Characterization and Modeling of Hysteresis in Ferromagnetic Material

Charakterisieren und Modellieren der ferromagnetischen Hysterese

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Notations and Abbreviations

Notations

$A$ (Wb/m) magnetic vector potential
$B, B$ (T) magnetic flux density or magnetic induction
$B_t$ (T) retentivity or residual induction
$B_s$ (T) saturation value of magnetic flux density
$\chi$ (volume) magnetic susceptibility
$D$ (As/m$^2$) electric flux density
$\delta$ (m) skin depth
$\varepsilon$ (relative error between measured and simulated data
$\bar{\varepsilon}$ mean relative error
$E$ (V/m) electric field strength or intensity
$e$ unit vector
$\varepsilon$ (As/Vm) electric permittivity
$\varepsilon_0$ (As/Vm) dielectric constant, $\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/Vm
$E(\alpha, \beta)$ inverted Everett function
$E$ inverted Everett matrix
$F(\alpha, \beta)$ new defined function for the inverted Preisach model
$F$ F function matrix
$f$ (Hz) frequency
$J$ (A/m$^2$) current density
$H, H$ (A/m) magnetic field strength or intensity
$H_c$ (A/m) coercivity or coercive field strength
$H_{\alpha}$ increasing magnetic field input of the forward Preisach operator
$H_{\beta}$ decreasing magnetic field input of the forward Preisach operator
$H^{-1}$ inverse processing of the hysteresis models
$J$ (T) magnetic polarization
$L$ (H) electric inductance
$l_m$ (m) length of magnetic circuit
$M, M$ (A/m) magnetization
$M_{\alpha}$ increasing magnetization input of the inverted Preisach operator
$M_{\beta}$ decreasing magnetization input of the inverted Preisach operator
$M_s$ (A/m) saturation value of magnetization
$\mu$ (Vs/Am) permeability
$\mu_0$ permeability of vacuum, $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am
$\mu_r$ relative permeability
Notations and Abbreviations

\( n \)  
\text{discretization size of the Preisach triangle}

\( N \)  
\text{size of magnetization input}

\( N_d \)  
\text{demagnetization factor}

\( \nu \)  
\((\text{Am/Vs})\) \text{magnetic reluctivity}

\( \nabla \)  
\text{nabla operator}

\( \nabla \times \)  
\text{curl}

\( \nabla \cdot \)  
\text{divergence}

\( \nabla \)  
\text{gradient}

\( P_c \)  
\((\text{W/kg})\) \text{core loss}

\( \Phi \)  
\((\text{Vs})\) \text{magnetic flux}

\( Q_k \)  
\text{trapezoids divided in the} \( S^+ \) \text{region in Preisach plane}

\( \sigma \)  
\((1/\Omega \text{m})\) \text{electric conductivity}

\( \rho \)  
\((\text{kg/m}^3)\) \text{material density}

\( S \)  
\text{switch matrix}

\( t \)  
\((\text{s})\) \text{time}

\( T \)  
\text{time period or one cycle of the magnetization process}

\( T_c \)  
\((\text{K})\) \text{Curie temperature}

\( \Theta \)  
\((\text{A})\) \text{magnetic ampere turns}

\( \Gamma \)  
\text{forward Preisach operator}

\( \Upsilon \)  
\text{inverted Preisach operator}

\( \hat{\gamma} \)  
\text{forward (relay) switch operator}

\( \hat{\upsilon} \)  
\text{inverted switch operator}

\( \mu(\alpha, \beta) \)  
\text{forward weight function}

\( \nu(\alpha, \beta) \)  
\text{inverted weight function}

\( \mathbf{V} \)  
\text{weight function matrix}

Abbreviations

AC  \text{alternating current}

CPM  \text{classical Preisach model}

DC  \text{direct current}

EML  \text{effective magnetic path length}

FE  \text{finite element}

FORCs  \text{first-order reversal curves}

GO  \text{grain-oriented}

HT  \text{heat treatment}

MSE  \text{mean squared error}

RD  \text{rolling direction}

SST  \text{single sheet tester}

TD  \text{transverse direction}

VSM  \text{vibrating sample magnetometer}

VVSM  \text{vector vibrating sample magnetometer}
Abstract

This thesis proposes a new inverted hysteresis model for including the ferromagnetic hysteresis in electromagnetic field calculations.

Numerical techniques have been commonly utilized to solve electromagnetic field problems. In finite element (FE) formulations, the magnetic vector potential is introduced as unknown, by which the magnetic flux can be directly obtained. The nonlinear hysteretic relation is usually approximated by a single-valued magnetization curve. This neglect of the hysteresis effect may lead to serious inaccuracies in calculation. To solve this problem, hysteresis model is required to be implemented in FE codes. However, most hysteresis models employ the magnetic field as the input variable and take the magnetic flux (or magnetization) as the output. Models have to be inverted for implementation. As the local inversion with iterative methods costs considerable computation, in recent years, the possibility of developing inverted hysteresis model with the roles of input and output being exchanged has gained great attention.

The inverted hysteresis model proposed in this thesis is based on the classic Preisach model. The inversion is realized by introducing a modified switch operator and the corresponding weight function. By using this new switch operator, the inverted model remains the wiping out and congruency properties. It also guarantees the total positiveness of weight function in the Preisach plane. This makes it possible and easy to model the weight function distribution. A differentiable weight function is proposed for the inverted model. To determine the function parameters, a quasi-Newton method algorithm is applied to optimize the mean squared error (MSE) between the measured and simulated data. This inverted model can be identified directly from major loops, which are more accessible than a set of first-order reversal curves required in the pseudo-inverse approach. Besides major hysteresis loops, minor loops and reversal curves can also be simulated. The simulation results show good approximation to the measurement data.

The inverted model was verified for both soft and hard magnetic materials. The hysteresis loops of steel sheets and thin films samples were measured by applying three different characterization methods, the Epstein frame method, double C-yoke SST and vibrating sample magnetometer. The measurements can be carried out to evaluate different influence factors on hysteresis loops, such as frequency, temperature, anisotropy and plastic deformation. The experimental data can be used for further expansion of the model.

In order to improve the calculation efficiency, the double integrations of the weight function were numerically approximated in precalculation stage. The integral expression of the inverted model was rewritten in a discrete form. In this way, it cost low memory usage and short calculation time during the calculation procedure.

**Keywords:** inverted hysteresis model, differentiable weight function, finite element method, numerical
Abstract

discrete formulation, vibrating sample magnetometer, double C-yoke, Epstein frame
Kurzfassung

In der vorliegenden Arbeit wird ein neues inverses Hystereses Modell entwickelt, um die ferromagnetische Hysterese in den elektromagnetischen Feldberechnungen zu beinhalten.


In den letzten Jahren versucht man die Möglichkeit, ein inverses Hysterese-Modell unter Austauschung zwischen Eingang- und Ausgangsgröße zu entwickeln.


Um die Effizienz zu verbessern, werden die doppelten Integrationen der Gewichtungsfunktion im Vorausberechnungsprozess numerisch approximiert. Die integrale Form des invertierten Modell wird in einer diskreten Form umgeschrieben. Auf diese Art und Weise kostet es
Kurzfassung

geringe Speicherauslastung und Rechenzeit während des Berechnungsprozesses.

Schlagwörter: inverses Hysterese-Modell, differenzierbare Gewichtungsfunktion, Finite-Elemente-Methode, numerische diskrete Formulierung, Vibrating-Sample-Magnetometer, Doppel-C-Joch, Epstein-Rahmen
1 Introduction

1.1 Motivation

Hysteresis is the central feature of ferromagnetic materials. It is present in much electromagnetic equipment, such as transformers, motors, actuators and magnetic recording, etc. With the development of computers, numerical techniques, such as the finite element (FE) method, have commonly been used to solve electromagnetic field problems. However, hysteresis is disregarded in most electromagnetic field calculations, the nonlinear relation is usually approximated by a single-valued magnetization curve [1, 2]. This neglect of the hysteresis effect may lead to serious inaccuracies in calculations involving hysteresis losses, especially when semi-hard or hard magnetic materials are involved in magnetic systems [3, 4].

In FE formulations the magnetic vector potential is introduced as an unknown, by which the magnetic flux can be directly obtained. However, most hysteresis models, including the Preisach model, employ the magnetic field as the input variable and take the magnetization or magnetic flux as the output quantity (forward hysteresis models). Therefore, in order to include hysteresis models in electromagnetic field calculations, the hysteresis models must be inverted to incorporate them into FE formulations.

Iterative methods are usually used in the local inversion of forward hysteresis models. Such methods require considerable computation to obtain converged solutions [5]. It is possible to circumvent this costly iterative procedure by applying an inverted hysteresis model, which employs the magnetization or magnetic flux as input. The magnetic field is then calculated directly without iteration.

This thesis presents an inverted model, which is accurate, effective and can be simply identified from less experimental data. The inverted model is based on the Preisach model, which is widely applied in electromagnetic computation. The identification can be simplified by proposing a differentiable weight function, whose parameters are determined from major hysteresis loops. With reference to the Everett function method, the integration of the weight function is approximated in a precalculation procedure, thus improving the efficiency of the calculation process.

1.2 State of the Art

In the past few years, the inclusion of hysteresis in FE formulations has attracted great interest, and much research effort has been put into this research field. Several techniques have been proposed to solve this problem [1, 6, 7, 5, 8]. However, these techniques have generally not been adopted in most FE software packages.
1 Introduction

In electromagnetic field computations the material capabilities are usually limited to the linear case (using a constant permeability) or approximated by a single-valued magnetization curve. The comparisons can be seen in Fig. 1.1. The inclusion of hysteresis is mostly used for prediction of power losses in electrical devices, which is related to the area surrounded in hysteresis loops [3, 4]. When the simulation involves soft magnetic materials with a small coercive field, the calculation error is acceptable when applying the single-valued nonlinear approximation. But in the case of hard or semi-hard materials, the nonhysteretic model may lead to considerable errors [9].

![Figure 1.1: Comparisons of the material capabilities approximated in calculation. (a) linear (constant permeability), (b) single-valued nonlinear, (c) inclusion of hysteresis.](image)

To incorporate hysteresis models in FE codes, several points need to be considered. A stable hysteresis model which can accurately describe hysteresis behaviour of materials is required. Furthermore, the inclusion method should be considered from a computational point of view. An efficient procedure for the handling of hysteresis nonlinearity is preferable. For FE users or industry designers, the identification process is another important factor, which seriously affects the calculation accuracy. If this process is too complex, interest in modelling hysteresis will decrease.

1.2.1 Models of Hysteresis

Until now, many hysteresis models have been proposed and applied in electromagnetic computation. Comparisons and evaluations have been made by Iványi in [10].

Attributing to accuracy, efficiency and robustness, the Preisach model of hysteresis is the most common choice in electromagnetic analysis [1]. Therefore, the inverted model proposed in this work is based on this model. It was first published by F. Preisach in 1935 and attracted great research interest during the next decades [11, 12, 13]. The basic properties of this model, such as return-point memory and wiping out property were stated in Madelung rules [14]. M. Kranoselskii proposed mathematical expression for this model and made it available for many physical hysteresis phenomena [15]. I. D. Mayergoyz researched this model in depth in [16]. He proposed the discrete formulation of the Preisach model and the identification method
with the Everett function. This work has made a great contribution and has benefitted the application of the Preisach model in numerical computation [17].

The classic Preisach model is a static scalar model. In the past years, many extensions and modifications attempts have been made and published in papers. These models are called Preisach type models [18]. Della Torre included the dynamic effect and proposed a moving Preisach model [19]. Zirka researched congruency problems and proposed a history-dependent model [20]. Adly introduced the artificial neural network technology to the identification of the Preisach model [21, 2]. The scalar model has also been extended to a vector one [16, 22, 6, 23].

The Preisach model is usually considered as a phenomenological and mathematical model rather than a truly physical one [18]. However, from calculation point of view, the application of the Preisach model provides great advantages in computation [3, 2].

Another frequently used hysteresis model is the Jiles-Atherton model (JA). Compared with the Preisach model, the JA model is considered to be based more closely on the physics of magnetism [24]. As the domain wall motion and spinning is taken into account, the magnetization process is determined from the energy analysis [25]. An important comparison was made between the JA model and the Preisach model by Philips in [26].

Other important models, such as the Stoner and Wohlfarth model and the Hodgdon model, are not as popular as the Preisach model. This may be due to their limitations of generality or computation speed.

1.2.2 Inclusion of Preisach Model

As has been introduced in the motivation, hysteresis models must be inverted to include them in a FE scheme. In FE formulations, the magnetic flux density \( B \) can be directly obtained from the magnetic vector potential, so that it is considered to be \( B \)-oriented in some publications [5]. On the other hand, like most of the hysteresis models, the Preisach model is considered to be \( H \)-based [5]. As it employs the magnetic field intensity \( H \) as input, it takes the magnetization \( M \) or magnetic flux density \( B \) as the output quantity. Thus, the \( H \)-based Preisach model has to be inverted for it to be coupled with FE codes.

