Characterization of Surface Roughness and Shape
Deviations of Aspheric Surfaces

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Zusammenfassung


- Frequenzanalyse der Nanorauhigkeit diamantgedrehter optischer Oberflächen
- Quasiabsolutmessung der Oberflächenabweichungen asphärischer Flächen mittels einer auf diffraktiver Optik basierenden interferometrischen Nullmethode


Die zweite Untersuchung erfolgte am IOIP. Ein 3-Positionen-Absoluttestverfahren für rotationssymmetrische Asphären mit Hilfe von „combined diffractive optical elements“ (combo-DOEs) wurde weiterentwickelt. Der Einfluß der DOE-Substratabweichungen auf die Kalibrierprozedur wurde untersucht und eine Zahl von Kriterien für das Design von optimierten combo-DOEs angegeben. Es wurde gezeigt, daß das optimierte Design die Gesamtkonsistenz des Verfahrens erhöht. Weiterhin wurde der
nicht rotationssymmetrische Anteil der Oberflächenabweichungen mit den entsprechenden Abweichungen verglichen, die durch ein N-Positionen-Rotationsmittelungsverfahren erhalten wurden; beide Resultate befanden sich in guter Übereinstimmung. Das Potential, die kompletten Abweichungen des Prüflings zu ermitteln, kennzeichnet das untersuchte Verfahren als ein vielversprechendes Hilfsmittel für die interferometrische Kalibrierung in der Asphärenmetrologie.

Es steht zu erwarten, daß die Ergebnisse dieser Untersuchungen die Genauigkeit von Produktions- und Testverfahren asphärischer Hochleistungsoptik deutlich steigern werden.


Abstract

This thesis is the result of the research work undertaken at Central Scientific Instruments Organisation (CSIO), Chandigarh, India and Max Planck Research Group, Institute of Optics, Information and Photonics (IOIP), University of Erlangen, Germany. The goal of the research work was the precise fabrication and characterization of aspheric surfaces. Aspheric surfaces are used in optical systems in order to improve the aberration correction and, often, without added weight and size. However, the major obstacle in using them lies in the difficulty in their fabrication and accurate testing. The thesis discusses the following two major investigations which have been undertaken to achieve the above said goal.

- Frequency analysis of nano-scale roughness formation on single point diamond turned optical surfaces.
- Quasi absolute measurement of aspheric surface deviations by using a diffractive optics based interferometric null method.

The first investigation was conducted at “National Aspheric Facility” in CSIO. The flat metal mirrors were fabricated and characterized by performing the frequency analysis (power spectral density and autocorrelation) of the surface roughness. The study focused on establishing a relationship between the manufacturing process and the surface roughness generation which is crucial for developing metal components/mirrors with the optical quality finish.

The second study was carried out at IOIP. A three position quasi absolute test procedure for rotationally invariant aspherics by using “combined diffractive optical elements” (combo-DOEs) is developed. The effects of the DOE substrate errors on the calibration procedure are investigated and a set of criteria for designing an optimized combo-DOE has been established. It is demonstrated that the optimized design enhances the overall consistency of the procedure. Furthermore, the rotationally varying part of the
surface deviations is compared with the rotationally varying deviations obtained by an N-position averaging procedure and both the results have been found to be in good agreement. The capability of extracting the complete deviations of the specimen signifies the investigated technique as a promising tool for interferometric calibration for aspheric metrology.

It is expected that the results of this study will substantially increase the accuracy of the production and testing of high-performance aspheric optics.

In the first chapter, a general overview to the aspheric optics is presented. It provides a review of fabrication and characterization techniques available to the optical engineers. The DOE based interferometric techniques with their relative advantages and limitations are discussed in the second chapter. The third chapter reports the investigations carried out on the formation of nano-scale surface roughness during single point diamond turning process. For this purpose, a flat mirror of 60 mm diameter from aluminium-6061 is machined with varying machining parameters. The power spectral density distributions are computed from the surface roughness data obtained by mechanical as well as optical roughness profilers.

The fourth chapter describes the measurement principle of the three position quasi absolute test procedure for rotationally invariant aspherics. The critical design parameters are discussed and then supported by simulation results. Based on the design conditions established here, the combo-DOEs for the specimen under test are developed. The fifth chapter reports the experimental results of the quasi absolute test applied to a rotationally invariant aspheric surface. The extent of the “quasi condition” is also discussed while presenting the first results for the measurement a toric surface. A comparison with an N-position rotational averaging test is also made. Current limitations of the test procedure, its advantages and the scope for future studies are discussed in the sixth chapter as conclusions.
Chapter 1

Introduction to Aspheric Optics

In this chapter a general overview of aspheric optics will be presented. The mathematical description of aspherics and their advantages in optical systems will be discussed. The fabrication techniques, single point diamond turning and grinding, will then be presented. In the last Section, testing methods for surface roughness and shape deviations of aspherics will be reviewed in brief.

1.1 Aspheric Optics

The majority of optical systems are based on the use of spherical components because of the ease in their fabrication and characterization. However, the progress in precision engineering technology is continuously demanding the use of aspheric components to improve the system performance by enhancing the aberration correction capabilities. The use of aspherics instead of spheres also reduces the number of optical surfaces in the optical system. There are cases where aspheric optical components have a significant advantage over the spherical ones. In photographic optics, head-up-displays and helmet mounted displays, where volume and weight constraints are significant, aspherics are often the only option. Semiconductor, consumer electronics, and aerospace systems depend on aspheric optics to increase the product performance. In astronomical optics, reflective aspheric components are widely used. With the significant development of new plastic materials, low-cost molded aspheric refractive optics is finding their place in the large consumer market. Precision diamond grinding and compression grinding of glass aspheric lenses is also becoming more common, especially in the optical lithography for producing integrated electronic circuits.
An aspheric surface is, by definition, an optical surface that is not spherical in form. It has more free parameters for the optimization than a spherical surface does (Born and Wolf [1999]). The traditional monochromatic aberrations, including spherical aberration, coma, astigmatism, and distortion can be eliminated by using aspherics. The ultimate purpose of an aspheric surface in a lens system is to assist in redirecting the rays to achieve a near-perfect image. In other words, aspheres introduce a wave front deformation which compensates deviations from sphericity of the wave front.

### 1.1.1 Mathematical Description of Aspherics

The term aspheric, as it is used in this thesis, implies a non-spherical and non-plane surface, which retains rotational symmetry around its optical axis (Shannon [1980]). A sphere is having only one shape parameter, the radius of curvature $R$ (the limiting case is the plane $R=\infty$). An aspheric, however, will have an infinite (in practice a large) number of additional shape parameters. Mathematically an aspheric is generated by the rotation of an axis-symmetrical plane curve around its axis. Generally, a rotationally invariant aspheric can be represented by the following mathematical relation between its sag ‘$z$’ and the aperture coordinate ‘$h$’:

$$z = \frac{ch^2}{1 + \sqrt{1 - (1+k)c^2h^2}} + A_4h^4 + A_6h^6 + \ldots \quad (1.1)$$

Here, the first term is the conic surface. ‘$z$’ is the sag of aspheric surface, ‘$c$’ is the curvature of base sphere and ‘$h$’ is the distance from the optical axis.

- $k$ is the conic constant, which is
  - $= 0.0$ for sphere
  - $= -1.0$ for parabola
  - $< -1.0$ for hyperbola
  - $> 0$ for oblate ellipsoid (ellipsoid rotated around minor axis)
  - $-1 < k < 0$ for prolate ellipsoid (ellipsoid rotated around major axis)

- $A_i = \text{higher order aspheric coefficients}$

In the most general case certain aspheric surfaces can be defined only through a table of numbers that defines the sag or shape of the surface locally at each coordinate. In some modern designs also odd terms in ‘$h$’ are added if for example an axicon-type form should be added.
1.1 Aspherics Optics

1.1.2 **Challenges in using Aspherics**

Despite of all the advantages and applications of aspheric optics, their use in optical systems remains a severe challenge. Aspherics are difficult to fabricate and characterize with conventional methods and machines. This is the reason why aspherics have been restricted to domains such as high performance or leading-edge systems, or high-volume molded elements. Manufacturing tools for aspheres are completely different from spherical or flat optics. Grinding operation of aspheres requires a point-contact contouring. Polishing of aspheres, without disturbing the shape, is one of the time consuming and painstaking processes. Particularly challenging to the precision manufacturing industry is the capability to measure complex shapes. The departure of an asphere from sphere is the determining parameter of the difficulty in metrology. Metrology of such components requires a dedicated null optics. Another testing requirement is that the method must be able to measure non-specular surfaces, because it is easier to correct an asphere in the grinding stage than in the polishing stage.

1.2 **Fabrication of Aspherics**

There have been manyfold improvements in the fabrication technology of aspheric optics. The standard single point diamond turning (SPDT) and the grinding machines capable of producing high quality optical surface are now commercially available.

1.2.1 **Single Point Diamond Turning (SPDT)**

Diamond turning of optics (Saito [1975], [1976],[1978]) is a process of very accurately cutting away a thin chip or layer of the surface under precisely controlled conditions. One of the most extensively applied techniques for aspheric fabrication is computer-numerically controlled (CNC) machining and grinding. For instance, the commercially available Nanoform 250 (Fig. 1.1) diamond turning (for non-ferrous metals) and grinding machine (for ferrous and brittle material) can generate rotationally invariant surfaces with high accuracy.

Fabrication of infrared (IR) optical elements compared to those for the visible region offer higher tolerances in both surface shape and roughness because of longer wavelengths in IR region. Additionally, the surfaces generated by diamond turning for the
IR region do not require post polishing as do the optics used in visible and ultra-violet regions.

Figure 1.1: *Nanoform-250 single point diamond turning machine (Taylor-Hobson make).*

In diamond turning, the intended shape and surface produced depend on the machine tool accuracy and other machining parameters. In traditional optical fabrication, lapping and polishing with an abrasive-loaded lap produce the final shape and surface of optical elements. The fundamental difference between diamond turning and traditional optical fabrication is that diamond turning is a *displacement-controlled* (point wise) process while conventional optical fabrication is a *force-controlled* (area wise) process (Rhorer and Evans [1995]).

1.2.1.1 *Advantages of SPDT*

There are several important advantages of using diamond turning, including the ability to produce good optical surfaces to the edge of the element, to fabricate soft ductile materials difficult to polish such as off-axis parabolas, to eliminate alignment requirement in some systems, and to fabricate shapes difficult to generate by other methods (Riedl [2001]).
When combined with an ultra-precision vibration-free machine, a compact rigid tool holder, stable and well-balanced fixturing, single crystal natural diamond cutting tools will remove material from workpiece cleanly and effectively. Because of the extreme level of sharpness on the diamond-cutting tool, very small forces are needed during the machining process. The end result is a surface that exhibits optical quality in both surface finish and form accuracy.

Extreme level of sharpness, minimal tool wear, cutting edge quality, edge waviness control, low coefficient of friction and thermal expansion, and high thermal conductivity make diamond the best cutting tool for ultra precision machining. There are two types of diamond tools: with controlled waviness and non-controlled waviness at the tool tip. Controlled waviness dictates that the radius shape of the tool tip deviates from a true circle by a guaranteed value ranging between 0.05-1.0 microns.

1.2.1.2 *Operating Parameters of SPDT*

The operating parameters of a precision machining process on a given piece of material will vary considerably depending on the production rates required, workpiece and machine characteristics, and all other process variables such as coolant, tool condition, depth of cut, tool feed rate (Sanger [1987]). Tool feed rates, cutting speeds, and depths of cut are typically much lower in diamond turning process compared to turning with conventional machining tools. For a given material, under different combinations of machining parameters (within their optimum range), similar surface figure and finish results can often be obtained. The tool feed rate is normally expressed in terms of either distance traveled by the tool per unit time (mm per min) or distance traveled per unit rotation (mm per revolution). It is most common to use the distance per revolution as it is directly related to the anticipated theoretical surface finish. For a given tool feed rate, larger the tool nose radius, the better the optical surface finish. The surface quality depends to a great extent on the material characteristics like: grain size, microstructure of crystal boundary, crystal uniformity, annealing procedures adopted etc.

1.2.1.3 *Diamond Turnable Materials*

Many non-ferrous materials such as aluminium, copper, electroless nickel lend themselves nicely to diamond turning. Additionally, several polymers and crystals are
also suitable for diamond turning. Concerning polymers/plastics, the common diamond turnable materials are polycarbonate, polymethyl methacrylate, polystyrene and nylon. The crystalline materials which can be successfully diamond turned are: germanium, zinc selenide, lithium niobate and silicon.

1.2.2 Grinding and Polishing of Aspherics

Spherical optics made from glass and crystalline materials have traditionally been ground and polished to deliver the specified form and surface finish. Grinding is a mechanism wherein a controlled network of small fractures is introduced onto glass surface such that the glass particles fall out of the surface when the fractures intersect. Material removal by grinding is a much slower process than machining because the brittle material removed must be disintegrated into micrometer sized particles.

In the case of aspheres, the only choice for grinding is some form of point-contact contouring. The contouring requires CNC technology to achieve the required precision. Deploying a bonded diamond grinding head on a diamond turning machine in place of the diamond tool, hard brittle materials like glasses and ceramics can be machined. CNC technology also enables the generation of precision surfaces that are precentered to final diameter and ready to polish. Every grinding operation introduces some surface and subsurface damage.

To remove the subsurface damage caused in the grinding process, a polishing cycle is mandatory. For spherical surfaces, a typical polishing tool is approximately of the same size as the lens surface and is the perfect match for the lens. Cushioned by slurry of water and soft polishing abrasives, the two surfaces are rotated and oscillated against each other to yield a very smooth and specular optical surface.

The simple lapping model does not work for aspheres since the radius of curvature is different in the different regions and the lapping tool and the workpiece no longer have the proper matching surfaces. The principal problem in attaining a polished aspheric surface is retaining the basic accuracy of the ground surface during the polishing operation. To solve this problem, a computer controlled sub-aperture polishing approach has been employed where the polishing tool is much smaller than the workpiece.
The requirements for extreme ultraviolet optics have taken the specifications of accuracies to new levels. These specifications pose some limitations on sub-aperture tool technologies. A promising technology for polishing aspheres up to these limits is magneto-rheological finishing (MRF) (Golini [1995, 2001]). This technology employs a magneto-rheological (MR) fluid as a polishing tool. The method is best understood by thinking of the MR fluid as a compliant replacement for the conventional rigid lap in loose abrasive polishing. The MR fluid stiffens as it passes through a magnetic field, thus forming a temporary polishing pad. The MR fluid carries the polishing slurry that is acting on the lens surface in a precisely controlled pattern by varying the magnetic field strength and direction. Since the tool is fluid-based, it conforms to any shape and eliminates the need of dedicated tooling. This process reduces cycle times and is capable of producing sub-wavelength surface accuracy.

1.2.3 Fabrication Errors: Form, Figure and Finish

SPDT allows high precision aspheric optics to be manufactured quickly and efficiently. However, there are three types of errors that may occur on the machined surface i.e. form, figure and finish or roughness. (Fig.1.2). It is impossible to say, at what point does finish error becomes figure error. It is better to separate finish, figure and form error according to their cause, as this relates to the performance factors. Roughness is due to the irregularities which are inherent in the production process (e.g. cutting tool, and feed rates). The roughness also depends on the material composition and heat treatment. Figure error or waviness may result from vibrations, chatter or workpiece deflections and strains in the material. Form error is the general shape deviation of the surface from the intended shape, neglecting variations due to roughness and figure error.

