d\omega} = -\frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda}
$$

(2.36)

From the above equation the group delay $\tau$ in terms of $V$ and $b_{eff}$ can be obtained by differentiating Eq. 2.27 with respect to $\lambda$, where $d\Delta/d\lambda = 0$ is assumed.

$$
\tau = \frac{N}{c} + \frac{N\Delta}{c} \frac{d(Vb_{eff})}{dV}
$$

(2.37)

where $N$ is the group index of the material and is defined as

$$
N = d(n_m k)/d\lambda = n_m - \lambda(dn_m/d\lambda)
$$

(2.38)

with $n_m \approx n_{co} \approx n_{cl}$ standing for either the core or the cladding refractive index. In the above Eq 2.37, the first term is the group delay introduced by the material dispersion, and the second term is the group delay introduced by the waveguide dispersion.

The dispersion parameter $D$ is formally obtained by deriving the group delay with respect to $\lambda$. Following Eq. 2.37, we define the derivative of the first term with respect to $\lambda$ as the material dispersion $D_m$, and the derivative of the second term with respect to $\lambda$ as waveguide dispersion $D_w$ [47, 49].

$$
D = d\tau/d\lambda = D_m + D_w
$$

(2.39)

where

$$
D_m = \frac{1}{c} \frac{dN}{d\lambda} = -\frac{\lambda^2}{c} \frac{dn_m}{d\lambda}
$$

$$
D_w = \frac{\Delta}{c} \left[ \frac{dN}{d\lambda} \frac{d(Vb_{eff})}{dV} - \frac{VN^2}{n_m \lambda} \frac{d^2(Vb_{eff})}{dV^2} \right]
$$

(2.40)
The material dispersion $D_m$ is governed by the Sellmeier equation [59]. Sellmeier equation reveals the empirical relation between the wavelength and the refractive index of the materials. Several publications have given out the Sellmeier coefficients for different materials under different environments [59–62]. In some cases $D_m$ for all the modes can be regarded as the same. The waveguide dispersion $D_w$ is in general different for different modes. Furthermore, waveguide dispersion $D_w$, which can be tailored through an optimization of the refractive-index profile of the fiber, plays a crucial role in defining the exact location of the overall zero dispersion wavelength.

In this section some basic equations for an optical fiber, especially for a step-index fiber, have been introduced. Different fiber modes correspond to different solutions of propagation constants. For a small index step ($\Delta \ll 1$), weakly-guiding approximation can be used. Accordingly, LP modes are obtained. Two important parameters for modes in an optical fiber are normalized propagation constant $b_{\text{eff}}$ and normalized group delay $b_g$. The dispersion properties for different modes can be derived from them. It is the refractive-index profile of an optical fiber that determines the optical properties of the guided modes. In the next several known specialty optical fibers will be reviewed. Note that among them dispersion-tailored higher-order-mode fibers have recently attracted a great of interest.

### 2.3 Specialty optical fibers

Two major fields of optical fiber technology are: opto-telecommunication and specialty non-communication fibers. Optical fiber communication demands new generations of ultra low loss, isotropic and broadband dispersion compensating, transmission fibers for dense wavelength division multiplexing (DWDM) technology. The optical fiber communication uses optimization technologies for current, more efficient usage of older fibers installed in the last century. The conventional step-index fiber discussed previously belongs to this class.

Optical fibers for applications outside the transmission field are called specialty or tailored ones [63]. These latter fibers are often used in optical communication as short-length, functional pho-
2.3. SPECIALTY OPTICAL FIBERS

tonic components. They are also used in photonic systems for information processing and hybrid optoelectronic integrated circuits as well as in optical fiber sensors. The name of “Tailored (or Specialty) optical fibers” extracted itself in the bibliography at the beginning of the eighties. A considerable increase in the research on tailored optical fibers is observed toward the end of this decade. Many applications appeared of fibers with increased or decreased (optimized) sensitivities to external fields (measurands). Commercial availability of tailored optical fibers is however a domain of the late nineties. According to the optical and geometrical fiber parameters, the guiding regime and the application of the fibers, specialty optical fibers can have various kinds of classifications. Here we briefly review several popular types of specialty fibers.

2.3.1 Polarization-maintaining fibers

In an ideal single-mode fiber, because of the circular symmetry, two perpendicularly polarized waves have the same propagation constants. Therefore, the polarization state of the wave remains unchanged during the propagation. This degeneracy of the two polarization modes is easily broken by any imperfection in the cylindrically symmetric geometry of the fiber, including mechanical stresses on the fiber. These effects can either be introduced intentionally during the fiber manufacturing process or inadvertently after the optical fiber has been installed. The effect results in two orthogonally polarized modes with slightly different propagation constants (note that the two modes need not be linearly polarized, and in general they are elliptically polarized). Because each mode experiences a slightly different refractive index, the modes will drift in phase relative to each other. At any point in time, the light at the exit facet of the fiber has a state of polarization that is a superposition of the two orthogonally polarized modes. Two parameters are commonly used to describe the polarization properties of a birefringent fiber: phase birefringence and group birefringence [45, 64]. The phase birefringence is specified by the difference of the effective refractive indices of the two polarized modes.

\[ \Delta n_{\text{eff}} = n_{\text{eff}(x)} - n_{\text{eff}(y)} \] (2.41)
where $n_{\text{eff}(x)}$ and $n_{\text{eff}(y)}$ are the effective indices of the two orthogonally polarized modes. The net polarization evolves as the light propagates through various states of ellipticity and orientation. After some distance, the two modes will differ in phase by a multiple of $2\pi$, resulting in a state of polarization identical to that at the fiber input. This characteristic length is known as the beat length $L_B$.

$$L_B = \frac{\lambda}{\Delta n_{\text{eff}}}$$ (2.42)

The second term used to describe the polarization property of a birefringent fiber is the group birefringence $\Delta n_g$.

$$\Delta n_g = n_g(x) - n_g(y)$$ (2.43)

where $n_g(x)$ and $n_g(y)$ are the group indices of the two orthogonally polarized modes. From the definition of the group index in Eq. 2.32. The relation between the phase birefringence and group birefringence is obtained [45, 64].

$$\Delta n_g = \Delta n_{\text{eff}} + \omega \lambda \left[\frac{d(\Delta n_{\text{eff}})}{d\omega}\right]$$

$$= \Delta n_{\text{eff}} - \lambda \left[\frac{d(\Delta n_{\text{eff}})}{d\lambda}\right]$$ (2.44)

The group birefringence is closely related to the polarization mode dispersion (PMD) and the differential group delay (DGD). PMD and DGD are often used interchangeably, but sometimes with slightly different meanings. DGD characterizes the group delay difference between the two fiber modes, in this context, between the two orthogonally polarized modes. DGD = $\Delta n_g/c$. Because of random coupling between the two modes, induced by random perturbations of birefringence occurring along the fiber, PMD is often used. PMD is defined by the mean value of the DGD distribution and is proportional to the square root of propagation distance $L$. DGD and PMD are very useful concepts when a low-coherence light source is used in the system.

A stable polarization state can be ensured by deliberately introducing birefringence into an optical fiber: this is known as polarization-preserving fiber or polarization-maintaining fiber. Fibers
with an asymmetric core profile will be strongly birefringent, having a different effective index and group velocity for the two orthogonal polarizations. When the birefringence is large enough (on the order of $10^{-4}$), coupling from one polarization to another is difficult. Therefore, the incident polarization which has to be aligned to the fast or slow axis of a polarization-maintaining fiber, is preserved. Nowadays three popular kinds of polarization-maintaining fibers are elliptical-core, bow-tie, and PANDA. Their cross-section images are shown in the Fig. 2.6. The elliptical cores provide a simple form of polarization-maintaining fiber. Different polarization beat lengths are achieved by precisely controlling silica dopant concentration, while keeping core dimensions intact. In bow-tie and PANDA polarization-maintaining fibers mechanic stress is created by inserting stress-inducing elements near the fiber core. The beat length of the commercial polarization-maintaining fiber is several millimeters while that of a standard single-mode fiber is 10 m-100 m.

There are other ways to make polarization-maintaining fibers, although they are not widely used in commercial products. For example, a double-core geometry will also introduce a large birefringence. Low-birefringence fibers can be made by very careful control of the fiber profile.

**2.3.2 Dispersion-tailored fibers**

It is now widely known that the total dispersion in an optical fiber is composed of two components, a material dispersion term and a waveguide dispersion term. The material dispersion is governed by the Sellmeier equations. The waveguide dispersion can be controlled by tailoring the
Figure 2.7: Typical wavelength dependence of the dispersion parameter $D$ for standard, dispersion-shifted, and dispersion-flattened fibers.
refractive-index profile of the fiber. Tailoring the dispersion characteristics in the region 1300-1550 nm has been challenging the fiber designers, especially during the 1980s. Now although the needs have evolved during this period, the efforts have been essentially centered on two main objectives: shifting the zero-dispersion point toward an optimized wavelength and flattening the dispersion curve. The corresponding fibers are called dispersion-shifted and dispersion-flattened fibers. Typical wavelength dependence of the dispersion parameter $D$ for standard, dispersion-shifted, and dispersion-flattened fibers is shown in Fig. 2.7.

![Figure 2.7: Typical wavelength dependence of the dispersion parameter $D$ for standard, dispersion-shifted, and dispersion-flattened fibers.](image)

Figure 2.8: Refractive-index profiles of dispersion-tailored fibers: (a)-(c) dispersion-shifted fiber; (d)(e) dispersion-flattened fiber; (f) dispersion-compensating fiber.

The basic idea of dispersion-tailored fibers is to depart from the rather simple step-index profile in order to obtain desired dispersion characteristics. In the case of the dispersion-shifted fiber, the zero-dispersion point is usually shifted to 1550 nm where the silica glass has an attenuation minimum. The index profile of a dispersion-shifted fiber generally has a triangular shape and can be improved with the additional annular structures around the nominal core or with the use of a depressed inner cladding [47, 55], as shown in Fig. 2.8(a)-(c). In case of dispersion-flattened fibers the general W-profile or multilayer W-profiles [47, 55] shown in Fig. 2.8(d)(e) have been adopted. There are also dispersion-compensating fibers. The refractive-index profile of such fibers is tailored for a large negative dispersion in order to compensate the accumulated dispersion induced by the standard single-mode fiber. The typical refractive-index profile of dispersion-compensating fibers is shown in Fig. 2.8(f). Generally, fibers with modified dispersion characteristics are more
expensive and require precise controlling the amount of the dopant concentration in the manufacturing process.

2.3.3 Photonic crystal fibers

A big research effort is now with photonic crystal fibers (PCF). PCFs guide light by confining it within a periodic array of microscopic air holes that run along the entire fiber length. PCF is now a proved technology, which is competing with conventional fibers in many applications and is opening others that are not accessible to all-glass fibers. Conventional optical fibers are limited to rather small differences in refractive index between core and cladding—a few percent at most for fibers made from doped silica. The comparatively large index contrast between air and glass in PCFs, combined with the ability to vary the sizes and positions of the air holes means that a much broader range of index profiles becomes possible, resulting in fibers with very unusual optical characteristics. PCFs can be single-mode at all wavelengths or at any given wavelength, up to large core diameters. They can also be highly nonlinear, can possess unusual dispersion, or can be highly birefringent.

PCFs come in two basic varieties: solid-core PCFs and hollow-core PCFs. While both types contain an arrangement of tens to several hundreds of air holes in an otherwise usually uniform material, operating principles, geometry, and optical properties of these fibers are quite different.

Like conventional fibers, solid-core PCFs guide light by total internal reflection at the boundary between a low-index cladding and a high-index core. In solid-core PCFs, as in all total-internal-reflection fibers, the vast majority of light propagates in the glass, i.e. the solid core. In most all-solid fibers, the required index difference is created by doping either the core or the cladding glass. In a PCF the same is achieved by incorporating holes into the cladding, causing the weighted average refractive index “seen” by the mode to be lower than that of the core. By altering the arrangement of the holes or the shape of the core, optical properties such as mode shape, non-linearity, dispersion, and birefringence can be varied over a range, often far exceeding what is
possible with conventional fiber technology.

Hollow-core PCFs employ a fundamentally different guiding mechanism. A photonic bandgap in the cladding acts as a virtually loss-free mirror confining light to a core that does not necessarily consist of solid material. This makes it possible to create low-loss waveguides with gas-filled or even evacuated cores, similar to the familiar hollow waveguides from microwave technology. Photonic bandgaps can be formed in any material with a periodically structured refractive index. In PCFs, this is achieved by incorporating holes into a glass matrix. What makes this concept so interesting is that the interaction between light and glass can be surprisingly small. Hollow-core PCFs can, therefore, have extremely low nonlinearity, high breakdown threshold, zero dispersion at any design wavelength, and negligible interface reflection. Furthermore, it becomes possible to fabricate low-loss fibers from comparatively high-loss materials, extending the range of materials that can be considered for fiber fabrication.

2.3.4 Dispersion-tailored higher-order-mode fibers

Since nineties of the last century, the optical fiber technology has been developed rapidly. Optical fibers play an important role not only in optical communication, but also in manufacturing different fiber devices for many applications. In-fiber devices have become a mainstay in fiber-optic communication systems, because they provide the advantages of low loss, polarization insensitivity, high reliability, and compatibility with the transmission line. They have found several applications, ranging from signal conditioning (amplification, dispersion control, etc.) to network management (add/drop multiplexing, optical monitoring, etc.). In these fiber devices usually the single-mode fiber is used, which serves as a platform to propagate the signal. The device effect itself is due to some extraneously introduced material or structure (dopants for amplification, gratings for phase matching, etc.).

However, there remain some challenging fields, for instance, generation of visible supercontinua, soliton propagation, pulse compression, and building fiber laser cavities, require fibers with
anomalous dispersion (e.g. group velocity dispersion $D > 0$) for certain spectral range. For example, in the wavelength range below 1300 nm, where silica material dispersion is negative, this has remained an unfulfilled aim of conventional silica-based fibers. Small-core microstructured optical fibers or bandgap fibers seem to be able to have positive waveguide dispersion in this spectral range and solve the above problem. However, the cost for this is relatively small mode effective area or complexity in fabrication.

There exists another, relatively less explored degree of freedom afforded by fibers—the ability to co-propagate more than one mode, in so-called higher-order-mode fibers. Recently a series of novel fibers which support several spatial modes have been invented by Prof. Ramachandran and his colleagues in the OFS lab in America [6–15, 17, 42, 65, 66]. In these fibers each mode may be designed to have a uniquely defined dispersion and propagation characteristics for desired application.

Consequently, varieties of new fiber devices have been invented, where few-mode fibers—fibers that typically support two to four modes with suitably tailored dispersive properties, are used, for instance, polarization-insensitive fiber gratings, broadband polarizers, tunable dispersion compensators and fiber mode converters [6, 7, 9, 10, 12, 14, 15, 42]. The unique dispersive properties of various modes in conjunction with the mode conversion technique lead to devices that offer novel solutions for dispersion compensation [10], spectral shaping and polarization control [9, 12, 42], to name a few. A unique feature of few-mode fiber devices is that it takes the complexity out of the device fabrication process, and introduces it into the fiber design process. Dispersion optimized few-mode fibers can be manufactured using techniques used for conventional transmission fibers. Thus, few-mode fiber devices can potentially be highly cost-competitive, since the most complex part of the device (the dispersive fibers) can be mass manufactured.
2.4 Fiber fabrication methods

The major design issues of an optical fiber are related to the refractive-index profile, i.e. the amount of the dopant and the core and cladding dimensions. In a conventional optical fiber, both core and cladding use silica as the base material. The difference in the refractive indices is realized by doping the core, or the cladding, or both. Dopants such as GeO$_2$ and P$_2$O$_5$ increase the refractive index of silica. On the other hand, dopant such as B$_2$O$_3$ and fluorine decrease the refractive index of silica. The fabrication of a conventional optical fiber involves precision control of the glass composition. The desired refractive-index profile is firstly fabricated by selectively doping a large glass preform, typically several centimeters in diameter and about a meter long, which maintains the relative dimensions and doping profiles for the core and cladding [47, 55]. The preform is later heated in an electric resistance furnace until it reaches its melting point over the entire cross section. Thin fibers are then drawn upward from the preform in a drawing tower. The pulling force controls the fiber diameter, which is monitored by a laser interferometer. Bare fiber is then drawn through a vat of polymer and receives a protective coating. Finally the fibers are spooled evenly onto a mandrel. If the preform is uniformly heated, then the cross section and index profile of the drawn fiber will be exactly the same as in the preform. In this manner, fibers with very complex refractive-index profile can be produced.

The primary technology used in fiber preform manufacturing is chemical vapor deposition (CVD), in which submicron silica particles are produced through one or both of the following chemical reactions, carried out at temperatures of around 1800°C to 2000°C:

\[
\text{SiCl}_4 + O_2 \rightarrow \text{SiO}_2 + 2\text{Cl}_2 \uparrow \\
\text{SiCl}_4 + 2\text{H}_2\text{O} \rightarrow \text{SiO}_2 + 4\text{HCL} \quad (2.45)
\]

This deposition produces a high-purity silica soot that is then sintered to form optical-quality glass. Two basic manufacturing techniques are commonly used, namely inside process and outside
In the so-called inside process, a rotating silica substrate tube is subjected to an internal flow of reactive gases. Two variations on this approach are modified chemical vapor deposition (MCVD) and plasma-assisted chemical vapor deposition (PCVD). In both cases, layers of material are successively deposited, controlling the composition at each step, in order to reach the desired refractive index. MCVD, which accounts for a large portion of the fiber produced today, especially in America and Europe, accomplishes this deposition by application of a heat source, such as a torch, over a small area on the outside of the silica tube. Submicron particles are deposited at the leading edge of the heat source. As the heat moves over these particles, they are sintered into a layered, glassy deposit. By contrast, the PCVD process employs direct radio-frequency excitation of a microwave-generated plasma. Because the microwave field can be moved very quickly along the tube (since it heats the plasma directly, not the silica tube itself), it is possible to traverse the tube thousands of times and deposit very thin layers at each pass, which makes for very precise control of the preform index profile. A separate step is then required for sintering of the glass. All inside vapor deposition processes require that a tube be used as a preform. Minor flaws in the tube can induce corresponding dips and peaks in the fiber index profile.