Iteration Method

To deal with this nonlinear problem, an iterative method is usually used to locally inverse the hysteresis models. Saitz and Dlala have made important reviews of this method, especially for the FE analysis of the electromagnetic field in electrical machines [27, 3]. The most commonly applied are the fixed-point method and the Newton-Raphson method [27, 8, 3, 6]. The fixed-point method converges stably but slowly. The Newton-Raphson method uses derivatives to accelerate its convergence, hence, the existing continuous derivative is crucial to obtain converged solutions. To ensure accuracy and convergence of the results, a large amount of computation is always required in FE analysis, which may reach 90% of the total CPU time [28, 5].
Inverted Preisach Model

In order to circumvent the iterative procedure, Takahashi et al. proposed applying an inverse weight function method [29]. Essentially, it is an attempt to construct an inverted ($B$-based) Preisach model, which employs the magnetic flux density $B$ as the input. The magnetic field can be directly calculated as output quantity. In this work the inverted model still uses the relay switch operator in the forward Preisach model. The inverted Preisach weight function is calculated from measured reversal curves. A large amount of experimental data and calculation is required for this approach. To simplify this method, an attempt was made to find suitable analytical weight function for the inverted Preisach model. However, it is a difficult task because the distribution of the weight function is quite complicated [1].

Dlala offered a definition for the inverted Preisach model in [5]. In some publications, the role of $H$ and $B$ has been exchanged in the inverted Preisach model [5, 7, 1]. However, this approach is called the pseudo-inverse of the Preisach model, because it used the same Preisach operator and retains all the basic properties of the Preisach model [3]. This feature offers the pseudo-inverse approach great advantages in calculation. The problem regarding the weight function can be solved by applying the Everett function method, which is commonly applied in the identification of the forward Preisach model [16]. The pseudo-inverse of the Preisach model has been extended to a vector model in [5, 1].

To evaluate the computation efficiency, Dlala made a comparison between the iteration method and the pseudo-inverse approach. As no iteration is required in FE simulation incorporating with the $B$-based model, the calculation time was 10 times shorter than the simulation with iteration method [5].

1.2.3 Identification of the Preisach Model

The identification of the model plays a significant role in the accuracy of the results. The weight function of the $H$-based Preisach model has been analytically approximated by both Gaussian [30] and Lorentzian functions [31]. However, the results produced by applying analytical weight functions do not approximate well to the measurement data [32]. The Everett function method is usually applied in identification [16]. It requires a family of first-order reversal curves as experimental data [see Fig. 1.2]. However, the measurement of FORCs is difficult and tedious [33], and this method is criticized because experimental errors may lead to inaccuracy of simulation results [1].

The high requirement for experimental data may lead to a lack of attraction for FE users. To simplify the identification process, Naidu proposed an approach applying a major loop to identify the forward Preisach model [34]. Dlala proposed an algorithm to identify the $B$-based model [1]. The major loop was firstly approximated by material data written in catalog, such as coercivity and saturated magnetic flux. By calculation, FORCs were generated with reversal points controlled by $B$. This identification method uses less experimental data but the calculation procedure is relatively complex.

In this thesis, reference was made to the approach proposed by Sutor in [35, 23]. A continuous analytical weight function was proposed for a forward Preisach model. Limited amount of parameters in the function were identified by a quasi-Newton method algorithm to optimize
1.2 State of the Art

Figure 1.2: Experimental data for identification. (a) FORCs (b) Major hysteresis loop.

the mean squared error (MSE) between the measured and simulated hysteresis loops.

1.2.4 Characterization Methods for Magnetic Hysteresis

The identification of hysteresis models requires reliable experimental data. There are many different characterization methods. As magnetic performance is strongly shape dependent and could be influenced by different measuring conditions [24], the applied methods and sample shapes should be indicated for validation.

The Epstein frame is a standardized method for determining the magnetic properties of electrical steels. According to standards (e.g., IEC 60404-2) this method shows excellent repeatability and reliability. To decrease the sample amount, a 25cm Epstein frame is applied instead of the classic one (50cm frame) [36]. However, this method is frequently criticized because the effective magnetic path length (EML) is approximately determined for all kinds of materials. When the steels under test are grain oriented, the difference to the real EML can be significant [24].

Nowadays the single sheet tester (SST) method plays a more important role in the characterization of electrical steels. Antonelli conducted numerical analysis on the field distribution in Epstein frame and SST [37]. The simulation results proved that the field distribution is more uniform in SST. The magnetic properties determined with the Epstein frame are considered to be the average magnetic properties of the sample material.

Vibrating sample magnetometer (VSM) is applied to sensitively detect the magnetic moment. It was developed by Van Oosterhout and Foner in 1950s [38, 39]. The approach has been greatly improved by applying Mallinson pickup coils and lock-in amplifier [40, 41]. Nowadays, VSM has become a standard instrument and is most widely used in professional magnetism laboratories. In the modern magnetic materials research field, especially in magnetic recording technology, the investigation of magnetization vectors is attracting more and more attention. By adding more pickup coils systems, VSM can be extended to 2D or 3D vector VSM. What’s more, rotational field measurements can be carried out with VSM by
1 Introduction

mechanically rotating the tested sample in magnetic field [24, 39].

1.3 Objectives and Achievements in Thesis

The objective of this thesis is to propose a new hysteresis model which can be implemented in FE formulations. The model should be capable of accurately describing hysteresis phenomena in ferromagnetic materials, and efficiency is a further important factor. The model should be constructed and optimized from the view of computation and application.

• Inverted Preisach Model

Iterations can be circumvented by directly applying the inverted model in FE analysis. The inverted model was constructed by exchanging the input and output relation in the classic Preisach model (CPM). Unlike the pseudo inverse of the Preisach model, the proposed inverted hysteresis model utilizes a new switch operator, which was modified from the relay operator in the CPM.

By applying the modified switch operator, the inverted model retain the wiping out and congruency properties. The corresponding weight function has a completely positive distribution in the Preisach plane. This allows the weight function distribution to be modelled. A differentiable weight function was proposed for the inverted model, which can be identified from less experimental data.

• Simple Identification Process

The inverted model is identified from major loops. Based on the quasi-Newton method, an algorithm is applied to optimize the mean squared error (MSE) between the measured and simulated data. With this method, parameters in the differentiable weight function can be rapidly determined. The inverted model was verified for both soft and hard magnetic materials. Besides symmetric hysteresis loops, asymmetric hysteresis curves, such as minor loops or FORCs can also be simulated. By comparison, the simulation results produced by the inverted hysteresis model show good approximation to the measurement data.

• Efficient Calculation Procedure

The calculation procedure has been optimized by applying a numerical approximation method, by which the double integration of the weight function can be avoided.

Similar to the calculation in pseudo inverse method, the Preisach triangle is discretized by uniform grid in preprocessing stage. Integrations of the weight function over grid elements are approximated and stored in a table. The integral expression of the inverted model is rewritten in a discrete form, according to which the output is calculated by adding or subtracting the values interpolated in the precalculated table. During calculation it is not necessary to store or update the state of all switch operators in the Preisach plane. In this way, much memory space is saved and the calculation efficiency is improved.
1.4 Structure of Thesis

- Reliable Experimental Data

Characterization of the electrical steels was made by means of an Epstein frame apparatus. The obtained results show good consistency with the standard material data. Measurements can be carried out at different temperatures and frequencies. To evaluate the influence caused by plastic deformation, local magnetic properties were determined with a double C-yoke SST. The characterization of thin films was realized by applying VSM. In addition to scalar determination, magnetization components both parallel and perpendicular to the applied field were determined by applying another pickup set. Furthermore, vector investigation was carried out in rotational field measurements by mechanically rotating the tested sample.

1.4 Structure of Thesis

In the whole thesis, the SI system of units is applied. The thesis is structured as follows,

- In chapter 2, the theoretical background of ferromagnetism is generally introduced. In this thesis, most research efforts were focused on modeling and the characterization of ferromagnetic materials. As the symbol of magnetism, hysteresis is discussed from its physical origin, which is the foundation for many important hysteresis models, such as the Preisach model. To include the Preisach model in FE analysis, the FE formulations for electromagnetic field calculation are shown in the last section.

- In Chapter 3, the proposed inverted Preisach model is described with details, e.g. the modification of the relay operator and the analysis of the weight function distribution. For comparison, the pseudo inverse is introduced in this chapter.

- In Chapter 4, three applied characterization methods, the Epstein frame, double C-yoke SST and VSM, are separately introduced. In addition to their main measurement principle, various measurement results are shown to illustrate the sample properties.

- In Chapter 5, the identification method and procedure are described. The accuracy of the model can be evaluated from comparisons between measurement and simulation results.

- In Chapter 6, calculation of the inverted model is discussed from a computational point of view. The applied numerical method and procedures are introduced in this chapter. The efficiency improvement is evaluated on the optimized procedure.

- In Chapter 7, a summary and outlook is made with regard to this thesis.
2 Theoretical Background

2.1 Ferromagnetism

Ferromagnetic materials are widely applied attributing to their strong magnetism. To characterize and simulate the magnetic behavior of ferromagnetic materials, it is necessary to understand the physical background of ferromagnetism.

2.1.1 Classification of Magnetism

In physics, according to different magnetic behavior, substances are classified as diamagnetic, paramagnetic, or ferromagnetic, etc [42]. As shown in Fig. 2.1, magnetic orders and temperature dependency vary in different types of substances. In comparison with strong ferromagnetism and ferrimagnetism, paramagnetism and diamagnetism exhibit a weak response to an external magnetic field, which can only be detected by sensitive instruments.

![Classification of magnetism](image)

Figure 2.1: Classification of magnetism. Magnetic orders are shown on the left, temperature dependence of magnetic behavior is shown on the right. $T$ is temperature, $\chi$ is the magnetic susceptibility, $M_s$ is the spontaneous magnetization. In (c) and (d) substances are in zero applied field within a single domain (below $T_c$), and in a group of atoms (above $T_c$) [42].
Table 2.1: Curie Temperatures for Some Ferromagnetic and Ferrimagnetic Materials

<table>
<thead>
<tr>
<th>Ferromagnetic Materials</th>
<th>Ferrimagnetic Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substance</td>
<td>$T_c$(K)</td>
</tr>
<tr>
<td>Ni</td>
<td>627</td>
</tr>
<tr>
<td>Fe</td>
<td>1043</td>
</tr>
<tr>
<td>Co</td>
<td>1388</td>
</tr>
<tr>
<td>Gd</td>
<td>292</td>
</tr>
<tr>
<td>Dy</td>
<td>88</td>
</tr>
</tbody>
</table>

Diamagnetic materials exhibit negative magnetism, which is independent of temperature [see Fig. 2.1(a)]. As another weak magnetism, paramagnetism exhibits relatively strong temperature dependency [see Fig. 2.1(b)]. According to the Curie law,

\[ \chi = \frac{M}{H} = \frac{C}{T}, \]  

where $C$ is the Curie constant per gram [42]. The distribution of magnetic moments are disordered, which is due to the disturbance of thermal energy [24]. When temperature increases, the susceptibility decreases linearly.

Compared with paramagnetic material, ferromagnetic material exhibits strong magnetism even in the absence of an external magnetic field. It is due to spontaneous magnetization resulting from interatomic exchange interaction [43, 24]. As is shown in Fig. 2.1(c), spontaneous magnetization ($M_s$) decreases with the increase of temperature. Above a certain temperature ($T_c$), ferromagnetic materials begin to exhibit paramagnetic behavior, where the susceptibility $\chi$ of ferromagnetic material is described in Curie-Weiss law

\[ \chi = \frac{C}{T - T_c}, \]  

where $T_c$ is the boundary temperature and called Curie temperature. In Table 2.1 the Curie temperature of some ferro- and ferrimagnetic materials are listed. It is an important parameter in design of equipment and metal processing.

In Fig. 2.1(d), it can be seen that like ferromagnets, ferrimagnetic materials exhibit spontaneous magnetization below the Curie temperature, but become paramagnetic above it. However, there are two different sublattices in ferrimagnetic materials shown in magnetic orders of Fig. 2.1(d). On different sublattices the magnetic moments are opposed but not equal, which leads to a spontaneous magnetization in ferrimagnetic materials.

In industry, ferrits are important materials alongside ferromagnets. In addition to their economic advantage, they are mostly applied in high-frequency conditions due to their high resistivity.
2.1.2 Magnetization and Domain Distribution

Magnetic Domains

As mentioned in the previous chapter, ferromagnetic materials still exhibit magnetization even in the absence of an external magnetic field. However, ferromagnetic materials can be demagnetized despite the internal “molecular field” [44, 24]. To explain this phenomenon, Pierre Weiss assumed that material is divided into many small regions (about 1-100 µm), which exhibits spontaneous magnetization in different directions. The small regions are known as domains, and the boundaries between different domains are called domain walls. When material is demagnetized, multiple domains are formed, which results in zero net magnetization [see Fig. 2.2(c)]. In saturation state, most of the domains are oriented along the external field direction.

![Diagram of magnetic domains with minimization of magnetostatic energy.](image)

Figure 2.2: Diagram of magnetic domains with minimization of magnetostatic energy. $H_s$ in (a) is the stray field caused by spontaneous magnetization in magnet, (b) the formation of domain structure reduces the magnetic energy, (c) with multiple domains the whole magnetization is zero, the magnet is in minimum energy state [45].

When a ferromagnet or a ferrimagnetic material is heated above its Curie temperature, the uniform magnetization within a domain spontaneously disappears. Each atom has its own direction of magnetic moment, which leads to paramagnetic behavior.