![Figure 1.2: Three types of errors arising from a turning operation: form, figure and finish](image)
1.3 **Metrology of Aspherics**

The testing of diamond turned aspheric surfaces includes the analysis of Form, Figure and Finish (roughness) errors. The form and figure error are often collectively termed as surface deviations (or shape errors). The most commonly accepted single-value parameters to evaluate surface deviations and surface finish are peak-to-valley value (P-V) and root-mean-squares value (rms) respectively. For the surface deviations more sophisticated parameters like polynomial coefficients have to be used additionally to characterize the deviations.

Characterization of aspheric surfaces can be divided into two tasks: Surface roughness measurement and the measurement of surface deviations. There are a multitude of roughness measurement techniques available such as: total scattering, angle resolved scattering, mechanical and optical profiling, atomic force microscopy, white light interferometry, confocal laser scanning microscopy (Dupparre et al. [2002]). Every method has its own advantages and limitations (Bennett [1985]). A nanoscale surface profiler is the most commonly used instrument to characterize the surface roughness while an interferometer is used to measure the surface deviations in the whole area of the aspheric surface.

1.3.1 **Surface Roughness by Profilometric Methods**

The surface roughness is generally measured by profilometers; mechanical as well as optical one. Mechanical profilers are based on a touch probe method where a stylus makes physical contact with the surface under test. The optical profilers use optical beam to probe the surface and hence are non-contact by nature. Both probes, mechanical and optical, have some advantages and disadvantages depending on the situation of their use (Whitehouse [1988], Church [1985]).

1.3.1.1 **Stylus Type Mechanical Profilers**

The stylus based profilers have long been in use for measuring statistical properties of smooth optical surfaces (Bennett and Dancy [1981]). They rely on a small-diameter stylus moving along a surface either by movement of the stylus or movement of the surface
under test. A true stylus profiler moves linearly to obtain the measurement result (Fig. 1.3). As the stylus traces across the surface features, it records the heights and valleys of the surface encountered along the scan path. They are widely used for measuring MEMS, semiconductor devices, optical thin films, optical surfaces and surface finish.

A typical instrument will consist of a stylus with a small tip (fingernail), a gauge or transducer, a traverse datum and a processor (see for example www.taylor-hobson.com). As the stylus moves up and down along the surface, the transducer converts this movement into a signal which is then fed to a processor which converts this into a visual profile with roughness numbers.

These touch-probe profilers have the disadvantage that the tip of the stylus traverses on the specimen’s surface and leaves a mark on it; hence they are destructive by nature. Moreover, they offer one linear scan of data allowing only rotationally invariant error to be detected. Another limitation is that the contact techniques smoothes the surface data which reduces the bandwidth of the resulting data. Additionally, stylus methods cannot test for index inhomogeneities of glass or assembly errors of a lens system.

1.3.1.2 Optical Profilers

The use of non-contact surface measurement instruments is becoming more and more common. The non-contact profiler gives 3D data sets with high speed without damaging the surface. These methods provide higher bandwidth of the data. Here an optical beam probes the surface under test. Many optical techniques are based on focusing a spot on the surface to be measured. The surface texture is derived from an “average” signal obtained
from the area of this spot. Typically the spot will be several microns across, and may vary in width through the vertical range of the detector.

![Figure 1.4: A typical optical profilometer based on the Mirau interferometry principle.](image)

There are different types of optical profilometers based on Nomarski and Mirau microscopes. Figure 1.4 shows a typical optical profilometer based on the Mirau interferometry principle (Wyant and Prettyjohns [1986]). A white light source with a filter is used as an illumination device. The light reflected from the surface interferes with the light reflected from the reference and the resulting interference pattern can be captured by a CCD array. The Mirau interferometer is mounted on a piezoelectric transducer so that it can be moved by applying voltage. During this movement, the test surface remains fixed, and a phase shift is introduced into the reference arm of the interferometer.

1.3.2 **Surface Deviations by Interferometric Methods**

The profilometric methods have the drawback that one cannot get the information about the figure or shape error of the optical surface. One has to apply stitching methods while using optical profilers, but such methods are prone to mapping errors. The
interferometric methods have been the standard means of testing conventional spherical optics and give the information about the whole surface: local as well as global deviations.

Interferometric test methods rely on the comparison of the wave front generated by the surface under test and an ideal reference wave front. For spheres and flats, it is easy to get a reference spherical or flat wave front. But for aspheric surfaces with large departures, interferometry is much more complex, because closely fitting comparison surfaces are not available. Generating a reference aspheric wave requires additionally a null element in the optical setup. The methods where null elements have been used are known as null methods. The aspherics which deviate weakly from a spherical surface can be tested by using a best matching spherical wave front. Such methods are known as non-null methods.

1.3.2.1 Null Methods

A null element is a compensating element used in the optical testing of a steep aspheric surface. It converts a spherical or plane wave front into one that precisely matches the aspheric surface under test. When the wave front is reflected from that surface, it retraces its path. If the surface is perfect, it will result in a perfect emerging spherical or plane wave front, which is then easily evaluated. These methods measure the departure from the desired aspheric shape. The null element can be of three types: (1) a reflective or refractive optics (Hilbert and Rimmer [1970]), (2) a holographic null optics (computer generated hologram or diffractive optical element) (Pastor [1969], MacGovern and Wyant [1971], Schwider et al. [1975, 1976, 1979], Schulz and Schwider [1976]) and (3) a combination of conventional null optics and computer generated hologram (Wyant and O’Neill [1974]). These null elements can be deployed in standard interferometric test setups such as Fizeau type or Twyman-Green type. The details of these methods are given in Section 2.4.

1.3.2.2 Non-null Methods

For aspherics having deviations of the order of few μm from a best fitting sphere, non-null test can be employed. In case of a non-null test, non-equally-spaced fringes will be
obtained even if the optics under test happens to be perfect, as asphericity has not been cancelled out. With computer analysis being available and fringe spacing sufficiently large so that the fringes can be detected, non-null tests are nearly as good as null tests, and they are easy to perform. In testing of a general aspheric surface, the complexity of the test is determined by the slope of the aspheric departure from the best-fitting spherical wave front. Since it is the slope of the departure, and not the actual departure itself that is important, the best-fitting wave front is the spherical wave front that minimizes the maximum slope of the departure. That is, the best reference is the wave front that minimizes the maximum spatial frequency of the fringes.

Non-null tests can be divided into two categories: (1) tests that measure the slope, such as, shearing interferometry (Hariharan et al. [1984]) and (2) tests that measure the wave front directly, such as, normal monochromatic interferometry, two-wavelength interferometry (Cheng and Wyant [1984], Creath et al. [1985]), long wavelength interferometry (Kwon et al. [1980], Wyant [1987]), sub-nyquist interferometry (Greivenkamp [1987]) and moiré interferometry (Peters and Ivaldi [1971], Andreev [1990]).
Chapter 2

DOE based Interferometry

The aim of this chapter is to explain the basic concepts of diffractive optics based interferometry. For this purpose, the interference phenomenon between monochromatic waves is briefly described in the first Section. Phase shifting interferometry and the different algorithms to accurately detect the phase will then be presented. Finally, the DOE based interferometric setups with their advantages and limitations will be discussed.

2.1 Interference

The metrology of high-precision optical systems is generally made using optical interferometric methods based on the interference (Hariharan [1987], Schulz and Schwider [1976]). The fundamental principle behind interference is that when two or more coherent electromagnetic waves exist at the same point in space they superimpose constructively or destructively. Two-beam interferometry is the most common tool for evaluating optical surfaces.

The electric field vector due to an electromagnetic field at a point in space is composed of amplitude and phase

\[ \mathbf{E}(x, y, z, t) = \tilde{\mathbf{A}}(x, y, z, t)e^{i\Phi(x, y, z, t)} \]  

(2.1) or

\[ \mathbf{E}(\mathbf{r}, t) = \tilde{\mathbf{A}}(\mathbf{r}, t)e^{i\Phi(\mathbf{r}, t)} \]  

(2.2)

where \( \tilde{\mathbf{A}} \) is the complex amplitude function and \( \Phi \) represents the phase term. Both are the functions of the spatial coordinate \( \mathbf{r} \) and time \( t \). The expression can be simplified if a linearly polarized monochromatic wave with angular frequency \( \omega \) is assumed. Then
only the component along the direction of polarization has to be considered and a scalar notation can be used

\[ E(x, y, z, t) = A(x, y, z)e^{i[ωt + φ(x, y, z)]} \] (2.3)

where \( ω = 2πν \), is the angular frequency of the monochromatic wave. ‘\( A \)’ and ‘\( φ \)’ are the real-valued functions. Typical values of the optical frequencies in visible region are in the range of \( 10^{15} \) Hz.

The time dependence can be separated and only the position-dependent complex amplitude ‘\( U \)’ has to be considered.

\[ U(x, y, z) = A(x, y, z)e^{iφ(x, y, z)} \] (2.4)

If we have more than one wave then the net complex amplitude is the sum of all the component fields:

\[ U(x, y, z) = \sum_i U_i(x, y, z) \] (2.5)

and since detectors are only sensitive to the time averaged intensity, the resulting intensity is

\[ I(x, y, z) = U(x, y, z)U^*(x, y, z) \] (2.6)

In two beam interference the resulting intensity becomes

\[ I = |U_1|^2 + |U_2|^2 + U_1U_2^* + U_2U_1^* \] (2.7)

Substituting eq. 2.4 into 2.7

\[ I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos[Δφ] \] (2.8)

where \( Δφ = φ_1(x, y, z) - φ_2(x, y, z) \). This is the basic equation for two-beam interferometry. The only measurable quantity is the intensity ‘\( I \)’. The third term contains the interference information in terms of a phase difference.

When two monochromatic waves are interfered, the interference fringes exist throughout the whole space. In all practical cases, the observation of interference is
confined to a detector plane, and this plane is usually assumed to be perpendicular to the 
z-axis. Therefore, the z dependence is often not stated explicitly.

By defining
\[ \gamma = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = 2 \sqrt{\frac{I_1 I_2}{I_1 + I_2}} \]  
\[ \text{(2.9)} \]

it becomes
\[ I(x, y) = I_0(x, y) [1 + \gamma(x, y) \cos \Delta \phi(x, y)] \]  
\[ \text{(2.10)} \]

where \( I_0(x, y) = I_1(x, y) + I_2(x, y) \) would be the total intensity for incoherent addition of 
the two waves, and \( \gamma \) is the local fringe contrast or visibility or modulation intensity. The 
only measurable quantity is \( I(x, y) \). The phase difference can be extracted from eq. (2.10). 
The fringe visibility will have a value between 0 and 1. For maximum visibility to occur, 
both the waves must have equal intensities. The eq. (2.10) defines an interferogram 
obtained in the \( x-y \) plane at a certain \( z \).

2.2 Fringe Evaluation Techniques

The fringes can be analyzed by converting the intensity information obtained from such 
interferograms into a phase map. Because measuring the intensity at a particular point and 
time is not sufficient for measuring the phase, various techniques have been developed for 
extracting the phase values from one or more interferograms.

These methods can be broadly divided into four classes (Schwider [1990]): (1) 
Fringe evaluation (2) Phase-lock interferometry (3) Heterodyne interferometry and (4) 
Phase-shifting interferometry (Bruning et al. [1974]. All require analysis of intensity 
variations in order to remove the uncertainties due to the unknown parameters in eq. 2.10 
and to extract the phase information precisely. Each technique has its own advantages and 
disadvantages for particular situations and none are universally applicable. The 
technological advances in detector technology and the growth in the real-time computing 
capabilities contributed immensely in the development of interferometric methods.
2.3 **Phase Shifting Interferometry**

The phase difference between the two interfering beams can be determined by measuring the intensity of the interference fringes as the phase difference between the two interfering beams changes in a known manner ([Schreiber and Bruning [2007]]). The wave front phase is encoded in the variations of the intensity pattern of the recorded interferograms and can be recovered. Phase shifting interferometry is a powerful technique for the high-precision measurement of many physical quantities. They have been applied to almost all types of interferometers, such as, Twyman-Green, Fizeau, Mach-Zehnder. The basic concept of phase shifting interferometry is that the phase in the interferogram can be determined by acquiring at least three frames of interferograms with different phase steps because there are also the three unknown parameters \( I_0, \gamma \) and \( \Delta \phi \) to be determined.

\[
I = I_0 [1 + \gamma \cos(\Delta \phi + \phi_i)]
\]

The phase steps ‘\( \phi_i \)’ can be introduced by phase shifters such as PZT (piezo-electric transducer) and LCD (liquid crystal device) either in the object or in the reference beams.

A standard scheme for PSI is depicted in Fig. 2.1. The light from a stabilized laser source enters a beam-splitting cube, dividing the light into a reference and test arm. The reference arm is equipped with a computer controlled piezo-electrically driven plane reference mirror, allowing for the adjustment of the reference phase values. A CCD array detects the intensity pattern which is the interference pattern between the phase shifted reference and test wave.

So far many algorithms for phase shifting interferometry have been developed for precision phase measurement. The algorithms, which are often used, include three step, four step, five step, and multi step (Creath [1988]). These algorithms are different from each other mainly in the frame number and the accuracy obtained in their applications (Creath [1986]). It is assumed that the phase steps are equal and precise as required in the phase calculations of the algorithms. However, some algorithms tolerate for example linear errors of the phase shifter. In practice, the phase steps are not only determined by the movement of phase shifters but also by any change of the optical path difference.
Therefore, the desired phase steps cannot be achieved, and the error of phase measurement is introduced due to the phase step errors caused by air turbulence, mechanical vibration, thermal drift, and miscalibration and nonlinearity of the phase shifter.

2.3.1 Three Step Algorithm

Interferometric data as per eq. 2.10 contains three unknowns: $I_0$, $\gamma$, and $\Delta\phi$. Three unknowns require making at least three intensity measurements to determine the phase. When the phase steps are $\pi/4$, $3\pi/4$, and $5\pi/4$, the following are the recorded intensities:

$$I_1(x, y) = I_0(x, y)\{1 + \gamma \cos[\Delta\phi(x, y) + \frac{\pi}{4}]\}$$  \hspace{1cm} (2.11)

$$I_2(x, y) = I_0(x, y)\{1 + \gamma \cos[\Delta\phi(x, y) + \frac{3\pi}{4}]\}$$  \hspace{1cm} (2.12)

$$I_3(x, y) = I_0(x, y)\{1 + \gamma \cos[\Delta\phi(x, y) + \frac{5\pi}{4}]\}$$  \hspace{1cm} (2.13)

The required phase can be computed from the following relation.

$$\phi(x, y) = \arctan\left(\frac{I_3(x, y) - I_2(x, y)}{I_1(x, y) - I_2(x, y)}\right)$$  \hspace{1cm} (2.14)
The three step algorithm requires only three measurements and is simplest to use. However, this algorithm is the most sensitive to errors in the phase shift between frames.