In the so-called outside process, a rotating, thin cylindrical mandrel is used as the substrate. The mandrel is then removed before the preform is sintered. An external torch fed by carrier gases is used to supply the chemical components for the reactions. Two outside processes are the outside vapor deposition (OVD) and vapor axial deposition (VAD) methods. OVD is basically a flame hydrolysis process in which the torch consists of discrete holes in a pattern of concentric rings, each of which provides a different constituent element for the chemical reactions. The VAD process is similar in concept, using a set of concentric annular apertures in the torch. In this case, the preform is pulled slowly across the stationary torch.
Chapter 3

Doughnut beams and LP_{11} modes

In the earlier time, higher-order modes inside the fiber, which introduced the intermodal dispersion, have limited the applications of step-index fibers in optical communications. Recently it has been found that the dispersion of higher-order modes can be tailored by a special fiber design so that higher-order modes explore some new applications in many regions [6, 7, 10]. However, traditional step-index fibers are low cost and higher-order modes inside them can also find some applications in some fields [29, 67–72]. In this chapter radially polarized and azimuthally polarized beams will be introduced. In some publications they are often called doughnut beams because of their doughnut shapes in intensity. It will be presented that they are equivalent with the vector TM_{01} and TE_{01} modes inside the LP_{11} mode group of a perfectly circular optical fiber and can be generated using a simple setup with a step-index fiber that operates at the appropriate wavelength and is subject to the appropriate pressure and torsion. The polarization of the obtained doughnut beams can be evaluated using two parameters derived from Stokes parameters.

3.1 Introduction

These years beams with radial polarization and azimuthal polarization are becoming popular and often used in investigating some basic physical effects [73]. For example, when a beam with radial polarization is focused with high-NA system, an intense longitudinal (i.e. along the optical axis) component of the electric field results [19]. This has several ramifications: (a) super-resolved focal spot size [20–26], with applications in microscopy [74] and lithography; (b) laser-cutting of
metals that is twice as efficient compared to conventional beams [75]; (c) optical tweezers that are significantly more efficient, since the strong longitudinal electric field creates large gradient forces but no scattering forces [76]; (d) a focused pattern resembling emissions from an atomic dipole, thus enabling their excitation, with applications in quantum information [77]. After summarizing the applications of the doughnut beams, the characteristics and the mathematical description of the doughnut beams as well as several methods for generating them will be reviewed next.

**Figure 3.1:** (a) An azimuthally polarized beam as a coherent summation of a y-polarized TEM$_{10}$ mode and an x-polarized TEM$_{01}$ mode. (b) A radially polarized beam given by the coherent summation of an x-polarized TEM$_{10}$ mode and a y-polarized TEM$_{01}$ mode.

The main feature of the doughnut beams is that their amplitude and polarization are cylindrically symmetric. The electric field $\mathbf{E}$ of an azimuthally polarized beam is oriented everywhere along the azimuthal direction given by the unit vector $\phi$ in cylindrical coordinates $(r, \phi, z)$. For a radially polarized beam, the electric field $\mathbf{E}$ is directed along the radial direction defined by the direction of the unit vector $r$. Figure 3.1 shows the formation of the two kinds of beams. Both radially and azimuthally polarized beams can be expressed as a coherent summation of the two Hermite-Gaussian modes TEM$_{10}$ and TEM$_{01}$. A y-polarized TEM$_{10}$ mode and an x-polarized TEM$_{01}$ give an azimuthally polarized beam whereas an x-polarized TEM$_{10}$ mode and y-polarized TEM$_{01}$ mode produce a radially polarized beam. The scalar field distributions of the TEM$_{10}$ and
3.1. INTRODUCTION

TEM₀₁ in cylindrical coordinates are described as

\[
\text{TEM}_{10} : E_{10}(r, \phi) = E_0 \frac{r}{\omega_r} \exp\left(-\frac{r^2}{\omega_r^2}\right) \cos(\phi) \\
\text{TEM}_01 : E_{01}(r, \phi) = E_0 \frac{r}{\omega_r} \exp\left(-\frac{r^2}{\omega_r^2}\right) \sin(\phi)
\]

where \( r, \phi \) are the cylindrical coordinates, \( E_0 \) is the magnitude of the field. Therefore, the vectorial electric fields of a radially and an azimuthally polarized beam in free space are often written as [18, 78]

\[
\text{radial} : E_r(r, \phi) = x E_{10}(r, \phi) + y E_{01}(r, \phi) \\
= r E_0 \frac{r}{\omega_r} \exp\left(-\frac{r^2}{\omega_r^2}\right)
\]

\[
\text{azimuthal} : E_\phi(r, \phi) = -y E_{10}(r, \phi) + x E_{01}(r, \phi) \\
= \phi E_0 \frac{r}{\omega_r} \exp\left(-\frac{r^2}{\omega_r^2}\right)
\]

where \( x, y, r, \phi \) are unit vectors.

New applications of the doughnut beams stimulate the invention of different methods to generate them. Several experiments have been reported by various authors. Following the above mathematical description for radially or azimuthally polarized beams, a straightforward way to generate such beams is the interferometric way, as shown in Fig. 3.2. This method involves combining interferometrically two orthogonally polarized TEM₁₀ and TEM₀₁ modes with the intensity and phase tailored in an appropriate way [79]. The initial light field in this experiment is a TEM₀₁ mode generated inside the cavity of an Ar⁺ ion laser. This mode is discriminated from the common fundamental Gaussian mode and higher-order modes by inserting a thin wire and an aperture into the cavity. In this way the threshold of the unwanted modes is made higher than for TEM₀₁ mode which therefore gets to oscillate and is emitted by the laser. The emitted TEM₀₁ mode is spatially filtered and split with a 50% beam splitter to form two arms of a Mach-Zehnder inter-
When the input is a y-polarized TEM$_{01}$ mode, the output is a radially polarized doughnut beam. When the input is an x-polarized TEM$_{01}$ mode, the output is an azimuthally polarized doughnut beam. M: reflecting mirror; BS: 50% beam splitter; PS: a folding periscope that rotates the beam by 90°. Some optics for matching the path lengths and the intensities of the two arms are not shown.

Figure 3.2: Interferometric method for generating a radially or azimuthally polarized beam [79]. One arm includes a folding periscope to rotate the beam by 90° before transmission to the recombining beam splitter where radially and azimuthally polarized beams are separately generated.

Instead of generating the TEM$_{01}$ mode in a laser resonator and interferometrically producing radially and azimuthally polarized beams externally, the interferometric process can be accomplished inside the laser resonator [78], as shown in Fig. 3.3. Such an experiment has been demonstrated which involved the coherent summation of the orthogonally polarized TEM$_{10}$ and TEM$_{01}$ modes generated within a Nd:YAG resonator. The authors used a birefringent beam displacer inserted into the resonator to spatially separate the x and y polarization components. Each of these components passes through a discontinuous phase element aligned appropriately to introduce a phase of 0 or π. In order to add the TEM$_{10}$ and TEM$_{01}$ modes coherently with the appropriate phase between them, an additional aligning plate was used in one of the paths. The generated orthogonal modes are combined within the resonator to generate either a radially or an azimuthally polarized laser output.
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Figure 3.3: Generating a radially or azimuthally polarized beam interferometrically in a laser cavity [78]. The two discontinuous phase elements are fabricated on the same substrate. The upper one produces a y-polarized \( \text{TEM}_{10} \) mode. The lower one produces an x-polarized \( \text{TEM}_{01} \) mode. A radially polarized beam is generated. After realign the two discontinuous phase elements, an azimuthally polarized beam can be obtained.

Besides the above two interferometric methods, a setup using two kinds of polarization converters has been used recently. A \( \lambda/2 \) polarization converter, which has been used in our experiments, is made from segments cut from a \( \lambda/2 \) plate [18, 80–82]. The second polarization converter, which has been used instead of the \( \lambda/2 \) polarization converter these years, is a liquid crystal cell operating in a metastable state [83]. The two kinds of polarization converters will be explained in details later in this chapter. Both converters produce a pseudo radially or azimuthally polarized beam, which can be regarded as a mixture of a pure radially or azimuthally polarized beam and some higher-order modes. Therefore, after the Gaussian incident beam passes through the polarization converter, a non-confocal Fabry-Perot resonator has to be used as a mode cleaner.

There are other methods reported in the literatures that include: computer generated space variant subwavelength metal stripe gratings [84], mode selection inside an optical fiber [15, 16, 27–31], polarization selective mirrors [85] etc [86–90]. By using two \( \lambda/2 \) plates with their fast axes subtending an angle of \( 45^\circ \) with each other, one can transform an azimuthally polarized beam to a
radially polarized beam and vice versa [28, 30, 91]. Details can be found in these references.

3.2 Doughnut beams and fiber modes

It is seen that the linearly polarized Hermite-Gaussian beams in free space, which are used to form radially or azimuthally polarized beams, have the same two-lobe intensity patterns and polarization as the well-known $LP_{11}$ modes in an optical fiber, while the radially or azimuthally polarized beams have the same doughnut intensity patterns and polarization as the $TM_{01}$ or $TE_{01}$ vectorial mode.

However, it has been shown in section 2.2 the field distribution of a fiber mode is described by the Bessel function of the first kind $J(Ur/a)$ inside the core and by the modified Bessel function of the second kind $K(Wr/a)$ outside the core. Furthermore, it is now well known that the radial distribution of the fundamental mode appears to be very close to Gaussian. This suggests that we can approximate the exact field distribution of the $LP_{01}$ mode by the Gaussian function

\[ E_g(r, \varphi) = E_0 \exp\left(-\frac{r^2}{\omega_g^2}\right) \]  

(3.3)

Such a Gaussian function is more practical to use than the exact Bessel function and it also allows us to define a mode spot size $\omega_g$ which is very useful in problems related to mode matching, either between two fibers, or between a fiber and a Gaussian laser beam.

Next the energy coupling coefficient between the incident radially polarized beam and the $TM_{01}$ mode of an optical fiber will be calculated. According to [46], the radial field distribution for TM modes $e_r$ can be expressed as,

\[ e_{co} = r J_1(Ur/a)/J_1(U) \]
\[ e_{cl} = r n_{co}^2 K_1(Wr/a)/[n_{co}^2 K_1(W)] \]

(3.4a) \hspace{1cm} (3.4b)
3.2. DOUGHNUT BEAMS AND FIBER MODES

where \( \mathbf{r} \) is the unit vector. \( \mathbf{e}_{co} \) and \( \mathbf{e}_{cl} \) satisfy the eigenvalue equation for TM modes,

\[
\frac{n_{co}^2 J_1(U)}{U J_0(U)} + \frac{n_{cl}^2 K_1(W)}{W K_0(W)} = 0
\]

(3.5)

where \( U \) and \( W \) can be expressed in terms of \( \beta \) using Eqs. 2.13.

The commercial step-index fiber 1060XP from Nufern is considered for the calculation of the energy coupling coefficient between the incident radially polarized beam and the TM\(_{01}\) mode of an optical fiber. 1060XP is a single-mode fiber at 1060 nm wavelength. The main parameters provided by the manufacturer are: the mode field diameter MFD, which is determined by the points where the power of the fundamental Gaussian mode is down by \( e^{-2} \) or the amplitude is down by \( e^{-1} \), equal to 6.2 \( \mu \text{m} \) for 1060 nm, \( \text{NA} = 0.14 \) and second-mode cutoff wavelength equal to 920 nm.

The fiber is used as a multimode fiber by sending light whose wavelength is below the second-mode cutoff wavelength of the fiber, i.e. 633 nm, which is the typical radiation from a He-Ne laser. Assuming the cladding of the fiber is pure fused silica, the refractive index of the cladding \( n_{cl} \) and the refractive index of the core \( n_{co} \) at 633 nm are derived according to the known parameters and Sellmeier equation. Furthermore, in order to get the core radius of the fiber \( a \), the following empirical formula is used [49].

\[
\frac{\omega_g}{a} = 0.65 + 1.619V^{-1.5} + 2.879V^{-6}
\]

(3.6)

where the beam waist of the fundamental Gaussian beam \( \omega_g = \text{MFD}/2 \) is known and knowing \( \lambda = 633 \text{ nm} \) and \( \text{NA} = 0.14 \), \( V \) can be expressed in terms of the core radius \( a \) according to Eq. 2.6. Therefore, \( a = 2.6 \mu \text{m} \) is easily obtained. Another method to get \( a \) is to use the second-mode cutoff wavelength and Eq. 2.6, where \( V = 2.405, \lambda = 633 \text{ nm} \) and \( \text{NA} = 0.14 \). Now all parameters of the fiber are known, and consequently, \( 2.405 < V = 3.65 < 3.83 \) is obtained at 633 nm. It means that for this step-index fiber only LP\(_{01}\) and LP\(_{11}\) mode are supported at 633 nm. Furthermore, the propagation constant of the TM\(_{01}\) mode \( \beta \) can be obtained by solving the
CHAPTER 3. DOUGHNUT BEAMS AND LP\textsubscript{11} MODES

eigenvalue equation 3.5. Consequently, the field distribution of TM\textsubscript{01} mode inside the core \( e_{co} \) and inside the cladding \( e_{cl} \) can be obtained according to Eqs. 3.4a and 3.4b.

The electric field of the radially polarized light in free space \( E_r \) can be written according to Eq. 3.2a with the unknown \( \omega_r \). Then the coupling coefficient between the TM\textsubscript{01} mode of the fiber and radially polarized beam generated in the free space \( P \) can be calculated as

\[
P = \int_{0}^{a} E_r(\omega_r) \cdot e_{co} dr + \int_{a}^{\infty} E_r(\omega_r) \cdot e_{cl} dr \tag{3.7}
\]

It is obvious that \( P \) is dependent of \( \omega_r \). It is determined that for the optimal \( \omega_r = 2.2 \mu m \) (0.85\( a \)), \( P > 99\% \). Therefore, the TM\textsubscript{01} mode of the fiber can be regarded to have the same electric field distribution as the radially polarized beam in free space. This is also true for the TE\textsubscript{01} mode and the azimuthally polarized beam.

3.3 The propagation of doughnut beams through step-index fibers

Knowing that the radially polarized beam is equivalent to the TM\textsubscript{01} mode of an optical fiber. The propagation of the doughnut beam through a step-index fiber will be investigated in this section. Using the setup shown in Fig. 3.4 a radially or azimuthally polarized beam was generated using a \( \lambda/2 \) polarization converter and then optimized using a non-confocal Fabry-Perot interferometer. The doughnut beam of high quality was launched into a step-index fiber. A CCD camera was used to capture the beam pattern after the tested fibers.

3.3.1 Experimental setup

The light source was a stabilized He-Ne laser operating at 633 nm. This laser supports only one longitudinal mode because that the mode-stabilization technique is employed. The polarization-
3.3. THE PROPAGATION OF DOUGHNUT BEAMS THROUGH STEP-INDEX FIBERS

Figure 3.4: Experimental setup for generation of radially polarized or azimuthally polarized beams and for study of their transmission through a step-index fiber: L: lens, PMF: polarization-maintaining fiber, NCFP: non-confocal Fabry-Perot interferometer as the mode cleaner, M1, M2: reflecting mirrors, P: $\lambda/2$ polarization converter. Pictures in the insets show beam patterns taken by the CCD camera. Inset 1: pseudo-doughnut beam generated by the $\lambda/2$ polarization converter; inset 2: the high-quality doughnut beam obtained by the non-confocal Fabry-Perot interferometer; inset 3: typical asymmetric beam from the step-index fiber with bending; inset 4: typical antisymmetric beam from the straight step-index fiber.
maintaining fiber works as a mode cleaner to get a pure linearly polarized Gaussian beam. The beam then goes through the $\lambda/2$ polarization converter, which comprises four $\lambda/2$ plate segments and converts the Gaussian input beam to the so-called pseudo-doughnut beam, shown in the inset 1 in Fig. 3.4. The pseudo-doughnut beam is then coupled into the non-confocal Fabry-Perot interferometer for mode cleaning so that a doughnut beam of high quality (shown in the inset 2 in Fig. 3.4) is obtained. Then the doughnut beam was focused by an aspheric lens with 11 mm focal length into the fiber under test. The focusing lens has been chosen to maximize mode coupling efficiency. The beam out of the fiber was the strongly attenuated and projected onto a charge-coupled device (CCD) image sensor by means of a lens with 10 mm focal length.

Next several important elements used in the setup: the $\lambda/2$ polarization converter, the non-confocal Fabry-Perot interferometer, and the optical fibers, will be introduced. The $\lambda/2$ polarization converter is a key element used in the setup. The four pieces of multi-order $\lambda/2$ plates, one in each quadrant, were mounted between two quartz plates. As shown in Fig. 3.5(a), the optical axis of each quadrant was oriented in such a way that the electric vectors are rotated by $90^\circ$ in adjacent segments. A $\lambda/2$ plate before the converter was used to properly adjust the polarization of the input Gaussian beam. Hence, a pseudo-radially or pseudo-azimuthally polarized beam was created. The mode conversion efficiency of such a converter depends on the number of the segments used. Our converter is based on four segments. The diffraction artifacts in the beam are due to the edges where the segments are joined to each other. The far-field intensity distributions of the pseudo-doughnut beam measured by the CCD camera without lens and with a weak focusing lens are shown in Fig. 3.5(b).

The $\lambda/2$ polarization converter can be replaced by another type of mode converter—the liquid crystal polarization converter [83]. The liquid crystal polarization converter is made of one unidirectional and one circularly rubbed alignment structure on two opposing glass plates, as shown in Fig. 3.6(a). The space separating the plates is filled with a nematic liquid crystal. The unidirectional alignment layer defines the cell axis. Because of its combined linear and circular symmetry the liquid crystal cell is called $\theta$ cell. The local liquid crystal orientation in a $\theta$ cell is that of a
Figure 3.5: (a) Four-segments $\lambda/2$ polarization converter which converts a Gaussian beam with vertical polarization to radially polarized beam, or a Gaussian beam with horizontal polarization to azimuthally polarized beam. (b) Intensity distributions measured in the far field behind the polarization converter and at the focal plane of a weak focusing lens (NA < 0.1).
Figure 3.6: (a) The glass plates showing linearly and circularly rubbed alignment layers [83]. (b) Orientation of the liquid crystal molecules in the ground state [83] when the incident linearly polarized light has the same polarization direction as the cell axis. The two defect lines can be seen. (c) The orientation of the liquid crystal molecules in the metastable state [83] when the incident linearly polarized light has the same polarization direction as the cell axis. An azimuthally polarized light is obtained.