In 1935, it was mathematically proved that domains form to ensure the lowest magnetostatic energy [46]. In 1949 the domain structure in silicon-iron single crystals was published by H. J. Williams, R. M. Bozorth, and W. Shockley [47]. This was considered as direct experimental evidence of the domain theory. Since that time, domain theory has been essential in explaining the shape of a magnetization curve or the mechanism of magnetic hysteresis [42, 24].

Magnetization and Domain Distribution

When a ferro- or ferrimagnetic material is placed in an increasing magnetic field, its net magnetization is increased up to a limited value called the saturation magnetization. This process is called technical magnetization as the increase in net magnetization is not the same as a
change in intensity of the spontaneous magnetization in Fig. 2.1(c) and (d). During the technical magnetization the net magnetization is changed by domain wall displacement and by rotation of domain magnetization [43].

Fig. 2.3 presents domain arrangements in different states of magnetization. Suppose the magnetization starts from a fully demagnetized state, then it is at the origin 0 in Fig. 2.3. When a small positive magnetic field is applied, as shown at points 1 and 2, the domains whose direction of spontaneous magnetization is close to the direction of the applied field start to grow. At the same time, the other domains are gradually eliminated. When the applied magnetic field is further increased, most of magnetizations are lined up almost in parallel as shown at point 3. Finally the net magnetization changes to a lesser degree with the increase of an applied field. The magnetization is realized by rotation of the direction of magnetization. As shown at point 4, it is at saturation state. The curve from the demagnetized state to saturation (path 0 – 4) is called the primary magnetization curve [24] or normal induction curve [42].

![Diagram of domain arrangements](image)

Figure 2.3: Domain arrangements for various status of magnetization [24]. The material is in demagnetized state at 0. When the applied magnetic field increases, the magnetization increases along the path 0-1-2-3-4 until saturation state.

2.1.3 Hysteresis Curves and Magnetic Properties

Major Hysteresis Loops

After saturation has been reached in Fig. 2.3, if the applied magnetic field is removed, the magnetization will not return back to zero. Instead, there is still some magnetization left in the material. To drive the magnetization back to zero, a field in the opposite direction should be applied to the material. This behavior is called magnetic hysteresis. It is considered as typical symbol of magnetism and an important character of magnetic properties [18].
A hysteresis loop is a plot showing the variation of magnetization with magnetic field. In material catalogues it is normally in terms of the flux density $B$ (or the polarization $J$) and the magnetic field strength $H$. According to (2.3), $H$ forms part of $B$, after saturation, a constant $M_s$ is reached, but $B$ continues to increase with magnetic field $H$ [42].

$$B = \mu_0(H + M) \quad (2.3)$$

$$B = \mu_0H + J \quad (2.4)$$

where $\mu_0$ is the permeability of free space.

As shown in Fig. 2.4, the magnetic flux density increases from the demagnetized state to saturation along the path 0-1. It is similar to the magnetization curve shown in Fig. 2.3. By measuring the primary magnetization curve, the permeability of material $\mu$ can be obtained by

$$\mu(H) = \mu_0\mu_r(H) = \frac{B}{H}. \quad (2.5)$$

where $\mu_r$ is known as relative permeability.

If we start decreasing the magnetic field strength, due to an irreversible change of domain walls’ locations, the return path is along the path 1-2 but not along the path 1-0. At zero magnetic field (point 2), there is a retentivity or residual induction $B_r$ in the magnetic material. To decrease the flux density back to zero (point 3), a magnetic field is applied in the opposite direction. This field is called coercivity $H_c$. If we continue increasing the magnetic field in a negative direction, the negative saturation $-B_s$ will be reached (point 4). If the magnetic field returns back to zero then increases to reach the positive saturation state (point 1), the path 1-2-3-4-5-6 forms a closed loop known as the major hysteresis loop.

**Minor Hysteresis Loops and FORCs**

In the primary magnetization process, if a reverse happens at some intermediate point before saturation, such as point a in Fig. 1.5(a), a small closed loop can be obtained inside the major loop. This can be achieved by symmetrically reversing the corresponding field. As shown in Fig. 2.5(a), the small loop whose tip is at point a is called a minor hysteresis loop. On this minor loop, the rest induction at point b is called remanence. The magnetic field at point c is called coercive field. They are named differently to the retentivity and coercivity of major loops [42]. Besides symmetrical (symmetrical to origin) minor hysteresis loops (loop abc), asymmetrical minor loops can also be formed at points d and e inside the major loop in Fig. 2.5(a).

In identification of hysteresis models (e.g. Preisach hysteresis model), as shown in Fig. 2.5(b), first-order reversal curves (FORCs) are usually measured by decreasing the field at some points on the ascending major loop (or increasing the field at some points on the descending major loop) [48].

In AC field measurements, primary magnetization curves are produced by joining tips of minor loops. To achieve the demagnetized state, material is firstly magnetized to saturation. By applying a series of alternating fields with slowly decreasing amplitude, the induction of
Figure 2.4: A typical major hysteresis loop. From demagnetized state at 0 to the saturation state $B_s$ at 1, the 0–1 path forms as a primary magnetization curve. When $H = 0$, the magnetic flux at 2 is the remanent flux density $B_r$. To decrease the magnetic flux back to zero, coercive field strength $H_c$ in negative direction is applied. The path 1-2-3-4-5-6-1 forms a major hysteresis loop.

The material becomes smaller and smaller. Another demagnetization method is called thermal demagnetization. As mentioned in Chapter 2.1.1, when ferro- or ferrimagnetic material is heated above its Curie point, it becomes paramagnetic. After cooling it in the absence of a magnetic field, the material can be demagnetized [42].

**Magnetic Properties Determined from Hysteresis curves**

The characterization of magnetic materials is usually performed by measuring hysteresis loops, from which much information can be extracted regarding the magnetic properties. E.g. the coercivity, retentivity, permeability, core loss, etc. According to these parameters, magnetic materials are applied in various fields. Normally these parameters are presented in material catalogues.

Of these parameters, the core loss $P_c$ is particularly important, especially for electrical steels. It describes the energy irreversibly transformed into heat during one magnetization cycle. According to engineering concept, hysteresis loop areas are applied to estimate the loss per unit volume $V$ injected into a magnet in a magnetization cycle [18]. Normally, the core loss $P_c$ is determined as specific power loss $P$ related to the mass $m$ of the sample [24]. By introducing the material density $\rho$, the core loss $P_c$ can be determined by

$$P_c = \frac{P}{m} = \frac{V}{Tm} \oint H \, dB = \frac{1}{T\rho} \oint H \, dB,$$

(2.6)

where $T$ is one cycle of the magnetization process. There are various reasons for such
losses, eddy current is a significant factor. Therefore, core losses is usually specified at a specified frequency and maximum flux density.

According to different magnetic properties, ferromagnetic materials are divided into three major application categories. As shown in Fig. 2.6, hard magnetic material has a relatively high coercive field and residual magnetization. It is desirable for permanent magnets because it is difficult to magnetize (or demagnetized) and it remains a largely saturated magnetization when the driving field is removed. In contrast, soft magnetic materials have a very low coercive field but high permeability. They are mostly used in AC applications such as magnetic cores of transformers, motors and inductors, as low hysteresis loss is preferred and required in order to minimize energy dissipation. Occupying the middle ground between the former two materials, semi-hard magnetic materials have a moderately high coercive field and residual magnetization. For this reason, they are mostly applied in magnetic recording.

The limit to distinguish between the three kinds of materials is not so rigid. Typically, the materials with coercivity less than 1000A/m belong to soft magnetic materials. For hard
magnetic materials, coercivity is greater than 50000A/m. Sometimes the same material may be either magnetically soft or hard, depending on its physical condition [43, 42].

2.1.4 Magnetic Anisotropy

There are several factors which affect the shape of magnetization curves or hysteresis loops, including temperature and mechanical stress. One strong factor is known as magnetic anisotropy, which means the magnetic properties depend on the direction of magnetization.

Consider a single crystal of iron with body centered cubic structure. As shown in Fig. 2.7, magnetization is applied along [100], [110] and [111] crystallographic directions. The shapes of the magnetization curves obviously differ according to the three crystallographic directions. Along the [100] axis the magnetization is easiest, in other words, the saturation can be reached with a low magnetic field. This direction is called “easy axis”. In contrast, the [111] axis is called the “hard axis” because along this axis the magnetization is most difficult. Such dependence of magnetic behavior on crystallographic directions is known as magnetocrystalline anisotropy [42]. It is intrinsic to material and assumed to be the result of spin-orbit-lattice interaction [24].

From an energy point of view, the magnetization or demagnetization along the easy axis cost less energy than along noneasy axis. This can be applied to minimize the energy loss of electrical steel in transformers. As it is expensive to produce the single crystal alloys, the practical way is to develop anisotropy in polycrystalline alloys. Goss patented an invention of textured silicon-iron electrical steels [24]. By applying rolling treatments to steels, one easy axis of the crystals is positioned close to the rolling direction (RD). In transformers magnetic field is applied parallel to the RD of the electrical steels.

Furthermore, anisotropy can be inducted by plastic deformation, annealing, irradiation, etc [42]. For magnetic thin films, uniaxial anisotropy can be induced by applying a magnetic field during the deposition process. If the magnetic behavior is the same in all directions, the material is characterized as isotropic. However, due to the demagnetization field, isotropic material
with different sample shapes may also exhibit anisotropy phenomenon, which is called shape anisotropy. This is the reason why sample shapes should be indicated in magnetic characterization.

2.2 Preisach Model of Hysteresis

Various models have been proposed for simulating hysteresis behavior [10, 16, 48]. Some of them are generally based on real physics of the magnetization process, while some are purely mathematical or phenomenographic models. Many hysteresis models are evaluated by Ivanyi in [10].

Among these hysteresis models, the Preisach model is widely applied, due to its accuracy and generality. The classic Preisach model is a static (rate-independent) scalar model, ferromagnetic material is assumed to be assembly of independent domains. The model can trace minor loops and other complex magnetizing processes [48] but it is usually described as a phenomenological rather than physics-based model.

2.2.1 The Relay Switch Operator

The classic Preisach model was initially proposed by F. Preisach in 1935 [11]. It assumes that the magnets are composed of separate magnetic “particles” (or elementary domains). These particles are assumed to exhibit simplest hysteresis behavior. As shown in Fig. 2.8(a), these particles are driven by the same external magnetic field \( H \) and have rectangular hysteresis loops. Each hysteresis loop has the same saturation magnetic moment \( m_s \) but different switch magnetic fields \( H_\alpha \) and \( H_\beta \). The switching happens when

\[
\hat{m}(H_\alpha, H_\beta) = \begin{cases} 
+m_s & H > H_\alpha, \\
-m_s & H < H_\beta.
\end{cases}
\]

When \( H \) is between \( H_\alpha \) and \( H_\beta \), the state of the magnetic moment depends on the history of applied field \( H \). To describe this simplest hysteresis behavior mathematically, the relay switch operator \( \hat{\gamma}_{\alpha\beta} \) is applied as an elementary hysteresis operator. It can be represented by a rectangular loop shown in Fig. 2.8(b). The switching happens at \( \alpha \) and \( \beta \) values of the input. The relay switch operator \( \hat{\gamma}_{\alpha\beta} \) is defined as

\[
\hat{\gamma}_{\alpha\beta}[H](t) = \begin{cases} 
+1 & \text{if } H(t) \geq \alpha, \\
\hat{\gamma}_{\alpha\beta}[H](t^-) & \text{if } \beta < H(t) < \alpha, \\
-1 & \text{if } H(t) \leq \beta.
\end{cases}
\]

2.2.2 The Classic Preisach Model

The classic Preisach model was generalized in a mathematical idea by Krasnoselskii in the 1970’s. It can be applied as a mathematical tool to any physical hysteresis phenomenon.
2 Theoretical Background

It was significantly improved by Mayergoyz in [16]. The Preisach model is defined as a superposition of relay switch operators $\hat{\gamma}_{\alpha\beta}[H]$ to calculate the overall magnetization. Each relay operator is statistically weighted by a weight function $\mu(\alpha, \beta)$ dependent on $\alpha$ and $\beta$. The Preisach model is expressed as:

$$M(t) = \hat{\Gamma}[H](t) = \int_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta}[H](t) d\alpha d\beta,$$

where $\hat{\Gamma}$ is applied as a notation of the Preisach hysteresis operator, which takes the magnetic field $H$ as input, the magnetization $M$ as the output.$^1$

The mechanism of the Preisach model can be simply illustrated by a diagram shown in Fig. 2.9(a). Relay operators are connected in parallel. Depending on the input $H(t)$ at a certain time $t$, some relay hysterons switch up ($+1$), while others switch down ($-1$). Each output is multiplied by their own weight $\mu(\alpha, \beta)$. Finally, these individual outputs are integrated over the whole valid domain.