2.3.2 Four Step Algorithm for the Elimination of Errors due to Nonlinear Detector Error

The three step algorithm calculates the desired phase but does not work properly in the presence of phase shifting error. The major sources of errors in the calculated phase values are phase-shifter miscalibration, phase-shifter nonlinearity, detector nonlinearity, quantization, and vibration. More than three measurements of intensities provide better accuracy especially when the error in the amount of phase shift is present. In recent years, many papers have dealt with this topic to diminish or eliminate the phase step errors. In the phase step error compensating algorithms, only one or two of the phase step errors can be theoretically reduced or suppressed in particular cases, so that they are limited in their applications.

A common algorithm for phase calculation is the four step method. In this case the four recorded sets of intensity measurements can be written as

\[
\begin{align*}
I_1(x,y) &= I_o(x,y)\{1 + \gamma \cos[\Delta \phi(x,y)]\} \\
I_2(x,y) &= I_o(x,y)\{1 + \gamma \cos[\Delta \phi(x,y) + \frac{\pi}{2}]\} \\
I_3(x,y) &= I_o(x,y)\{1 + \gamma \cos[\Delta \phi(x,y) + \pi]\} \\
I_4(x,y) &= I_o(x,y)\{1 + \gamma \cos[\Delta \phi(x,y) + \frac{3\pi}{2}]\}
\end{align*}
\]

(2.15)

The phase at each point is

\[
\phi(x,y) = \tan^{-1}\left\{ \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \right\}
\]

(2.16)

The four step algorithm eliminates the phase errors which arise due to a nonlinear relationship between the irradiance incident upon a detector and the voltage as an output. Due to the nonlinear response, the measured optical irradiance \( I' \) can be written in terms of the incident optical irradiance \( I \) as
where $\varepsilon$ is the nonlinear coefficient.

To obtain the accurate results in the presence of second and third order detection nonlinearities, a minimum of four measurements is necessary. Three step algorithms give noticeable errors.

### 2.3.3 Five Step Algorithm for Elimination of Errors due to Linear Phase Shift Error

Phase shifting errors are also caused by inaccurate phase-shifter calibration. This can be minimized by adjusting the interferometer for a single fringe. However, with large amount of aberration present, it may not be possible to adjust single fringe. A linear phase shift error may be written as

$$\alpha' = \alpha(1 + \varepsilon)$$

Where $\alpha$ is the desired phase shift, $\alpha'$ is the actual phase shift, and $\varepsilon$ is the normalized error. It has been shown that the errors in the phase resulting from a calibration error or nonlinearity in the phase shifter will decrease as the number of measurement increases (Schwider et al. [1983]). They proposed a five step algorithms which removes the linear phase shifting errors. This algorithm uses intensity data with relative phase shifts of $-\pi$, $-\pi/2$, 0, $\pi/2$, and $\pi$. The phase is calculated using

$$\phi(x, y) = \tan^{-1}\left(\frac{2[I_4(x, y) - I_2(x, y)]}{2[I_3(x, y) - I_5(x, y)] - I_1(x, y)}\right)$$

### 2.3.4 Phase Unwrapping

Phase unwrapping is an integration of the wrapped phase differences. The general assumption is a fulfillment of the sampling theorem; at least two samples should be taken from one period of the interference pattern (Nyquist limit), i.e., the phase difference of adjacent pixels should lie in the region

$$-\pi \leq \phi_n - \phi_{n-1} \leq +\pi$$
where $\phi_n, \phi_{n-1}$ are the phase values of the adjacent pixels. If there is such a phase difference modulus of more than $\pi$ between two pixels, the wrapped phase has to be unwrapped by adding or subtracting (depending on the sign of the difference) $2\pi$ relative to the neighbored pixel.

### 2.4 DOE Based Null Interferometry

Optical interferometry is a useful tool for testing the optical components where the surface under test is compared with a reference (Malacara [2007]). For testing spherical surfaces, the illuminating wave front has to be spherical whose radius of curvature should coincide with the center of curvature of the surface under test. In this case the wave front impinges perpendicularly onto the test surface providing constant measuring sensitivity over the whole aperture. In addition the reflected wave front is again a spherical wave having on the average the same radius of curvature as the impinging wave front but with the opposite sign. The deviations of the surface under test are imprinted onto the reflected wave front as small deviations. As the interference plane is the detector plane, a sharp imaging of the surface under test onto the detector should be guaranteed.

Interferometers for optical testing are generally of either the Twyman-Green or Fizeau type. In Twyman-Green configuration (Fig. 2.2), a beam-splitter divides the expanded laser beam into test and reference beams. Upon returning through the beam shaping optics, the test wave front is a plane wave again. It is recombined with the plane reference wave front at the beam splitter to form an interferogram, which may be viewed by a video camera.

In Fizeau interferometer (Fig. 2.3), the reference wave front is provided by the partially reflecting last spherical surface of the objective in the test arm. The last surface of the aplanatic meniscus lens fulfills the sine condition. The Twyman-Green interferometer requires high quality optics for its beam shaping optics, beam-splitter, and reference mirror. Fizeau interferometer optics can be of lower quality, with the exception of the reference surface, since the test and reference beams follow nearly identical paths.
2.4 DOE based Null Interferometry

Figure 2.2: Twyman-Green interferometer for testing of spherical surfaces.

Figure 2.3: Fizeau interferometer for testing of spherical surfaces.
In the case of aspherics (Schwider [1999]), the departure of the aspheric surface under test from the best fitting spherical surface is large in many cases as measured in wavelengths. This makes it difficult to measure the aspheric surfaces. This large departure from the spherical or plane reference surfaces in aspheric testing result in too many interference fringes for interpretation. Therefore, it is necessary to seek other forms of interferometers using reference surfaces that more closely match the surface under test.

A null corrector has to be introduced in the standard interferometers (Offner [1992]). In null tests, the accessory optics is included in the test set-up to cancel the asphericity of the wave front of the aspheric optics under test. If the aspheric optic under test is perfect, an interferogram will show straight, parallel and equally spaced fringes. Each different aspheric surface to be tested requires a different null lens. Null tests can be divided into three categories:

- Reflective or refractive null optics
- Computer generated hologram (CGH) nulls
- Combination of null optics and CGH

These methods have been introduced first in chapter 1 (Section 1.3.2.1). Here they are explained in more detail.

2.4.1 Reflective or Refractive Null Optics

Null tests are preferable for surfaces with very large deviations from a sphere. One has to design a beam shaping optics (null optics) that converts the incoming plane or spherical wave front to an aspheric wave front, which exactly matches the aspheric surface under test. The aspheric wave front is incident perpendicularly on the aspheric surface. When the beam reaches the asphere, then still the null condition can be achieved. On return path it becomes again a plane or spherical wave and interferes with the reference wave.

The null optics has to be in general a quite complex system of several spherical lenses which produces this aspheric wave front. Of course, these lenses have to be themselves without errors and they have to be aligned relative to each other very accurately. Testing of the null lens is a serious problem because the resulting wave front does not show the symmetry of a spherical wave front. For general aspherics lacking rotational symmetry, i.e., free form optics, even null lenses cannot be a solution. Here, the
2.4 DOE based Null Interferometry

diffractive wave front generator in the form of a computer generated hologram or
diffractive optical element comes in to play. Such elements can provide any wave front
and are therefore ideal as beam shaping optics.

2.4.2 Computer Generated Hologram (CGH) Nulls

Holography is extremely useful for the testing of optical components and systems. If a
perfect aspheric element is present, a hologram can be made of the wave front produced
by the component. The front stored by the hologram can be used later in the
interferometric testing of other supposedly identical aspheric elements. In many situations
when master optics is not available for making a real hologram to serve as a holographic
test plate, a synthetic CGH can be made (Lohmann [1967]). CGHs have been used
extensively as null elements to test aspherics (Creath and Wyant [1995], Burge [1995],
Dörband B and Tiziani [1985], Fercher [1976]). It is a more flexible alternate to the
conventional null optics comprised of multiple refractive and / or reflective spherical
elements. Figure 2.4 shows a typical master CGH.

A CGH null corrector is a diffractive optics written by e-beam or laser lithography
that yields a nominally flat or null interferogram (Schwider [1998]). This provides an
additional advantage of using CGH. The difficulty of designing and fabricating a CGH
null is largely independent of the detailed shape of the test asphere. CGH nulls do not
become more difficult to produce as more aspheric terms are added, and moreover, they
can be designed to null an off-axis segment rather than its entire parent. Additional
structures can be fabricated on the CGH as an aid in the alignment.

Figure 2.4: Master computer generated hologram.
MacGovern and Wyant [1971] demonstrated perhaps the earliest detailed application of CGHs for optical testing. Several Twyman-Green interferometers have since built to use CGH nulls (Emmel and Leung [1979]).

2.4.3 Combination of Null Optics and CGH

There is a limit to the amount of asphericity, which may be encoded in a hologram because the maximum grating frequencies, which can be fabricated, are limited. To test very steep aspherics - either complicated reflective / refractive null lenses or high frequency CGHs are required. Such high frequency CGHs are difficult to produce and are expensive. It is possible to replace a high frequency CGH to test deep aspherics with a combination of a relatively low frequency CGH and a relatively simple null optics. This method reduces the complexity of fabricating the compensation system and it will be cost-effective also. Especially for high numerical aperture aspherics, the combination of a spherical condenser with a CGH is necessary for reducing the grating frequency of the CGH.

Wyant and O’Neill [1974] recognized the advantages of combining a null optics with a CGH. They showed that the results of a test in which a Matsutov spherical mirror is used to reduce the departure of an aspheric mirror from 910 waves to 45 waves and the slope from 3000 waves per radius to 70 waves per radius. A hologram was then used to remove the remaining asphericity. They demonstrated that an expensive null optics can be replaced by a combination of relatively inexpensive null optics and a CGH.

2.4.4 Different Architectures with CGH as Null Element

There are many positions where the CGH can be placed in the interferometer set-up. Arnold [1989, 1990] has reviewed the use of CGHs in aspheric testing. Merits and limitations of various Twyman-Green and Fizeau interferometer configurations have been discussed and some guidelines for designing and specifying a CGH are presented.

Figure 2.5 shows a configuration, in which the CGH is located in the viewing arm of the Twyman-Green interferometer. The advantage of this configuration is that both test and the reference beams pass through the CGH which itself is imaged onto the final interferogram. This offers few advantages. First, the CGH substrate is common to both
test and the reference paths so its wave front aberrations do not have significant effect to the test accuracy. Second, the CGH is used in single pass so its diffraction efficiency need not be high. However, these types of CGH null test are not autostigmatic-the test wave front does not retrace its path after reflection. Consequently, the CGH null must be designed by taking all the errors of interferometer optics into consideration which requires ray tracing of CGH, test optics, and interferometer optics.

The use of CGH in the viewing arm can be regarded as a Moire technique. A conventional interferogram is created at the CGH plane. The CGH is a simulation of the expected interferogram. When the CGH is properly aligned, a Moire pattern may be observed representing the difference between the actual interferogram and the CGH. If spatial filtering is used the first order diffracted reference wave and zero order of the object wave can be superposed to give directly the deviations of the aspheric from the ideal surface. The first order of the object wave is off-axis and can be filtered out.

Figure 2.5: Twyman-Green interferometer with CGH in viewing arm.

Figure 2.6 depicts a configuration where null lens is positioned in the test arm of a Twyman-Green interferometer. The architecture with the CGH in the object arm just before the specimen, compensating the aspheric wave front, provides robust
interferometer architecture. Additionally, the DOE can be designed independently of the setup since in that case the entire interferometric optics needs not to be included. It can be used with commercial interferometer without any need for modifying the interferometer. However, in this configuration the quality of the DOE substrate plays a critical role and requires high substrate quality. Therefore, it is imperative either to know the DOE deviations beforehand so that they can be subtracted from the final results or to apply suitable calibration procedures. Another disadvantage is that because of the double-pass mode a high diffraction efficiency of the CGH is required.

Figure 2.6: Twyman-Green interferometer with CGH in test arm.

2.4.5 Production of CGH

A CGH null is first designed as a component of a test configuration using ray tracing. It models a CGH as a diffractive surface having a polynomial phase function. The production of CGH is limited in spatial resolution and space-bandwidth product provided by optical recording devices. There are two approaches mainly used in today’s technology for the development of CGHs, one is e-beam lithography (Leung [1980]) and the other one is laser lithography (Dresel and Schwider [1996]). E-beam lithography
produces CGHs of high spatial resolution and large space-bandwidth product. Laser lithography system is much faster with a resolution up to 1 μm.

The first step is the exposure of the structure into photoresist. In many cases the photoresist is deposited on top of a thin chromium layer. After development process the resist structure is transferred into the substrate by means of reactive ion etching. Finally, the remaining resist and chromium are removed so that a transparent binary phase only hologram is achieved. A suitable substrate material is fused silica which enables also the production of sub-μm structures during the etching process.

There are two types of laser lithographic systems depending on the geometry of the machine: \((r,\phi)\)-system and \((x,y)\)-system. In \((r,\phi)\)-system, the substrate is positioned on a rotating table being equipped with an angle encoder. The laser is focused on to the substrate. By moving the laser beam across the frame of the rotating table starting at the axis of rotation for a zero position, any \(r\) and \(\phi\) position can be reached. In \((x,y)\)-system, the substrate is positioned on an \(x,y\)-stage with laser interferometric length measuring units for both coordinates. The laser beam is switched on/off by an acoustic-optic modulator being synchronized to the laser interferometer units of the stage. The details of one such system is given in Appendix A. This system has been used to develop the null elements used in the research work of this thesis.
Chapter 3

Frequency Analysis of Nano-scale Roughness Formation on Single Point Diamond Turned Optical Surfaces

3.1 Introduction

Surface errors can be classified into two classes: localized errors and continuous errors. Continuous errors are global errors of surface topographic features and comprise roughness, waviness and form errors. Surface roughness significantly affects the quality of optical systems, as it causes scattering and stray light, degrading the contrast and sharpness of the optical images (Vorburger [2002]). In general, the roughness includes the tool feed marks found in turning and grinding process and irregularities within the process produced by micro fracture, build-up edge on the tool etc. The smoother the surface the lesser will be the scattering, resulting in enhanced functioning of the optical component, whether in reflection or transmission mode.

With surface quality specifications for diamond turned optics reaching nanometric levels, the measurement of roughness of an optical surface is of great significance during as well as after the fabrication. Before quantifying roughness, one should look into the measuring technique as well as the parameters considered in the metrology. Roughness is generally characterized by two types of parameters: vertical and horizontal. Among vertical parameters are the average height variations from the mean height and its root-mean-square equivalent. Among horizontal parameters is the power spectral density, which characterizes roughness in terms of its spatial wavelength components.

The next Section provides an overview of the parameters to evaluate surface roughness and limitations of height dependent parameters. Section 3.3 presents how the
surface roughness is generated in the turning operation of the fabrication. Sections 3.4 and 3.5 discuss power spectral density and autocorrelation evaluation respectively for the complete representation of the surface roughness. Section 3.6 contains the experimental results of single point diamond turned surfaces.

3.2 **Statistical Parameters for Roughness Evaluation**

If the surface is measured with an instrument that produces a profile such as by mechanical profiler, several different statistical methods can be used to describe the surface. Historically, the most common method of surface measurement has been to mechanically measure the variation of height of the surface along a line across the surface (Whitehouse [2003]).