Figure 3.7: (a) A horizontally polarized beam incident onto a liquid crystal polarization converter to produce a radially polarized doughnut beam [83]. (b) A vertically polarized beam incident onto a liquid crystal polarization converter to generate an azimuthally polarized doughnut beam [83].
3.3. THE PROPAGATION OF DOUGHNUT BEAMS THROUGH STEP-INDEX FIBERS

twisted cell with a variable twist angle defined by the local alignment layers.

Assume that linearly polarized light is incident upon the unidirectionally oriented cell substrate and propagates parallel to the cell normal. The optical properties of the \( \theta \) cell for generating a radially or azimuthally polarized beam can be simply understood as follows: the angle between the electric vectors of the light and the aligned the liquid crystal molecules remains constant. When the incident linearly polarized light has the same polarization direction as the cell axis, the orientation of the liquid crystal molecules in ground state (no voltage supplied) is shown in Fig. 3.6(b). In the ground state the twist angles are always smaller than \( \pm 90^0 \) and minimize the elastic twist energy. In order to obtain an azimuthally polarized light at the output of the liquid crystal cell, the plates are connected to transparent electrodes which supply an alternating voltage of about 16 V peak to peak. If the electric field is switched off, the liquid crystal cell is left in a metastable state. Consequently, an azimuthally polarized light is obtained, as shown in Figs. 3.6(c) and 3.7(b). In the similar way, a radially polarized light is obtained when the incident linearly polarized light has a polarization perpendicular with the cell axis, as shown in Fig. 3.7(a). The liquid crystal molecules in metastable state will slowly twist as they go back to the ground state. Therefore in order to ensure the output of the liquid crystal device remains a radially or azimuthally polarized beam in experiments, the applied voltage has to be switched on and off every half an hour. Compared to the \( \lambda/2 \) polarization converter, the liquid crystal polarization converter has the advantages of high mode conversion efficiency and large bandwidth.

An improved version of the liquid crystal polarization converter is now available commercially [92]. It has three elements assembled in one housing. It consists of a phase shifter, polarization rotator and the liquid crystal cell \( \theta \)-cell described above. The phase shifter generates a \( \pi \) phase step between the upper and the lower half of the incident beam. It prepares a Hermite-Gaussian beam for the \( \theta \)-cell. Consequently, an azimuthally or radially polarized beam can be generated by the \( \theta \)-cell in ground state. Therefore, no voltage shaking is needed for the converter any more, which allows the long-term continuous measurements. The polarization rotator is capable to rotate the polarization direction by \( 90^0 \) by applying an alternating voltage larger than 5 V (RMS, root mean
square). Therefore, the output light of the converter can switch between the radially polarization and azimuthally polarization easily.

Figure 3.8: The pseudo-doughnut beam is cleaned by a non-confocal Fabry-Perot interferometer comprising of one plano-concave mirror and a spherical mirror. P: the λ/2 polarization converter, L: the coupling lens. Inset 1: incident Gaussian beam; inset 2: pseudo-doughnut beam behind the polarization converter; inset 3: doughnut beam of high quality obtained using the non-confocal Fabry-Perot resonator. R: the radius of the curvature of both mirrors, d: the distance between the two mirrors, n: the refractive index of the plano-concave optical plate. ω1 is the beam waist inside the cavity when taking into account the lens effect of the entrance mirror. The incident Gaussian beam has a beam waist of 3 mm. The calculated optimal focal length of the coupling lens L is about 90 cm.

In order to obtain a radially or azimuthally polarized beam with high quality, the beam behind the polarization converter has to be focused into the non-confocal Fabry-Perot interferometer for the purpose of the mode cleaning, as shown in Fig. 3.8. The interferometer is formed by a plano-concave mirror as the entrance to the cavity and a spheric mirror with the same curvature as an exit from the cavity. Both mirrors were anti-reflection coated on the backside and the reflectivity is larger than 95%. The plano-concave mirror acts as a negative lens and changes the characteristics of the input beam, which has to be taken into account for the calculation of the optimal focal length.
of the focusing lens [25]. The spheric mirror preserves the wavefront of the selected mode as it emerges from the cavity.

The beam waist inside the Fabry-Perot resonator with two spheric mirrors is given by \( \omega_0 \). \( \omega_0 \) is determined by the radius of the mirror curvature \( R \) and the separation between the mirrors \( d \) [93].

\[
\omega_0^2 = \frac{\lambda}{2\pi} [d(2R - d)]^{1/2}
\]  

(3.8)

However, for the optimal coupling into the resonator, it is necessary to consider the behavior of the entrance mirror as that of a lens. The equivalent focal length of the plano-concave mirror is derived from the lensmaker’s equation and is given by

\[
f = \frac{-R}{n - 1}
\]  

(3.9)

where \( n \) is the refractive index of the plano-concave optical plate. Then the equivalent beam waist inside the cavity \( \omega_1 \) can be obtained [93].

\[
\omega_1^2 = \frac{\lambda R}{\pi} [d(2R - d)]^{1/2} \left[\frac{2R + d(n^2 - 1)}{n^2 - 1}\right]
\]  

(3.10)

Next the optimal focal length of the coupling lens \( L \) will be calculated. It is shown by calculation in [81] that if a Gaussian beam with \( \omega_g \) shines the four-segments \( \lambda/2 \) polarization converter, a pseudo doughnut beam with \( \omega_r = \omega_g / \sqrt{2} \) will be obtained. This relation is very important for the calculation of the optimal focal length of the coupling lens \( L \) for the doughnut beam. Furthermore, in the experiments a resonator with \( R = 50 \) mm, \( d = 85 \) mm and \( n = 1.5 \), and a laser source with \( \lambda = 633 \) nm are used. The beam radius of the collimated Gaussian mode in front of the polarization converter is about 3 mm. Consequently, it is calculated the optimal focal length of the coupling lens is calculated to be 90 cm [93]. In practice a plano-concave singlet with 75 cm focal length was used to couple the pseudo-doughnut beam into the resonator.

Note that the size of the incident Gaussian beam, the focal length of the coupling lens, and
the resonator parameters $R$ and $d$ have to satisfy the certain relation to achieve the maximal mode coupling efficiency and hence a stable doughnut mode. However, the $\lambda/2$ polarization converter needs a relatively large beam size to minimize the effect of the misalignment and the diffraction at the edges. This requires a relatively large focal length of the coupling lens, which may cause the setup to occupy more space. This is the drawback of this doughnut generation technique.

The distance of the mirror spacing is detuned from the confocal distance to make sure that the different transverse modes can be selected individually due to Gouy phase shift [93]. The resonator length is stabilized for maximum transmission of the doughnut mode by using a lock-in technique. For this purpose the length is periodically modulated by a piezo element with an amplitude of less than 3 nm, so that the intensity varies by 1%.

Behind the Fabry-Perot mode cleaner a high-quality doughnut mode is obtained, as shown in the inset 2 in Fig. 3.4. The radially or azimuthally polarized beam is then coupled into the tested fiber. Three types of bare fiber of about 1 meter length have been tested individually. The fiber is mounted on the 3D stage with the standard fiber clamp and might suffer bending with a curvature bigger than 10 cm. Table 3.1 shows all the information of the three used fibers. 1060XP and 780HP are commercial single-mode fibers. LB800 is a low-birefringence fiber. It is designed to have a special low-stress coating and, therefore, maintains the low birefringence properties. The parameters regarding the fiber profile, e.g. $a$ and $\Delta$ are usually not provided by the manufacturer. They are calculated following the methods mentioned in section 3.2. The last row of the table gives

<table>
<thead>
<tr>
<th>Fiber type</th>
<th>1060XP</th>
<th>780HP</th>
<th>LB800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer</td>
<td>Nufern</td>
<td>Nufern</td>
<td>YORK V.S.O.P</td>
</tr>
<tr>
<td>Single-mode operating wavelength (nm)</td>
<td>980-1600</td>
<td>780-970</td>
<td>850</td>
</tr>
<tr>
<td>Second-mode cutoff wavelength (nm)</td>
<td>920</td>
<td>730</td>
<td>800</td>
</tr>
<tr>
<td>Mode field diameter MFD ($\mu$m)</td>
<td>6.2@1060</td>
<td>5@850</td>
<td>5@800</td>
</tr>
<tr>
<td>Numerical aperture NA</td>
<td>0.14</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>Core radius $a^*$ ($\mu$m)</td>
<td>2.6</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Relative refractive index $\Delta^*$</td>
<td>0.0047</td>
<td>0.0040</td>
<td>0.0023</td>
</tr>
<tr>
<td>Normalized frequency $V^*$ at 633 nm</td>
<td>3.64</td>
<td>2.58</td>
<td>2.98</td>
</tr>
</tbody>
</table>

Table 3.1: Three fiber types used in the setup shown in Fig. 3.4. * indicates the derived parameters.
the calculated $V$ values at 633 nm. $V$ is important for estimating the number of modes supported by the fiber. The $V$ values for all the three fibers satisfy the cutoff condition $2.405 < V < 3.83$. According to the above cutoff condition all three fibers support only two LP mode groups, $LP_{01}$ and $LP_{11}$. Furthermore, for the fiber 1060XP $V = 3.64 >> 2.405$, $LP_{11}$ mode is far from the cutoff, while for the fiber 780HP $V = 2.58 \approx 2.405$, the $LP_{11}$ mode is close to the cutoff. In fact three fibers with different $V$ are used to investigate whether the propagation of the doughnut beam through an optical fiber depends on the normalized frequency $V$.

3.3.2 Results and discussion

The corresponding experimental results are shown in inset 3 and inset 4 in Fig. 3.4. The typical intensity distributions of the beam from the three fibers, shown in the inset 3 in Fig. 3.4, are similar. The asymmetric two-lobe pattern is a typical coherent superposition of the two $LP_{01}$ modes and four $LP_{11}$ modes, as shown in Fig. 2.3. The weights of the $LP_{01}$ mode inside the outgoing beam can be roughly evaluated by the symmetry of the resultant beam, because the $LP_{01}$ mode, once excited by the fiber, will interfere with the antisymmetric $LP_{11}$ modes and cause the asymmetry [94].

Furthermore, it has been determined that the $LP_{01}$ mode is caused by the bending of the fiber using the following measurements. A short piece of the fiber with about 10 cm length was mounted straightly in the setup, replacing the one-meter fiber used previously. One part of the fiber coating was removed and immersed into a drop of glycerol in order to get rid of the cladding modes because the fiber is too short. The incident beam of the fiber remains a high-quality radially or azimuthally polarized doughnut beam. An antisymmetric beam, shown in the inset 4 in Fig. 3.4, appears at the end facet of the fiber, which is the superposition of the $LP_{11}$ modes and is an evidence that nearly no $LP_{01}$ mode is excited inside the fiber.

However, it is seen in the inset 4 in Fig. 3.4 that even without bending the output beam does not possess the same doughnut intensity pattern as the radially or azimuthally polarized incident
beam. The fact that the fiber prefers an antisymmetric two-lobe pattern is due to the impairment of the fiber, for instance, ellipticity, eccentricity or stress of the fiber core. The axis of the pattern follows the orientation of the fiber impairment. The electric field of such a beam can be described as the superposition of the four scalar \( \text{LP}_{11} \) modes with two intensity patterns and two polarization directions. The superposition gives a variety of modal patterns and polarizations [95, 96].

In conclusion, it has been investigated in this section whether a radially or azimuthally polarized light can be guided through a conventional step-index fiber. A radially or azimuthally polarized doughnut beam of high quality has been generated using a \( \lambda/2 \) polarization converter and a non-confocal Fabry-Perot interferometer. The doughnut beam is then launched into three types of step-index fibers individually. Experimental results show that the doughnut beam does not propagate through the fiber because of the anisotropy of the fiber. This problem can not be overcome by using a step-index fiber with different normalized frequency \( V \).

### 3.4 Generation of doughnut beams using step-index fibers

In the earlier setup, a step-index fiber with \( 2.405 < V < 3.83 \) is used. The cutoff condition guarantees that the fiber will strip all the higher-order modes within the incident beam and only \( \text{LP}_{01} \) and \( \text{LP}_{11} \) modes are guided. From this point of view, launching a pseudo-doughnut beam directly into an optical fiber by removing the non-confocal Fabry-Perot mode cleaner (the dashed box in Fig.3.4) would be more convenient for obtaining a doughnut beam from the fiber. Consequently, a very simple technique for doughnut beam generation is proposed, shown in Fig. 3.9. In this setup, an additional device, a commercial polarization controller, is used.

#### 3.4.1 Experimental setup

In fact launching the pseudo-doughnut beam from the polarization converter directly into the fiber offers one more degree to optimize the beam shape of the final outgoing beam. For a good mode cleaning the fiber with one-meter length is used in the setup. Therefore the \( \text{LP}_{01} \) may be intro-
Figure 3.9: Experimental setup for generation of radially polarized or azimuthally polarized beams using a step-index optical fiber: L: lens, PMF: polarization-maintaining fiber, P: λ/2 polarization converter. Pictures in the insets show beam patterns taken by the CCD camera. A pseudo-doughnut beam is generated by the λ/2 polarization converter. A radially or azimuthally polarized beam is obtained behind the optical fiber and one or two λ/2 plates.
duced by the bending of the long fiber according to the previous investigations. However, the polarization converter in front of the fiber and the entrance facet of the fiber can be adjusted using a three-dimension stage to cancel the $\text{LP}_{01}$ mode. The typical resultant beam is an antisymmetric beam which contains only $\text{LP}_{11}$ modes. The intensity pattern is similar to the beam shown in inset 4 in Fig. 3.4 when a short straight fiber is used.

Next a commercial polarization controller is applied for the selective excitation of $\text{LP}_{11}$ modes so that the radially or azimuthally polarized doughnut beam is achieved at the exit facet of the fiber. In the experiment a mechanical fiber-squeezer polarization controller F-POL-IL [97] from General photonics is used. Compared to the bulk phase retarder and the “fiber coil” polarization controller, it has the features of no intrinsic back reflection, low cost and compactness. Fig. 3.10 shows its operating principle [97]. The center portion of a single mode fiber is sandwiched in the squeezer.

![Figure 3.10](image)

**Figure 3.10:** (a) Schematic diagram of the polarization controller using the mechanical fiber squeezer mechanism; (b) cross-section images of the effective part of the squeezer showing the introduced birefringence.
3.4. GENERATION OF DOUGHNUT BEAMS USING STEP-INDEX FIBERS

The knob on the squeezer will apply a pressure to the fiber and produce a linear birefringence. Therefore, the rotatable fiber squeezer causes fiber center portion to act as a waveplate of variable retardation with its slow axis in the direction of applied pressure similar to a Babinet-Soleil compensator.

When the rotatable fiber squeezer is rotated by firstly releasing the pressure, the net change of the incident angle between the slow axis of the fiber center portion and the input polarization is $\theta$ degrees as shown in 3.10(b). This is called rotate-without-twist procedure. It is used for the coarse adjustment of polarization. The rotate-with-twist procedure can be used to fine tune the output polarization. When the squeezer is rotated while the pressure is applied, the rotation will cause the segments of the fiber at both sides of the squeezer to twist. Because this twist will also rotate the incident polarization, the net incident angle between the slow axis and the input polarization will differ from the rotate-without-twist case.

The above polarization controller is usually used for a single-mode fiber to convert an arbitrary input polarization to any desired output polarization. However, in our experiment it is used for selective excitation of $LP_{11}$ modes so that a radially or azimuthally polarized beam is obtained from the fiber.

3.4.2 Results and discussion

The beam pattern from the fiber is observed using a CCD camera as the fiber is pressed or twisted by the polarization controller. The appropriate pressure or twist would cause the beam to switch from the two-lobe pattern to the two-lobe pattern oriented in the perpendicular direction, as shown in Fig. 3.11. In between a ring (doughnut shape in intensity) beam is obtained. Then a linear polarizer is used to analyze the polarization state of the ring beam, as shown in Fig. 3.9. If the polarization state is not satisfying, the procedure is repeated by altering the strength of the pressure or twist.

Next the polarization state of of the beam will be discussed. The combination of the four $LP_{11}$
Figure 3.11: Typical evolution of the intensity distribution of LP_{11} modes from the fiber under pressure or twist in experiments. Pattern A stands for a typical antisymmetric beam from the fiber, which contains only LP_{11} modes. It is the start point of the procedure. Pattern B stands for a ring-shape beam. It is the end of one iteration.
modes can have arbitrary field distribution, which results in various beam patterns and polarizations. However, when the outgoing beam has a ring shape, only several combinations of $LP_{11}$ modes are possible. In this case it is more convenient to use the vector modes, $TM_{01}$, $TE_{01}$ and $HE_{21}$ as the bases of the fiber, because they have circularly symmetric electric field as the fiber output.

The concept of polarization order is useful for understanding the spatial polarization distributions of the fiber beams. The polarization order number is the number of complete polarization rotations per full azimuthal rotation. $TM_{01}$ mode has a polarization order number of +1 because that for a full azimuthal rotation, the same electric vector appears only one time and varies in the same sense with the rotation, while $HE_{21}^{even}$ and $HE_{21}^{odd}$ have a polarization order -1 because the electric vector varies in the opposite sense. In particular when the $TM_{01}$ and $TE_{01}$ superpose in phase (phase difference 0 or out phase (phase difference $\pi$), a beam with circular shape and cylindrically symmetric polarization is obtained. It still has the order +1. For the $HE_{21}^{even}$-$HE_{21}^{odd}$ superposition, a -1 order circular beam will be produced. Figure. 3.12 shows the polarization distributions of the above combinations when the two components have the same amplitude. Each point within the beam cross section has linear polarization.

![Figure 3.12](image)

**Figure 3.12:** (a) Polarization distributions of +1 order beams as coherent superpositions of $TM_{01}$ and $TE_{01}$ with the same amplitude and phase difference 0 and $\pi$; (b) polarization distributions of -1 order beams as the coherent superpositions of $HE_{21}^{even}$ and $HE_{21}^{odd}$ with the same amplitude and phase difference 0 and $\pi$.