Each elementary hysteresis unit can be represented by a point $(\alpha, \beta)$ in the Preisach plane $P = \{(\alpha, \beta) \in \mathbb{R}^2 : \alpha \geq \beta \}$ [16, 49]. Depending on the output of the relay operator at different points, the Preisach plane can be separated into two subsets $S^+(t)$ and $S^-(t)$ for hysteresis units in the up or down states [see Fig. 2.9 (b)].

$$(\alpha, \beta) \in \begin{cases} S^+(t) & \text{if } (\alpha, \beta) \in P \cap \hat{\gamma}_{\alpha\beta}[H](t) = +1, \\ S^-(t) & \text{if } (\alpha, \beta) \in P \cap \hat{\gamma}_{\alpha\beta}[H](t) = -1. \end{cases} (2.10)$$

$^1\hat{\Gamma}$ is distinguished from the inverted Preisach hysteresis operator $\hat{\Upsilon}$, which takes the magnetization $M$ as input, the magnetic field $H$ as the output. The inverted Preisach model is introduced in the following chapters.
2.2 Preisach Model of Hysteresis

Figure 2.9: Schematic interpretation of the Preisach model. (a) is the block diagram describing the Preisach model in (2.9). (b) shows the Preisach plane.

In calculation, (2.9) can be expressed as

\[
\hat{\Gamma}[H](t) = \int \int_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta - \int \int_{S^-(t)} \mu(\alpha, \beta) d\alpha d\beta.
\] (2.11)

As shown in Fig. 2.9 (b), the subsets \(S^+(t)\) and \(S^-(t)\) are divided by an interface \(L(t)\). At a certain time \(t'\), the interface \(L(t')\) reflects the state of the Preisach operator \(\hat{\Gamma}[H](t')\). During a period of time \(t \in [t_0, t_n]\), the movement of \(L(t)\) reflects the system’s dynamics [49].

2.2.3 Geometric Interpretation of the Preisach Model

The Preisach model can be geometrically interpreted with an example shown in Fig. 2.10. The input \(H(t)\) starts with a value \(H_0\) less than \(-H_s\) [see Fig. 2.10 (a)]. All relay operators are equal to \(-1\). In other words, \(S^-(t)\) occupies the whole Preisach plane \(S\). This corresponds to the negative saturation state [16]. The input \(H(t)\) is subdivided into four segments with local extrema \(\{H_1, H_2, H_3, H_4\}\).

S-I. When \(H(t)\) increases from \(H_0\) to \(H_1\), according to (2.19), the relay operators in area \(\alpha \leq H_1\) switch up [see \(S^+\) in Fig. 2.10 (c)- I]. At the same time, the operators in area \(\beta \geq H_1\) switch down. In the area \(\beta < H_1 < \alpha\), which can be called the memory area, the relay operators keep the old value at previous instants of time. As it starts from a negative saturation state, the operators in memory area keep \(-1\) state. The interface \(L(t)\) is horizontal and moves upwards as \(H(t)\) is increased.

S-II. When \(H(t)\) decreases from \(H_1\) to \(H_2\), in the memory area \(\beta < H_2 < \alpha\), the relay operators should keep the state in Fig. 2.10 (c)- I]. Geometrically, this decrease forms a vertical link of \(L(t)\) and it moves from right to left. In this way, the area of \(S^+(t)\) becomes smaller, the reduction area is occupied by \(S^-(t)\) area. This results in a reversal curve at \(H = H_1\) [see Fig. 2.10 (b)- II].
2 Theoretical Background

Figure 2.10: Geometric interpretation of the Preisach model. (a) shows the process of input $H(t)$, the whole process is subdivided into four segments I-IV. (b) shows the corresponding output $\tilde{\Gamma}[H(t)]$. (c) are the Preisach planes in different segments I-IV, where $L(t)$ is the interface between $S^+$ and $S^-$.

S-III. In this segment, $H(t)$ increases from $H_2$ to $H_3$. The mechanism of memory formation is the same as mentioned above. With the increase of $H(t)$, on the right of dashed line $\beta = H_2$, the horizontal interface moves upwards until $\alpha = H_3$. The interface $L(t)$ is a staircase line whose vertices have $\alpha$ and $\beta$ coordinate as $(H_1, -H_4)$, $(H_1, H_2)$, $(H_3, H_2)$, and $(H_3, H_3)$. They are local extrema of the input history. By this method, the Preisach model keeps and updates the memory (local extrema) in the interface line $L(t)$.

S-IV. In this segment $H(t)$ continues to increase until $H(t) = H_4$. This monotonic increase of input $H(t)$ results in a horizontal final link of $L(t)$ which moves upwards until $H_4$ is reached. As $H_4 > H_1 > H_3 > H_2$, all vertices whose $\alpha$-coordinates are below $H_4$ have been wiped out [see Fig. 2.10 (c)- IV]. In other words, the memory kept in $L(t)$ is erased. This wiping out behavior also occurs in the monotonically decreasing condition. It is known as the wiping out property of the Preisach model.
2.3 Finite Element Analysis of Magnetic Field

As can be seen in Fig. 2.10 (b)- IV, a minor loop is formed at $H = H_1$. It is assumed that the minor loops developed by back and forth variations between two input values should be congruent, which is considered as the congruency property of the Preisach model.

The above-mentioned wiping out and congruency properties are the necessary and sufficient conditions for a hysteresis nonlinearity to be described by the Preisach model [16].

2.3 Finite Element Analysis of Magnetic Field

Accurate analysis of magnetic field is essential for the evaluation and design of electrical devices. The finite element (FE) method is one of the most widely used numerical techniques in computational electromagnetism. It has been successfully applied to solve magnetic fields in complicated geometries such as electrical machines and transformers [1].

In the first part of this section, the partial differential equation (PDE) of electromagnetic fields is introduced. In the second part, common methods for the approximation of nonlinear hysteretic relation are presented. The description provided indicates why hysteresis models need to be inverted when it is included into the FE analysis.

2.3.1 Partial Differential Equations of Electromagnetic Analysis

The problem of numerical analysis on a macroscopic level is solving partial differential equations (PDE) under prescribed boundary conditions [2]. The FE method is widely applied to approximate the solution of the partial differential equations. The PDEs for magnetic field analysis are derived from Maxwell’s equations, which were proposed by James Clerk Maxwell in 1862. Based on the work and experiments of Ampere, Faraday and Gauss, Maxwell’s equations state the relationships between the fundamental electromagnetic quantities [50].

In static and quasistatic (eddy current) magnetic fields, the displacement current can be neglected. The governing parts of Maxwell’s equations are expressed as follows

\[
\nabla \times \mathbf{H} = \mathbf{J}, \tag{2.12}
\]
\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2.13}
\]
\[
\nabla \cdot \mathbf{B} = 0, \tag{2.14}
\]

where $\mathbf{E}$ is the electric field intensity. The electric current density $\mathbf{J}$ is applied here as the source of electromagnetic field. In addition, the constitutive relations are appended to Maxwell’s equations to describe the properties of the media.

\[
\mathbf{J} = \sigma \mathbf{E} = \sigma (\mathbf{E}_i + \mathbf{E}_s), \tag{2.15}
\]
\[
\mathbf{B} = \mu \mathbf{H} = \mu_0 (\mathbf{H} + \mathbf{M}). \tag{2.16}
\]
Here it is assumed there are no moving bodies in the simulation area. \( E_i \) is the impressed (or irrotational) electric field intensity, \( E_s \) is the solenoidal electric field intensity. Normally, the magnetic reluctivity \( \nu = 1/\mu \) is applied instead of the permeability \( \mu \) in calculation. \( \sigma \) is the electric conductivity of simulated media.

By introducing scalar or vector magnetic potentials, the Maxwell’s equations can be transformed into the desired PDEs [2]. As commonly be applied in FE method, the vector magnetic potential \( A \) is defined by

\[
B = \nabla \times A. \tag{2.17}
\]

Applying (2.16) and (2.17) to the first part of the above mentioned Maxwell’s equations, the PDE for the quasistatic electromagnetic fields is

\[
\sigma \frac{\partial A}{\partial t} + \nabla \times \nu \nabla \times A = J_i. \tag{2.18}
\]

In electromagnetic modeling, \( J_i \) refers to impressed current density due to a given electric potential difference (the current or voltage loaded coil) [50]. It can be obtained by \( J_i = \sigma E_i \).

### 2.3.2 Nonlinear Electromagnetic Field Computation

In the PDE applied in FE analysis (2.18), the magnetic vector potential is employed as the unknown. According to (2.17), the magnetic flux density can be directly obtained as an output quantity. In order to calculate the magnetic field strength, the constitutive relationship is required for application.

Most paramagnetic and diamagnetic materials, such as air, aluminum and copper, are considered as magnetically linear mediums. The permeability or reluctivity is set to be constant. However, ferromagnetic materials are usually involved in electromagnetic simulation, the calculation become more complex because of their nonlinear characteristics. Normally the nonlinearity is approximated by a single-valued magnetization curve [see Fig. 2.11 (a)] in the FE analysis. As shown in Fig. 2.11 (b), the nonlinear relation \( \nu(b) \) can be obtained from this magnetization curve for simulation purposes. With this method, the hysteresis characteristic is completely neglected. To achieve more accurate results, hysteresis must be taken into consideration, especially in calculating the hysteresis losses in electrical machines. This problem can be solved by including hysteresis models in the FE analysis.

Most hysteresis models, such as the Preisach model and the Jiles-Atherton model, employ magnetic field \( H \) as an input variable and take magnetic flux \( B \) (or magnetization \( M \)) as the output quantity. These models can be called \( H \)-based or forward hysteresis models. To include hysteresis models in \( B \)-oriented FE analysis, hysteresis models need to be inverted before they are implemented in FE formulations. Here we introduce a operator \( H^{-1} \) to represent the inverse processing of the hysteresis models.

According to the mentioned conditions, the constitutive relationship is summarized as

\[
H = \begin{cases} 
\nu B & \text{in magnetically linear medium,} \\
\nu(b)B & \text{in magnetically nonlinear medium, hysteresis is ignored,} \\
H^{-1}(B) & \text{in magnetically nonlinear medium, hysteresis is considered.} 
\end{cases} \tag{2.19}
\]
2.3 Finite Element Analysis of Magnetic Field

Figure 2.11: The single-valued magnetization curve (a) and obtained reluctivity curve (b). Measurement was carried out on electrical steel M33035A with Epstein frame method at 50Hz.

The block diagram in Fig. 2.12 illustrates the necessary and two main approaches to inverse hysteresis models. Iterative methods are typically employed in the local inversion of forward hysteresis models. The fixed point method and the Newton-Raphson method are the most commonly applied techniques. However iteration methods always require much computation to obtain converged solutions, especially in transient simulations. It is more efficient to find a inverted hysteresis model, which applies \( B \) as input to calculate \( H \). This inverted model can be directly implemented in FE analysis without iterations.

![Inverse a hysteresis model](image)

Figure 2.12: Block diagram of including hysteresis models into electromagnetic FE analysis.

In this chapter, general introductions have been made to the physical origin of ferromagnetic hysteresis, definition of the classic Preisach model and the FE analysis of electromagnetic field. These theories have been applied in the characterization and modeling of the ferromagnetic materials in this work. In the next chapter, an inverted Preisach model, which is based on the classic Preisach model, is proposed and introduced with details.
3 The Inverted Preisach Model

Most hysteresis models employ the magnetic field as the input variable and take the magnetization or magnetic flux as the output quantity. These models are considered as forward (or \(H\)-based) hysteresis models. The previous chapter explained why (forward) hysteresis models need to be inverted in order to include them in the \(B\)-oriented FE analysis \([1, 5]\). Usually iterative methods are utilized in the local inversion of forward hysteresis models \([5]\). However, such methods require much computation for converged solutions.

In order to circumvent the iterative procedure, much research effort has been made to find a hysteresis model, which takes the magnetization or magnetic flux as input. With this model the magnetic field can be directly calculated without iteration. In comparison with the forward hysteresis models, the roles of input and output are exchanged in this model. Therefore, this type of model is considered as inverted hysteresis model.

As a result of its accuracy, simplicity and efficiency, the classical Preisach model (CPM) has been widely applied within FE applications. The inverse of the CPM is defined by Dlala \([5]\) in a simplified manner as

\[
H(t) = \hat{\Upsilon}[M](t) = \hat{\Gamma}^{-1}\hat{\Upsilon}[H](t),
\]

where \(\hat{\Gamma}\) is the CPM operator in \((2.9)\) introduced in section 2.2. \(\hat{\Upsilon}\) is defined as the inverted Preisach operator. Mathematically, \(\hat{\Upsilon}\) can be considered as the inverse operator \(\hat{\Gamma}^{-1}\). This chapter introduces two different inverted Preisach models. One uses the relay switch operator of CPM and known as the pseudo-inverse approach. The other one is a new inverted Preisach model using a modified switch operator\(^1\).