3.2.1 **Arithmetic Average Roughness ($R_a$)**

The arithmetic average roughness ($R_a$) is the best known and most widely used amplitude parameter to evaluate the surface roughness. It is the average arithmetic departure of the profile from the mean reference line over the evaluation length ‘$L$’, defined as

$$R_a = \frac{1}{L} \int_0^L [z(x)] dx$$  \hspace{1cm} (3.1)

When evaluated from the digital data, eq. 3.1 becomes

$$R_a = \frac{1}{N} \sum_{i=1}^{N} |z_i(x)|$$  \hspace{1cm} (3.2)

where $N$ is the number of data points and $z(x)$ is the height as a function of distance ‘$x$’ above and below the mean line.

3.2.2 **Root Mean Square Roughness ($R_q$)**

The root mean square (rms) roughness ($R_q$) is more statistical in nature and is one of the important parameters calculated from the surface profile as given below:

$$R_q = \sqrt{\frac{1}{L} \int_0^L z^2(x) dx}$$  \hspace{1cm} (3.3)

The digital equivalent is
3.2 Statistical Parameters for Roughness Evaluation

\[ R_q = \left[ \frac{1}{N} \sum_{i=1}^{N} \left[ z_i(x) \right]^2 \right]^{1/2} \]  

(3.4)

\( R_q \) is useful in optical applications where it is more directly related to the optical quality of a surface. The value of the rms roughness depends on the bandwidth of the surface spatial wavelengths that the instrument can sense. It means that two important parameters should be specified for the rms roughness: the lowest spatial wavelength (lateral resolution) and the longest spatial wavelength (sampling length). It is important to know the values of these two limiting factors for any measuring condition and roughness analysis.

3.2.3 Peak-to-Valley Roughness (\( R_t \))

Peak-to-valley (\( R_t \)) is the value of the difference of highest peak to the lowest valley in the length of the profile. This parameter is useful in finding unusual conditions such as a sharp spike or burr on the surface, or a crack or scratch that might be indicative of inhomogeneities of the material.

3.2.4 Limitations of Height Dependent Parameters

Each of these parameters represents the roughness by a single number. A single characteristic analysis of a surface may be sufficient for some applications. However, for accurate analysis of a surface, a much more specific description that measures the spatial interaction is required. The profile descriptions discussed for the rms roughness ignore any information about spatial wavelengths. The limitation with either of these roughness parameters is that they look at only the heights and depths of the peaks and valleys, and not at their spatial distribution, i.e. only amplitude parameters are delivered, resulting in an incomplete representation of roughness features of the surface (Coursey [2001]). Figure 3.1 shows a situation where two very different surfaces are represented by the same value of \( R_a, R_q \) and \( R_t \).

![Figure 3.1: Two very different surfaces having the same value of \( R_a, R_q, R_t \).](image)
To obtain an accurate representation of the roughness with a random distribution of peaks and valleys, it would be worthwhile to see how the roughness ‘power’ is distributed among the different surface frequencies, i.e., by analysing the spectral distribution (Fig. 3.2). To correct the anomaly of an incomplete presentation of roughness by a single parameter (e.g. rms roughness), the power spectral density (PSD), a comprehensive roughness distribution parameter, has been introduced. The detailed calculation procedure for PSD is given in Section 3.4.

Figure 3.2: Distribution of power of frequencies by power spectral density analysis.

3.3 Surface Roughness Generation during Single Point Diamond Turning

A diamond turned surface is produced by moving a cutting tool across the surface of the component, as illustrated in Fig. 3.3. A detailed description of the single point diamond turning process is given in section 1.2. Surface roughness of the turned component is a culmination of large number of periodic components of different frequencies, which are correlated to the process and machine parameters. The tool feed rate, relative vibration between the cutting tool and the workpiece, and material induced vibrations are the prominent effects generating the surface roughness.

3.3.1 Theoretical Surface Finish during Turning

The resultant roughness produced during the turning operation is due to the combination effect of two independent quantities such as ideal roughness and natural roughness. Ideal roughness is due to the tool geometry and its feed. It is a geometrical phenomenon and is the minimum possible magnitude of the unevenness, which results from a machining operation.
Movement of a cutting tool across the surface of the turning component produces the diamond turned surface; therefore it always has some periodic surface roughness (Rhorer and Evans [1995]). The surface texture is directly related to the combination of the tool shape and radius, and the tool’s path over the surface as depicted in Fig. 3.4, where ‘f’ is the feed rate, ‘R’ the tool nose radius and ‘h’ is the unevenness or surface finish. The theoretical surface finish is mathematically represented by the equation given as:

\[
\text{Maximum Height of Unevenness} = \frac{(\text{feed/revolution})^2}{8 \times (\text{tool nose radius})}
\]  

(3.5)

Figure.3.3: Diamond turning of an optical element.

In addition to the theoretical finish based on the ‘cusp’ structure (Fig. 3.4), the measured surface finish on diamond turned parts is influenced by other factors such as: slides straightness error, spindle rotation error, external and self-induced vibrations, material impurities and roughness of the edge of the tool.

Natural roughness in an actual turning operation is the result of the formation of a built up edge and vibration, which adversely affect the surface finish. When the cutting conditions are properly chosen, the vibration may be avoided. Since the built up edge formation depends whether coolant is being used or dry machining is performed and the proper cutting speed is used or not, it is expected that natural roughness to vary with actual cutting speeds.
Cutting speed can be defined as the speed by which the tool moves over the length of the job per unit time and can be represented as:

\[
\text{Cutting Speed} = \pi \cdot D \cdot N
\]

where \( D \) is the diameter of the workpiece and \( N \) is the spindle’s rotational speed (in revolutions per unit time).

In the fabrication process, as the cutting tool is approaching nearer to the centre, the cutting speed becomes lower as the diameter is approaching zero. It is expected that for a given set of cutting conditions, the natural roughness will vary with the cutting speed. It is also expected that except for low cutting speed, the intensity of built up edge formation decreases with the cutting speed, so the maximum height of surface unevenness is also expected to decrease with the cutting speed.

### 3.3.2 Effects of Tool Feed Rate

The spectrum of the roughness profile depends on tool feed rate and its geometry. The feed components appear at spatial frequencies of \( v(\text{feed}) = (n+1)/\text{feed rate} \), where \( n \) denotes the number of harmonics. The fundamental feed components (\( n=0 \)) appear at frequencies of \( v(\text{feed}) = 1/\text{feed rate} \). Since an ideal surface produced by the tool nose is
3.3 Surface Roughness Generation during….  35

periodic but not sinusoidal, its FFT spectrum would have power spectral density at the harmonics of feed frequency. But the fundamental frequency of the feed component is predominant and is used to characterize the effects of tool feed rate and tool geometry in the FFT spectrum analysis.

3.3.3 Effects of Relative Vibration between the Tool and the Workpiece

The small amplitude and low frequency vibration exists relatively between the tool and the workpiece. These vibrations cause waviness not only in the cutting direction but also in the tool feed direction. This effect has been termed as tool-workpiece vibrations (Kim et al. [2002], Takasu et al. [1985]). It causes the sinusoidal variation in height of the spiral groove of the tool path. Fig. 3.5 shows the scheme of undulations formed on the surface machined in the presence of vibration.

If the vibration between the tool and workpiece is having the frequency $f_0$ with amplitude $A$, then the height variation along the radial direction will be

$$z(r) = A\left(1 - \cos\left(2\pi \frac{f_0 r}{n s}\right)\right)$$

(3.7)

where ‘$n$’, ‘$s$’ and ‘$r$’ are the spindle rotational speed in rpm, tool feed rate and radius respectively.

Figure 3.5: Undulations formed by relative vibration between tool and workpiece.
The ratio between the frequency \( f_0 \) of the vibration to the spindle speed \( n \) of the workpiece is

\[
\frac{f_0}{n} = a + b
\]  

(3.8)

where ‘\( a \)’ is an integer and ‘\( b \)’ is a decimal fraction in the range of \(-0.5\) to \(0.5\). The phase shift ‘\( \phi \)’ of the vibration to the one revolution of the workpiece is defined as

\[
\phi = 2\pi b
\]  

(3.9)

The wavelength ‘\( \lambda \)’ of the undulation formed on the surface in the tool feed direction can be written in terms of ‘\( \phi \)’ and feed rate’s’ as follows:

\[
\lambda = \frac{2\pi s}{\phi}
\]  

(3.10)

When the surface is seen in macroscopic scale, several ‘flutes’ are observed of radial wavelength much larger than the given feed rate. The flutes are a 3-D result of surface modulation induced by the tool vibration in combination with the spindle rotational speed.

### 3.3.4 Material Induced Vibrations

Material induced vibrations are caused due to the reaction of the material to the thrust force applied to cut the material. It causes the surface undulations of the frequency higher than the tool-workpiece vibrations. These variations are caused because of variation in the material properties and the ductility of the material. As the chosen material for this study is polycrystalline (aluminium alloy 6061), the variation of surface roughness is independent of orientation angle.

### 3.4 Power Spectral Density (PSD)

The surface roughness profile of a machined surface provides information not only about the cutting process but also about the tool-workpiece vibrations, material induced effects and other machining effects (Cheung [2000], [2001]). The usual roughness parameters, e.g., \( R_q \), \( R_a \) or \( R_t \) are inadequate to provide this information. Hence, for the comprehensive roughness analysis of an optical quality surface, PSD has been preferred (Marx [2002]). PSD represents the spatial frequency spectrum of the surface roughness measured in
3.4 Power Spectral Density

inverse-length units. It can be calculated from the surface profiles made by an optical or mechanical profiler (Elson and Bennett [1995]).

The power spectrum of a surface roughness profile is composed of several periodical components, which can be correlated to different process parameters and mechanisms of surface generation. The lower spatial frequencies represent the waviness, whereas higher spatial frequencies represent the actual roughness. The PSD is closely related to the amount and distribution of light scattered by a rough surface. One of the most interesting properties of PSD between two designated spatial frequency limits is, that it is equal to the square of the rms roughness of the original profile, when the profile is measured or filtered to have same spatial frequency limits.

3.4.1 Calculation of PSD

The one-dimensional PSD can be computed from discrete data obtained from the mechanical profilometer. If \( z(x) \) is the surface roughness height as a function of distance ‘\( x \)’, a finite-length Fourier transform may be written as:

\[
Z(k) = \frac{1}{L} \int_{0}^{L} z(x) \exp(-ikx) \, dx
\]  

(3.11)

where \( i = \sqrt{-1} \) and \( k \) is the wave number. In practice, one can measure only a finite number of values of \( z(x) \). The surface roughness data set consist of \( N \) values of \( z(x) \), which are measured at equally spaced intervals \( \Delta x \) over the total sample length \( L = N \Delta x \).

The discrete equivalent of eq. 3.11 is:

\[
Z_{j} = \frac{1}{N} \sum_{m=1}^{N} z_{m} \exp \left( - \frac{2\pi i(m-1)j}{N} \right) \]  

(3.12)

where \( z_{m} = z ((m-1) \Delta x) \) and \( Z_{j} \) is a function of the spatial frequency ‘\( f \)’, where \( f = 2\pi / k \).

The Fast Fourier transform (FFT) routine of MATLAB has been used to calculate the discrete Fourier transform (DFT) of the roughness data. The one-dimensional PSD is
the complex conjugate square of the DFT of the linear surface profile divided by the total number of discrete points, \( N \).

\[
PSD = \left( \text{Normalisation factor} \right) \frac{Z(f)^* \text{conj}(Z(f))}{N}
\]  

(3.13)

### 3.4.2 Parseval's Theorem

The frequency range of a computed PSD starts at the lower end with \( 1/L \) and goes up to \( 1/2\Delta x \). A suitable multiplicative normalization factor has been used to compute PSD. To find out this normalization factor, the PSD over the Nyquist bandwidth limits of surface spatial frequency has been integrated to obtain the area under the PSD curve. According to the Parseval’s theorem, the area under the PSD curve should be equal to the rms roughness:

\[
R_q^2 = \int_{f_{\text{min}}}^{f_{\text{max}}} PSD(f) df
\]  

(3.14)

This expression provides the correct Normalization factor, \( 1/2\Delta x \), which is used to compute PSD in eq. 3.13.

### 3.4.3 Average PSD

The PSD calculated from a single profile is exceedingly noisy and non-reproducible. To obtain a true PSD of a surface that has random roughness, the average of PSD estimates has to be calculated from the profiles made at different regions on a surface. A long profile can be subdivided into \( j \) (1, 2, … n) small profiles. The average PSD estimate is given by

\[
\text{Average PSD} = \frac{1}{N} \sum_{j=1}^{N} PSD_j
\]  

(3.15)

### 3.5 Autocorrelation Analysis

The autocorrelation is a mathematical tool used frequently in signal processing for analyzing functions or series of values, such as time domain signals. Informally, it is a measure of how well the signal matches a time-shifted or space-shifted version of itself, as a function of the amount of time shift or space shift.
The autocorrelation is useful for finding repeated patterns in a signal, such as determining the presence of a periodic signal which has been buried under noise, or identifying the missing fundamental frequency in a signal implied by its harmonic frequencies. If the surface is random, the plot drops rapidly to zero. If the plot oscillates around zero in a periodic manner, then the surface has a dominant spatial frequency. The correlation length is the length along the x-axis where the autocorrelation function first crosses zero (Fig. 3.6).

![Surface Profile](image)

**Figure 3.6: Correlation length of a periodic signal.**

### 3.6 Experimental Results

With an attempt to establish a relationship between the process of surface roughness generation and the manufacturing parameters, the workpiece is diamond turned with varying machine parameters and the frequency spectrum of the machined surface data is performed.

#### 3.6.1 Machining Details and Analysis

In this study, 60 mm diameter flat aluminium-6061 workpieces are machined on Nanoform-250, a precision diamond turning machine from Taylor-Hobson. Aluminium-6061 is chosen for this study, as aluminium alloys are extensively used in precision optical components for various applications. The material used is assumed to be isotropic. A single-crystal natural diamond tool (Fig. 3.7) is used having a 0.5 mm nose radius, 100° included angle, 0° rake angle with a designated waviness of 0.40 μm. The tool feed rate is varied between 0.3 to 30.0 μm/rev. The depth of cut was varied from 1 μm to 15 μm, and spindle’s rotational speed from 1000 to 4500 rpm. The depth of cut of 2 μm and spindle’s rotational speed 3000 is found to be optimum for 60 mm diameter aluminium workpiece to get the desired surface quality. A standard Form Talysurf Series 2 (PGI)
Profilometer from Taylor-Hobson and an interferometric optical surface profiler from Veeco have been used for the surface evaluation.

Figure 3.7: Diamond tool of 0.5 mm nose radius used for machining.

A spectrum analysis for varying tool feed rates has been conducted by computing the PSD distribution from the data extracted from the Form Talysurf profilometer. Throughout the analysis, the lowest spatial wavelength (lateral resolution) is chosen to be 0.0025mm and the longest spatial wavelength (sampling length) is selected to be 0.8mm. The power spectrum of the roughness profile is determined by the Fast Fourier Transformation (FFT) algorithm as explained earlier.

3.6.2 Results and Discussion

A single profile analysis delivered a noisy and non-repeatable spatial distribution. In order to compute a realistic PSD of a typical surface, the average of PSD estimates has been calculated from the series of small profiles. For this purpose, a profile of 20 mm length has been selected and this is further divided into 20 small profiles with 1000 data points each. This data is then used to calculate the average PSD. The un-averaged and averaged PSD’s at tool feed rate of 6 μm / rev are shown in Fig. 3.8.