With a linear polarizer, a two-lobe structure with a dark bar in between will be obtained on the detector. When the polarizer is rotated, for +1 order beam, the lobs pattern will rotate in the same sense with the rotation of the polarizer, for -1 order beam the lobs pattern will rotate in
the opposite sense with the rotation of the polarizer. The angle between the transmission axis of the polarizer and the dark bar of the lobs is $45^\circ$. The total intensity on the detector is constant. However, when the two components have different amplitude, the angle between the transmission axis of the polarizer and the dark bar of the lobs is not $45^\circ$ and depends on the amplitude of the two components. Furthermore, when they are not in phase or out of phase, the black bar might disappear for a certain polarizer orientation and the intensity on the detector will have a modulation during the rotation of the polarizer.

![Figure 3.13](image)

**Figure 3.13:** The first row: $\text{TM}_{01}$-$\text{TE}_{01}$ combination. The second row: the $\text{HE}_{21}^{\text{even}}$-$\text{HE}_{21}^{\text{odd}}$ combination. The third row and the fourth row are $\text{TM}_{01}$ and $\text{TE}_{01}$ converted from the $\text{HE}_{21}^{\text{even}}$-$\text{HE}_{21}^{\text{odd}}$ combination using one $\lambda/2$ plate. Column (a) without a polarizer in front of the camera. Column (b)-(e) after passing a polarizer orientated in the direction indicated by the white arrows. (f) The deduced polarization state.

In the experiment the $\text{TM}_{01}$-$\text{TE}_{01}$ combination and $\text{HE}_{21}^{\text{even}}$-$\text{HE}_{21}^{\text{odd}}$ combination are easily obtained, as shown in the first and the second row of Fig. 3.13. $\text{HE}_{21}^{\text{even}}$-$\text{HE}_{21}^{\text{odd}}$ can be converted to $\text{TM}_{01}$ or $\text{TE}_{01}$ simply using one $\lambda/2$ plate, as shown in the third and the fourth row of Fig. 3.13. $\text{TM}_{01}$-$\text{TE}_{01}$ can be converted to $\text{TM}_{01}$ or $\text{TE}_{01}$ using two $\lambda/2$ plates. One $\lambda/2$ plate changes the polarization order and two keep the polarization order. Depending on the pressure and twist onto
the fiber, the TM$_{01}$ or TE$_{01}$ mode can be also obtained directly from the fiber. Two $\lambda/2$ plates convert TM$_{01}$ to TE$_{01}$ or vice versa.

It is also noted that the incident beam does not have to be pseudo radially or azimuthally polarized beam. The Hermite-Gaussian laser beam TEM$_{10}$ or TEM$_{01}$ can be used as the incident beam because the mode selection or mode conversion within the LP$_{11}$ mode group is performed by the polarization controller. Hence, the four segments $\lambda/2$ polarization converter can be replaced by a simple two segments $\lambda/2$ polarization converter.

At the end the setup shown in Fig. 3.9 is proved to be a very easy low-cost setup for generation of the radially or azimuthally polarized beam. The setup comprises a commercial laser source, a four segments $\lambda/2$ polarization converter, a conventional step-index fiber operating below the second-mode cutoff wavelength, and a polarization controller using mechanical fiber squeezer. This method is non resonant and hence broadband. This has been verified using a tunable pulse laser source operating at 600-700 nm. A two segments $\lambda/2$ polarization converter made from cellophane, whose property as a $\lambda/2$ plate at 600-700 nm has been verified, is used. A radially or azimuthally polarized doughnut beam is obtained at 600-700 nm without changing any optics in the setup. This cannot be achieved by the previous technique using non-confocal Fabry-Perot interferometer.

However, at the exit of the fiber the TM$_{01}$ and TE$_{01}$ or HE$_{21}^{\text{even}}$ and HE$_{21}^{\text{odd}}$ may not have exactly phase difference 0 or $\pi$. The evidence for this is that behind the polarizer the dark bar in between the two lobs might be dim and the intensity on the detector has a small modulation as the polarizer is rotated. In the next section two parameters are used to characterize the polarization state of the generated doughnut beams quantitatively.

### 3.5 Characterization of doughnut beams

According to the previous discussion, the outgoing beam from the fiber is the combination of four LP$_{11}$ modes with slightly different prorogation constants. Hence, the beam from the fiber can have
complex polarization state. The selective excitation of those LP\(_{11}\) modes can be performed using a polarization controller so that the radially or azimuthally polarized beam is obtained. The purify of the radially or azimuthally polarized beam can be demonstrated qualitatively using a rotating polarizer (see Appendix C).

The aim of this section is to quantitatively characterize the polarization state of these non-uniformly totally polarized beams using polarization optics. A non-uniformly totally polarized beam is a beam with spatially inhomogeneous polarization states. Each point in the beam cross section is polarized. Radially and azimuthally polarized doughnut beams discussed above are non-uniformly totally polarized beams. These beams are different from common Gaussian beams which are uniformly polarized. Therefore, the traditional terms for polarization characterization are not valid any more. In this section two parameters proposed by Martínez-Herrero [98–100] are used to characterize the polarization state of the radially or azimuthally polarized beams produced by the Fabry-Perot mode cleaner and by the step-index fiber. The two parameters provide information, in a simple way, about the linear or circular polarization content of the beams.

### 3.5.1 Two parameters for characterization of non-uniformly totally polarized beams

The Stokes parameters \(s_0, s_1, s_2, s_3\) are commonly used to describe the polarization degree of an uniformly polarized beam. In this case the Stoke parameters do not depend on the position in the (x, y) plane transverse to the direction of z, and are proportional to the value obtained after integration throughout the transverse plane. Consequently, to measure them either a photodetector or a CCD camera can be used and placed at the observation plane. Using a polarizer and a \(\lambda/4\)
3.5. CHARACTERIZATION OF DOUGHNUT BEAMS

plate, one can write the well-known expressions for Stokes parameters

\[ s_0 = I_0 + I_{90} = I_{45, \lambda/4} + I_{135, \lambda/4} \]  
\[ s_1 = I_0 - I_{90} \]  
\[ s_2 = I_{45} - I_{135} \]  
\[ s_3 = I_{45, \lambda/4} - I_{135, \lambda/4} \]  

In Eqs. 3.11a-3.11d the subscripts indicate the angle that the transmission axis of the polarizer makes with the x-axis. To obtain \( s_3 \) the irradiance should be measured after the \( \lambda/4 \) plate whose fast axis makes an angle 0° with the x-axis and the polarizer oriented so as to transmit the component in the azimuths 45° and 135°.

In the case of non-uniformly totally polarized beams, the Stokke parameters depend on the transverse vector \( r = (x, y) \). The electric field vector of the beam, \( \mathbf{E}(r) \), can then be represented by means of the Jones vector

\[ \mathbf{E}(r) = \begin{pmatrix} E_s(r) \\ E_p(r) \end{pmatrix} \]  

where \( E_s \) and \( E_p \) denote the transverse field components orthogonal to the z-axis. Assuming that \( \mathbf{E}_r \) represents the field at the output of an optical device that only transmits the right-handed circularly polarized component of the input field, we define

\[ \| \mathbf{E}_r \|^2 \equiv \int |\mathbf{E}_r(r)|^2 \, dr \]  

The integration extends over the beam cross section. This expression can be understood as the beam power associated to the “right-handed circular content” \( \mathbf{E}_r \) of the field. In addition it follows at once:

\[ \| \mathbf{E}_r \|^2 = \frac{1}{2} \int s_0(r) \, dr + \frac{1}{2} \int s_3(r) \, dr \]  

(3.14)
CHAPTER 3. DOUGHNUT BEAMS AND LP\textsubscript{11} MODES

The total power would read

$$\|E\|^2 \equiv \int [\|E_s(r)\|^2 + |E_p(r)|^2]dr = \int s_0(r)dr$$

(3.15)

and we have

$$\frac{\|E_r\|^2}{\|E\|^2} = \frac{1}{2} + \frac{1}{2} \int \frac{s_3(r)dr}{s_0(r)dr}$$

(3.16)

This equation would then provide the ratio between the power associated to the right-handed circular content of the beam and its total power. Consequently, we introduce the following parameters

$$\bar{\rho} = \frac{\int s_3(r)dr}{\int s_0(r)dr} = \frac{\int \rho(r)s_0(r)dr}{\int s_0(r)dr}$$

(3.17a)

$$\sigma_{\rho} = \frac{\int (\rho(r) - \bar{\rho})^2s_0(r)dr}{\int s_0(r)dr}$$

(3.17b)

where

$$\rho(r) \equiv \frac{s_3(r)}{s_0(r)}$$

(3.18)

From Eqs. 3.16 and 3.17a note that the ratio $\|E_r\|^2 / \|E\|^2$ and $\bar{\rho}$ have the same physical meaning. The parameter $\bar{\rho}$ ranges from -1 (pure left-handed circularly polarized light) to +1 (pure right-handed circularly polarized light). The value $\bar{\rho} = 0$ corresponds to a pure linearly polarized field. As it is apparent from the definition, $\bar{\rho}$ can be obtained in a simple way by measuring the Stokes parameters $s_0$ and $s_3$ integrated over the full detection area, even though the beam is non-uniformly polarized.

The above values of $\bar{\rho}$ in particular cases are verified using Jones calculus in the following. The parameter $\sigma_{\rho}$ gives the dispersion of the values of $\rho(r)$ across the beam cross section. In fact, it represents the variance of $\rho(r)$. Therefore, this parameter globally characterizes the uniformity of the ratio $\rho(r)$ over the beam profile. In particular, when $\bar{\rho} = \sigma_{\rho} = 0$, it can be concluded that the beam is linearly polarized all over its transverse section.

The above values of $\bar{\rho}$ in particular cases are verified using Jones calculus in the following. The
3.5. CHARACTERIZATION OF DOUGHNUT BEAMS

Jones matrices for the three optical elements which have to be used to measure $s_3$ are

$$Q_H = e^{i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$ (3.19)

for a $\lambda/4$ plate with the fast axis pointing along the x-axis.

$$P_{45} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$ (3.20)

for a linear polarizer with the transmission axis at $45^\circ$.

$$P_{135} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$ (3.21)

for a linear polarizer with the transmission axis at $135^\circ$.

In the following derivation the integration is ignored for simplification. According to Eqs. 3.11d, 3.12, 3.15 and 3.17a one gets

$$\tilde{\rho} = \frac{s_3}{s_0} = s_3 = P_{45}Q_HE - P_{135}Q_HE$$ (3.22)

where $E$ is the normalized Jones vector of the incident light and hence $s_0 = 1$. In particular, when the incident light is a left-handed circular light, right-handed circular light and linearly polarized
light, we have the following Jones vectors

\[ E = E_l = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \]

\[ E = E_r = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \]

\[ E = E_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } E = E_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]  \hspace{1cm} (3.23)

respectively. The corresponding obtained values of \( \tilde{\rho} \) using Eqs. 3.19-3.23 are -1, +1 and 0. These \( \tilde{\rho} \) values for left-handed circular light, right-handed circular light and linearly polarized light are consistent with the previous derivation.

### 3.5.2 Results and discussion

The polarization distribution of a pure radially polarized beam after a \( \lambda/4 \) plate and a polarizer is shown in Fig. 3.14. Behind a \( \lambda/4 \) plate oriented along the x-axis (horizontal), a new kind of ring mode whose polarization locally switches between linear and circular, is produced. Through a rotating polarizer the beam intensity will switch between a ring mode and the two Hermite-Gaussian modes. This ring mode has linear polarization for each point within the cross section and

![Figure 3.14: Polarization distribution of a pure radially polarized beam after a \( \lambda/4 \) plate and a polarizer when \( s_3 \) is to be measured. A \( \lambda/4 \) plate orientated along the x-axis produces a local variation between linear and circular polarization. This beam switches between a linear polarized ring mode and the two Hermite-Gaussian modes behind a rotating polarizer. The black arrows indicate the electric field. The small circles on the black arrows indicate \( \pi/2 \) phase difference. The white arrows indicate the orientation of the polarizer axis.](image)
can be regarded as the overlap of the two TEM$_{01}$ and TEM$_{10}$ modes with the same polarization directions but 90° phase difference. To get $\rho(r) = s_3(r)/s_0(r)$ means to calculate the difference of the two ring modes behind the polarizer oriented at 45° and 135° and then to normalize the difference to the total intensity of the beam $s_0$ at each point on the screen. Thus $\tilde{\rho}$ and $\sigma_{\rho}$ can be calculated using Eqs. 3.17a and 3.17b. For a pure TM$_{01}$, TE$_{01}$, HE$_{21}$ and the in-phase or out-phase TM$_{01}$-TE$_{01}$ or HE$_{21}^{\text{even}}$-HE$_{21}^{\text{odd}}$ combination, the two ring modes have exactly the same ring intensity distribution because these beams all have linear polarization at each point. Therefore, $\tilde{\rho} = \sigma_{\rho} = 0$. The proposed parameters only evaluate the global polarization state of the beam with $\tilde{\rho}$ and the uniformity of the polarization with $\sigma_{\rho}$. They do not distinguish the compositions of the beam in terms of fiber modes. Therefore, the mode type would not be identified. The mode type can be determined by putting a polarizer behind the beam and checking the angle between the orientation of the resultant lobs structure and the transmission axis of the polarizer.

Next a $\lambda/4$ plate, a polarizer and a CCD camera are used to perform the above evaluation of polarization. The measured beams are an uniformly linearly polarized Gaussian beam from the laser source, the nearly perfect radially polarized doughnut beam from the non-confocal Fabry-Perot mode cleaner and the radially polarized doughnut beams generated by the optical fiber. For the measurement of the beams emerging from the fiber, we repeat the measurement by changing the pressure and the twist introduced via the polarization controller (The pressure and the twist were alway set to ensure a radially polarized doughnut.). The experimental results are shown in the Fig. 3.15, where $|\tilde{\rho}|$ and $\sigma_{\rho}$ are plotted.

For a pure uniformly linearly polarized Gaussian beam and a pure radially polarized doughnut beam, $\tilde{\rho} = \sigma_{\rho} = 0$ should be obtained. It is seen in Fig. 3.15 that the linearly polarized Gaussian beam from the laser source has the smallest $|\tilde{\rho}|$ and $\sigma_{\rho}$ as well as error bars, but still they are not equal to zero. The outgoing beam from the laser can be assumed to be a pure uniformly linearly polarized light (The stabilzed He-Ne laser SL02/1 from SIOS Meßtechnik GmbH is in use. The nominal polarization ratio$>500:1$). The small $|\tilde{\rho}|$ and $\sigma_{\rho}$ indicate the noise of the measurement system. They are introduced, for example, by the imperfection of the optics or the uncertainty of
Figure 3.15: The measured values of $|\hat{\rho}|$ and $\sigma_\rho$ for an uniformly linearly polarized Gaussian beam from the laser source, the nearly perfect radially polarized doughnut beam obtained from the non-confocal Fabry-Perot mode cleaner and the radially polarized doughnut beams generated by the optical fiber. $\hat{\rho}$ and $\sigma_\rho$ are defined in Eqs. 3.17a and 3.17b. For a pure uniformly linearly polarized Gaussian beam and a pure radially polarized doughnut beam, $\hat{\rho} = \sigma_\rho = 0$. 
the polarizer orientation. On the other hand, compared to the uniformly linearly polarized Gaussian beam, the radially polarized beam from the Fabry-Perot interferometer has a bit larger $\sigma_\rho$, because it is a non-uniformly polarized beam and the polarization state at each point might be slightly different. The beam from the fiber has the largest $|\tilde{\rho}|$ and $\sigma_\rho$ and relatively large error bars, for that the fiber is quite sensitive to the motion of the polarization controller onto the optical fiber, although a rotating polarizer was used to ensure a radially polarized doughnut output.

In conclusion, two parameters $\tilde{\rho}$ and $\sigma_\rho$ for the characterization of non-uniformly totally polarized beams have been introduced. They are used to characterize the purity of the radially polarized beams from the non-confocal Fabry-Perot mode cleaner and from the optical fiber. It is shown the “rotating-polarizer method” is not an efficient way to ensure a radially polarized beam of high quality. When using an optical fiber to generate the radially polarized doughnut beams, the two parameters $\tilde{\rho}$ and $\sigma_\rho$ can be obtained at the end facet of the fiber in real time. $\tilde{\rho}$ and $\sigma_\rho$ can be used as a feedback of the system so that the pressure and the twist onto the fiber can be optimized to ensure a radially polarized beam of high quality.
Chapter 4

Characterization of the special few-mode fiber using low-coherence interferometry

4.1 Introduction

In this chapter a Michelson-type low-coherence interferometry is proposed to characterize the modes of a few-mode fiber with special refractive-index profile. As the start of the chapter, the principle and the applications of an optical low-coherence interferometry are reviewed. The characteristics of the fiber under test are introduced. Then our low-coherence interferometry is detailed. The data processing using simplified phase-shifting algorithm is also described. The results on the mode profile, group-index difference of the modes of the fiber are compared to the results of numerical simulations. A very interesting finding in the presented results is that it is more efficient to use the group-index difference of the degenerate HE_{21} modes in the LP_{11} mode group to estimate the ellipticity of the fiber than to use the group-index difference of the fundamental modes. The ellipticity of the fiber is then be confirmed by another measurement technique: a frequency-domain optical low-coherence interferometry.

4.1.1 Principles of optical low-coherence interferometry

Interference is the superposition of two or more waves, resulting in a new intensity pattern. When two coherent light beams are combined, their fields add and produce interference. An optical low-coherence interferometry is an interferometric measurement technique using a low-coherence light
source. In an optical low-coherence interferometry, different configurations of the interferometer such as Michelson and Mach-Zehnder interferometer, and low-coherence light source with different spectral properties can be used. A simplified schematic of a low-coherence interferometer of a Michelson type is shown in Fig. 4.1 [101, 102]. In this case it is called optical low-coherence reflectometry. Optical low-coherence reflectometry is now a well-established tool for detecting, localizing, and quantifying the properties of the object such as roughness, thickness, 3-D shape and refractive index [103–105]. In the Fig. 4.1 the time delay between the two arms $\tau$ is

$$\tau = \frac{2}{c}(l_r - l_s)$$

$$= \frac{2}{c}(n_r L_r - n_s L_s)$$

(4.1)

where $l_r$ and $l_s$ are the optical path lengths for the reference arm and sample arm respectively; $L_r$ and $L_s$ are the corresponding geometric lengths; $n_r$ and $n_s$ are the corresponding refractive indices of the media; $c$ is the speed of the light.