3.1 The Pseudo-inverse of the Preisach Model

In several published inversion works \([29, 54, 1]\), the inverted model is still considered as a superposition of infinite relay operators \(\hat{\gamma}_{\alpha\beta}\). As shown in \((3.2)\), the magnetic field \(H\) is calculated from the magnetization \(M\). Except the exchanged roles of input and output, the expression looks similar to the forward Preisach model in \((2.9)\), the only difference is it uses a different weight function \(v(\alpha, \beta)\) to realize the inverse operation.

\[
H(t) = \hat{\Upsilon}[M](t) = \iint_{\alpha \geq \beta} v(\alpha, \beta) \hat{\gamma}_{\alpha\beta}[M](t) d\alpha d\beta,
\]

\(^1\)The inverted Preisach model introduced in this chapter has been published in \([51, 52, 53]\).
where the relay switch operator $\hat{\gamma}_{\alpha \beta}$ works in the same way as the one in the forward Preisach model (2.7). The integration is also fulfilled in the Preisach plane $P = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \geq \beta \}$.

$$\hat{\gamma}_{\alpha \beta}[M](t) = \begin{cases} +1 & \text{if } M(t) > \alpha, \\ \hat{\gamma}_{\alpha \beta}[M](t-) & \text{if } \beta < M(t) < \alpha, \\ -1 & \text{if } M(t) < \beta. \end{cases} \quad (3.3)$$

### 3.1.1 The Weight Function in the Pseudo-inverse

By using the relay switch operator $\hat{\gamma}_{\alpha \beta}$, the pseudo-inverse of the Preisach model maintains the basic properties of the Preisach model, such as the wiping out and congruency properties. The past extrema values of input are also stored in memory. These properties provide a strong advantage in numerical calculation. However, the modeling of the inverted weight function $v(\alpha, \beta)$ is difficult, as the “weight” of each relay operator is distributed in a complicated manner in the Preisach plane.

The distribution of weight function can be analyzed with hysteresis reversal curves. To analyze the inverted weight function $v(\alpha, \beta)$, a $H(M)$ reversal curve is used and shown in Fig. 3.1(a). The input $M(t)$ increases from the negative saturation to $\alpha_1$ then to $\alpha_2$. The corresponding outputs at $\alpha_1$ and $\alpha_2$ are $H_{\alpha_1}$ and $H_{\alpha_2}$. At the point $(\alpha_2, H_{\alpha_2})$, a reversal curve is formed by decreasing the input. Finally, the input decreases to $\beta_1$ ($\beta_1 = \alpha_1$). As the output at $\beta_1$ is on the reversal curve from $H_{\alpha_2}$, it is labeled as $H_{\alpha_2 \beta_1}$.

![Figure 3.1: The distribution of inverted weight function in the Preisach plane. (a) is a $H(M)$ reversal curve, (b) is the corresponding Preisach plane. The Inputs $M(t)$ firstly increase from negative saturation to $\alpha_1$, then decrease from $\alpha_2$ to $\beta_1$ ($\beta_1 = \alpha_1$).](image)

According to the work principle of the relay switch operator $\hat{\gamma}_{\alpha \beta}$ described in (3.3), when input increases from $\alpha_1$ to $\alpha_2$, the change of the output can be calculated by the integration of the weight function in the Preisach plane.

$$H_{\alpha_2} - H_{\alpha_1} = 2 \int_{S_1+S_2} v(\alpha, \beta) \mathrm{d}\alpha \mathrm{d}\beta > 0. \quad (3.4)$$
When input decreases from $\alpha_2$ to $\beta_1$, the operators $\hat{\gamma}_{\alpha\beta}$ in triangle $S_1$ switch from $+1$ to $-1$. The output decreases from $H_{\alpha_2}$ to $H_{\alpha_2\beta_1}$. As $\beta_1 = \alpha_1$ and $H_{\alpha_2\beta_1} < H_{\alpha_1}$, the integration of $\nu(\alpha, \beta)$ in area $S_2$ can be calculated as

$$\int_2 \int S \nu(\alpha, \beta) d\alpha d\beta = \int_1 \int S (\alpha, \beta) d\alpha d\beta - \int_1 \int S (\alpha, \beta) d\alpha d\beta$$

$$= \frac{H_{\alpha_2} - H_{\alpha_1}}{2} - \frac{H_{\alpha_2} - H_{\alpha_2\beta_1}}{2}$$

$$= \frac{H_{\alpha_2\beta_1} - H_{\alpha_1}}{2} < 0. \quad (3.5)$$

The integration of $\nu(\alpha, \beta)$ in area $S_2$ is negative, as the integration of $\nu(\alpha, \beta)$ in area $S_1 + S_2$ is proved to be positive in (3.4), it can be assumed that the integration in area $S_1$ should be quite large.

The weight function of Preisach model is supposed to be symmetric in the Preisach plane [16]. From the above analyzed example, we can deduce that the integration of the weight function has a sharp increase over the area near the line $\alpha = \beta$. In the area close to the point $(\alpha_0, \beta_0)$, the weight function has a negative distribution.

According to the analysis method applied above, six measured $H(J)$ reversal curves shown in Fig. 3.2(a) are used to calculate the weight function distribution. The resulted weight function distribution is presented in Fig. 3.2(b). It is obvious that the inverted weight function $\nu(\alpha, \beta)$ is partly positive and partly negative in the Preisach plane. The complicated distribution makes it difficult to find an analytical weight function. Improper modeling may cause inaccuracy and nonsmoothness of the results. As a result, it is necessary to find an alternative method for calculation with the pseudo-inverse of the Preisach model.
3 The Inverted Preisach Model

3.1.2 The Inverted Everett Function

The Everett function method has been widely applied in numerical implementation of the forward Preisach models [16, 1]. With this method, it is not necessary to determine the weight function nor to calculate the double integrals in (2.9). By introducing this approach to the pseudo-inverse of the Preisach model, the problems of the inverted weight function can be solved. To distinguish it from the Everett function for forward models, the Everett function for inverted models is called the inverted Everett function and is defined as

\[ E(\alpha, \beta) = \frac{1}{2}(H_\alpha - H_{\alpha\beta}) = \int \int_{T(\alpha, \beta)} v(\alpha, \beta) \, d\alpha \, d\beta. \tag{3.6} \]

Similarly to the integral over \( S_1 \) in Fig. 3.1, \( E(\alpha, \beta) \) is equal to the integral over a triangle \( T(\alpha, \beta) \), which is related to the output increments along reversal curves.

In numerical calculation, the integral expression of the Preisach model is usually rewritten in a discrete form. Mayergoyz has proposed a discrete expression of the forward Preisach model, which is calculated with the Everett functions [16]. As the pseudo-inverse of the Preisach model remains the basic properties of the forward Preisach model, its discrete expression is formed with reference to the Mayergoyz’s method.

As shown in Fig. 3.3, the Preisach plane can be separated into two subsets \( S^+(t) \) and \( S^-(t) \) by a staircase interface \( L(t) \). The relay switch operators \( \hat{\gamma}_{\alpha\beta} \) in the \( S^+(t) \) area are equal to \(+1\), while the ones in the \( S^-(t) \) area are equal to \(-1\). Therefore, the pseudo-inverse of the Preisach

\[ L(t) \text{ is the staircase interface dividing } S^+ \text{ and } S^- \text{ regions. (a) The input } M(t) \text{ is decreasing, it forms a vertical final link at last vertex of } L(t). \text{ (b) The input } M(t) \text{ is increasing, it forms a horizontal final link at last vertex of } L(t). \]

Figure 3.3: Geometric interpretation of the trapezoid areas in a Preisach plane. \( L(t) \) is the staircase interface dividing \( S^+ \) and \( S^- \) regions. (a) The input \( M(t) \) is decreasing, it forms a vertical final link at last vertex of \( L(t) \). (b) The input \( M(t) \) is increasing, it forms a horizontal final link at last vertex of \( L(t) \).
model in (3.2) can be expressed as

\[
H(t) = \int_S v(\alpha, \beta) d\alpha d\beta - \int_{S^0(t)} v(\alpha, \beta) d\alpha d\beta
\]

\[
= -\int_T v(\alpha, \beta) d\alpha d\beta + 2 \int_{S^+(t)} v(\alpha, \beta) d\alpha d\beta.
\]  

(3.7)

The integration of \(v(\alpha, \beta)\) in the whole Preisach triangle \(T\) can be calculated by an inverted Everett function

\[
\iint_T v(\alpha, \beta) d\alpha d\beta = E(\alpha_0, \beta_0)
\]  

(3.8)

To calculate the integration of \(v(\alpha, \beta)\) in \(S^+(t)\) area by the inverted Everett functions, as shown in Fig. 3.3, the \(S^+(t)\) area can be subdivided into trapezoids (and a triangle) [16]. All the vertices on the interface \(L(t)\) are extrema of the input. Their coordinates in the Preisach plane are represented by \(M_k\) and \(m_k\). The notation \(M_k\) represents the input maximums and \(m_k\) represents the input minimums. When the input \(M(t)\) is decreasing, as shown in Fig. 3.3(a), it forms a vertical final link at last vertex of \(L(t)\). The \(S^+(t)\) area can be subdivided into \(n(t)\) trapezoids \(Q_k\). When the input \(M(t)\) is increasing, as shown in Fig. 3.3(b), it forms a horizontal final link at last vertex of \(L(t)\). The \(S^+\) area can be divided into \(n(t) - 1\) trapezoids and 1 triangle.

When \(M(t)\) is decreasing, the integration of \(v(\alpha, \beta)\) in the \(S^+(t)\) area can be expressed as

\[
\iint_{S^+(t)} v(\alpha, \beta) d\alpha d\beta = \sum_{k=1}^{n(t)} \iint_{Q_k} v(\alpha, \beta) d\alpha d\beta.
\]  

(3.9)

When the input \(M(t)\) is increasing, the integration of \(v(\alpha, \beta)\) in the \(S^+(t)\) area can be expressed as

\[
\iint_{S^+(t)} v(\alpha, \beta) d\alpha d\beta = \sum_{k=1}^{n(t)-1} \iint_{Q_k} v(\alpha, \beta) d\alpha d\beta + \iint_{T_{m(t)}} v(\alpha, \beta) d\alpha d\beta.
\]  

(3.10)

Each trapezoid \(Q_k\) is geometrically a difference of two triangles \(T(M_k, m_{k-1})\) and \(T(M_k, m_k)\). As the integral over a triangle can be calculated by the inverted Everett function (3.6). The integral over a trapezoid \(Q_k\) can be calculated as

\[
\iint_{Q_k} v(\alpha, \beta) d\alpha d\beta = \iint_{T(M_k, m_{k-1})} v(\alpha, \beta) d\alpha d\beta - \iint_{T(M_k, m_k)} v(\alpha, \beta) d\alpha d\beta
\]

\[
= E(M_k, m_{k-1}) - E(M_k, m_k).
\]  

(3.11)

Combine (3.7), (3.8), (3.9) and (3.11), in the decreasing case, the output \(H(t)\) can be calcu-
lated as

\[ H(t) = -E(\alpha_0, \beta_0) + 2 \sum_{k=1}^{n(t)} [E(M_k, m_{k-1}) - E(M_k, m_k)]. \]  \hspace{1cm} (3.12)

In the increasing case, the output \( H(t) \) can be calculated as

\[ H(t) = -E(\alpha_0, \beta_0) + 2 \sum_{k=1}^{n(t)-1} [E(M_k, m_{k-1}) - E(M_k, m_k)] + 2E(M(t), m_{n(t)-1}). \]  \hspace{1cm} (3.13)

In FE applications, the inverted Everett functions can be precalculated at all discrete points of the Preisach plane. All \( E(\alpha, \beta) \) results are saved as an inverted Everett matrix. In the calculation procedure, (3.12) and (3.13) are calculated by interpolating in the inverted Everett matrix. It is relatively fast and saves considerable memory space.

From a computational point of view, this approach is an efficient way of including the Preisach model in FE calculation. However, the inverted model with relay switch operator is not considered as an exact inverse, it is called pseudo-inverse of the Preisach model in [1, 7]. To calculate the inverted Everett matrix, a group of FORCs are required for identification\(^2\). The measurement of FORCs is difficult and tedious. The errors caused in measurements can lead to inaccuracy of the simulation.

### 3.2 The Inverted Model with Modified Switch Operator

As analyzed in the previous section, using the relay switch operator in the inverted model leads to a complicated weight function. In this section, a modified switch operator \( \hat{\upsilon}_{\alpha\beta} \) is introduced to the inverted model in (3.14).

\[ H(t) = \hat{\Upsilon}[M](t) = \int \int_{\beta \geq \alpha} v(\alpha, \beta) \hat{\upsilon}_{\alpha\beta} [M](t) d\alpha d\beta. \]  \hspace{1cm} (3.14)

By means of the new switch operator, the weight function is positive in the whole Preisach plane and it can be modeled with an analytical weight function. Furthermore, the inverted model also keeps the wiping out and congruency properties of the Preisach model. More details are explained in the following subsections.

#### 3.2.1 The Modified Switch Function \( \hat{\upsilon}_{\alpha\beta} \)

In the forward Preisach model, \( H \) is applied as input, the overall magnetization \( M \) is calculated by superposition of infinite relay operators. Fig. 3.4(a) illustrates a typical \( M(H) \) loop. The relay switch operator \( \hat{\gamma}_{\alpha\beta} \) is applied to model the rectangular hysteresis loop of elementary domains. As shown in Fig. 3.4(b), \( \alpha \) and \( \beta \) correspond to “up” and “down” switching values

---

\(^2\)More FORCs, which are uniformly distributed inside the major loop, can improve the accuracy of simulation results. In this work at least 20 FORCs are measured to calculate the inverted Everett matrix.
3.2 The Inverted Model with Modified Switch Operator

[16]. As the switch “up” value $\alpha$ is always greater than the switch “down” value $\beta$, the Preisach plane is in area $P = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \geq \beta \}$ shown in Fig. 3.4(c).