Figure 3.8: Power spectral density a) Un-averaged PSD; b) Averaged PSD.
3.6 Experimental Results

3.6.2.1 Power Spectral Density Analysis with Varying Tool Feed Rate

Figures 3.9(a-f) represent the PSD distribution of the roughness for tool feed rates 6, 10, 12, 15, 18 and 25 μm / rev respectively. The feed frequencies appear at 166, 100, 83.3, 66.6, 55.5 and 40 mm⁻¹ respectively. In these figures, \( v(tw) \), \( v(mat) \) and \( v(feed) \) denote frequencies due to tool-workpiece vibrations, material induced vibrations and tool feed rate respectively.

![Power Spectral Density Analysis with Varying Tool Feed Rate](image)

Figure 3.9 (a-f): PSD distribution curves for varying tool feed rates. The unit of spatial frequency is cycles per mm.

The feed component contribution is found to be predominant in terms of amplitude and is used to characterize the effects of feed rate in the FFT spectrum analysis. As the tool feed rate increases the feed components shift to a lower spatial frequency range. It is observed that, at low feed rate (Fig. 3.9a), amplitude components of tool-
workpiece vibrations and material effects dominate over the feed component. At medium feed rates (Fig. 3.9b) the amplitude components of tool-workpiece vibrations and feed are comparable. But at higher feed rates (Figs. 3.9c-3.9f), the amplitude component due to feed dominates over contributions due to tool-workpiece vibrations and material effects.

It is also observed that the contributions of tool-workpiece vibrations and material induced vibrations gradually increase with the feed rate. The frequency corresponding to the relative tool-workpiece vibrations component shifts to the lower frequency range with increased feed rate. For the tool feed rate in the range of 1.5-8.0 μm/rev, the surface finish is mainly determined by low frequencies and the contribution of the feed component (high frequency) is very less. The amplitude of the low frequency component is almost constant within this range.

3.6.2.2 Identification of the Contributions of Different Effects on the Surface Roughness

Additionally, using Parseval’s theorem rms roughness values have been obtained by integrating the PSD curves. The rms roughness from the PSD have been calculated (rms-PSD) and compared with the rms roughness values (rms-scan) obtained from the Form Talysurf. The average values (Av.rms-PSD and Av.rms-scan) of the 20 small profiles are also computed. Figure 3.10 represents the comparison of rms-scan and rms-PSD values in un-averaged and averaged modes. The relative contributions of the tool feed rate, tool-workpiece vibrations and material induced vibrations to the rms roughness have also been computed by applying the Parseval’s theorem in their respective frequency bands. Figure 3.11 represents the relative average contributions of the tool feed rate (Av.rms\text{Feed}), material effects (Av.rms\text{Mat}) and of tool-workpiece vibrations (Av.rms\text{Vib}) to the rms roughness.
3.6 Experimental Results

Figure 3.10: Comparison of rms-scan and rms-PSD values in a) un-averaged and b) averaged modes.

Figure 3.11: Average contributions of the tool feed rate, material induced vibrations and tool-workpiece vibrations to the rms roughness
3.6.2.3 **Autocorrelation Study**

The autocorrelation function shows the common components of the surface structure. These surface components are a) the long-range waviness b) the medium-range waviness and c) the short-range periodicity. Figure 3.12 shows the autocorrelation function of the surface profile that is machined with 10 μm /rev tool feed rate. It clearly shows the 10 μm periodicity due to the chosen tool feed rate. In addition to this, a medium range periodicity of 40 μm and a long-range periodicity of 330 μm are also present, caused by material effects and vibrations between the tool and workpiece respectively. The autocorrelation length (ACL) at different tool feed rates is plotted in Fig.3.13. The ACL is found to be longer in the range of 2.0 –6.0 μm/rev feed rate. The higher value of ACL shows that the surface quality is better at this tool feed rate range.

![Autocorrelation Function](image)

**Figure 3.12:** Autocorrelation function (ACF) of the surface roughness at 10 μm/rev.

![Autocorrelation Length vs Tool Feed Rate](image)

**Figure 3.13:** Autocorrelation Length (in microns) vs tool feed rate.
3.6.2.4 Averaging Algorithms for PSD

In order to compute a realistic PSD of a typical surface, the ensemble average of PSD estimates has been calculated from the sections of the profiles. For this purpose, the Bartlett method (Bartlett [1948]) is adopted, where a single profile of having 2000 data points has been sectioned in 7 parts having 256 points. The computations performed indicate that the PSD estimate has become smoother and the noise has been eliminated, as it is clear from Fig. 3.14(b). With the Welch method (Welch [1967]), where these subdivisions are allowed to overlap and at the same time a Hanning window is also used, the PSD has become even cleaner (Fig.3.14c). The peaks related to tool feed rate, material effects and tool work piece vibration have become clearer. Figure 3.14(d) is the PSD estimate by using parametric based Burg algorithm (Burg [1968]). It is observed that this method gives the best-smoothed curve for PSD estimate.

Figure 3.14: PSD distribution vs spatial frequency for (a) conventional PSD, (b) Bartlett, (c) Welch and (d) Burg Averaged PSD.
3.6.2.5 *Effects of the Spindle Rotational Speed on Surface Quality*

Figure 3.15 shows the surface topography of the specimen at different spindle rotational speeds of the workpiece. A phase shifting interferometric profiler with 15.5 X magnification has been used. The field of view is 0.3x0.4 mm$^2$. The central part was viewed to understand the formation of undulations on the surface due to the relative vibration between the tool and the workpiece. The central pip defect appears because of the tool offset error.

Figure 3.16 shows a macroscopic view of the surface undulations occurred at spindle rotational speed of 4500. The numbers of flutes are observed of the radial wavelength much larger than feed rate. These are supposed to fulfill the eqs. 3.7 and 3.10. At present the frequency of relative tool-workpiece vibration at corresponding spindle rotational speeds are not known, and the results depicted here could not be verified with eqs 3.7 and 3.10. Therefore, it is left for future investigations, how the relative tool-workpiece vibration frequencies for this particular machine are affecting the surface topography.

![Figure 3.15: Surface topography of the specimen machined at different spindle rotational speeds. The field of view is 0.3x0.4 mm$^2$](image-url)
3.6 Experimental Results

Figure 3.16: *Observed flutes in macroscopic view of the surface undulations at 4500 spindle rotational speed.*

3.7 Conclusions

In this study, the optical surface roughness has been presented in terms of PSD rather than by the conventional single parameter such as rms roughness. The following conclusions can be drawn from this study:

1. The surface roughness is composed of several periodical components, which are: tool feed component, tool-workpiece vibrations and material induced vibrations.
2. The amplitude of the tool feed component dominates at higher feed rates, while at low feed rates the tool-workpiece vibrations dominate. At very low feed rate the feed component is almost negligible and the tool-workpiece vibrations component determines the surface roughness.
3. Surface roughness is found to be small and constant in the feed range of 1.5 to 8.0 μm/rev. This is the optimum range of tool feed rate where the surface quality is acceptable for the workpiece under investigation.
4. The frequency corresponding to the relative tool-workpiece vibrations component shifts to the lower frequency range with the increased feed rates.
5. A single PSD profile gives noisy and non-reproducible frequency estimates. Hence PSD estimates should be averaged before drawing any conclusion.
6. PSD analysis provides all the amplitude and frequency details of the surface and helps in establishing the relationship between manufacturing process and the surface roughness generation.
7. It is observed that the radial wavelengths of the surface undulations are different for different spindle rotational speeds. It gives an indication that the frequency of relative tool-workpiece vibration is dependent on spindle rotational speeds.
Chapter 4

Quasi Absolute Testing of Rotationally Invariant Aspherics: Simulation Studies

Interferometry is based on the principle of comparison of the component under test with a reference. The accuracy of a derived wave front is only as good as the reference surface. The reference is generally a flat or spherical mirror but perfect reference optics does not exist. The resulting errors of a single measurement are a combination of alignment errors of the test surface, systematic errors of the interferometer (including the errors of the reference surface) and actual test surface errors. The small misalignment errors can be eliminated by software via fitting of misalignment functions to the measured data. However, an absolute test is necessary to separate the systematic interferometer errors from the surface errors of the test sample.

In this chapter, first the absolute test for spheres will be recalled and thereafter the measurement principle of the quasi absolute three position test procedure will be described. The scheme for calculating the phase functions for the combined diffractive optical elements will be explained. In order to optimize the performance of the combined diffractive optical elements, simulation studies on their design criteria will finally be presented in detail in Section 4.3.

4.1 Absolute Test

Absolute testing eliminates errors of the reference and other systematic deviations. For spherical and flat surfaces there are absolute tests available which require different rotated or displaced measurements. For flats, three flat surfaces can be tested in many combinations in order to obtain the absolute departure of the three surfaces with respect to an ideal flat (Schulz G and Schwider J [1976]).
4.1.1 Three Position Test for Spheres

A three position test has been developed to measure the spherical surfaces in an absolute sense (Jenson [1973], Bruning [1974]). Figure 4.1 shows the three measurements required for the absolute measurement. The three positions are a normal position \((W_0^\circ)\) of the test surface, a 180° rotated position \((W_{180^\circ})\) of the test surface and the cat’s-eye position \((W_{\text{focus}})\) with interferometer focused on the vertex of the sphere. If \(W_{\text{ref}}\) is the error due to the reference arm of the interferometer, \(W_{\text{obj}}\) is the error due to the object arm, and \(W_{\text{surf}}\) is the error due to the spherical surface under test, the three recorded interferograms would be:

\[
W_0^\circ(x, y) = W_{\text{ref}}(x, y) + W_{\text{obj}}(x, y) + W_{\text{surf}}(x, y) \tag{4.1}
\]

\[
W_{180^\circ}(x, y) = W_{\text{ref}}(x, y) + W_{\text{obj}}(x, y) + W_{\text{surf}}(-x, -y) \tag{4.2}
\]

\[
W_{\text{focus}}(x, y) = W_{\text{ref}}(x, y) + \frac{1}{2}[W_{\text{obj}}(x, y) + W_{\text{obj}}(-x, -y)] \tag{4.3}
\]

By suitably combining these three measurement results the absolute surface deviations can be derived:

\[
W_{\text{surf}} = \frac{1}{2}[W_0^\circ(x, y) + W_{180^\circ}(-x, -y) - W_{\text{focus}}(x, y) - W_{\text{focus}}(-x, -y)] \tag{4.4}
\]

Figure 4.1: Absolute measurement of spherical surface: (a) Basic position \((0^\circ)\), (b) Rotated position \((180^\circ)\) and (c) cat’s-eye position.

The success of the three position test depends on alignment accuracy of the surface. The errors occurred due to the misalignment of the surface in different measurements have been examined by Traux [1988] and Elssner [1989].
4.1 Absolute Test

4.1.2 Quasi Absolute Test of Aspherics: Measurement Principle

A procedure, analogous to the three position test for spheres explained above, is applied here for the absolute measurement of aspherics. For the measurement of the aspheric surface, an aspheric wave has to be used where the rays impinge perpendicularly onto the surface. An aspheric wave has no sharp focus (excluding the special case that the aspheric is a sphere) and therefore the cat’s-eye measurement cannot be made. However, a combined diffractive optical element (combo-DOE) can be deployed as a beam shaper. The combo-DOE carries the information for the aspheric as well as for the best fit sphere with a slight linear offset, where the spherical wave front is used for the cat’s-eye measurement. The linear offset is introduced to be able to separate the two wave fronts. This is further explained in next chapter, section 5.1. The basic idea behind this procedure is that the structural errors due to the limited accuracy of the lithographic tools are nearly identical for the aspherical and spherical structure of the DOE if both waves have similar curvature. In addition, it can be assumed that the variations in the optical thickness of the DOE substrate impair both wave fronts in the same manner.

![Image of measurement principle](image)

Figure 4.2: Measurement principle: Three positions for the quasi absolute measurement of aspheric surfaces.

Figure 4.2 depicts the required three positions for the absolute measurement of a rotationally invariant aspheric. The measured phase in the first two positions contains the systematic errors and errors of the surface under test. In the measurement of the cat’s-eye position, a mirror is placed at the focal point to invert the wave front. If the aberrations from the reference arm, object arm and surface errors under test are denoted by \( W_{\text{ref}} \), \( W_{\text{obj}} \), and \( W_{\text{Asph}} \) respectively, the first two measurements are:
\[ W_{\theta'}(x, y) = W_{\text{ref}}(x, y) + W_{\text{obj}}(x, y) + W_{\text{Asph}}(x, y) \] (4.5)

\[ W_{180'}(x, y) = W_{\text{ref}}(x, y) + W_{\text{obj}}(x, y) + W_{\text{Asph}}(-x,-y) \] (4.6)

where \( W_{\text{obj}}(x, y) \) contains the lithographic errors for aspheric wave front including the DOE substrate errors and also the systematic errors of the object arm.

The cat’s-eye position is measured by using the spherical wave front of the combo-DOE.

\[ W_{\text{focus}}(x, y) = W_{\text{ref}}(x, y) + \frac{1}{2} \left[ W'_{\text{obj}}(x, y) + W'_{\text{obj}}(-x,-y) \right] \] (4.7)

where, \( W'_{\text{obj}}(x, y) \) contains the lithographic errors for spherical wave front including the DOE substrate errors and also the systematic errors of the object arm.

Under the assumption, that the lithographic errors for the aspheric and spherical wave fronts are identical within the allowed tolerances, \( W'_{\text{obj}}(x, y) \) becomes approximately equal to \( W_{\text{obj}}(x, y) \).

\[ W'_{\text{obj}}(x, y) \approx W_{\text{obj}}(x, y) \] (4.8)

Under the above approximation the three measurements yield the absolute deviations of the aspheric under test:

\[ W_{\text{Asph}} = \frac{1}{2} \left[ W_{\theta'}(x, y) + W_{180'}(-x,-y) - W_{\text{focus}}(x, y) - W_{\text{focus}}(-x,-y) \right] \] (4.9)

Nevertheless, the small differences between \( W'_{\text{obj}}(x, y) \) and \( W_{\text{obj}}(x, y) \) cause that this test is only “quasi absolute”.

### 4.2 Combined Diffractive Optical Elements

First the general procedure for designing the combo-DOE for testing aspheric surfaces is developed. Then the simulation studies for optimisation of design parameters of the combo-DOE are presented in Section 4.3. As already mentioned the combo-DOE generates two wave fronts, aspheric and a best fit spherical. Therefore, the DOE must
have two phase functions encoded simultaneously. These phase functions are calculated separately and then encoded on the DOE substrate (Lindlein [2004]). The incoming wave front with phase $\phi_{in}$ should transform to the outgoing wave front with phase $\phi_{out}$, so that it hits the aspheric everywhere perpendicularly. It can be expressed mathematically as

$$\phi_{out} = \phi_{in} + m\phi$$  \hspace{1cm} (4.10)

where ‘$\phi$’ is the phase function of the DOE and ‘$m$’ is the diffraction order. The task is to calculate the phase function of the DOE to produce the desired aspheric wave front.