Assuming the scalar electric field of the reference arm and the sample arm are $E_r$ and $E_s$, the

\[ 
\text{Figure 4.1: Simplified diagram of a Michelson interferometer. BS: beam splitter.} \]
total optical field at the detector plane is

\[ E(\tau) = E_r(t) + E_s(t + \tau) \]  \hspace{1cm} (4.2)

The intensity of the interference detected by the detector at the exit of the interferometer relative to the time delay \( \tau \), i.e. reflectogram \( I(\tau) \), can be written as

\[ I(\tau) = \langle [E_s(t) + E_s(t + \tau)][E_r(t) + E_s(t + \tau)]^* \rangle \]  \hspace{1cm} (4.3)

where the angular brackets denote the time average over the integration time at the detector. The asterisk symbol means the complex conjugate operation. Because intensities \( I_s = \langle E_s(t)E_s^*(t) \rangle \) and \( I_r = \langle E_r(t + \tau)E_r^*(t + \tau) \rangle \), the resultant intensity becomes

\[ I(\tau) = I_s + I_r + 2\text{Re} \langle E_r(t)E_r^*(t + \tau) \rangle \]  \hspace{1cm} (4.4)

where the first two components \( I_s \) and \( I_r \) are the detected intensities reflected by the sample and by the reference mirror, respectively, and the last term gives the amplitude of the interference fringes, which depends on the optical time delay set by the position of the reference mirror and which carries the information about the structure of the sample.

For many applications, the reflectivity of the sample can be considered wavelength independent in the spectral range of the broadband light source. Assuming the reflectivity of the reference mirror and the reflectivity of the sample are equal, we get \( E_r = E_s \), \( I_r = I_s \) and

\[ I(\tau) = 2I_r + 2\langle E_r(t)E_r^*(t + \tau) \rangle \]  \hspace{1cm} (4.5)
The autocorrelation function of the broadband light source $I_a(\tau)$ is known to be proportional to the oscillating part of the intensity obtained by the detector.

$$I_a(\tau) \propto \langle E_r(t)E^*_r(t + \tau) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} E_r(t)E^*_r(t + \tau) dt$$  \hspace{1cm} (4.6)

Furthermore, according to the Wiener-Khintchine theorem, the spectral density function of the light source $S(\omega)$ and the autocorrelation function $I_a(\tau)$ have the Fourier-transform relation [102]

$$I_a(\tau) \propto \int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau} d\omega$$ \hspace{1cm} (4.7a)

$$S(\omega) \propto \int_{-\infty}^{\infty} I_a(\tau)e^{-i\omega\tau} d\tau$$ \hspace{1cm} (4.7b)

It is often easier to obtain, experimentally or theoretically, $I_a(\tau)$ rather than $S(\omega)$, so that $S(\omega)$ is derived by a Fourier transformation of $I_a(\tau)$. This is termed Fourier-transform spectroscopy.

Following the above time-frequency Fourier-transform relation, if we assume the spectral density of the light source $S(\omega)$ is a Gaussian function, the autocorrelation function of the light source

**Figure 4.2:** A typical intensity distribution detected by the detector at the exit of the optical low-coherence reflectrometry. The waveform versus the position of the reference mirror represents the autocorrelation function of the low-coherence light source.
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is then expressed by a Gaussian function multiplied by a frequency carrier. The autocorrelation function detected in an optical low-coherence reflectometry is shown in Fig. 4.2 [102]. The coherence length of the light source $L_c$ is defined by the full width at half maximum (FWHM) of the interference envelop detected by the detector in the optical low-coherence reflectometry. Assuming the medium is air, $L_c$ is expressed as follows

$$L_c = A \frac{\lambda^2}{\Delta \lambda} \quad (4.8)$$

where $\lambda$ is center wavelength of the light source; $\Delta \lambda$ is the bandwidth (FWHM) of the light source; $A$ is a coefficient depending on the spectral shape of the source. For a Gaussian-shaped spectrum, $A = 0.44$. The above equation reveals the relation between the coherence length of the light source $L_c$ and its spectral characteristic $\Delta \lambda$. This is the so-called Fourier-transform limit in ultrafast optics, which is understood as the lower limit of the pulse duration which is possible for a given optical spectrum of a pulse.

In an optical low-coherence interferometry any type of optical source with wide spectral width can be used. Typical sources are edge-emitting light-emitting diodes, superluminescent diode, and EDFAs. White light such as supercontinuum generated by an optical fiber, and a monochromator are often used as the light source and wavelength scan device for the optical low-coherence reflectometry. Compared to other high-resolution reflectometry techniques, optical low-coherence reflectometry currently offers substantial advantages in both theoretical performance and practical implementation.

The use of optical fibers brings an optical low-coherence reflectometry some interesting features. The typical configuration of a fiber optical low-coherence reflectometry is shown in the Fig. 4.3, where the 3 dB $2 \times 2$ fiber-optic bidirectional coupler is used in stead of the 50/50 beam splitter. The fact that an optical fiber offers a longer optical path length increases the measurement range of the system and helps to build a compact system. However, the fiber-induced effect, for example, the temperature dependence, has to be considered in such systems.
CHAPTER 4. CHARACTERIZATION OF THE SPECIAL FEW-MODE FIBER

Additionally, an optical fiber can be also used in an optical low-coherence reflectometry as a sensor. The applications involve the absolute remote measurement of quasi static parameters, such as displacement, temperature, pressure, strain and refractive index [106]. In particular, an additional so-called sensing interferometer and a local-receiving interferometer can be used in the sample arm and in the detection path respectively. The sensing interferometer is located inside the measured field whilst the local-receiving interferometer is normally placed in a controlled environment far from the field.

4.1.2 Optical fiber characterization using low-coherence interferometry

Optical fiber characterization is a very important branch of the applications of the optical low-coherence interferometry. This is also the main topic of this thesis. Using different configurations, an optical low-coherence interferometry can be used to characterize nearly all the optical properties of an optical fiber, for instance, intermodal dispersion, chromatic dispersion, polarization mode dispersion, mode type, mode number, mode profile, backscattering and loss [102, 107, 108]. The tested object of the interferometry can be not only a single piece of fiber, but also complex fiber devices inside the fiber link. The work of the thesis will focus on the intermodal-dispersion and mode-profile characterization using an optical low-coherence interferometry. Here the previous techniques in this field are reviewed.

Optical low-coherence interferometry has been used to characterize the dispersion properties of an optical fiber since many years. The idea is that the excited modes in an higher-order-mode
fiber can be separated in time owing to their group-delay difference. By sweeping the wavelength the group velocity dispersion can be derived. These technique can be classed into two catalogs: time-domain interferometry and frequency-domain interferometry.

In the time-domain interferometry the optical delay line in the reference arm of the interferometer is introduced to get the interference between the reference beam and the respective mode of the fiber in the other arm. Several variations of this technique have been reported [5, 32–34]. In these technique either complex detection systems had to be used to distinguish the fiber modes or it had to be assumed that the dispersion properties of the fiber modes were well known according to the prior knowledge of fiber profile.

In the traditional frequency-domain interferometry [43–45] the experimental setups were relatively simple because the interference patterns were observed by an optical spectrum analyzer and no reference arm was required. The theoretic background of the technique is that different frequency components in the light source propagate with different speed inside the fiber, which results in a frequency-dependent time delay. However without the mode identification system, only intermodal dispersion of limited number of modes, i.e. the fundamental modes [43, 45, 64] or the lowest two LP modes [44], has been measured. In the new types of the frequency-domain interferometry, the Fourier transform [109–112] and the pulse stretching technique [113] were used to extract the mode information and to obtain a high temporal resolution. However in these techniques, no accurate mode profiles were obtained. In [114] the optical spectrum analyzer and the mode detection system were simultaneously applied for the intermodal dispersion measurement and the mode identification respectively. A fiber pigtail had to be used to pick up the intensity distribution of the transverse fiber modes and the spatial accuracy of the mode profile detection was limited.

In the present work a time-domain low-coherence interferometry using the simplified phase-shifting method for direct measurement of group-index difference and of the mode intensity profile of a short piece of special few-mode glass fiber (about 50 cm) is proposed. Furthermore, the polarization distribution of each mode was analyzed. The measurement technique is also feasible
for the characterization of a few-mode photonic crystal fiber, which will be shown in the chapter 5. The method can be used for the characterization of any type of optical fibers without any prior knowledge of the fiber profile.

### 4.1.3 Methods of fiber-mode decomposition

The proposed optical low-coherence interferometry technique in the thesis can be used to decompose the different fiber modes in terms of mode weights (the fractional power in each guided mode) besides to obtain the intermodal dispersion and mode profile. Here publications dealing with fiber-mode decomposition [115–117] are reviewed.

Skorobogatiy has presented an algorithm to characterize the modal content excited in a step-index fiber which supported only LP\textsubscript{01} and LP\textsubscript{11} modes [116]. He performed a least-squares fit of the measured intensity (the far-field intensity pattern of the fiber outgoing beam measured by a CCD camera) to predicted intensity, with the modal weights and phases as fit parameters. The multidimensional Newton method has been found to be an efficient minimization method. No degenerate vectorial modes were considered.

Afterwards Shapira proposed a similar, but more efficient and accurate method [117]. Using the proposed mode decomposition method the author examined the interactions of 16 modes in a hollow-core photonic bandgap fiber. The method was similar to the earlier one. Along two orthogonally polarization directions the near field and far field of the outgoing beam from the fiber were obtained by a CCD camera. A more complicated algorithm based on a phase-retrieval algorithm [118–120] was used to minimize the error function. The phase information of each guided mode and the vectorial modes were considered.

Using a coherent light source both above methods are noninterferometric methods and thus the performed measurements were very easy. However, the precondition of the methods is that the fiber profile is known and the electric fields of the modes are ideal. But in practice, the profile of the fiber under test is often not well known and the electric fields of the modes are in general far
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from perfect. It will be shown later in the thesis that mode profiles, especially profiles of higher-
order modes, are quite sensitive to the imperfection of the fiber core inadvertently introduced in
the manufacturing process. Using the proposed optical low-coherence interferometry and the sim-
plified phase-shifting algorithm the decomposition of fiber modes can be performed without any
prior knowledge of the fiber.

4.2 The special few-mode fiber

In this section the main features of the few-mode fiber under test will be introduced. The fiber
is designed to form the matrix for polarization-insensitive microbend fiber gratings [8, 12, 42,
121], more particularly, to form polarization-insensitive microbend fiber gratings by intentionally
separating the HE and TE/TM vector modes in an uniquely configured fiber such that only the
HE modes are coupled to the resonant wavelength of interest. The theoretical background of the
fiber design will be introduced firstly. Secondly, the fiber design will be described. At the end of
the section the most important applications of the fiber, i.e. the polarization-insensitive microbend
fiber gratings and devices, will be reviewed.

4.2.1 Theoretical background

Long-period gratings have been developed since many years. They have been used to offer wave-
length-selective attenuation of a wave-division-multiplexed (WDM) communication signal. A va-
riety of dynamic tuning technique of the spectral properties of long-period gratings have been
proposed. A microbend-induced fiber grating is one type of dynamic long-period grating where
the perturbation in refractive index is induced by periodic “microbending” of the fiber [122–125].

Microbend fiber gratings have several important applications. For example, when one of the
co-propagating modes is the fundamental mode of a single-mode fiber and the other is a cladding-
guided mode, microbend fiber gratings yield wavelength-dependent loss spectra when broadband
light is transmitted through the single-mode fiber. This is known to be useful for applications, such
as gain-equalization filters, spectral shapers for broadband lightwave devices, amplified spontaneous emission filters [126], spectral filters for stabilizing the operating wavelength of fiber lasers, etc. On the other hand, if both of the co-propagating modes are guided in the core region of a fiber, microbend fiber gratings can be used to realize efficient mode conversion which has application such as higher-order mode conversion [122], variable optical attenuation, etc.

However, microbend fiber gratings have one drawback of their inherent polarization sensitivity, even when the grating is induced in a perfectly circular fiber. For example, a microbend-induced fiber grating will couple a circularly symmetric and polarization degenerate LP_{01} mode with antisymmetric LP_{1m} modes. Here we use \( m = 1 \), which is the radial order of the antisymmetric mode. In a traditional step-index fiber, the LP_{11} mode, as has been shown already, possesses a four-fold degeneracy including the vector TM_{01}, TE_{01} and the odd and even HE_{21} modes. These four modes exhibit slightly different propagation constants, even in a perfectly circular fiber. Thus, coupling with a microbend fiber grating of a given grating period results in exhibiting slightly different resonant wavelength for each of the modes. Since different polarization orientations of the fundamental mode will result in different excitation levels for the four modes, the resulting spectrum will also be polarization dependent, which causes the inherent problem of devices based on a microbend fiber grating, where a “polarization insensitive” response is necessary. Different techniques have been proposed to reduce the polarization sensitivity of microbend fiber gratings. But they are either too lossy or involve in complex and high-cost grating machining technique or fiber manufacturing technique, and, therefore, are not practical.

These years a polarization-insensitive microbend fiber grating has been invented by Prof. Ramachandran and his colleagues [8, 12, 42, 121]. A fiber is designed such that the propagation constants of the TM_{0m} and TE_{0m} modes are substantially separated from the propagation constant of the odd/even HE_{2m} mode. As a result, the resonant wavelengths associated with the TM_{0m} and TE_{0m} modes will be significantly separated from the resonant wavelength of the HE_{2m} mode, where the resonant wavelength of the HE_{2m} mode falls into the wavelength range of interest for the optical system. For the remainder of the discussion, it will be presumed that \( m = 1 \). It is to be
understood that the principles are equally applicable to higher-order modes.

It is well known that the \( \text{LP}_{11} \) modes are actually linear combinations of the four distinct vector modes. When only the scalar wave equation is considered, these modes have identical propagation constants. However, when the full vectorial equation is considered, the four modes have distinct propagation constants. That is,

\[
\beta_{TE_{01}} \neq \beta_{TM_{01}} \neq \beta_{HE_{21}}, \text{ and } \beta_{HE_{21}^{\text{even}}} = \beta_{HE_{21}^{\text{odd}}}
\] (4.9)

which shows that in a perfectly circular fiber the \( TM_{01} \), \( TE_{01} \) and \( HE_{21} \) modes have different propagation constants, while the even and odd \( HE_{21} \) modes are degenerate. The difference in propagation constants for these various modes is relatively small, and can be obtained by firstly solving the scalar wave equation with \( \nabla_t \ln n^2 = 0 \) in Eq. 2.12, which yields the propagation constant for the \( \text{LP}_{11} \) mode, and then applying the first-order perturbation theory to obtain the first-order vector corrections to this field [41], as shown below:

\[
\beta_{\text{vector}} = \beta_{\text{scalar}} + \delta \beta_{\text{pert}}
\] (4.10)

This difference \( \delta \beta_{\text{pert}} \) has serious implications for the performance of microbend fiber gratings, leading to polarization insensitivity as a result of mode splitting. According to the resonant condition of long-period gratings, the resonant wavelength is also different for each of the modes. Further, the \( TM_{01} \) and \( TE_{01} \) modes are, by definition, polarization sensitive and are excited by the grating only for the certain input states of polarization of the \( \text{LP}_{01} \) mode. On the other hand, the degenerate \( HE_{21} \) pair is polarization insensitive and thus, can be excited by any incident polarization of the \( \text{LP}_{01} \) mode. This result implies that a microbend fiber grating has three different resonant wavelengths, two of which are strongly polarization dependent. Theory gives that the vectorial corrections in propagation constants for the four modes of a fiber with arbitrary refractive-index
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profile are [12, 41, 42, 46, 127]:

\[
\begin{align*}
\delta \beta_{TE_{01}} &= 0 \\
\delta \beta_{TM_{01}} &= 2(I_1 + I_2) \\
\delta \beta_{HE_{21}^{even}} &= \beta_{HE_{21}^{odd}} = I_1 - I_2
\end{align*}
\] (4.11)

where the quantities \( I_1 \) and \( I_2 \) are related to the refractive-index profile of a fiber by the following relations:

\[
\begin{align*}
I_1 &\propto \int r F_1(r) \frac{\partial F_1(r)}{\partial r} \frac{\partial f(r)}{\partial r} dr \\
I_2 &\propto \int F_2^2(r) \frac{\partial f(r)}{\partial r} dr
\end{align*}
\] (4.12)

where \( f(r) \) is the normalized refractive-index profile, which is defined in Eq. 2.1 at the beginning of the thesis. \( F_1(r) \) is the scalar field distribution that satisfies the ordinary differential equation

\[
\left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} + k^2 n^2(r) - \frac{l^2}{r^2} - \beta_{\text{scalar}}^2 \right] F_1(r) = 0
\] (4.13)

where \( n(r) \) is the refractive index. This equation can be obtained from the scalar wave equation.

For \( LP_{11} \) mode, the azimuthal order of the mode \( l = 1 \).

Equations 4.11 and 4.12 indicate that the refractive-index profile of the optical fiber determines the splitting of propagation constants of the four vector modes, and therefore, the splitting of the resonant wavelengths of the grating. A fiber design with widely spaced propagation constants for the \( TE_{01}, HE_{21}, \) and \( TM_{01} \) modes will yield a microbend fiber grating that has only one resonance over a broad wavelength range. When the coupled mode in such a fiber is the \( HE_{21} \), the resonance is polarization insensitive.