In the inverted Preisach model, $M$ is applied as input, similarly, the overall magnetization $H$ is calculated by superposition of infinite switch operators. Fig. 3.4(d) shows a typical $H(M)$ loop. Compared with the $M(H)$ loop, the $H(M)$ loop has different ascending and descending paths. A new switch operator $\hat{\upsilon}_{\alpha\beta}$ is presented to model the $H(M)$ loop. As shown in Fig. 3.4(e), in the rectangular loop of $\hat{\upsilon}_{\alpha\beta}$, the switch “up” value $\alpha$ is smaller than the switch “down” value $\beta$. This switch behavior is similar to the ascending and descending path of $H(M)$ loop. The corresponding Preisach plane is located in area $P' = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \leq \beta \}$ shown in Fig. 3.4(f).

![Figure 3.4: Forward and inverted Preisach models.](image)

The switch operator $\hat{\upsilon}_{\alpha\beta}$ can be mathematically expressed as

$$
\hat{\upsilon}_{\alpha\beta}[M](t) = \begin{cases} 
+1 & \text{if } M(t) \geq \beta, \\
-\hat{\gamma}_{\alpha'\beta'}[M](t^-) & \text{if } \alpha < M(t) < \beta, \\
-1 & \text{if } M(t) \leq \alpha.
\end{cases} 
$$

(3.15)

It can be noticed that $\hat{\upsilon}_{\alpha\beta}$ is equal to the negative value of $\hat{\gamma}_{\alpha'\beta'}$. Here a coordinate conversion is required.

$$
\alpha' = M_{\text{max}} - \beta + M(t), \\
\beta' = M(t) - \alpha - M_{\text{max}},
$$

(3.16)

where $M_{\text{max}}$ is the maximum of the input (saturation magnetization). As $\hat{\upsilon}_{\alpha\beta}$ derives from the modification of $\hat{\gamma}_{\alpha\beta}$, in this work it is called the modified switch operator.
The work principle of $\hat{\upsilon}_{\alpha\beta}$ can be explained clearer with the geometrical interpretation shown in Fig. 3.5. In the region $M(t) \geq \beta$, $\hat{\upsilon}_{\alpha\beta}$ is equal to $+1$, which is represented with dark color in the Preisach plane shown in Fig. 3.5. In the region $M(t) \leq \alpha$, $\hat{\upsilon}_{\alpha\beta}$ is equal to $-1$, which is represented with light color in the Preisach plane. In the region $\alpha < M(t) < \beta$ in the Preisach plane, this region is called “memory area”. The output of $\hat{\upsilon}_{\alpha\beta}$ depends on the history of $\hat{\gamma}_{\alpha'\beta'}$. As shown in Fig. 3.5, the regions with skew lines are the memory areas for switch operators ($\hat{\gamma}_{\alpha\beta}: \beta < M(t) < \alpha$; $\hat{\upsilon}_{\alpha\beta}: \alpha < M(t) < \beta$).

![Figure 3.5: Geometrical interpretation of $\hat{\upsilon}_{\alpha\beta}$ in the Preisach plane. In (a) the input $M(t)$ increases from negative saturation to $M_1$. In (b) the input $M(t)$ decreases from $M_1$ to $M_2$. The regions with skew lines are the memory areas.](image)

When the input $M(t)$ increases from negative saturation to $M_1$, as shown in Fig. 3.5(a), all the $\hat{\gamma}_{\alpha\beta}$ in the memory area remain $-1$ state. According to (3.15), all the $\hat{\upsilon}_{\alpha\beta}$ in memory area switch to $+1$.

When the input $M(t)$ decreases from $M_1$ to $M_2$, as shown in Fig. 3.5(b), $\hat{\gamma}_{\alpha\beta}$ in the memory area keeps the old state value as in Fig. 3.5(a), whereas $\hat{\upsilon}_{\alpha\beta}$ negates the state value of $\hat{\gamma}_{\alpha'\beta'}$ according to (3.16).

Like the interpretation of the classic Preisach model, here an input $M(t)$ with local extrema $\{M_1, M_2, M_3, M_4\}$ is applied to the inverted model, the corresponding output $H(t)$ and the state of the switch operator $\hat{Y}$ in the Preisach plane are shown in Fig. 3.6. The input starts with a value less than $-M_s$. The whole process is subdivided into four subsections (I-IV). As the value of $\hat{\upsilon}_{\alpha\beta}$ in the memory area is obtained according to $\hat{\gamma}_{\alpha'\beta'}$, the state of the switch operator $\hat{\gamma}_{\alpha\beta}$ is also shown as a reference in Fig. 3.6(d).

In all subsections, the work principle of $\hat{\gamma}_{\alpha\beta}$ is the same as introduced in the classic Preisach model (Fig. 2.10). The interface $G(t)$ keeps geometrically reversed from $L(t)$. In the last subsection IV, when $M(t)$ increases to $M_4$, a minor loop is formed at $M(t) = M_1$ [see Fig. 3.6(b)-IV]. The minor loops are assumed to be congruent, which is similar to the congruency property described in the Preisach model. Furthermore, as $M_4 > M_1 > M_3 > M_2$, in the Preisach plane shown in Fig. 3.6(d)- IV the past extrema $M_1$, $M_2$ and $M_3$ are wiped out from $G(t)$. This is consistent with the wiping out property of the Preisach model.

From the interpretation, it can be concluded, by means of the modified switch operator $\hat{\upsilon}_{\alpha\beta}$, that the inverted model remains the congruency and wiping out properties of the Preisach.
3.2 The Inverted Model with Modified Switch Operator

Figure 3.6: Geometric interpretation of the inverted Preisach model. (a) shows the process of input $M(t)$, the whole process is subdivided into four segments I-IV. (b) shows the corresponding output $\hat{\upsilon}(t)$. (c) are $\hat{\upsilon}_{\alpha\beta}$ in the Preisach planes, where the magenta line $G(t)$ is the interface between $S^+$ and $S^-$. (d) are $\hat{\gamma}_{\alpha\beta}$ in the Preisach planes, where the green line $L(t)$ is the interface. $G(t)$ and $L(t)$ geometrically related.

model. By means of coordinate conversion in the Preisach plane, the staircase interface $G(t)$ can be obtained from the past input extrema. These properties can be utilized in the numerical implementation of inverted model in an FE calculation process.

It is simpler and clearer to use the staircase interface to obtain the output of $\hat{\upsilon}_{\alpha\beta}$. As
shown is Fig. 3.7, for \( \tilde{\gamma}_{\alpha\beta} \), the interface \( L(t) \) starts from the top-left point\(^3\) and ends at the \( \alpha = \beta = M(t) \). For \( \hat{\upsilon}_{\alpha\beta} \), the interface \( G(t) \) starts from the \( \alpha = \beta = M(t) \) and ends at the bottom-right point. Geometrically \( L(t) \) and \( G(t) \) are completely related to each other. When the shape of \( L(t) \) is known, it is easy to obtain \( G(t) \).

Figure 3.7: The geometrical relation between \( L(t) \) and \( G(t) \). (a) \( L(t) \) is the \( S^+ (t) \) and \( S^- (t) \) interface by applying the relay switch operator \( \tilde{\gamma}_{\alpha\beta} \). (b) \( G(t) \) is the \( S^+ (t) \) and \( S^- (t) \) interface by applying the relay switch operator \( \hat{\upsilon}_{\alpha\beta} \). (c) illustrates the geometrical relation between the two interfaces.

By applying \( \tilde{\gamma}_{\alpha\beta} \), the extrema of input history are saved in the vertices coordinates of \( L(t) \). As the vertices of \( G(t) \) can be obtained from \( L(t) \) by coordinate conversion in (3.16), the modified switch operator \( \hat{\upsilon}_{\alpha\beta} \) also keeps the input history in memory. The past extrema values of input shape the staircase interface \( G(t) \).

### 3.2.2 Modeling of the Inverted Weight Function

In order to find a suitable weight function for the inverted model, analysis was made by means of the \( H(M) \) reversal curves. As shown in Fig. 3.8, the magnetization between \(-M_s \) and \( M_s \) is uniformly discretized into \( n \) parts with interval \( \Delta M \). The Preisach plane shown in Fig. 3.8(b) can be discretized into \( n \) cells in both \( \alpha \) and \( \beta \) directions. In each cell, \( \alpha_k = \beta_k = -M_s + k \Delta M \). Here a simple example \( n = 4 \) is explained.

Fig. 3.8(a) shows the \( H(M) \) reversal curves. \( H_{\alpha_k} \) is the output when \( M \) increases from \(-M_s \) to \( \alpha_k \). All \( H_{\alpha_k} \) are distributed along the ascending branch of the \( H(M) \) major loop. \( H_{\alpha_k \beta_j} \) \((j < k)\) is the output when \( M \) decreases from \( \alpha_k \) to \( \beta_j \). They are distributed along the reversal curves starting at \( H_{\alpha_k} \).

The change of output \( H \) can be applied to analyze the integral of the weight function in the Preisach plane. Owing to the symmetry of the hysteresis loops, the weight function is supposed to be symmetric with respect to the line \( \alpha = -\beta \). In Fig. 3.8(b), the weight function integrals in cells \( S_1 \sim S_6 \) are analyzed.

\(^3\)It is different from the interface introduced in the classic Preisach model in section 2.2. The interface \( L(t) \) in the CMP separates the \( S^+ (t) \) and \( S^- (t) \) subsets in the Preisach plane. Here the interface \( L(t) \) is applied to obtain \( G(t) \). It always starts from the top-left point of the Preisach plane.
3.2 The Inverted Model with Modified Switch Operator

Figure 3.8: Analysis of weight function with $H(M)$ reversal curves. The input $M$ is uniformly discretized into 4 parts. (a) shows the $H(M)$ reversal curves, analysis required $H$ outputs are noted on the curves. The Preisach plane shown in (b) is discretized into 10 cells. The weight function integral over cells $S_1$-$S_6$ are analyzed. The integrals over red cells are greater than the integrals over the gray cells.

When $M$ decreases from $\alpha_4$ to $\beta_1$, the output is $H_{\alpha_4\beta_1}$. According to the work principle of the inverted switch operator, only cell $S_1$ is included in the subset $S^+$ of the Preisach plane. If $M$ keeps decreasing until $-M_s$, all the switch operators switch to $-1$. The weight function integral over $S_1$ can be calculated as

$$H_{\alpha_4\beta_1} - (-H_s) = 2 \int\int_{S_1} v(\alpha, \beta) d\alpha d\beta.$$  \hfill (3.17)

When $M$ decreases from $\alpha_3$ to $\beta_1$, the output is $H_{\alpha_3\beta_1}$. In the Preisach plane cells $S_1$ and $S_2$ are included in the subset $S^+$. The weight function integral over $S_1 + S_2$ can be calculated as

$$H_{\alpha_3\beta_1} - (-H_s) = 2 \int\int_{S_1+S_2} v(\alpha, \beta) d\alpha d\beta.$$  \hfill (3.18)

By subtracting (3.17) from (3.18), the weight function integral over $S_2$ can be calculated as

$$\int\int_{S_2} v(\alpha, \beta) d\alpha d\beta = \int\int_{S_1+S_2} v(\alpha, \beta) d\alpha d\beta - \int\int_{S_1} v(\alpha, \beta) d\alpha d\beta = \frac{1}{2} (H_{\alpha_3\beta_1} - H_{\alpha_4\beta_1}).$$  \hfill (3.19)

Using this method, the weight function integrals over different cells can be calculated with the output values on reversal curves. The calculations of the weight function integrals over cells $S_1$-$S_6$ are listed in Table 3.1. With reference to the $H(M)$ reversal curves shown in Fig. 3.8(a), the weight function integrals over cells $S_1$ and $S_4$ are relatively greater than other cells. As weight function is symmetric with respect to the line $\alpha = -\beta$, the cells with red color in
Table 3.1: Integrals of $v(\alpha, \beta)$ in regions $S_1$-$S_6$ in Fig. 3.8

<table>
<thead>
<tr>
<th>Regions</th>
<th>Integrals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$\int_{S_1} \int v(\alpha, \beta) , d\alpha d\beta = \frac{1}{2} (H_{\alpha_4\beta_1} + H_s)$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$\int_{S_2} \int v(\alpha, \beta) , d\alpha d\beta = \frac{1}{2} (H_{\alpha_3\beta_1} - H_{\alpha_1\beta_1})$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$\int_{S_3} \int v(\alpha, \beta) , d\alpha d\beta = \frac{1}{2} (H_{\alpha_2\beta_1} - H_{\alpha_3\beta_1})$</td>
</tr>
<tr>
<td>$S_4$</td>
<td>$\int_{S_4} \int v(\alpha, \beta) , d\alpha d\beta = \frac{1}{2} (H_{\alpha_1} - H_{\alpha_2\beta_1})$</td>
</tr>
<tr>
<td>$S_5$</td>
<td>$\int_{S_5} \int v(\alpha, \beta) , d\alpha d\beta = \frac{1}{2} (H_{\alpha_4\beta_2} - H_{\alpha_3\beta_1})$</td>
</tr>
<tr>
<td>$S_6$</td>
<td>$\int_{S_6} \int v(\alpha, \beta) , d\alpha d\beta = \frac{1}{2} (H_{\alpha_3\beta_2} - H_{\alpha_4\beta_2} - H_{\alpha_2\beta_1} + H_{\alpha_3})$</td>
</tr>
</tbody>
</table>

Fig. 3.8(b) represent greater integral results.