### 4.2.1 Calculation of the Aspheric Phase Function

Following are the steps for calculation of the aspheric phase function:

1) A grid of N rays perpendicularly emitting from the aspheric surface are traced to the plane of the DOE (Fig. 4.3) and the optical path lengths OPD from the aspheric to the DOE are calculated. The combo-DOE is tilted by a small angle to avoid on-axis reflections from its front and back surface. The phase is calculated at this tilted plane. Since due to symmetry considerations the phase of all rays has to be the same on the aspheric surface the desired phase $\phi_{out}$ on the tilted plane is directly proportional to the calculated optical path lengths. However, this phase is $\phi_{out} = \frac{2\pi}{\lambda}(OPD)$, because rays take the opposite direction.

2) The phase of the incident wave front $\phi_{in}$ is calculated in the plane of the DOE. The incident wave front is a plane wave front in this case. Therefore, $\phi_{in}$ is just a linear function due to the tilt (without tilt it would be a constant function).

3) Using eq. 4.10 the phase function $\phi$ of the DOE is calculated for the diffraction order $m=1$.

4) Due to the discrete sampling with rays the phase is computed only at some points of the DOE. But, it has to be known at all points of the DOE. To achieve this we fit a polynomial to the phase values using a least squares fit.
Figure 4.3: A grid of N rays perpendicularly emitting from the aspheric surface are traced to the plane of the DOE. The parallel rays coming from the right side are just auxiliary rays due to the software implementation and determine the starting points on the aspheric surface where the rays are now perpendicular to the aspheric and have all phase zero.

4.2.2 Calculation of the Spherical Phase Function

The N rays diverging from an off-axis point source are traced to the plane of the DOE (Fig. 4.4). The distance between the point source and the DOE is chosen such that the diverging wave front represents a spherical wave front best fitted to the aspheric. The rest of the procedure is the same as for the aspheric wave front. The linear offset is introduced to be able to separate the spherical wave front from the on-axis aspheric wave front.

Figure 4.4: The N rays diverging from an off-axis point source are traced to the plane of the DOE.
4.2.3 **Encoding the Two Phase Functions**

There are two possibilities for encoding these two phase functions, aspheric and spherical, simultaneously on the DOE substrate (Beyerlein[2002]). The aperture can be sliced in stripes (sliced-DOE), which are alternatively assigned to the spherical and aspheric waves. The width of the stripes has to be so small that they cannot be resolved on the final detector. The other method is to add the complex amplitude of the hologram functions of both the waves (superposed-DOE). Figure 4.5 shows the structure of the combo-DOE in both the cases. Such a combo-DOE will produce the desired aspheric and spherical wave fronts. However, there are additional diffraction orders in both the cases.

![Figure 4.5: Structure of the combo-DOE (a) Sliced (b) Superposed.](image)

The correct tilt alignment of the DOE is important and to achieve it, a reflection grating structure is provided outside the test structure of the DOE [Fig. 4.6]. The -1\textsuperscript{st} order of the reflected beam of this alignment grating has to be adjusted to fluffed-out fringes in order to have the correct tilt of the DOE.

![Figure 4.6: Alignment structure in the combo-DOE.](image)
4.3 **Design Considerations for the Combo-DOE: Simulation Studies**

4.3.1 **Sources of Error**

There can be many sources of errors such as (i) a DOE substrate error, (ii) a systematic error of the empty interferometer without DOE, (iii) a DOE misalignment error, (iv) insufficient stability of the interferometer (air turbulence, vibration, drift etc), and (v) lithographic writing errors. With the aim to design an optimized combo-DOE and, furthermore, to have an estimate of the achievable accuracy of the procedure, we performed simulation studies in the presence of the systematic errors (i) – (iii) by using our software package ‘RAYTRACE’ (Lindlein [1998]).

To be very close to the real experimental situation, the measured systematic errors of the empty interferometer and the substrate deviations have been used as input in the simulations. Figure 4.7 displays typical aberrations in a double pass configuration. A noisy environment and the lithographic writing errors have not been considered in the simulations. For the aspheric under test, the paraxial radius of curvature was 94.84 mm [see Fig. 5.5]. An additional small astigmatism has been added to the specimen to represent the surface deviations. The main reason for performing the following simulations is that a DOE being a flat element violates the sine-condition. So, in situations where the DOE is misaligned or the incoming wave front is not ideal, aberrations (mainly coma) will be present.

![PV: 0.241 λ, Line spacing: λ/30]

![PV: 0.435 λ, Line spacing: λ/30](a) (b)

Figure 4.7: **Typical aberrations:** (a) Systematic error of the empty interferometer in double pass configuration (b) DOE substrate error in double pass configuration. The contour line spacing is λ/30. The wavelength ‘λ’ is 633 nm.
4.3 Design Considerations for the Combo-DOE

4.3.2 Consistency Test

When there is no systematic error present in the system, the surface deviations are perfectly reconstructed by the simulated three position absolute test. However, in the presence of systematic errors of the empty interferometer and the DOE substrate in particular, some remaining error is observed. Since a single absolute measurement does not give a direct clue on the accuracy of the procedure, we applied a consistency test to obtain an indication on the achievable accuracy.

A consistency test is simulated in the following way: (1) two absolute results are simulated, one by having the surface measurements at 0° and 180° as the basic and rotated positions, and the second one at 90° and 270°. The cat’s-eye position is the same in both the cases. (2) rotate the second absolute result by -90° to ensure the same orientation of the coordinate system for the absolute deviations, and then (3) take the difference between them. The three most critical design parameters that influence the accuracy of the procedure are presented.

4.3.3 Relation between the Tilt Orientation of the DOE and the Offset Direction of the Spherical Wave Front

As already mentioned the spherical wave front has a small tilt angle which has initially been chosen as 1°. There can be four choices for the offset direction of the spherical wave front as shown in Fig. 4.8. The optical axis is taken along the Z-direction and the combo-DOE is tilted relative to the XY plane, where the Y-axis shall be the rotation axis. The offset of the spherical wave can be along the +X, -X, +Y, and –Y directions.

In principle, it is possible to use any of these offset directions irrespective of the tilt orientation of the combo-DOE, provided there are no systematic errors of the setup. But in the presence of a DOE substrate and/or interferometer deviation, which in reality is the case, it becomes necessary to consider the relation between the tilt direction of the combo-DOE and the offset direction. The DOE surface deviations include a typical wedge of 10 arcsec in the direction of the global tilt of the combo-DOE.
Figure 4.8: Four choices of the linear offset for the spherical wave front with respect to the optical axis of the interferometer: (a) +X direction, (b) +Y direction, (c) –X direction and (d) –Y direction. The DOE has been tilted around the y-axis.

Figure 4.9 shows the results of a consistency test for each of the above mentioned four cases using the same substrate error for all offset situations. These results indicate a remaining systematic error of the absolute testing procedure, depending on the relation between tilt and offset direction. It can be observed that the best situation is selected if the offset of the spherical wave front has been chosen in the direction of the surface normal of the DOE.

The amount of the linear offset in the focal plane of the spherical wave front is also chosen corresponding to the tilt angle of the DOE, i.e., the tilt angle of the spherical wave should be the same as the tilt of the DOE in the same direction. In this case, the focus of the spherical wave front lies on a straight line through the center of the DOE having the direction of the normal of the DOE. Since this is the condition where the cat’s-eye measurement follows the symmetry with respect to the DOE plane, it is denoted as the “symmetry condition” for the cat’s-eye position. In all the other situations, a higher
amount of astigmatism is present because of the broken symmetry of the cat’s-eye measurement.

![Diagram]

**Figure 4.9:** Simulation results of the described consistency test in all the four possible cases of the linear offset for the spherical wave front with respect to the optical axis of the interferometer: (a) +X direction, (b) +Y direction, (c) –X direction and (d) –Y direction. The contour line spacing is \( \lambda/300 \).

### 4.3.4 Influence of the Tilt Amount of the DOE in the Presence of a Systematic Error of the Setup

Even in the figure 4.9(a), which obeys the “symmetry condition”, one can still see systematic astigmatism. This may be due to the influence of a systematic error of the setup, in particular a wedge error in the DOE substrate. Since the DOEs are flat elements and do not fulfill the sine-condition, in the presence of a wedge error they introduce coma in a single measurement. The amount of comatic error increases linearly with the increase...
in the global tilt of the DOE. The primary coma gets removed as a misalignment error once the misalignment elimination algorithm is applied, but the higher orders remain. To better understand the effects of the wedge error, we simulated a case where no error has been introduced into the setup, except a wedge of 0.002°. Even the specimen has been chosen to be an ideal surface.

Figure 4.10 shows the results of the consistency tests for a varying amount of the DOE tilt from 0.25° to 2°, whereby the symmetry condition discussed in the earlier section has been followed. One can notice a linear increase in the error with the DOE tilt. This situation demands that one should design a DOE with a minimum tilt angle. But the amount of tilt depends on two other parameters. One is that some amount of tilt is required to avoid the back reflections from the DOE surfaces going into the interferometer. In the experimental setup, at least 0.25° tilt angle is needed to avoid them.

The minimum tilt angle can be determined from the space bandwidth product which is dictated by the CCD-camera and the optical system as a whole. The space bandwidth product should not be restricted too much because then only slowly varying aberrations can be corrected. Another parameter is that the offset of the spherical wave front should be large enough to separate it from the on-axis aspheric wave front. This trade-off limits us to select the tilt angle as much as 0.5° with respect to the interferometer axis.

4.3.5 Matching Condition and Quasi Nature of the Procedure

The basic idea behind the combo-DOE test relies on the fact that the wave aberrations caused by lithographic errors are only proportional to the ratio of the positioning error of the lithographic writing machine and the local period of the DOE. But in the close lateral neighborhood two DOE structures show similar aberrations. Therefore, it is possible to use different wave fronts as the difference of the aberrations will be small enough to make the philosophy worthwhile.

The design fulfilling the “symmetry condition” as well as the least possible tilt angle (see Fig. 4.10b) situation presents the least error. Nevertheless, it still shows astigmatism of approx. \( \lambda/160 \) in the consistency test. The reason for this can be that a spherical wave front, for the measurement in the cat’s-eye position, different from the aspheric one introduces the “quasi nature” in the three position test. This quasi nature has
two components: 1) a different calibrating wave front from the aspheric one and 2) an additional offset to separate it from the aspheric wave front. Since these are the basic procedural limitations, we call the whole procedure as “quasi absolute”.

![Simulation results of the consistency test for the varying amount of global tilt of the DOE from 0.25° to 2° with respect to the interferometer while following the symmetry condition discussed in the last Section (see Fig.4.8a) (a) 0.25° tilt, (b) 0.5° tilt (c) 1.0° tilt, and (d) 2.0° tilt. The contour line spacing is λ/1000.

This procedure requires that the two wave fronts should match in the best possible way, which is here termed as “matching condition”. In this Section, the criticality of this condition and efforts to minimize the quasi absolute extent of the procedure are explored. Minimizing the tilt angle of the DOE (see Section 4.3.4) guarantees a minimum offset of the spherical wave front, thus a better matching of both the wave fronts. So, the effects shown in the last section are also related to the quasi nature. The smaller the offset angle
the better the matching condition is fulfilled. Furthermore, the spherical wave front can either be chosen as a base fit or a best fit to the aspheric. In the base fit case just the paraxial radius of curvature of the aspheric wave is taken. In the best fit case, the integral of the squares of the wave front differences to a spherical wave front has to be minimized.

Figure 4.11 compares the consistency results for the two conditions in the presence of general interferometric and substrate errors. The best fit spherical wave front is a better choice, because of the simple argument that the aspheric wave front departure is the least in this case. But the lithographic writing errors become more critical in the high spatial frequency region, where writing periods become finer and finer. So in the fitting algorithm it would be better to have more weight in the high frequency region, which is in turn based on the minimization of the relative difference of the writing periods rather than the departure of the wave front itself. However, this cannot be done unless the exact behavior of the fabrication errors is known.

![PV: 0.0082 \( \lambda \)
Line spacing: \( \lambda/1000 \)](a) ![PV: 0.0069 \( \lambda \)
Line spacing: \( \lambda/1000 \)](b)

Figure 4.11: Consistency results when the calibrating spherical wave front is chosen as (a) base fit to the aspheric, and (b) best fit to the aspheric. The contour line spacing is \( \lambda/1000 \).

4.4 Conclusions

The quasi absolute three position test procedure has been simulated under the presence of different error sources. The design procedure of the combo-DOE has been discussed in full detail. The simulation studies suggest that in the presence of substrate errors, the design of the combo-DOE must follow the following three conditions: a) symmetry...
condition (i.e. chief ray of the spherical wave perpendicular to the tilted substrate plane), b) minimum tilt condition and c) the matching condition (using for example a best fit sphere).
Chapter 5

Quasi Absolute Testing of Rotationally Invariant Aspherics and Toric Surfaces: Experimental Results

This chapter presents the experimental results of quasi absolute three position test for aspherics. In the first Section, the experimental setup and the specifications of the specimen under test will be presented. After discussing the interferometer’s performance in terms of repeatability and reproducibility, the experimental results of the quasi absolute test will be presented. A comparison with an N-position rotational averaging test will also be made. Furthermore, the quasi absolute test procedure will also be applied to toric surface. Limitations, advantages, and possible improvements will be discussed in the last Section.

5.1 Experimental Setup

5.1.1 Twyman-Green Interferometer

The Twyman-Green phase shifting interferometric setup was built up (Fig 5.1) using combo-DOEs as null optics. A linearly polarized He-Ne laser beam (633 nm) passes through a λ/2 wave plate and falls on the polarising beam splitter. Depending on the incident linear polarisation angle, which can be adjusted by the rotation of the λ/2 wave plate, the incident beam splits in two polarized components, hence generating test and reference wave fronts in the respective arms. To obtain the maximum contrast in the interferogram, the light intensity of these two beams is controlled by rotation of the λ/2 wave plate. These two beams pass totally twice through λ/4 wave plates deployed in both arms with their axes oriented by 45° relative to the incident linear polarization. Totally, the double passage of the λ/4 wave plates rotates the respective polarization of object or
reference arm by 90° allowing unattenuated passage through the polarising beam splitter into the viewing arm. Of course, between the two passages of the \( \lambda/4 \) wave plates the light is circularly polarized.

The analyzer situated in the viewing arm selects the common polarization component from the two orthogonally polarized return beams, thus allowing them to interfere. This interferogram is recorded by a CCD camera. The telescopic systems in test and viewing arms are used to expand the beam diameter and for the proper imaging of the DOE plane onto the detector respectively. The phase shifting is introduced in the reference arm by a piezo-driven reference mirror controlled by software.

Figure 5.1: Schematic of Phase-Shifting Twyman-Green Interferometer with combo-DOE as null element.

Figure 5.2 depicts the photo of the interferometer developed in the lab. The tilted combo-DOE provides the designed aspheric and spherical wave fronts simultaneously. Because of the striped nature, the combo-DOE will generate additional diffraction orders that may disturb the measurements. To avoid the direct overlapping of the different orders of both wave fronts, the off-axis direction of the spherical wave front is selected to be
perpendicular to the grating vector of the grating produced by the stripes. Upon reflection, while retracing the path, the disturbing diffraction orders are blocked by employing a stop at the confocal plane of the telescopic system in the object arm.

The combo-DOE is tilted by a small angle to avoid on-axis reflections from its front and back surface to enter back into the system. These reflections are also blocked by the stop in the confocal plane of the telescopic system in the object arm. The distance between the combo-DOE and the aspheric under test should be on the one hand as small as possible to have an unequivocal correspondence between points on the aspheric and on the DOE which may be disturbed by the free-space propagation between the DOE and the aspheric. On the other hand, there has to be a certain distance between DOE and aspheric so that the different diffraction orders of a binary DOE can be separated (Lindlein [2001]). So, in practice a compromise has to be found, which is in this case 10 mm.