Figure. 4.4 shows the tailored refractive-index profile of the fiber used to test this concept along with the mode intensity profile for the \( LP_{11} \) mode in arbitrary linear unit. It is noted that the fiber
Figure 4.4: Index profile (solid line) and LP$_{11}$ mode intensity profile (dashed line). The LP$_{11}$ mode has high intensity near large index steps leading to large $I_1$ and $I_2$. This yields a large vector correction for the HE$_{21}$ and TM$_{01}$ modes. Hence the three vector components of the LP$_{11}$ mode are substantially separated in resonant wavelength.
design yields high mode intensities close to the waveguide transition regions. This is indeed what Eqs.4.11 and 4.12 demand—large separation in propagation constants require large $I_1$ and/or $I_2$ values, which can be obtained when large LP$_{11}$ power resides close to a sharp index step. This yields fiber designed to have a large degeneracy splittings. The refractive-index profile of the fiber will be described in the next section.

4.2.2 The fiber design

The cross-section image of the few-mode fiber is shown in Fig. 4.5. The fiber core includes a central core and a ring core surrounding it. The two cores are separated by a trench. Each portion of the core is defined by its radius $r$ and refractive-index step $\Delta n$, being a “refractive-index difference” with respect to the refractive index of the cladding which is pure silica.

![Figure 4.5: Cross-section image of the tested few-mode fiber.](image)

The ring area has a refractive-index difference $\Delta n_r$ greater than 0.015 and a radius $r_r$. $\Delta n_r$ and $r_r$ are configured to provide a sufficiently large gradient in refractive index as a function of distance from the fiber core center, to provide a resonant mode splitting between the TE$_{0m}$ and TM$_{0m}$ modes with respect to the polarization insensitive odd/even HE$_{2m}$ modes.

The trench surrounding the central core region has a refractive-index difference $\Delta n_t$ and radius
4.2. THE SPECIAL FEW-MODE FIBER

The refractive index of the central core $\Delta n_c$ is maintained to be approximately three-quarters the value of the refractive-index difference of the ring core, i.e. $\Delta n_c \approx 0.75 \Delta n_r$. The central core radius $r_c$ and the refractive index $\Delta n_c$ have to be large enough to support the propagation of the $LP_{01}$ mode.

According to the above design rules, fibers with different refractive-index values and core radii can be manufactured to build a microbend fiber grating. In our measurement the few-mode fiber with

\[ r_r = 5 \mu m, \quad r_t = 3.7 \mu m, \quad r_c = 2 \mu m \tag{4.14a} \]
\[ \Delta n_r = 0.024, \quad \Delta n_t = -0.003, \quad \Delta n_c = 0.017 \tag{4.14b} \]

was used. According to Sellmeier equations the refractive-index values of the fiber at 1550 nm are given by:

\[ n_{cl} = 1.444, \quad n_r = 1.468, \quad n_t = 1.441, \quad n_c = 1.461. \tag{4.15} \]

4.2.3 Applications of the fiber design

Using the above fiber design, any well-known method for inducing microbend can be used to form a microbend fiber grating. A corrugated plate and a rubber pad, and a pair of corrugated blocks offer the easiest low-cost ways to produce a microbend fiber grating. A piezoelectric transducer driven by radio-frequency power source can be also used to produce a microbend fiber grating, where both the grating period and amplitude may be adjusted with the acousto-optic arrangement. A microbend fiber grating can be also permanently formed by using ultraviolet laser light, periodic arcing with a splicer, or periodic ablation with a CO$_2$ laser.

There exists a variety of different systems or subsystems that may utilize a polarization-insensitive microbend fiber grating [42]. A cascaded set of such gratings may be used to create a
spectrally flat gain profile of an EDFA whose gain spectrum is highly wavelength dependent. A polarization-insensitive microbend fiber grating can also be used as an optical switch, since it is capable of moving light between two spatial modes in a polarization-insensitive fashion. A four-port 2×2 optical switch can be formed by a microbend fiber grating in conjunction with a fused fiber coupler. The microbend fiber gratings may be used as either static or dynamic mode converters, which are key elements in building static or tunable dispersion compensator and delay lines that operate in a higher-order mode of a fiber.

4.3 Experimental setup

In the current work we investigate the mode properties of the few-mode fiber using our novel low-coherence interferometry [35]. It bases on a generalized time-domain low-coherence interferometry for the intermodal dispersion measurement and phase-shifting algorithm for the mode-profiles calculation.

In the experimental setup as the broadband low-coherence light source an EDFA was used to generate unpolarized continuous light in infrared, whose typical spectrum is shown in Fig. 4.6(a). It has 8 nm bandwidth (FWHM) centered around 1531 nm. The relative spectral width \( \Delta \lambda / \lambda \) is about 0.005. A tunable spectral filter was then used to flatten the spectrum of the EDFA radiations in order to improve the accuracy and the resolution of the measurements. The spectral filter consists of two cascaded all-fiber acousto-optic tunable filters. Each filter is driven by three electronic signals. Each determines the central wavelength and rejection efficiency of the corresponding notch filter. The optimized spectrum, shown in Fig. 4.6(b), has a quasi-Gaussian shape with 27 nm bandwidth centered around 1547 nm. The relative spectral width is about 0.017 and corresponds to 120 fs resolution of group delay in the measurements.

In the measurements a Michelson interferometer composed of the few-mode fiber in the fiber arm and free air path in the reference arm, as shown in Fig. 4.7, was used. The unpolarized beam from the EDFA was collimated with an achromatic doublet with 19 mm focal length and then was
Figure 4.6: Spectrum of the light source: (a) radiation from the EDFA; (b) after the tunable spectral filter.
split into two by means of a non-polarizing beam splitter. In the reference arm a Newport step-motor translation stage (M-UTM25pp.1) was used to control the path length. In order to improve the interference visibility a neutral attenuator was used in the reference arm. In the fiber arm, the light was coupled into the tested fiber using an achromatic doublet with 10 mm focal length. At the end of the fiber the light was reflected back by a gold layer produced in a sputtering machine. The gold layer of about 50 nm thickness had reflectivity over 99% in the infrared region. The length of the fiber was 58 cm. The interference pattern of the beams at the output of the interferometer was recorded by an InGaAs camera which had a resolution of 320×256.

Figure 4.7: Low-coherence interferometer for intermodal-dispersion measurement with mode-profile detection. NPBS—non-polarizing beam splitter; OSA—optical spectrum analyzer; L1, L2—lenses; M1—gold mirror; M2, M3, M4—dielectric mirrors.

As the variable delay was changed under computer control, an interference pattern appeared when the optical path of the reference beam matched the optical path of the fiber beam for a certain transversal fiber mode. Other modes excited in the fiber produced a background which just decreased the interference contrast. In Fig. 4.8 the light intensity measured at one point on the
Figure 4.8: The interferogram corresponding to intensity of one single pixel detected by the InGaAs camera for \( \text{LP}_{01} \) mode (solid lines) and the envelopes reconstructed using Eqs. 4.18(a)(b) (dotted lines).
screen as a function of delay (mirror position in µm) is shown. This measurement corresponds to the LP$_{01}$ mode of the few-mode fiber interfering with the Gaussian reference beam. Using an incident Gaussian beam and a simple offset-launching technique, together with a camera for interference pattern detection and a two-dimensional phase-shifting algorithm, as described below, all modes of the fiber were resolved without any additional frequency-filtering electronic circuit to enhance the visibility of the interference fringes [34].

Next the algorithm used to reconstruct the mode profile and to calculate the mode intensity and the intermodal dispersion will be described. The intensity of the interference signal $I_{nr}$, of the $n$th fiber mode $I_n$, and of the reference beam $I_r$ have following relation

$$I_{nr} = I_0 + I_r + I_n + 2\sqrt{I_r I_n} \cos \varphi$$

(4.16)

where $I_0$ is the intensity of all fiber modes, the optical paths of which do not match that of the reference arm and produce the background; the intensity of the reference beam $I_r$ was obtained at the beginning of the measurement by blocking the fiber arm; $\varphi$ is the relative phase between the two beams which might depend on space, giving rise to the appearance of fringes in the detected image; the intensity of the interference signal $I_{nr}$ was obtained by the camera in real time.

For N-bucket phase-shifting method [36–39], the $\varphi$ dependence of the interference signal is

$$I_{nr(i)} = I_0 + I_r + I_n + 2\sqrt{I_r I_n} \cos (i2\pi/N)$$

(4.17)

where $I_{nr(i)}$, i = 0, 1, 2,..., N-1, is the intensity of the interference signal for $ith$ step of the translation stage in reference arm. For each step the step motor introduces a phase shift of the reference beam $2\pi/N$. For simplification, the following algorithm was used to calculate the intensity of each fiber mode $I_n$ at each pixel of the camera.
4.3. EXPERIMENTAL SETUP

\[ I_{nr(max)} = \max_{i=0}^{N-1}(I_{nr(i)}) = I_0 + I_r + I_n + 2\sqrt{I_rI_n}, \] 
(4.18a)

\[ I_{nr(min)} = \min_{i=0}^{N-1}(I_{nr(i)}) = I_0 + I_r + I_n - 2\sqrt{I_rI_n}, \] 
(4.18b)

\[ M = \frac{(I_{nr(max)} - I_{nr(min)})}{2} = 2\sqrt{I_rI_n}, \] 
(4.18c)

\[ I_n = M^2/4I_r \] 
(4.18d)

In the measurement the used step size was 0.1 \(\mu\)m and \(N = 9\), accordingly. \(I_{nr(max)}\) and \(I_{nr(min)}\) correspond to the maximal and minimal intensity of the interference signal within one oscillating period (9 steps). \(I_{nr(max)}\) and \(I_{nr(min)}\) for the LP\(_{01}\) mode at one pixel are shown by the dotted lines in Fig. 4.8. The total intensity of the \(n\)th fiber mode \(P_n\) is equal to the sum of the intensity of all the pixels within the transverse plane:

\[ P_n = \sum_j \sum_k I_{n(jk)} \] 
(4.19)

where \(j\) and \(k\) indicate the coordinate of a pixel within the CCD detection area. \(I_{n(jk)}\) can be obtained using Eqs. 4.18. Thus, it was possible to reconstruct both the mode profile and the intensity of each fiber mode. The measurement software in LabVIEW accomplishes the step motor control, the image acquisition, processing the interference fringes and the real-time results display. The software is described in details in Appendix D.

Furthermore, the delay of each peak corresponds to the group delay of each mode, so the relative position of those peaks was used to calculate the intermodal dispersion of the fiber. The one-way optical path for \(n\)th fiber mode \(OP_n\), the optical path difference between \(n\)th mode and LP\(_{01}\) mode \(OPD_n\), the group-index difference between \(n\)th mode and LP\(_{01}\) mode \(\Delta n_g(n)\) and the
CHAPTER 4. CHARACTERIZATION OF THE SPECIAL FEW-MODE FIBER

group delay difference $\Delta \tau_g(n)$, i.e. intermodal dispersion, can be described as follows:

\[
OP_n = n_g(n) \cdot L + l_0,
\]
\[
OPD_n = OP_n - OP_0 = (n_g(n) - n_g(0)) \cdot L,
\]
\[
\Delta n_g(n) = n_g(n) - n_g(0) = OPD_n/L,
\]
\[
\Delta \tau_g(n) = \Delta n_g(n)/c = OPD_n/(L \cdot c) \tag{4.20}
\]

where $OP_0$ stands for the optical path for LP$_{01}$ mode; $n_g(n)$ and $n_g(0)$ are the group indices for $n$th mode and LP$_{01}$ mode; L is the length of the fiber; $l_0$ is the free air optical path in the fiber arm, which is the same for all the modes; $OPD_n$ is specified by the relative position of the mirror in the reference arm.

4.4 Results and discussion

In this section the experimental results of the few-mode fiber obtained using the proposed low-coherence interferometry technique will be shown. Then the results of numerical simulations of dispersion properties and of intensity profiles of the fiber modes will be presented. A very good agreement with experimental results has been obtained. At the end the effect of fiber ellipticity on intermodal dispersion and mode profiles will be discussed. It is shown birefringence of the fiber affects the degenerate LP$_{11}$ modes much stronger than the fundamental modes.

4.4.1 Group-index difference and mode intensity pattern

At 1547 nm four LP mode groups have been observed and identified as illustrated in Fig. 4.9. In the figure the intermodal dispersion together with the mode intensity profiles as insets is shown. The delay was normalized to ps/m using Eq. 4.20. The mode intensity profiles were obtained by interpreting the interferogram for each pixel using Eqs. 4.18. The mode intensity at each pixel was normalized to the maximal intensity within the transverse plane. Additionally, the maximum
Figure 4.9: Intermodal dispersion and mode intensity profiles (insets) of the tested few-mode fiber at 1547 nm under unpolarized beam excitation.
of each peak corresponds to the total intensity of each fiber mode (mode weight). It is obtained using Eq. 4.19. The measured group indices of $LP_{11}$, $LP_{21}$ and $LP_{01}$ mode groups relative to the $LP_{01}$ mode are: $\Delta n_{g11} = 0.0025$, $\Delta n_{g21} = 0.0055$, $\Delta n_{g02} = 0.0087$. Furthermore, all four modes in the $LP_{11}$ mode group were resolved. They are almost degenerate in a conventional step-index fiber [40].

As it has been already discussed there are four vector modes within the $LP_{11}$ mode group. These are $TM_{01}$, $TE_{01}$ and the two $HE_{21}$ modes. In an ideal circular optical fiber these modes possess a ring-shape mode profile and are not uniformly polarized. In a step-index fiber they typically almost degenerate to four linearly polarized $LP_{11}$ modes because they have nearly similar propagation constants.

In order to check the polarization distribution of the four modes within the $LP_{11}$ mode group of the few-mode fiber the launching condition was adjusted for excitation of all the four modes with significant magnitude. As shown in Fig. 4.10(a) they were clearly resolved without using any polarizing component in the setup. Then a polarizer was used in front of the camera in order to check the polarization properties of the modes. The red curve in Fig. 4.10(b) was obtained by orienting the polarizer along the vertical direction corresponding to the direction of the dark bar which appeared in the mode with the largest magnitude. Two modes with the same vertical polarization but with orthogonal intensity patterns were observed, as shown in Fig. 4.10(b). They correspond to the two $HE_{21}$ modes. The other two modes, $TM_{01}$ and $TE_{01}$, are suppressed. As also shown in Fig. 4.10(b) by the blue curve for the horizontal polarizer orientation, the $TM_{01}$ and $TE_{01}$ modes have a polarization orthogonal to that of $HE_{21}$ modes. They have the same linear polarization and orthogonal intensity patterns as the $HE_{21}$ mode pair.

It is seen in the experiments that the four modes in the $LP_{11}$ mode group of the few-mode fiber have the same linear polarization and two-lobe intensity pattern as the $LP_{11}$ modes in a step-index fiber, where the scalar approximation is valid. However, the four modes were well distinguished in the measurements while those of a step-index fiber with the same length would not be distinguishable [5, 34]. The large mode splitting is indeed the advantage of the fiber design,
4.4. RESULTS AND DISCUSSION

Figure 4.10: Intermodal dispersion and intensity profiles of the four vector modes within the $LP_{11}$ mode group under unpolarized beam excitation: (a) no polarizer in front of the camera; (b) with a polarizer in front of the camera. The polarizer axis (indicated by the black arrow) was oriented vertically or horizontally for red or blue curve, accordingly.
Figure 4.11: Results of numerical simulations for lowest 12 modes of the tested few-mode fiber: (a) dependence of normalized propagation constant \( b_{\text{eff}} \) on normalized frequency \( V \); (b) dependence of the normalized group delay \( b_g \) on \( V \). Vertical line: \( V = 5.9 \); Points: experimental data.
4.4. RESULTS AND DISCUSSION

as described in section 4.2.1.

Next the the results of the numerical simulations done on the tested few-mode fiber will be presented and be compared with the experimental results. Figure 4.11 shows the normalized propagation constant $b_{\text{eff}}$ and the normalized group delay $b_g$ of the lowest 12 modes. Following the discussion on fibers with ring-index profiles by Stolen [128], the normalized frequency $V$ is still used as the variable. The $V$, $b_{\text{neff}}$ and $b_{ng}$ have been defined in Eqs. 2.6, 2.24, 2.35. The calculations of mode indices have been performed in the same way which was used in chapter 2 for the calculation of modes properties of a step-index fiber. In the calculations the wavelength $\lambda = 1547$ nm, and the refractive index of the cladding $n_{cl} = 1.444$ were used and were kept constant. The normalized frequency $V$ was changed by varying the refractive index of the core $n_{co}$. For the tested fiber, $n_{co}$ refers to the refractive index of the outer ring core. The core radius $a$ refers to the radius of the outer ring core. Material dispersion was not taken into account as before.

In Fig. 4.11 the points are the experimental data. The vertical line $V = 5.9$ corresponds to the fiber design used in the measurements. It has been checked numerically that in this region the effect of the material dispersion which was not taken into account in the calculations on the group-index difference for different modes is about only 1%. Comparing Fig. 2.5 and Fig. 4.11 it is seen that the dispersion properties of such a fiber are very different from conventional step-index fibers, especially for HE$_{21}$ mode. The splitting of TE$_{01}$ and HE$_{21}$ modes, or HE$_{21}$ and TM$_{01}$ modes in group index is about $2 \times 10^{-4}$, while in a step-index fiber the four modes are almost degenerate. Such a splitting results in that TE$_{01}$, or TM$_{01}$ mode once excited is robust against stress and bending. However, the even mode and odd mode within one HE or EH hybrid mode should be degenerate in perfectly circular fibers.
4.4.2 Group-index difference between higher-order modes in an anisotropic fiber

Next the effect of fiber ellipticity on the group-index difference between the two \( \text{HE}_{21} \) modes in the \( \text{LP}_{11} \) mode group will be considered. It is seen in the experimental results the \( \text{HE}^{\text{even}}_{21} \) and \( \text{HE}^{\text{odd}}_{21} \) modes are clearly separated by a delay of 260 fs/m. This can be explained by the anisotropy of the fiber [41, 46, 127]. This interpretation is confirmed by the mode profiles of the \( \text{LP}_{21} \) mode and the \( \text{LP}_{02} \) mode displayed in Fig. 4.9. In practice, both geometric ovality and stress may contribute to this problem. However, in order to simplify the model it was assumed that the fiber anisotropy is only due to geometric ovality of the fiber core and the central core and the outer ring core have the same ellipticity. Using the fiber design \( V = 5.9 \), the dependence of group indices of modes on the ellipticity of the fiber core was studied. The calculated results are shown in Fig. 4.12. The ellipticity is defined as \( e = (1 - b^2/a^2)^{1/2} \), where \( a \) is the semi-major axis of the elliptical core and \( b \) is the semi-minor axis. The measurement shows that the splitting of the \( \text{HE}^{\text{even}}_{21} \) and \( \text{HE}^{\text{odd}}_{21} \) modes in group index is \( \Delta n_{g HE_{21}} = 7 \times 10^{-5} \). Based on this value and the results of the numerical simulations the estimated ellipticity of the tested fiber is \( e^2 = 0.068 \). In Fig. 4.12 the measured group indices of all modes are shown by the points. As one can see, ellipticity leads to not only the splitting of the degenerate modes in each modes group, but also influences the group-index difference between mode groups. This is why for \( V = 5.9 \) the experimental data do not fit the calculated group-index values any more, as to be seen in Fig. 4.12(a). In order to fit the simulation results to the measurement data, the refractive-index profile of the fiber \( V \) was changed to \( V = 6.0 \) and the group indices of modes depending on the ellipticity of the fiber core were calculated accordingly. The results are shown in Fig. 4.12(b). As one can see in Fig. 4.12(b), the measured group indices fit the simulation results very well with \( V = 6.0 \) and \( e^2 = 0.068 \).