According to the analysis example shown in Fig. 3.8, it can be assumed that the weight function $v(\alpha, \beta)$ becomes very large in areas near vertices of the Preisach triangle. From the vertices to the center of the Preisach triangle, $v(\alpha, \beta)$ gradually decreases to a minimum. Fig. 3.9(b) presents the approximated weight function $v(\alpha, \beta)$ calculated from the $H(J)$ reversal curves shown in Fig. 3.9(a). According to the shape of this surface, the weight function $v(\alpha, \beta)$ can be modeled based on a hyperbolic paraboloid function.

![Figure 3.8](image1.png)

![Figure 3.9](image2.png)

Figure 3.9: (a) shows six $H(J)$ reversal curves measured with VSM on 1.9 µm Fe$_{49}$Co$_{49}$V$_2$ thin film sample. (b) shows the calculated weight function distribution from measured reversal curves in (a).
To find an analytical function to approximate the inverted weight function, it is necessary to make reference to the forward Preisach weight function. Until now, many analytical functions, such as the Gaussian and Lorentzian functions, have been applied to approximate the forward Preisach weight function [1].

An analytical weight function has been proposed by Sutor for a forward Preisach model, which can be identified by a major hysteresis loop [35, 55]. This analytical weight function derives from the fact that the general shape of major $M(H)$ loops are often similar to the arc tangent function, the forward weight function is designed to be similar to its derivative [35].

$$
\mu(\alpha, \beta) = \frac{A}{1 + \left\{((\alpha + \beta)\sigma)^2 + ((\alpha - \beta - h)\sigma)^2\right\}^{\eta}} 
$$

(3.20)

In the same way, as the general shape of major $H(M)$ loops are often similar in shape to the tangent function, the form of the weight function should be similar to its derivative [51]. The differentiable weight function $\nu(\alpha, \beta)$ is proposed as

$$
\nu(\alpha, \beta) = A \left\{ [(\alpha + \beta - 1)\sigma_1]^{\eta} - [(\alpha - \beta - h)\sigma_2]^{\eta} \right\}^2.
$$

(3.21)

![Figure 3.10: Normalized weight function examples. The forward weight function $\mu(\alpha, \beta)$ is shown in (a) (b). The inverted weight function $\nu(\alpha, \beta)$ is shown in (c) (d).](image)
Figure 3.11: (a) shows the ascending branch of a $M(H)$ major loop and its derivative $dM/dH$. (b) shows the ascending branch of a $H(M)$ major loop and its derivative $dH/dM$. Both $M$ and $H$ are normalized for comparison. The $M(H)$ major loop was measured on a 1.9 µm FeCo thin film by vibrating sample magnetometer.

Fig. 3.10 shows the shapes of $\mu(\alpha, \beta)$ and $\nu(\alpha, \beta)$. The forward weight function $\mu(\alpha, \beta)$ in the Preisach plane $P = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \geq \beta \}$ is shown in Fig. 3.10(a). The inverted weight function $\nu(\alpha, \beta)$ in Preisach plane $P' = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \leq \beta \}$ is shown in Fig. 3.10(c). By applying the modified switch operator, $\nu(\alpha, \beta)$ remains positive in the whole Preisach plane. As analyzed in Fig. 3.8(b), $\nu(\alpha, \beta)$ becomes large near the vertices of the Preisach triangle. It can be observed in contour maps shown in Fig. 3.10(b) and (d), that both the forward and inverted weight functions are symmetric with respect to the line $\alpha = -\beta$.

The forward weight function $\mu(\alpha, \beta)$ has an obvious peak near the center of the Preisach triangle. This is due to the big slopes of $M(H)$ curves near the coercivity $H_c$. Fig. 3.11(a) shows the ascending branch of a normalized $M(H)$ major loop. The slope can be observed by means of derivatives $dM/dH$ in Fig. 3.11(b). The fast change of the magnetization $M$ results in a maximum slope near coercivity. The identification of $\mu(\alpha, \beta)$ near coercivity should be carefully concerned. Compared with $M(H)$ curves, the magnetic field $H$ changes smoothly with respect to $M$. It can be seen in Fig. 3.11(d), that the derivative $dH/dM$ derives from the ascending branch of a $H(M)$ major loop shown in Fig. 3.11(c). The smooth distribution offers a great advantage in approximating the inverted weight function.

This Chapter describes the definition of this inverted Preisach model. To identify this model, hysteresis curves are required as experimental data. Three characterization methods have been applied in this work and they are introduced with details in the next chapter.
4 Experimental Techniques

In order to precisely model the magnetic hysteresis behavior, reliable experimental data is required for identification. As introduced in the section on ferromagnetism theory, the magnetic properties of various materials may differ greatly. The difference can be reflected in the shape of hysteresis curves. Even for the same magnetic material, the shape of the hysteresis curves can be greatly influenced by many factors, such as frequency, temperature, anisotropy, deformation or even the shape of the samples.

Until now, many characterization methods have been proposed for the determination of the magnetic properties. Some of these methods have been described by international standards, e.g., the Epstein frame and the single sheet tester (SST) methods. Some of them have been developed as commercial instruments and widely applied in professional laboratories, such as vibrating sample magnetometer (VSM). These methods apply different measurement techniques. Magnetic materials are processed in different shapes, measurements are carried out with DC or AC excitation, in closed or open magnetic circuits, etc. Therefore, it is advisable to mention the characterization method and sample shape in measurement results. In this work, three characterization methods were applied to determine the magnetic properties in different conditions.

4.1 Epstein Frame Method

The Epstein frame is the most commonly used method for characterization of electrical steels. The measurement results show great repeatability and reliability according to standards (e.g., IEC 60404-2). As shown in Fig. 4.1, a frame is formed by four coil sets. Each coil set comprises an outer winding and an inner winding. The outer windings are connected in series as primary winding ($N_1$ is the number of turns). The magnetizing current $I_{prim}$ is applied to the primary winding for magnetizing specimens. The magnetic field $H$ is calculated from $I_{prim}$ as

$$H = \frac{N_1 I_{prim}}{l_m},$$  \hspace{1cm} (4.1)

where $l_m$ is the effective magnetic path length (EML). The inner windings are connected in series as the secondary winding ($N_2$ is the number of turns). The voltage induced in the secondary winding $U_{sec}$ is measured to obtain the flux density by

$$\frac{dB}{dt} = -\frac{U_{sec}}{N_2 A},$$  \hspace{1cm} (4.2)

where $A$ is the cross sectional area of the specimens, it is defined from the weight of specimens.
Specimens are cut as strips from electrical steels. They are placed inside the coil sets and overlapped at corners. Due to the overlapping, the effective magnetic path length (EML) is approximately determined as 0.94m for all kinds of materials. This is criticized because when the steels under test are grain oriented, the difference to the real EML can be significant [24].

Fig. 4.2 shows an automated measurement system with Epstein frame\(^1\). During the measurement process, the excitation signal is fed to a power amplifier to ensure the required power for magnetizing. The magnetizing current \(I_{\text{prim}}\) is measured through the voltage of a shunt resistor \(U_R\), which is connected in series with the primary winding of the Epstein frame. The voltages \(U_R\) and \(U_{\text{sec}}\) are measured using an oscilloscope. To compensate for the air flux generated in the secondary winding, a mutual inductor \(M\) is applied here. The adjustment of the mutual inductor is made in absence of the specimen. Electrical steels are widely applied

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\(^1\)The automation method is introduced in the published paper [56]
in transformers or electric motors. In order to reduce energy loss caused by the eddy current, electrical steels are usually made of SiFe alloys and produced as thin lamination. The addition of silicon can increase the electrical resistivity of the steel. However, high silicon content makes the steels hard and brittle, which leads to difficulties in mechanical treatments. Normally the standard steels are produced with 3 to 4.5% silicon [24, 57].

Based on the measured parameters, electrical steels are classified according to international standards. Table 4.1 presents the classification of non-oriented electrical steels according to European standard EN 10106. The steel names are designated with the nominal thickness and core loss is measured at 1.5T, 50Hz. The letter M represents electrical steel and the letter A stands for the annealing process.

<table>
<thead>
<tr>
<th>Steel name</th>
<th>Nominal Thickness (mm)</th>
<th>Core loss at 1.5T, 50Hz (W/kg)</th>
<th>Polarization for 5000A/m (T)</th>
<th>Polarization for 10000A/m (T)</th>
<th>Conventional density kg/dm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>M330-35A</td>
<td>0.35</td>
<td>3.30</td>
<td>1.60</td>
<td>1.70</td>
<td>7.65</td>
</tr>
<tr>
<td>M350-50A</td>
<td>0.50</td>
<td>3.00</td>
<td>1.60</td>
<td>1.70</td>
<td>7.65</td>
</tr>
<tr>
<td>M600-50A</td>
<td>0.50</td>
<td>6.00</td>
<td>1.66</td>
<td>1.76</td>
<td>7.75</td>
</tr>
<tr>
<td>M800-100A</td>
<td>1.00</td>
<td>8.00</td>
<td>1.66</td>
<td>1.75</td>
<td>7.70</td>
</tr>
<tr>
<td>M1300-100A</td>
<td>1.00</td>
<td>13.00</td>
<td>1.7</td>
<td>1.78</td>
<td>7.80</td>
</tr>
</tbody>
</table>

The parameters specified in the standard can be applied to evaluate the instrument accuracy. Table 4.2 presents the material parameters measured with the Epstein frame setup shown in Fig. 4.2. For specimens with thickness not more than 0.50mm, the measured core losses are quite close to the standard ones. The average relative difference is about 8.5%.

<table>
<thead>
<tr>
<th>Steel name</th>
<th>Nominal Thickness (mm)</th>
<th>Core loss at 1.5T, 50Hz (W/kg)</th>
<th>Polarization for 5000A/m (T)</th>
<th>Polarization for 10000A/m (T)</th>
<th>Conventional density kg/dm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>M330-35A</td>
<td>0.35</td>
<td>2.856</td>
<td>1.631</td>
<td>1.746</td>
<td>7.650</td>
</tr>
<tr>
<td>M350-50A</td>
<td>0.50</td>
<td>3.091</td>
<td>1.651</td>
<td>1.760</td>
<td>7.650</td>
</tr>
<tr>
<td>M600-50A</td>
<td>0.50</td>
<td>5.490</td>
<td>1.744</td>
<td>1.851</td>
<td>7.850</td>
</tr>
<tr>
<td>M800-100A</td>
<td>0.98</td>
<td>6.090</td>
<td>1.667</td>
<td>1.773</td>
<td>7.714</td>
</tr>
<tr>
<td>M1300-100A</td>
<td>0.98</td>
<td>8.248</td>
<td>1.740</td>
<td>1.836</td>
<td>7.795</td>
</tr>
</tbody>
</table>
4 Experimental Techniques

4.1.1 Frequency Dependence of Hysteresis Loops

AC measurement is carried out here with a frequency up to 400Hz. Specimens are magnetized with an alternating sinusoidal magnetic field, whose peak value is gradually increased. A family of hysteresis loops are measured as shown in Fig.4.3, the primary magnetization curves are obtained by joining tips of these loops.

![Figure 4.3: A family of hysteresis loops and the primary magnetization curve obtained by connecting the tips of these loops.](image)

The frequency dependency of the hysteresis loops can be described by permeability and core loss curves at different frequencies. The relative permeability $\mu_r$ is calculated at each tip point of the primary magnetization curves according to (2.5). Fig. 4.4 shows the permeability $\mu_r$ relative to the magnetic field $H$ measured at different frequencies. It can be seen that the permeability decreases with the increasing of the frequency. When the specimen is magnetized to saturation, the relative permeability $\mu_r$ tends to be one. At higher frequencies, as more power is required for magnetization, measurement is limited by instrument range.

The core loss $P_c$ introduced in (2.6) can be calculated from the area of the hysteresis loops.

$$P_c = \frac{f}{\rho} \oint H dB.$$  \hspace{1cm} (4.3)

As shown in Fig. 4.5, the core loss increases with the increasing of the frequency. The thicker the specimen is, the more obvious this tendency is. E.g. at the same magnetic flux $B = 0.4T$, compare the core loss at 10Hz and 400Hz, the core loss of the specimen M1300-100A (thickness 1 mm) increases from 0.11W/kg to 15.32W/kg, the core loss of the specimen M330-35A (thickness 0.35 mm) increases from 0.05W/kg to 3.93W/kg.

In an electromagnetic field with high frequency, the loss caused by eddy currents plays an important role in the total core loss in specimen. Based on skin effect [50], the penetration depth in the specimen $\delta$ can be evaluated as

$$\delta = \frac{1}{\sqrt{\pi \sigma \mu f}},$$  \hspace{1cm} (4.4)

where $\sigma$ is the electric conductibility of the specimen, and $\mu$ is the absolute permeability of
4.1 Epstein Frame Method

The eddy currents can be reduced by decreasing of lamination thickness and increasing of the resistivity of the material.