Figure 5.2 Phase shifting Twyman-Green interferometer developed at laboratory.

Figure 5.3(a) shows the photo for the situation of the surface measurement while figure 5.3(b) is for the cat’s-eye position. A mirror is placed at the focus of the spherical wave where the aspheric wave and other diffraction orders are blocked by using a pinhole aperture.
Figure 5.3: Measurement positions: (a) Surface measurement situation for the basic position and 180° rotated position (b) cat’s-eye measurement situation where mirror is placed at the focus of the spherical wave.

5.1.2 Combo-DOE

Depending on the specimen under test the combo-DOE is designed by following the procedure explained in section 4.2. The phase function of the element is encoded with in-house developed program CAHD (computer aided holographic design). The combo-DOE is manufactured with laser based lithography system at the Institute of Optics, Information and Photonics. Figure 5.4 shows a photo of one of the combo-DOE which is used for the experimental studies.

Figure 5.4: Photo of one of the combo-DOE used for the experimental studies.

5.1.3 Specimen Under Test

A rotationally invariant convex aspheric of diameter 50 mm has been chosen as a test specimen. The maximum aspheric deformation from the best fit sphere is approx. 0.25 mm (i.e. ~ 400 waves) at the edge (Fig.5.5).
5.1 Experimental Setup

Figure 5.5: Specimen under test: (a) rotationally invariant aspheric surface and best fit sphere of the chosen specimen (b) departure of the aspheric surface from its best fit sphere.

5.2 Interferometer’s Performance

Before presenting the absolute test results, it is necessary to discuss the repeatability, reproducibility, and interferometer drift of the setup and the misalignment removal capabilities of the evaluation process. Repeatability, reproducibility and drift analysis give an idea about the overall stability status of the interferometer. In all cases the software routines to remove the misalignment errors are applied (Young [1986]).

5.2.1 Interferometer Stability

5.2.1.1 Repeatability

The repeatability can be derived from the difference between two measurements, one right after the other, without any nominal change in the system. This provides information about the sensitivity of the setup against vibrations and air turbulence. The repeatability for a single measurement is $\frac{\lambda}{500}$ waves (Fig.5.6), where $\lambda$ is 633 nm, the wavelength of a He-Ne laser.

Figure 5.6: Repeatability of a single surface measurement.
5.2.1.2 Reproducibility

Since the specimen has to be rotated by 180° during the test procedure, the effect of readjustment should also be evaluated. These effects are captured by the reproducibility where the difference between two measurements involving a readjustment has been taken. The reproducibility depends on the precision of the mechanical components in the specimen holder and the capabilities of the experimenter himself to readjust it to the “fluffed out fringe” condition. Since it is impossible to readjust the specimen exactly, coherent noise is introduced, limiting the reproducibility in practice to a value of about double the repeatability. The observed reproducibility of the setup for single measurements is $\lambda/200$ waves as shown in figure 5.7.

![Figure 5.7: Reproducibility of a single surface measurement.](image)

5.2.1.3 Drift Analysis

If the series of measurements is going to take some time, which is the case in the three position test procedure, then it is necessary to know the stability of the interferometer during this time span. This can be investigated by performing a drift analysis, which consists of looking at the difference between the two measurements after a necessary time gap. This has to be carried out without touching or changing anything in the setup. The difference between the drift analysis and the repeatability is that the repeatability is a measure of short time disturbances, while the drift analysis shows how these deviations evolve with time. It depends on thermal gradient in the laboratory and the amount of space between the test part and the reference.

Since the N-position rotational averaging test is also presented (Section 5.4), where 36 measurements take approximately one hour, the drift analysis is also performed...
for the same duration. The rms values of the difference of the measurements with a time gap varying from 30 seconds to one hour are presented in Table 5.1. These are within the range of the reproducibility of a single measurement. This ensures that the drift taking place while performing the three position test does not limit its accuracy. The reproducibility of a single measurement is the dominating parameter.

<table>
<thead>
<tr>
<th>Duration</th>
<th>30 sec.</th>
<th>1 min.</th>
<th>3 min.</th>
<th>5 min.</th>
<th>10 min.</th>
<th>20 min.</th>
<th>40 min.</th>
<th>60 min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>difference rms ((\lambda))</td>
<td>0.0022</td>
<td>0.0027</td>
<td>0.0043</td>
<td>0.0039</td>
<td>0.0044</td>
<td>0.0056</td>
<td>0.0063</td>
<td>0.0072</td>
</tr>
</tbody>
</table>

Table 5.1: Drift analysis for a duration of one hour. Difference has been taken from the initial measurement.

5.2.2 Removal of Misalignment Aberrations

The misalignment of the specimen relative to the interferometer’s coordinate system plays an important role in interferometric metrology in general, but it becomes more crucial in aspheric metrology because of its reduced degree of symmetry. In the presence of misalignment, the interference pattern is always a superposition of the wave aberrations due to the surface errors of the test piece, the systematic errors of the interferometer, and the aberrations caused by the movement of the aspheric relative to the interferometer frame. The specimen can never be adjusted to the perfect positions, the extent to which the misalignment errors can be removed must be considered.

The alignment errors due to tilt, decentre and defocus of the test surface can be analyzed empirically. The misalignment functionals are described by representing five degrees of freedom of the aspheric within the interferometer (e.g. three displacements and two tilts) by five misalignment vectors. Rotation around the Z axis is not considered as a degree of freedom since the surface under test is rotationally invariant. These errors have been computed mathematically and were then removed from the measuring data by a least squares fit to find out the actual surface errors plus systematic errors of the interferometer.

To estimate the accuracy of the misalignment elimination, several misalignments have been simulated with RAYTRACE program (Lindlein [1998]) and have subsequently
been eliminated. The misalignment parameters were chosen to have a peak-to-valley value of the simulated aberrations close to one wavelength, a value that we could achieve in the experiment. The same has been performed in the experiment with all five degrees of freedom. Table 5.2 and 5.3 show the results of the elimination process in simulations and in experiment, respectively. Both studies clearly show that the misalignment in the Z-direction is the most critical one and requires much more careful alignment compared to the rest of the degrees of freedom.

<table>
<thead>
<tr>
<th>Simulated</th>
<th>P-V Simulated</th>
<th>Residual Aberration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misalignment</td>
<td>Field (λ)</td>
<td>rms (λ)</td>
</tr>
<tr>
<td>X-shift</td>
<td>0.9982</td>
<td>0.0004</td>
</tr>
<tr>
<td>Z-shift</td>
<td>0.9996</td>
<td>0.0013</td>
</tr>
<tr>
<td>Y-rotation</td>
<td>1.0670</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Table 5.2: Residual aberrations after misalignment elimination in simulations.

<table>
<thead>
<tr>
<th>Misalignment in λ (approx.)</th>
<th>Residual aberrations after misalignment of approximately</th>
</tr>
</thead>
<tbody>
<tr>
<td>1λ</td>
<td>2λ</td>
</tr>
<tr>
<td></td>
<td>rms (λ)</td>
</tr>
<tr>
<td>X-shift</td>
<td>0.0015</td>
</tr>
<tr>
<td>Y-shift</td>
<td>0.0019</td>
</tr>
<tr>
<td>Z-shift</td>
<td>0.0051</td>
</tr>
<tr>
<td>X-rotation</td>
<td>0.0016</td>
</tr>
<tr>
<td>Y-rotation</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Table 5.3: Residual aberrations after misalignment elimination in experiments.

For a well-aligned specimen (less than two fringes), the residual misalignment aberrations are below the reproducibility of the absolute results. The residual aberrations increase linearly with the increase in misalignment. In contrast to the simulation results, the experimental results are worse by approximately a factor of 4. A possible reason is the micro roughness of the wave fronts, which is different for different alignment positions of the specimen (Schwider [1983]).
5.3 Quasi Absolute Measurement

5.3.1 Measurements with Striped and Superposed Combo-DOE

A single measurement of the specimen, after subtracting tilt and defocus, is shown in Figure 5.8 with a rms of ~0.30 waves and a P-V of ~1.80 waves for both types of the combo-DOEs, striped and superposed. Wave front errors were analysed in terms of Zernike polynomials. Contour line spacing is $\lambda/10$. A mirror placed at the focus of the spherical wave front in the cat’s-eye position exhibits the errors present in the interferometer, including the DOE substrate.

![Contour plots showing wave front errors](image)

**Figure 5.8:** *Single measurement results after removing tilt and power (a) for sliced combo-DOE (b) for superposed combo-DOE.*

Using the three position procedure, the surface errors of the test specimen is determined, thus eliminating the systematic errors. Figure 5.9 shows the contour plots of absolute deviations of the surface, with contour line spacing of $\lambda/20$.

5.3.2 Substrate Quality

Figure 5.10 depicts the wedge error of the glass substrates which have been used for sliced and superposed combo-DOEs. These substrates are commercially available mask plates from HOYA Corp. The four fringes over 50 mm of aperture turns out to be a wedge error of 0.003°. The effects the wedge errors on the consistency of the procedure have already been discussed in the Section 4.3.4.
PV: 1.768 \lambda, 
s rms: 0.256 \lambda, 
Line spacing: \lambda/10

PV: 1.729 \lambda, 
s rms: 0.263 \lambda, 
Line spacing: \lambda/10

Figure 5.9: Contour plots of absolute deviations extracted from three position test using (a) sliced combo-DOE (b) superposed combo-DOE.

Figure 5.10: Substrate quality of the (a) sliced (b) superposed combo-DOE.

5.3.3 Disturbing Diffraction Orders

There are disturbing diffraction orders present in both types of combo-DOEs, striped and superposed. In the first case, the additional diffraction orders are obvious as they are caused by the striped nature of the aperture [Fig. 5.11 (a)]. To avoid the direct overlapping of the different orders of both wave fronts, the off-axis direction of the spherical wave front is selected to be perpendicular to the grating vector of the grating produced by the stripes. Upon reflection, while retracing the path, the disturbing diffraction orders are blocked by employing a stop at the confocal plane of the telescopic system in the object arm. In the second case, the diffraction orders are more overlapped with the desired aspheric wave front [Fig. 5.11(b)]. The diffraction orders are much less
disturbing in the case of striped DOE compared to the superposed one. Because of this advantage, only the striped structure of the DOE has been used for further experiments.

![Disturbing diffraction orders](image)

**Figure 5.11:** Disturbing diffraction orders (a) sliced combo-DOE (b) superposed combo-DOE.

### 5.3.4 Measurement with Optimized Striped DOEs

Using the three position procedure described previously, the surface deviations of the test specimen are determined. Figure 5.12 (a) shows the contour plot of the absolute deviations of the surface by using the optimized combo-DOE. The P-V value is 1.031 waves and the rms value is 0.244 waves. A x-y polynomial fit of degree 12 has been applied to eliminate the noise in the final absolute result. Figure 5.12(b) shows the reproducibility of the three position test itself, whereby the difference of two such absolute measurements at the same orientation of the specimen was taken. The rms value is $\lambda/200$, which is in the range of the reproducibility of a single measurement. The interferometric errors have been extracted by subtracting the absolute measurement from a single measurement and are shown in figure 5.12 (c).

### 5.3.5 Consistency Results with Different Striped Combo-DOEs

As discussed in the simulation Section of chapter 4, the consistency test is applied as an indication for the accuracy of the procedure. Figure 5.13 compares the results of the consistency test when a) the combo-DOE does not fulfill the symmetry condition and has a tilt of 1°, b) the combo-DOE satisfies the symmetry condition and has 1° tilt, and c) an optimized combo-DOE with 0.5° tilt is taken. These results conform to the predictions of the simulation studies. The consistency improves by a factor of 10 if the symmetry condition is followed (Fig. 5.13 b) and further improves by a factor of 2 with an
optimized design, where 0.5° tilt angle and the best fit spherical wave have been selected
(Fig. 5.13 c).

\[
\text{PV: } 1.031 \lambda, \text{ rms: } 0.244 \lambda \\
\text{Line spacing: } \lambda/5 \\
rms: 0.0048 \lambda
\]

\[
\text{PV: } 0.13 \lambda, \text{ rms: } 0.023 \lambda \\
\text{Line spacing: } \lambda/50
\]

Figure 5.12: Quasi absolute three position test procedure. (a) Absolute deviations of the
aspheric surface. The contour line spacing is \( \lambda/5 \). (b) Reproducibility of the three position
test as a whole. (c) Interferometric errors extracted by subtracting the absolute
measurement from a single measurement. The contour line spacing is \( \lambda/50 \).

The occurrence of residual astigmatism of P-V value 0.028 waves in the
consistency test even in the case of the optimized combo-DOE is because of the fact that
the procedure is still limited by its quasi nature and the flat DOEs do not fulfill the sine-
condition. Further simulations yield that the consistency error depends on the errors of the
empty interferometer and on those of the DOE substrate. Therefore, it is expected that the
lesser these errors are, the better the consistency will be. In particular, the wedge error in
the substrate is very critical as discussed in chapter 4. Currently, commercially available
mask plates are used as substrates, which have a wedge error of 10 arcsec. This value can
5.3 Quasi Absolute Measurement

definitely be improved by using a higher quality substrate. It is, therefore, left for further investigations to use a better quality custom made substrate and to demonstrate the effects of substrate quality on the procedure.

Figure 5.13: Results of the consistency test when the combo-DOE (a) does not fulfill the symmetry condition (b) follows the symmetry condition (c) is optimized by having the offset angle of the spherical wave front as 0.5°.

5.4 N-Position Test

While the three position test method delivers the absolute errors of the surface, there is a possibility to measure the rotationally varying deviations with another method. The surface shape deviations can have rotationally invariant as well as rotationally varying errors. The rotationally varying errors can be extracted from measurements with the surface being rotated in a stepwise manner; a method known as N-position rotationally averaging test (Evans [1996]). The average of N such measurements at the interval of 360/N degrees removes all rotationally varying errors of the surface whereby the systematic errors of the interferometer are fixed.

\[ W_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} W_i \]  

(5.1)

Subtracting this average from a single measurement which also has the same systematic errors of the interferometer will give the rotationally varying errors \(W_{\text{NR}}\) of the surface:
To validate the results obtained by the quasi absolute three position test, the independent procedure of N-position rotational averaging testing has been performed. For this purpose, 36 measurements are recorded while the specimen was rotated in 10º-steps. By subtracting a single measurement from the average of these multiple measurements, the rotational invariant errors of the specimen and the systematic errors of the interferometer get cancelled out leaving behind the rotationally varying errors of the surface. Figure 5.14 compares the rotationally varying errors obtained by the N-position rotationally averaging test with that of the three position test, where the later were calculated numerically from the full absolute errors. There is a good agreement between them. The difference between both methods, shown in Fig. 5.14 c), is of 0.029 waves P-V and 0.0038 waves rms, which is believed to be a good agreement keeping in mind that the N-position test procedure also has several additional error sources as discussed in (Freimann [1999]). The trifoil shape in the difference (Fig 5.14 c.) is an indication that the interferometric error, which is also trifoil in this case, has not been eliminated identically in the two methods. Needless to mention, that none of the methods is perfect, since both of them have their limitations and sources of errors.

\[ W_{NR} = W_i - W_{avg} \]  

(a)                  (b)             (c)

PV: 0.680 \( \lambda \),  
PV: 0.683 \( \lambda \),  
PV: 0.029 \( \lambda \),  
rms: 0.122 \( \lambda \)  
rms: 0.121 \( \lambda \)  
rms: 0.0038 \( \lambda \)  
Line spacing: \( \lambda/10 \)  
Line spacing: \( \lambda/10 \)  
Line spacing: \( \lambda/200 \)

Figure 5.14: Comparison of the rotationally varying deviations of the aspheric surface extracted by using (a) the quasi absolute three position test procedure (b) the N-position rotationally averaging procedure. (c) The difference of rotationally varying deviations obtained from both the procedures (a-b).
5.5 **Quasi Absolute Test for Toric Surfaces**

A toric surface is a special case of an aspheric which has two different curvatures perpendicular to each other. The industrial demand for such surfaces has increased because of their use in astigmatic systems like laser scanning devices and electronic printers. Testing of these surfaces in an absolute manner is more complicated than that of flats, spheres or even rotationally invariant aspherics because of the reduced degree of symmetry.