The calculations show that the group-index difference between the \( \text{LP}_{01} \) modes \( \Delta n_{g LP_{01}} \) is about \( 7 \times 10^{-6} \). It is ten times smaller than the group-index difference of the \( \text{HE}^{\text{even}}_{21} \) and \( \text{HE}^{\text{odd}}_{21} \) modes in the \( \text{LP}_{11} \) mode group \( \Delta n_{g HE_{21}} \) of \( 7 \times 10^{-5} \). Thus, it is more efficient to use the value
4.4. RESULTS AND DISCUSSION

Figure 4.12: Results of numerical simulations on ellipticity dependence of the group index $n_g$ for modes of the tested few-mode fiber: (a) for $V = 5.9$; (b) for $V = 6.0$. Points are the experimental data.
of $\Delta n_{gHE_{21}}$ to determine the anisotropy of the fiber because the birefringence effect is about one order of magnitude stronger on HE$_{21}$ modes than on the fundamental modes. Furthermore, HE$_{21}$ modes belong to the lowest higher-order-mode group LP$_{11}$ and using a Gaussian-beam excitation with a slight offset, it is possible to excite them with large magnitude. It is also noted in simulations that the splitting of HE$_{21}$ modes in group index is very sensitive to the ellipticity of the fiber, but not that sensitive to the V value of the fiber. This allows to estimate the anisotropy of the fiber even if the fiber profile is not well known.

In Fig. 4.13 the calculated mode intensity profiles of the tested fiber with $V = 6.0$ and $e^2 = 0.068$ and the measured mode profiles are displayed. As one can see, the measured intensity profiles and polarization distributions of the LP$_{01}$ mode and four LP$_{11}$ modes match the results of the numerical simulations very well. For the LP$_{21}$ mode, three modes are observed. Two of them agree with the results of the simulations very well. The mode pattern, which can not be found in the simulations, might be the coherent superposition of two LP$_{21}$ modes which have similar group velocity. According to the simulations the mode intensity patterns of the two LP$_{02}$ modes possess

![Mode Intensity Profiles](image)

**Figure 4.13**: Intensity profiles for all guided modes in the tested fiber at 1547 nm: (a) calculated mode profiles of the fiber with $V = 6.0$ and $e^2 = 0.068$; (b) measured mode profiles. Each row shows one mode group; each column shows vector modes in one mode group. White arrows indicate the dominant polarization of the modes.
a clear anisotropy, while in the measured profiles, one has clear anisotropy, the other one remains circular. This can be explained by the fact that only geometry ovality was taken into account when the fiber ellipticity was determined. In practice stress can also contribute to the anisotropy of the fiber, which causes the error of the determined ellipticity.

It is noted that using the above time-domain interferometer and 58 cm fiber the group-index difference between the two $LP_{01}$ modes $\Delta n_{g,LP_{01}}$ is too small to measure. In order to experimentally confirm the estimated ellipticity of the fiber, $\Delta n_{g,LP_{01}}$ was measured using frequency-domain low-coherence interferometry shown in Fig. 4.14 [43–45, 64] and was compared with the corresponding calculated value. The measurement principles of frequency-domain interferometry have been introduced in 4.1.2 and details can be found in literatures. A longer tested fiber (about 8 m) was used in the measurement. The spectral properties of the used light source were similar to that used in the previous setup. The linear-polarized Gaussian beam was coupled into the tested fiber, whose each end was spliced to a 0.5 m single-mode fiber so that only $LP_{01}$ modes were guided in the fiber (The amplitude ratio of the $LP_{01}$ mode to other higher-order modes is larger than 50:1, measured by the commercial instrument Luna Technologies’ Optical Vector Analyzer (OVA).). The output beam from the fiber was coupled to an in-line fiber polarizer. The light was then sent to the optical spectrum analyzer. Thus, the interference between the two $LP_{01}$ modes was obtained on the optical spectrum analyzer. At the exit of the tested fiber the two $LP_{01}$ modes have

![Figure 4.14: The frequency-domain low-coherence interferometry for the measurement of the group-index difference between the two $LP_{01}$ modes. L: lenses; OSA: optical spectrum analyzer.](image-url)
the relative phase

$$\phi = \Delta\beta L = 2\pi L \Delta n_{\text{eff}} / \lambda$$

where $\Delta\beta$ is the propagation-constant difference of the two $LP_{01}$ modes and $\Delta n_{\text{eff}}$ is the corresponding effective-index difference. $\phi$ changes with $\lambda$:

$$d\phi/d\lambda = -2\pi L (\Delta n_{\text{eff}} - \lambda d\Delta n_{\text{eff}}/d\lambda) / \lambda^2 = -2\pi L \Delta n_g/\lambda^2$$ (4.21)

Assuming that $\phi$ locally changes linearly with $\lambda$ and that $\phi$ changes by $2\pi$ when the wavelength is turned over $\Delta\lambda$. $\Delta\lambda$ is the periodicity of the interference fringes on the optical spectrum analyzer. Using $d\phi/d\lambda \approx 2\pi/\Delta\lambda$ and above equation, the group-index difference of the modes are given by

$$\Delta n_g = -\lambda^2/(L\Delta\lambda)$$ (4.22)

The modulation of the output spectrum is shown in Fig. 4.15. Only one fringe was obtained with the modulation depth close to 100%. The modulation depth is dependent on the relative power in each of the polarization modes. The polarizer in front of the tested fiber and the in-line fiber polarizer at the exit of the tested fiber were rotated to optimize the observed fringe contrast. On the spectrum analyzer $\Delta\lambda = 25$ nm was obtained and the group-index difference between the two $LP_{01}$ modes $\Delta n_{gLP_{01}} = 10^{-5}$ was estimated, accordingly. This value is close to the previous prediction of $0.7 \times 10^{-5}$ in the numerical simulations.

In summary it has been shown that the group-index difference between the $HE_{21}$ modes in the $LP_{11}$ mode group $\Delta n_{gHE_{21}}$ is much larger than that between $HE_{11}$ modes in the $LP_{01}$ mode group $\Delta n_{gLP_{01}}$ both in simulations and in experiment. In practice the fiber ellipticity can be characterized simply using the group-index difference between the $HE_{21}$ higher-order modes. This makes the work easier, especially when the fiber is short and the group-index difference between the fundamental modes is too weak to measure.
4.5 Conclusion

In this chapter two optical low-coherence interferometry techniques have been used to investigate the mode properties of an all-glass few-mode fiber with tailored refractive-index profile.

A time-domain low-coherence of Michelson type is used to characterize the intermodal dispersion and the mode intensity profiles of a short fiber sample (58 cm). The optical delay line in the reference arm of the interferometer is introduced to get the interference between the Gaussian reference beam and the respective mode of the fiber in the fiber arm. The phase-shifting algorithm is used to interpret the interferogram. Four LP mode groups are resolved without any prior knowledge of the fiber properties. Additionally, the group-index difference between the degenerate \( HE_{21} \) higher-order modes is obtained and used for characterizing the ellipticity of the fiber core. The group-index difference between the two fundamental \( LP_{01} \) modes is predicted in the numerical simulations.

However, in the above experiments the two fundamental \( LP_{01} \) modes are not resolved because

![Output spectrum from the 8 m tested fiber after the fiber polarizer.](image)
the fiber sample is too short and the group-index difference between the two modes are too small. According to the theoretical prediction, a longer fiber (>8 m) should be used. However, a time-domain low-coherence interferometer with such long optical paths would be huge. A conventional single-mode fiber can be used to replace the free air path in the reference arm in order to make the system compact. But the alignment of the system would be more complicated. Additionally, the phase-shifting algorithm is not necessary any more for calculating the mode intensity profile, because the mode profiles for the LP_{01} modes are well known.

Based on the above considerations, the traditional frequency-domain interferometry technique instead of the above time-domain interferometry technique is used to measure the group-index difference of the LP_{01} modes in an 8 m fiber. The experimental setup is relatively simple because the interference patterns are observed by an optical spectrum analyzer and no reference arm is required. The theoretic background of the technique is that different frequency components in the light source propagate with different speed inside the fiber, which results in a frequency-dependent time delay. The mode profiles can not be obtained in such a system. However, the mode identification is not necessary at this point. The measured group-index difference between the LP_{01} modes agrees very well with the theoretical predictions. Consequently, the estimated ellipticity of the fiber core is confirmed.
Chapter 5

Characterization of the few-mode solid-core photonic crystal fiber

5.1 Introduction

There are two main types of photonic crystal fiber (PCF): a solid-core PCF and a hollow-core PCF [129, 130].

One of the attractive properties of the solid-core PCF is their possibility to be single mode over a wide wavelength range, surpassing the ordinary single-mode fibers which become multimode for wavelength below their single-mode cutoff wavelength. Therefore, solid-core PCFs are often called endlessly single-mode PCFs. However, this naming can be a bit misleading because an endlessly single-mode PCF may support more than one mode. Also in PCFs, the number of modes supported by the fiber core depends on the ratio of the fiber core diameter to the wavelength and the refractive-index step between the core and the cladding. Accordingly, several papers have been published on the criterion of the endlessly single-mode operation of solid-core PCFs [50, 51, 131–134]. A solid-core PCF can be highly nonlinear. The selective excitation of higher-order modes in these fibers can be used for supercontinuum generation and spectral transformation, and hence, a peculiar spectrum of the output radiation can be obtained [135, 136].

Similar to a solid-core PCF, a hollow-core PCF may also support a large number of modes. Several techniques for the selective excitation of a single higher-order mode of a hollow-core PCF have been reported [137, 138]. These techniques are based on the shaping capabilities of a spatial
light modulator. The results are relevant to applications in which the intensity distribution of the light inside the fiber is important, such as particle- or atom-guidance, and in optimizing nonlinear processes such as high-harmonic generation.

For the above applications of PCFs, the characterization of modes of PCFs is of great importance. In this chapter a low-coherence interferometry of Mach-Zehnder type is used to characterize a few-mode solid-core PCF. \( \text{LP}_{01} \) and \( \text{LP}_{11} \) modes are resolved. The group-index difference between the two LP modes and the chromatic dispersion of the \( \text{LP}_{01} \) modes agree very well with the results of the numerical simulations. However, the four modes in the \( \text{LP}_{11} \) mode group can not be distinguished due to the limitation of the measurement resolution.

5.2 The few-mode solid-core photonic crystal fiber

The few-mode solid-core PCF used in the experiments was fabricated using the conventional stack-and-draw process. Silica tubes were first drawn to thin-walled capillaries of the desired diameter. The capillaries were stacked and inserted into a jacket tube to create the preform, which was then drawn down to a cane. Different pressures were applied to the core and the cladding regions of the cane to allow independent control of core diameter and air-filling fraction in the cladding. Figure. 5.2 shows a scanning electron micrograph of the microstructure of the few-mode solid-core PCF. The cladding has a pitch (center-center spacing of the holes) of 3.5 \( \mu \text{m} \) and the hole diameter is 2.37 \( \mu \text{m} \).

It has been shown that similar to a step-index fiber, the effective V-parameter can be used for a PCF to determine whether the fiber is single mode [50, 51, 130–134].

\[
V_{PCF} = k\Lambda F^{1/2}(n_0^2 - n_a^2)^{1/2}
\]

(5.1)

where \( k = 2\pi/\lambda \), \( \Lambda \) is the pitch, \( F \) is the air-filling fraction, \( n_0 \) is the index of silica (the core material) and \( n_a \) is the index of air (or whatever is in the holes). The resulting curves of \( V_{PCF} \)
5.2. THE FEW-MODE SOLID-CORE PHOTONIC CRYSTAL FIBER

Figure 5.1: Cross-section image of the few-mode solid-core PCF under test.

Figure 5.2: Variation of $V_{PCF}$ with $\Lambda/\lambda$ for various relative hole diameters $d/\Lambda$ [130]. Numerical modeling shows that the endlessly single-mode behavior is maintained for $V_{PCF} \leq 4$ or $d/\Lambda \leq 0.43$. 
Figure 5.3: (a) Results of the numerical simulations on mode properties of the tested few-mode solid-core PCF: (a) the wavelength dependence of group indices of the modes; (b) the chromatic dispersion of the modes. The fiber supports only two LP mode groups: \( LP_{01} \) and \( LP_{11} \), in the spectral range.
against $\Lambda/\lambda$ for $n_0 = 1.45$ and $n_a = 1.00$ are shown in Fig. 5.2 for various relative hole sizes $d/\Lambda$ (d is the hole diameter) [130, 132]. Note that the single-mode operation is not defined by $V < 2.405$ which is valid for a step-index fiber. Numerical modeling shows that the endlessly single-mode behavior is maintained for $V_{PCF} \leq 4$ or $d/\Lambda \leq 0.43$. For the few-mode PCF used in the experiments $\lambda = 1550$ nm, $\Lambda = 3.5$ $\mu$m, $n_0 = 1.45$, $n_a = 1.00$, and the average hole diameter $d = 2.37$ $\mu$m. It was calculated that the air-filling fraction $F = 0.37$ and $V_{PCF} = 8.8 > 4$. Therefore, it is multimode. However, the rigorous information regarding the mode type and the dispersion properties of the modes requires more complex numerical simulations. The results of numerical simulations on the group index $n_g$ and the chromatic dispersion of the fiber modes $D$, shown in Fig. 5.2, are obtained using the commercial software “JCMwave” and the plane-wave-expansion method.

It is seen in Fig. 5.2 that this fiber supports only two lowest mode groups: LP$_{01}$ and LP$_{11}$. Furthermore, it is obtained from the results of the numerical simulations that the group-index difference between the LP$_{01}$ mode and LP$_{11}$ mode $\Delta n_{g11}$ is 0.020 at 1550 nm. In the LP$_{11}$ mode group the TE$_{01}$ and TM$_{01}$ modes have relatively small group-index difference of 0.0004. LP$_{01}$ and LP$_{11}$ modes have very different chromatic dispersion. The chromatic dispersion of the LP$_{01}$ mode: $D_{01} = 70$ $ps/(km \cdot nm)$ at 1550 nm. The four modes in the LP$_{11}$ mode group have similar group index and chromatic dispersion. Their average chromatic dispersion: $D_{11} = 134$ $ps/(km \cdot nm)$ at 1550 nm. These values are useful for later discussions.

5.3 Experimental setup

A Mach-Zehnder low-coherence interferometry was used to investigate dispersion properties of the modes experimentally [35]. The block scheme of the setup is shown in Fig. 5.4. It used the same EDFA source as the previous Michelson interferometer used for study of dispersion properties of modes in the special few-mode all-glass fiber. The unpolarized beam from the EDFA was collimated with a lens with NA = 0.24 and focal length equal to 37 mm. The 70 cm PCF was
Figure 5.4: Mach-Zehnder low-coherence interferometer for intermodal-dispersion measurement. The fiber under test is a few-mode solid-core PCF. NPBS—non-polarizing beam splitter; OSA—optical spectrum analyzer; L1-L3—lenses; M1-M5—dielectric mirrors.

placed in the fiber arm of the Mach-Zehnder interferometer. In the fiber arm, the light was coupled into the tested fiber using an aspheric lens with $NA = 0.25$ and focal length equal to 11 mm. At the exit end of the fiber the light was collimated using a $40 \times$ microscope objective and passed through two dielectric mirrors for the convenience of the alignment. In the reference arm the step-motor translation stage was used to control the path length. The two beams were combined by a non-polarizing beam splitter. The interference pattern was recorded by the camera.

The interference image obtained by the camera was processed using the phase-shifting algorithm described in section 4.3 of chapter 4. The optical path difference between the $n$th mode and $LP_{01}$ mode of the fiber $OPD_n$ can be described as follows:

$$OPD_n = 2\Delta d_n = \Delta n_{g(n)} L$$  \hspace{1cm} (5.2)  

where $\Delta d_n$ stands for the relative mirror position when the interference between the reference beam and $n$th fiber mode was obtained. Therefore, $2\Delta d_n$ gives the optical path difference intro-
duced in the reference arm. $\Delta n_{g(n)}$ is the group-index difference between the $n$th fiber mode and the LP$_{01}$ mode. $L$ is the length of the fiber ($L = 70$ cm). Thus, $\Delta n_{g(n)}L$ gives the optical path difference introduced in the fiber arm. Consequently, the group-index difference $\Delta n_{g(n)}$ can be calculated as

$$\Delta n_{g(n)} = 2\Delta d_n/L$$  \hspace{1cm} (5.3)

Furthermore, the group-delay difference $\Delta \tau_{g(n)}$ can be calculated as

$$\Delta \tau_{g(n)} = 2\Delta d_n/(Lc)$$  \hspace{1cm} (5.4)

where $c$ is the speed of the light.

### 5.4 Results and discussion

At 1547 nm two LP mode groups, LP$_{01}$ and LP$_{11}$, have been resolved. The intermodal dispersion as well as the mode intensity profiles as insets are shown in Fig. 5.5. The time scale is normalized to ps/m. The group-index difference between the LP$_{01}$ and LP$_{11}$ modes $\Delta n_{g11} = 0.021$ was obtained, which agrees very well with the predicted value of 0.020. In Fig. 5.5 the two peaks corresponding to the fundamental modes are introduced by the birefringence of the fiber because one single peak can be obtained using a polarizer in front of the camera, as shown in Fig. 5.6.

Furthermore, it has been predicted that the interferogram obtained using optical low-coherence interferometry can be used to determine the spectral property of the device under test [139]. However, this has not been investigated experimentally so far. Here it is demonstrated that in our intermodal dispersion measurement the width of the interferogram corresponding to one fiber mode can be used to estimate the chromatic dispersion of the mode. The broadening of the interferogram is the effect of the chromatic dispersion of the mode. Therefore, the measured FWHM of the peak
Figure 5.5: Intermodal dispersion of the few-mode PCF at 1547 nm using unpolarized beam excitation and mode intensity profiles (insets).
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Figure 5.6: Intermodal dispersion of fundamental LP_{01} modes of the fiber showing birefringence of the fiber. Mode intensity profile is shown in the inset. Arrows indicate the orientation of transmission axis of the polarizer in front of the camera. The envelopes have FWHM of 1.0 ps, which corresponds to 1.4 ps FWHM of the interferogram.
in time $\Delta \tau$ can be described as

$$
\Delta \tau = \Delta \tau_D + \Delta \tau_s \\
= DL \Delta \lambda + B \lambda^2 / (\Delta \lambda c)
$$

(5.5)

where $\Delta \tau_s$ is the coherence time of the used light source and is calculated to be 0.26 ps for $\lambda = 1547$ nm and $\Delta \lambda = 27$ nm; $B$ is a coefficient depending on the spectral shape of the source and for a Gaussian-shaped spectrum $B = 0.88$. $\Delta \tau_D$ is the broadening in time introduced by the chromatic dispersion of the fiber mode. Different fiber modes may have different values of chromatic dispersion $D$ and hence different $\Delta \tau_D$. It is seen in Fig. 5.6 the measured FWHM of the envelops indicating $\text{LP}_{01}$ mode intensity is 1.0 ps. According to Eq. 4.18d, the corresponding FWHM of the interferogram $\Delta \tau$ is 1.4 ps. Hence, the chromatic dispersion of the $\text{LP}_{01}$ mode $D_{01} = (\Delta \tau - \Delta \tau_s) / (L \Delta \lambda) = 74 \text{ ps}/(\text{km} \cdot \text{nm})$ is obtained. This is very close to the theoretical prediction of 70 ps/$\text{km} \cdot \text{nm}$.

Furthermore, it has been noted that in order to distinguish the modes the group-delay difference between the modes $\Delta \tau_g$ has to be larger than the width of the peak $\Delta \tau$, i.e. $\Delta \tau_g > \Delta \tau$ (Both are in ps.). $\Delta \tau_g$ is proportional to the length of the fiber. According to Eqs. 5.5 the coherence time of the light source $\Delta \tau_s$ is inverse proportional to the bandwidth of the source $\Delta \lambda$, while $\Delta \tau_D$ is proportional to $\Delta \lambda$. Therefore, for the fixed fiber length the bandwidth of the light source $\Delta \lambda$ can be basically optimized according to Eq. 5.5 to achieve the largest difference $\Delta \tau_g - \Delta \tau$, and hence, the best time resolution in the measurement.

The spectral-width dependences of $\Delta \tau_s$, $\Delta \tau_D$ and the sum $\Delta \tau$ for the $\text{LP}_{11}$ mode in fibers with different length were calculated according to Eq. 5.5 and are shown in Fig. 5.7, where the average theoretical chromatic dispersion of the four modes in the $\text{LP}_{11}$ mode group $D_{11} = 134 \text{ ps}/(\text{km} \cdot \text{nm})$ is used. The horizontal lines in the figure $\Delta \tau_g$ correspond to the group-delay difference of the $\text{TE}_{01}$ and $\text{TM}_{01}$ mode for different fiber length. $\Delta \tau_g$ was calculated according to the calculated group-index difference between the $\text{TM}_{01}$ and $\text{TE}_{01}$ modes of 0.0004. The vertical
Figure 5.7: Spectral-width dependences of the measurement resolution for LP_{11} mode for different fiber length. The dotted line indicates $\Delta\tau_s$, the coherence time of the light source. $\Delta\tau_s$ does not depend on the fiber length. The three dashed-dotted lines indicate $\Delta\tau_D$, the broadening in time introduced by the chromatic dispersion of the LP_{11} mode for different fiber length. $D_{11} = 134 \text{ ps}/(\text{km} \cdot \text{nm})$ (the average chromatic dispersion of the modes in LP_{11}) is used. The three solid lines indicate $\Delta\tau$, the total time delay: $\Delta\tau = \Delta\tau_D + \Delta\tau_s$. The vertical line corresponds to $\Delta\lambda = 27 \text{ nm}$, the spectral width used in the measurement. The three horizontal dashed lines correspond to $\Delta\tau_g$, the splitting of the LP_{11} modes in time for different fiber length. For the fiber under test, $L = 0.7 \text{ m}, \Delta\tau_g = 0.98 \text{ ps}$; for $L = 0.5 \text{ m}, \Delta\tau_g = 0.7 \text{ ps}$; for $L = 2 \text{ m}, \Delta\tau_g = 2.8 \text{ ps}$.
line corresponds to $\Delta \lambda = 27$ nm used in the measurement. It is shown in Fig. 5.7 that for $\Delta \lambda = 27$ nm and $L = 0.7$ m used in the experiments, $\Delta \tau_g \ll \Delta \tau$. This explains why in the measurement the four modes in the $LP_{11}$ mode group are not distinguished separately, as shown in Fig. 5.5. In this case we can not obtain correct chromatic dispersion for each mode. However, it is predicted that for $L = 2$ m and $\Delta \lambda \approx 5$ nm, $\Delta \tau_g \approx \Delta \tau$, as shown Fig. 5.7, and the $TE_{01}$ mode and $TM_{01}$ mode are expected to be distinguishable using the low-coherence interferometry.

In this chapter, the Mach-Zehnder low-coherence interferometry was used to characterize a few-mode solid-core PCF. The results of the numerical simulations on group-index difference and chromatic dispersion of the modes were shown. In the measurement two LP mode groups, $LP_{01}$ and $LP_{11}$ were resolved. It is shown that the group-index difference between the two LP mode groups and the chromatic dispersion of the $LP_{01}$ mode agree very well with the results of the numerical simulations. Furthermore, the optimization of the spectrum for the improvement of the measurement resolution, and hence better resolving the adjacent fiber modes, especially vector modes in one LP mode group, has been discussed. Besides the spectrum-optimization method, several techniques can be used to avoid the mode beating in the interferogram, for example, changing the power ratio guided in different modes to focus on one mode particularly [33]. Details are not shown in this thesis.
Chapter 6

Conclusion

Systematic investigations on dispersion properties as well as intensity profiles of different transverse modes in optical fibers have been performed. The thesis involves in introduction of general optical waveguide theory as well as numerical simulations and measurements of fiber modes properties. The tested fibers were conventional step-index fibers, a few-mode all-glass fiber with special refractive-index profile and a few-mode solid-core photonic crystal fiber.

Light beams possessing special polarization, for example, a radially or azimuthally polarized doughnut beam, have recently attracted a great deal of interest. Several fiber techniques have been developed to generate such beams while employing higher-order modes of respective shapes. In particular, a radially or azimuthally polarized beam corresponds to the TM\textsubscript{01} or TE\textsubscript{01} vector mode in the LP\textsubscript{11} mode group of a circular optical fiber. These doughnut beams can be obtained using a second-mode step-index fiber under adequate pressure and twist. However, these beams are not solid against disturbance of the environment because they are coherent superpositions of the almost degenerate LP\textsubscript{11} modes. Therefore, a full characterization of the mode dispersion and the mode profiles as well as the polarization state of these modes is essential to successfully apply these novel techniques.

In the thesis a novel time-domain low-coherence interferometry was proposed and used for the characterization of the dispersion properties and intensity profiles of higher-order modes in a few-mode all-glass fiber with special refractive-index profile. Four mode groups LP\textsubscript{01}, LP\textsubscript{11}, LP\textsubscript{21} and LP\textsubscript{02} as well as the vector modes in each mode group of the tested fiber were observed. The
polarization of the four vector modes in the LP_{11} mode group was analyzed. The large splitting of the vector modes in group index was shown, which is the intention of the fiber design. Accordingly, the application of the fiber can be a polarization-insensitive microbend fiber grating, where one of the vector modes is excited exclusively for the wavelength range of interest.

Furthermore, with the help of the numerical simulations the measured group-index difference between the four mode groups can be used to roughly characterize the refractive-index profile of the tested fiber. The measured group-index difference between degenerate HE_{21} higher-order modes in the LP_{11} mode group can be used for characterizing the ellipticity of the fiber. At the same time, a more precise refractive-index profile of the fiber was obtained. The estimated ellipticity was experimentally confirmed by a traditional frequency-domain low-coherence interferometry. The fiber ellipticity is important not only for the performance of the polarization-insensitive microbend fiber grating based on the tested fiber, but also for generation of light beams with interesting polarization distribution using higher-order fiber modes. The experimental results were compared with the results of the numerical simulations. A very good agreement was obtained.

Additionally, the technique was used to investigate the dispersion properties of the modes in a few-mode solid-core photonic crystal fiber. It has been shown that the absolute chromatic dispersion of the fiber modes can be estimated. The resolution of the system was discussed.
Appendix A

Operator definitions

The vector operator $\nabla_t$ and $\nabla_t^2$ are defined as follows. If $\Psi$ and $A$ are the separable scalar and vector function and decompose into transverse and longitudinal components using Eq. 2.11. The unit vectors $x$, $y$, $z$, $r$ and $\phi$ are parallel to the coordinate directions

$$\Psi = \psi(x, y) \exp(i\beta z)$$
$$A = a(x, y) \exp(i\beta z) \quad (A.1)$$

then

$$\nabla_t \Psi = x \frac{\partial \Psi}{\partial x} + y \frac{\partial \Psi}{\partial y}$$
$$= r \frac{\partial \Psi}{\partial r} + \phi \frac{\partial \Psi}{\partial \phi} \quad (A.2)$$

$$\nabla_t \cdot A_t = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}$$
$$= \frac{1}{r} \frac{\partial (r A_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} \quad (A.3)$$

$$\nabla_t \times A_t = z \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$
$$= z \left[ \frac{1}{r} \frac{\partial (r A_\phi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right] \quad (A.4)$$

$$\nabla_t^2 A = \nabla_t (\nabla_t \cdot A) - \nabla_t \times (\nabla_t \times A) \quad (A.5)$$
Appendix B

COMSOL Multiphysics for numerical simulations of fiber mode properties

COMSOL Multiphysics is a powerful interactive environment for modeling and solving all kinds of scientific and engineering problems based on partial differential equations. To solve the partial differential equations, COMSOL Multiphysics uses the finite-element method. Two of the three programming environments of COMSOL are used in the simulations: the graphical user interface and the MATLAB language. Using the application modules in COMSOL Multiphysics, various types of analysis can be performed. “Mode analysis” is used for the simulations. Firstly the 2D application module is created in RF module—Perpendicular waves—Hybrid-mode waves—Mode analysis. The modeling steps include:

1. Creating the geometry: the geometry of the fiber cross section is drawn using the “ellipse/circle” drawing tool.

2. Meshing the geometry: free meshes consisting of triangular elements are created by clicking the button “initialize mesh”.

3. Defining the physics on the domains and at the boundaries: the boundary condition at the core-cladding interface of the fiber is set to be continuity. The refractive index of the core and the cladding, and the wavelength in use are defined for this step.

4. Solving the model: the “Eigenvalue” solver is used. The desired number of mode effective indices of the solver is set to 15. The tested few-mode fiber is expected to have 12 core
modes. The redundant solutions need to be solved for the confirmation of the mode cutoff.

5. Postprocessing the solution: the 2D intensity profiles and the polarization of the modes are checked in this step.

6. Performing parametric studies: the fiber model is exported as a Model M-file, which means the entire modeling session is saved as a sequence of commands. Then the M-file is run directly in “COMSOL with MATLAB”. The codes are modified to study the parameters dependency. The refractive index of the core and the cladding, or the ellipticity of the fiber, is set to be a variable. Loop structures are used for the investigation of the dispersion properties of the modes.
Appendix C

Evaluation of the polarization of a doughnut beam using a polarizer

As it has been discussed in chapter 3, a radially or azimuthally polarized doughnut beam can be regarded as a coherent superposition of two Hermite-Gaussian modes \( \text{TEM}_{10} \) and \( \text{TEM}_{01} \). A \( y \)-polarized \( \text{TEM}_{10} \) mode and an \( x \)-polarized \( \text{TEM}_{01} \) give an azimuthally polarized beam whereas an \( x \)-polarized \( \text{TEM}_{10} \) mode and \( y \)-polarized \( \text{TEM}_{01} \) mode produce a radially polarized beam, as shown in Fig. 3.1.

However, when the two constituent Hermite-Gaussian modes of a doughnut beam do not have the same amplitude and right phase relation (0 phase difference), the obtained doughnut beam would not be perfectly radially or azimuthally polarized. For example, according to Eqs. 3.1 and Eq. 3.2a, the electric field of the superposition of an \( x \)-polarized \( \text{TEM}_{10} \) mode and \( y \)-polarized \( \text{TEM}_{01} \) mode can be described as

\[
E(x, y) = xE_{10}(r, \phi) + yE_{01}(r, \phi)
\]

\[
= \frac{E_0}{\omega_r} \exp\left(-\frac{x^2 + y^2}{\omega_r^2}\right) [x\hat{x} + \exp(i\varphi)\rho y\hat{y}]
\]

where \( \rho \) is the amplitude of the \( E_{01}(r, \phi) \) component relative to the \( E_{10}(r, \phi) \) component, \( \varphi \) is the relative phase. For \( \rho = 1 \) and \( \varphi = 0 \), the above equation describes an ideal radially polarized doughnut beam. When it passes through a polarizer, a two-lobe pattern with a dark bar in between will be obtained. When the polarizer is rotated the dark bar is always perpendicular to the trans-
mission axis of the polarizer. The lob pattern rotates in the same sense with the polarizer rotation and its intensity remains constant. However, when $\rho \neq 1$ or $\varphi \neq 0$, an imperfect radially polarized beam is obtained, and accordingly, the pattern and the total intensity of the lobs do not follow the above rules any more. Therefore, a rotating polarizer can be used to roughly evaluate the purity of radially or azimuthally polarized doughnut beams.

According to Eqs. C.1 the amplitude of the beam behind a polarizer whose axis is oriented by $\theta$ relative to the x axis can be described as

$$P(x, y) = \frac{E_0}{\omega r} \exp(-\frac{x^2 + y^2}{\omega^2}) [x \cos \theta + \exp(i\varphi)py \sin \theta]$$  \hfill (C.2)

Therefore, the beam intensity detected by the detector is

$$|P(x, y)|^2 = \frac{E_0^2}{\omega^2} \exp\left[-\frac{2(x^2 + y^2)}{\omega^2}\right] (x^2 \cos^2 \theta + \rho^2 y^2 \sin^2 \theta + 2xy\rho \cos \theta \sin \theta \cos \varphi)$$  \hfill (C.3)

If $X = x/\omega r$ and $Y = y/\omega r$, the above equation reduces to

$$|P(x, y)|^2 = E_0^2 \exp\left[-2(X^2 + Y^2)\right] (X^2 \cos^2 \theta + \rho^2 Y^2 \sin^2 \theta + 2XY\rho \cos \theta \sin \theta \cos \varphi)$$  \hfill (C.4)

In Fig. C.1, the intensity distributions of an imperfect radially polarized beam behind a polarizer oriented along $\theta = 45^\circ$ are shown with different $\rho$ and $\varphi$. It is seen in the figure that the ratio of the amplitude of the two Hermite-Gaussian modes $\rho$ influences the angle between the dark bar of the lobes and the polarizer orientation, while the phase difference $\varphi$ influences the clearness of the dark bar. In particular, when $\rho = 0$ the intensity pattern of the beam behind the polarizer does not rotate as the polarizer is rotated. When $\varphi = 90^\circ$ the dark bar disappears.
Figure C.1: Intensity distributions of an imperfect radially polarized beam behind a polarizer oriented along $45^\circ$ (shown by white dashed lines): (a) with $\varphi = 0$ and different $\rho$; (b) with $\rho = 1$ and different $\varphi$. 
Appendix D

LabVIEW implementation for low-coherence interferometry

The LabVIEW program is used to control the measurements. In the following, it will be described how the “Virtual Instruments” are organized for image acquisition, step-motor control, data processing and results display.

In the background the program executes a while-loop containing two frames in sequence, i.e. executed one after the other. One frame realizes the image acquisition of the camera. The other frame controls the position of the reference mirror. The obtained images are sent to the module of data processing written in MATLAB scripts. The module performs the interpretation of the interference images and sends the results to LabVIEW for data storage and data display.

The front panel of the program is shown in Fig. D.1. Before the program is started, several items in the “program control”, “step-motor control”, and “save” have to be set. The program can be stopped by setting the number of the acquired images through “frame controller”, or by setting the total moving distance of the step motor through “set moving range”. The program can be also terminated by clicking the button “STOP” at any time when the program is running. Furthermore, the serial port of the computer used by the step motor, the moving direction and the step size of the step motor should be set. File paths are specified as the destination where the raw data or the calculated results are saved. Additionally, two “numerical indicators” are used to display the number of the images taken by the camera and the total moving distance of the step motor in real time.

Two 2D intensity graphs are used to display the raw image obtained from the camera and the
real-time mode intensity distribution, respectively. Additionally, two charts are used to display the cross-section intensity distribution of the raw images. Two charts are used to display the total intensity of the fiber mode and the intensity at one pixel at the screen for checking the stability of the system.

When the beams of the interferometer have to be aligned, the program can work as a normal camera driver. In this case, the “write to port” has to be set to “off” so that the step motor is inactive. Only the “real-time image” intensity graph is active for the real-time image acquisition.

![Figure D.1: Front panel of the LabVIEW measurement program.](image-url)
Bibliography


Curriculum Vitae

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