One method for the absolute measurement of torics requires several relative measurements, including measurements in a shifted as well as in a rotated position of the specimen with respect to the interferometer, and in addition a measurement with an absolutely calibrated sphere to know the deviation of the toric wave front from the sphere (Bluemel [1997]).

The possibility of using the quasi absolute three position test for toric surfaces has also been explored. Since torics are invariant with respect to a rotation of 180°, the first two positions (basic and 180° rotated) can be taken in the similar manner as for the rotationally invariant aspherics. The spherical wave front for the cat’s-eye measurement is generated by two ways: 1) by using a combo-DOE and 2) by using a combination of spherical condenser and single wave front encoded DOE. Both the approaches have been implemented.

5.5.1 **Representation of the Toric Surface Under Test**

The toric surface is represented by means of the angles \( \beta \) and \( \psi \) as depicted in figure 5.15. The Cartesian coordinates \( x, y, \) and \( z \) in terms of the angles \( \beta \) and \( \psi \) are (Bode, priv. commun.):

\[
\begin{align*}
  x(\beta, \psi) &= (R_x - R_y + R_z \cos \beta) \sin \psi \\
  y(\beta, \psi) &= R_y \sin \beta \\
  z(\beta, \psi) &= (R_x - R_y + R_z \cos \beta) \cos \psi - R_z
\end{align*}
\]  

(5.3)

where the vertex of the toric lies in the plane \( z=0 \). \( R_x \) and \( R_y \) are the radii of principle curvature.
Chapter 5: Quasi Absolute….Experimental Results

Figure 5.15: Representation of the tyre type toric surface in terms of the angles $\beta$ and $\psi$.

The surface under test is a tyre type toric with two radii of curvatures: $R_x=213.84$ and $R_y=61.38$ mm (Fig. 5.16). The diameter of the surface is 48 mm. The mean of the two principal radii of curvatures (approx. 138 mm) is chosen as the curvature for the spherical wave front in the combo-DOE. The maximum departure from the mean spherical wave front is $\approx 3.0$ mm (i.e approx. 4800 waves) at the edge.

5.5.2 Measurement Techniques

5.5.2.1 Quasi Absolute Test by using Combo-DOE

A combo-DOE has been employed to generate the toric as well as spherical wave front simultaneously as illustrated in figure 5.17. Unlike for the rotationally invariant aspherics, the spherical wave front has also been kept on-axis. The mean of the two principal radii of curvatures of the toric has been chosen as the radius of curvature for the spherical wave front. This is approximately 138 mm and obviously quite different from the toric wave front so that the diffraction orders can be separated easily. The distance between the combo-DOE and toric specimen was kept as 20 mm. The DOE is tilted at $1^\circ$. 
5.5.2.2 **Quasi Absolute Test by using Combination of a DOE and Spherical Condenser**

A combination of a spherical condenser and a single diffractive optical element has also been explored. This method is being used at AGFA Gevaert AG, Munich. Here, a spherical wave front is generated by the condenser and passes the DOE in the 0\(^{th}\) order. The toric wave front is generated through diffraction of the spherical wave by the DOE in the 1\(^{st}\) order. The scheme is shown in figure 5.18. The spherical condenser is having a focal length of 300 mm and the distance between the DOE and the toric surface is 17 mm. The DOE is tilted at 1°. The advantage of this approach is that the DOE has lower spatial frequencies and hence less structural errors. Another advantage is that the DOE is having only one wave front encoded.
5.5.3 Experimental Results

5.5.3.1 Using Combo-DOE

Figure 5.19 shows the contour plot of the absolute deviations of the surface by using the quasi absolute test with combo-DOE as described above. Figures 5.19 (a) and 5.19 (b) display the measurements at the normal position (0°) and the rotated position (180°) of the toric surface respectively. The measurement in the cat’s eye position which is required for the three position test is shown in Fig. 5.19 (c). Figure 5.19 (d) shows the measured surface deviations by using these three positions. The contour line spacing is $\lambda/50$. The reproducibility of the three position test is $\lambda/250$ as shown in figure 5.20.

Figure 5.19: Contour plots of the measurement at three positions by using combo-DOE (a) normal position, (b) 180° rotated position, (c) cat’s-eye position and (d) absolute deviations extracted from three position test. The line spacing is $\lambda/50$.

Figure 5.20: Contour plots of two different measurements (a) and (b) of the surface deviations using the three positions test. (c) reproducibility of the test, i.e. (a-b).
5.5 Quasi Absolute Test for Toric Surfaces

5.5.3.2 Using Combination of a DOE and a Spherical Condenser

Figure 5.21 shows the contour plot of the absolute deviations of the surface by using the combination of a DOE and a spherical condenser. This DOE was supplied to us by AGFA Gevaert AG, Munich. Figures 5.21 (a) and 5.21 (b) display the measurements at the normal position (0°) and the rotated position (180°) of the toric surface respectively. Fig. 5.21 (c) shows the measurement in the cat’s eye position which is acquired by using the spherical wave front which passes through the DOE in the 0th order. Figure 5.21 (d) shows the measured surface deviations by using these three positions.

Figure 5.21: Contour plots of the measurement at three positions by using combination of a DOE and a spherical condenser (a) normal position, (b) 180° rotated position, (c) cat’s-eye position and (d) absolute deviations extracted from three position test. The line spacing is λ/50.

Figure 5.22 compares the absolute surface deviations obtained by both the techniques. Figure 5.22(a) shows the absolute deviations by using the combination of a DOE and a spherical condenser. Figure 5.22(b) presents the same while using the combo-DOE. Both the results show a qualitative agreement, however indicating a difference of λ/12 as shown in fig. 5.22(c).

Another motivation to conduct the study on toric surfaces was to explore the limits of the quasi condition, which is the basic assumption of this procedure. This condition assumes that the errors arising from surface deviation and lithographic writing inaccuracies are identical for both the wave fronts of the combo-DOE. The difference
between lithographic writing inaccuracies is proportional to the departure from the best fit sphere and the substrate quality.

\[
\begin{align*}
\text{PV: } & 0.407 \lambda \\
rms: & 0.066 \lambda \\
\text{PV: } & 0.367 \lambda \\
rms: & 0.058 \lambda \\
\text{PV: } & 0.081 \lambda \\
rms: & 0.010 \lambda \\
\end{align*}
\]

(a)      (b)    (c)

Figure 5.22: Comparison of absolute surface deviations by using (a) combination of DOE and spherical condenser, P-V; 0.4078\(\lambda\) (b) three position quasi absolute test with combo-DOE, P-V; 0.3671\(\lambda\) (c) the difference in the absolute surface deviations observed by both procedures.

In the case of the experiment with aspheric, the maximum departure from the best fit sphere was approx. 400 waves at the edge. The maximum departure from the best fit sphere to the toric surface is twelve times higher than in the case of the aspheric, i.e. 4800 waves. In the present experiment, the combo-DOE substrate has thickness deviations of 3\(\lambda\). Although the first comparative results using two different procedures indicates qualitative agreement, however, at this time sufficient results are not available to comment on the limits of the “quasi condition” of this procedure, but it is believed that experiments with better substrate quality will provide more insight into it.

5.6 Conclusions

We presented the experimental demonstration of the proposed quasi absolute three position test for aspherics by using a combo-DOE as null element. The accuracy of the procedure is enhanced by optimizing the design parameters of the combo-DOE, which could be shown by the consistency test. It is observed that the tilt of the DOE and its orientation, as well as the offset direction of the spherical wave front play a critical role in the presence of substrate deviations and systematic errors of the interferometer.
Furthermore, the rotationally varying deviations extracted by using the three position absolute procedure are compared with that of an N-position rotationally averaging procedure and are found to be in good agreement. The capability of extracting the complete deviations of the specimen signifies our proposed technique as a promising tool for interferometric calibration of aspheric metrology. Another major advantage of our procedure is that we need not know the DOE fabrication errors as long as they have a global character and are identical in both wave fronts. In principle, this situation decides about the degree of accuracy of the procedure. It is believed that the achieved consistency or accuracy is limited by the stability of the interferometer and the substrate quality at present. Simulation studies show that the consistency becomes better as interferometer errors and substrate deviations go down. Therefore, it is expected that the accuracy will enhance further, once high quality optical components and DOE substrates are used.

The quasi absolute test procedure is further applied for the precise metrology of toric surfaces also.
Chapter 6

Conclusions and Future Scope

The objective of this work was to characterize the surface roughness generated by single point diamond turning process and to measure precisely the shape deviations of the aspheric surface by interferometric methods. The first activity was performed at National Aspheric Facility, CSIO, India. The flat metal mirrors were fabricated and characterized by performing the frequency analysis (power spectral density and autocorrelation) of the surface roughness. The optimized machine parameters are found to achieve the desired surface figure and roughness in the desired frequency range. This study helps us establishing the relationship between manufacturing process and the surface roughness generation which is crucial for developing metal components with optical quality finish. It is also observed that relative tool-workpiece vibration is dependent on rotational spindle speed.

The second activity was conducted at IOIP, University of Erlangen-Nuremberg. A three position quasi absolute test for rotationally invariant aspherics by using combined-diffractive optical elements (combo-DOEs) is developed. The effects of the DOE substrate errors on the proposed calibration procedure are investigated and presented a set of criteria for designing an optimized combo-DOE. The simulation studies suggest that in the presence of substrate errors, the design of the combo-DOE must follow the following three conditions: a) symmetry condition (i.e. chief ray of the spherical wave perpendicular to the tilted substrate plane), b) minimum tilt condition and c) the matching condition (using for example a best fit sphere).

It is demonstrated that the optimized design enhances the overall consistency of the procedure. Furthermore, the rotationally varying part of the surface deviations is
compared with the rotationally varying deviations obtained by an N-position rotationally averaging procedure and is found to be in good agreement. The capability of extracting the complete deviations of the specimen signifies the proposed technique as a promising tool for interferometric calibration of aspheric metrology. Another major advantage of our procedure is that we need not know the DOE fabrication errors as long as they have a global character and are identical in both wave fronts.

Nevertheless, this procedure is still limited by its quasi absolute nature, as the calibrating spherical wave front, which has an additional angular offset, deviates from the aspheric one. It can further be enhanced by having both wave fronts on-axis, thus eliminating the contribution of the offset of the spherical wave front. But in that case, one has to devise a scheme to block one wave front while using the other. Future study will concentrate on to incorporate such a wave front selective mask, associated with the combo-DOE. But even after that the spherical wave front will remain different from the aspheric one. It is left for further research to examine to what extent this difference in the two wave fronts limits the overall performance of the technique. For that purpose one needs to know the localized fabrication errors of the DOE in both the wave fronts. Furthermore, it has been observed that the misalignment in Z-direction is very critical. It can further be improved by having better mechanical stability and alignment of the specimen, or by improving the misalignment elimination algorithm.
Appendix A

Laser Lithography System

The Institute of Optics, Information and Photonics is having one laser lithographic system DWL 400 from Heidelberg Instruments, Germany (Fig. A.1). The system is based on water cooled Ar-ion laser working at a wavelength of 363 nm.

Figure A.1: Photos of the laser lithography system.

The exposure intensity can be varied in 128 gray levels by using an acousto-optic modulator which also handles data to write. An acousto-optic deflector is used to increase and to control the writing speed. Switching between different writing heads allows the writing of minimum structure sizes from 1.6 μm down to 0.5 μm. Table A.1 presents the ranges of parameters for the different writing heads. The technical data of the system as a whole is summarized in table A.2.
Table A.1: The ranges of parameters for different writing heads.

<table>
<thead>
<tr>
<th>Writing Head</th>
<th>2 mm</th>
<th>4 mm</th>
<th>10 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length (mm)</td>
<td>2.0</td>
<td>4.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Depth of focus (μm)</td>
<td>1.0</td>
<td>1.8</td>
<td>8.0</td>
</tr>
<tr>
<td>Min. feature size (μm)</td>
<td>0.5</td>
<td>0.7</td>
<td>1.6</td>
</tr>
<tr>
<td>Pixel size (nm)</td>
<td>100</td>
<td>200</td>
<td>500</td>
</tr>
<tr>
<td>Writing speed (mm²/min)</td>
<td>12</td>
<td>45</td>
<td>265</td>
</tr>
</tbody>
</table>

Table A.2: The technical data of the laser lithography system (DWL 400).

<table>
<thead>
<tr>
<th>Minimum feature size</th>
<th>0.5 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge roughness</td>
<td>&lt; 80 nm</td>
</tr>
<tr>
<td>Overlay accuracy</td>
<td>&lt; 250 nm</td>
</tr>
<tr>
<td>Auto focus range</td>
<td>90 μm</td>
</tr>
<tr>
<td>Interferometric position control</td>
<td>&lt;10 nm</td>
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List of Publications

1. Refereed Journal Articles


2. Published Conference Proceedings


### 3. Other Conference Paper Presentations


R. S. Pangtey, G. S. Singh, G. S. Khan, S. V. RamaGopal and P. K. Jain “Design and development of Telemicroscope as a low vision aid for distant vision using plastic
Aspheric lenses” *International Conference on Optoelectronics Technology, ICOT-04, organized at North Maharashtra University, Jalgaon, January 12-14, 2004*

**G. S. Khan, S. V. RamaGopal, K. D. Chattopadhyay, P. K. Jain and V. M. L. Narasimham** “Effects of Tool Feed Rate in Single Point Diamond Turning of Aluminium 6061 Alloy” *Conference on Optics and Photonics in Engineering, COPE-03 at NSIT, New Delhi, January 6-8, 2003*


**G. S. Khan, S. V. RamaGopal, K. D. Chattopadhyay, P. K. Jain and V. M. L. Narasimham** “Job RPM vs Surface Quality in Single Point Diamond Turning of Aluminium 6061 Alloy for Optical Surface Generation” *Conference on Optics and Photonics in Engineering, COPE-03 at NSIT, New Delhi, January 6-8, 2003*

**R. S. Pangtey, R. Mandal, G. S. Khan, S. V. RamaGoal, P. K. Jain and V. M. L. Narasimham** “Optimum machining parameters vs optical surface quality for various materials during single point diamond turning” *Conference on Optics and Photonics in Engineering, COPE-03 at NSIT, New Delhi, January 6-8, 2003*

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