The frequency dependency can be analyzed with hysteresis loops. The hysteresis loops of three specimens M330-35A, M600-50A and M1300-100A are shown in Fig. 4.6. In Fig. 4.6 (a)(c)(e), hysteresis loops were measured with the same maximal magnetic field $H$. With the increase of frequency, the magnetic flux $B$ at the tip point decreases, which leads to a decrease in permeability. In other words, to magnetize the materials at high frequencies,
greater magnetic field is required.

It can be more clearly observed in Fig. 4.6 (b)(d)(f), where hysteresis loops are shown with the same maximum magnetic flux $B$. With the increase in frequency, greater magnetic field is required for magnetization. To change the magnetic flux back to zero, a greater coercive field $H_c$ in the opposite direction is needed. The hysteresis loops measured at high frequencies have an oval form, which leads to greater loop areas and an increase of the core loss. This can be utilized to explain the frequency dependency of the core loss shown in Fig. 4.5.

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4.1 Epstein Frame Method

Figure 4.6: Comparison of hysteresis curves at different frequencies (10Hz – 400Hz). (a)(b) M330-35A, steel thickness is 0.35mm (c)(d) M600-50A, steel thickness is 0.50mm, (e)(f) M1300-100A, steel thickness is 1.00mm.

4.1.2 Temperature Dependence of Hysteresis Loops

Temperature is another influencing factor on the shape of hysteresis loops. As introduced in Fig. 2.1, above the Curie temperature ferromagnetic materials behave in a paramagnetic manner. Below the Curie temperature the spontaneous magnetization of ferromagnetic materials decreases with an increase in temperature. In this work the measurements are carried
out at -40°C–140°C, which is much lower than the Curie Temperature of the electrical steels (appr. 740°C). The temperature has little influence on the saturation magnetization of electrical steels. But the temperature dependency can be reflected by the core loss curves.

Fig. 4.7 shows the core loss curves measured at 50Hz. The core loss decreases with an increase in temperature. For comparison, hysteresis loops with the same magnetic field are shown in Fig. 4.8. When the temperature increases, both the magnetic flux and the coercive field decreases. This may be due to the magnetic orderings. At high temperatures, random thermal motion makes it more difficult for electrons to maintain alignment [58]. This results in smaller loop areas and core loss.

Another possible explanation, may be the decrease of the electric conductivity at high temperatures. The eddy currents are reduced in the materials with lower electric conductivity, resulting in a decrease in core loss caused by eddy currents.

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Figure 4.7: Comparison of core loss curves at different temperatures (-40°C – 140°C). (a) M350-50A, (b) M600-50A, (c) M800-100A, (d) M1300-100A.
4.1 Epstein Frame Method

4.1.3 Measurements of Grain-oriented Steels

As mentioned in Section 2.1.4, anisotropy can be developed by applying rolling treatments to the electrical steels. Most crystals of these steels are positioned with one easy axis close to the rolling direction (RD). As the magnetization along the easy axis is easiest, the optimum magnetic properties are developed in the RD. Relatively, the direction perpendicular to RD is called the transverse direction (TD). The electrical steels manufactured with this texture are named grain-oriented (GO) electrical steel.

As shown in Fig. 4.9, a specimen is cut from GO M33035 steel along RD. Fig. 4.9(a) is the optical microscope image of the specimen surface. The photo is made by means of an optical microscope keyence vhx500. The texture is clearer in Fig. 4.9(b) and (c), which are surface topographies made by a confocal microscope Nanofocus μSurf explorer. The height of the surface in focus is detected and shown in colors.

Figure 4.8: Comparison of hysteresis curves at different temperatures (-40°C – 140°C). (a) M350-50A, (b) M600-50A, (c) M800-100A, (d) M1300-100A.
The difference in the magnetic properties in the RD and TD can be evaluated using measurements with the Epstein frame method. Specimens are individually cut from GO steels in the RD or TD. As the RD is close to one easy axis of the material, it is easier to magnetize the RD specimen. Therefore, when the RD specimen is under test, the effective magnetic path...
length (EML) should be shorter than the TD specimen case. As the EML in the Epstein frame
is approximated to be the same for all kinds of materials, it should be stated that the magnetic
properties characterized here are considered as the average magnetic properties.

Fig. 4.10 shows the primary magnetization curves and core loss curves of GO steels
35TW400 and 35Z155. From the primary magnetization curves shown in Fig. 4.10(a) and
(b), it can be concluded that with the same magnetic field, greater magnetic flux is detected
in RD specimen. To saturate the specimens, a smaller magnetic field is required for the RD
specimens. Therefore, the magnetization of TD specimens cost more energy. As shown in
Fig. 4.10(c) and (d), at the same magnetic flux, the core loss of TD specimen is higher than
the RD specimen. The difference is more obvious in the measurements of 35Z155 specimens.

This can be analyzed with the measured hysteresis loops. If the same magnetic field is
applied to RD and TD specimens, as shown in Fig. 4.11(a) and (b), great differences of the
magnetic flux exist at the tips of the hysteresis loops. As a result, higher permeability can
be achieved in the RD specimen. Comparing the hysteresis loops with the same maximum
magnetic flux, as shown in Fig. 4.11(c) and (d), the TD loop area may be more than double
the RD loop area, which results in a smaller core loss in RD specimens. Therefore, it is more
efficient to apply GO SiFe electrical steels in transformers. Normally the GO steels are more
expensive than non-oriented steels.

![Figure 4.11](image-url)

Figure 4.11: Hysteresis loops of GO steels 35TW400 and 35Z155 measured at 50Hz at room
temperature. (a)(b) are hysteresis loops with the same maximum magnetic field,
(c)(d) are hysteresis loops with the same maximum magnetic flux.
4 Experimental Techniques

4.2 Double C-yoke SST

In recent years single sheet testers (SST) have been widely applied for characterization of magnetic materials. In contrast to the Epstein frame method, the SST method does not have the problem of effective magnetic path length as the magnetic field is directly measured by applying a $H$-coil. According to numerical analysis, the field distribution is more uniform in the SST [37]. Normally less specimens are needed for measurements with SST. However, to carry out comparable determination, samples need to be produced in a large area, which is not appropriate for electrical sheets.

The double C-yoke measurement system is shown in Fig. 4.12. Steel samples are placed between two symmetrically furnished C-yokes. The excitation signal is fed to two excitation coils surrounding each C-yoke and connected in series. In this way, two parallel closed magnetic circuits are formed symmetrically. The eddy current can be reduced in AC measurements.

In the middle of the steel sample, a small $H$-coil is placed close to the specimen. Based on the Maxwell’s equations, near the interface of two adjacent media, the tangential component of magnetic fields in two media should be equal to each other [50, 24]. Therefore, the $H$-coil is applied to measure the tangential field component in the air. In the middle of the sample, it can be expressed as

$$H_{\text{air}} \parallel = H_{\text{sample}} \parallel \approx H_{\text{sample}}.$$  (4.5)

The magnetic field in the steel sample can be directly obtained from $H_{\text{air}} \parallel$. The $H$-coil in this measurement system is wound around a $50\text{mm} \times 15\text{mm} \times 1.6\text{mm}$ polyvinyl chloride core (PVC), which has a constant permeability equal to $\mu_0$.

![Figure 4.12: Overview of the double C-yoke measurement system. (a) is the diagram of the measurement system, (b) shows the double C-yoke SST constructed in laboratory.](image)

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4.2 Double C-yoke SST

By applying a $B$-coil, the magnetic flux of the sample is calculated from the induced voltage by (4.2). The $B$-coil is placed around the steel sample and the $H$-coil. Before measurements, the compensation for the air flux is made by applying numerical method.

To realize a small dimension, the $H$-coil is made of thin wires with a small number of windings. The voltage signal over the $H$-coil is quite weak, therefore an amplifier is utilized here for enlargement. At frequencies greater than 25Hz, there is a deformation in measured hysteresis curves. The measurements here are limited to 1Hz – 25Hz. Because of the low-frequency excitation signal, much effort has to be put into signal conditioning in frequency domain [59].

According to the Epstein frame method, the measured magnetic properties are the average magnetic properties of the samples. In the double C-yoke SST, the magnetic properties are determined in a specific part of the sample. Therefore, the magnetic properties characterized with the double C-yoke SST are the local magnetic properties. These can be applied to evaluate the plastic deformation influence on the local magnetic properties.

4.2.1 Plastic Deformation Effects

The plastic deformation may cause local magnetic degradation of the electrical steels. This is attributed to the generation of dislocations acting as pinning sites for the domain walls [60].

To observe the plastic deformation influence over local magnetic properties, cold plastic training is applied to the specimen. As shown in Fig. 4.13, the steel sample [Fig. 4.13(a)] was folded in the middle [Fig. 4.13(b)] and attended to be inserted into the yoke. The $H$-coil was put on the surface of the deformed section of the specimen [Fig. 4.13(c)] for measurement.

![Image of cold plastic training on electrical steel M33035A samples. (a) shows the steel sample before deformation, (b) shows the cold plastic training on the steel sample, (c) shows the deformed part of the steel sample.]

Figure 4.13: Cold plastic training on electrical steel M33035A samples. (a) shows the steel sample before deformation, (b) shows the cold plastic training on the steel sample, (c) shows the deformed part of the steel sample.

With the double C-yoke SST shown in Fig. 4.12, local magnetic properties were measured in both the deformed and the undeformed sections of the sample. Fig. 4.14 shows the determined permeability curves. Compared with the results measured before deformation, the permeability clearly decreases in the deformed section. The local magnetic property in the

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2This section as been published in [51, 61]
undeformed part is also influenced. These experiments were carried out both on RD and TD samples. The results indicate that the magnetic properties of soft magnetic materials are quite sensitive to plastic straining. Generally, plastic deformation causes changes in the dislocation density, which in turn affects the pinning strength of domain walls and consequently their movement [62].

![Image](image1.png)

Figure 4.14: Permeability curves of steel sample M33035A at 1Hz at room temperature. (a) RD steel sample, (b) TD steel sample.

For further observation, heat treatment (HT) in a nitrogen environment was carried out on the plastically deformed sample. After annealing in nitrogen environment, the local magnetic properties were again measured in the deformed and undeformed sections. It can be seen from the comparisons in Fig. 4.15, that the local magnetic properties exhibit an obvious recovery after thermal treatment. This may be due to the reduction of distortion within the particles during the heat treatment [63].

![Image](image2.png)

Figure 4.15: Comparisons of permeability curves. TD steel sample M33035A was tested after HT 800°C at 25Hz at room temperature.
4.3 Vibrating Sample Magnetometer

Since a thin film magneto resistor was applied by Hunt as a magnetic reading head in 1970 [24, 64], ferromagnetic thin films have attracted growing interest both from fundamental scientific researchers and in the computer industry [36]. The ferromagnetic thin films are commonly characterized in open circuit. In this case, magnetic moment is usually detected with sensitive magnetometers [65]. The vibrating sample magnetometer (VSM) is widely applied, which was developed by Van Oosterhout and Foner in 1950s [38, 39]. Nowadays, VSM has become a standard instrument and is most widely used in professional magnetism laboratories.

Fig. 4.16 presents an overview of the vibrating sample magnetometer measurement system. The sample is held by a rod and inserted in the gap of an electromagnet, which is applied to generate a static magnetic field. The rod is connected with a shaker to vibrate the sample with a specific amplitude and frequency.

![Diagram of the vibrating sample magnetometer measurement system.](image)

Figure 4.16: Diagram of the vibrating sample magnetometer measurement system.

The small magnetized sample can be considered as a vibrating dipole. Suppose the magnetic moment is directed along the axis of the pickup coils, then the induced voltage detected in the pickup coils is proportional to the magnetic moment $m$:

$$u(t) = -\frac{d\Phi(t)}{dt} = m \, G(z) \, \frac{dz(t)}{dt}.$$  \hfill (4.6)
where $G(z)$ is the geometry factor depending on the pickup coil geometry. In measurement, a sinusoidal vibration $z(t) = z_0 \sin \omega t$ is applied to the sample. The induced voltage is

$$u(t) = m \, G(z) \, z_0 \, \omega \, \cos \omega t.$$  \hspace{1cm} (4.7)$$

Appropriate pickup coil configurations can cause the induced voltage to be insensitive to sample position [41]. As shown in Fig. 4.17(b), a Mallinson four-coil system [40] is constructed between the electromagnet poles [see Fig. 4.17(a)]. The two pairs of coils are connected in series opposition, their axes are parallel to the magnetic field and perpendicular to the direction of sample vibration.

Due to the small sample size (film thickness less than 3 µm), the induced signal is very small. A lock-in amplifier is applied to detect the small voltage signal. In the measurement system, a reference signal is generated by the lock-in amplifier. The signal has a specific frequency (80Hz in this work) and fed to the shaker. By means of the accelerometer, the vibrating amplitude $z_0$ is monitored to keep it stable. The output of the lock-in amplifier, which is the RMS (root mean square) value of $u(t)$, is followed by

$$U_{\text{rms}} = K \, m,$$  \hspace{1cm} (4.8)$$

where $K = G(z) \, z_0 \, \omega$, which can be calibrated by a standard sample with known magnetic moment.

In the VSM, the sample is magnetized in an open magnetic circuit. A Hall sensor is placed near the sample to measure the magnetic field. In the open magnetic circuit, the demagnetization field of sample $H_d$ should be considered. If a magnetizing field $H$ is applied to the sample, the internal field of the sample $H_{\text{in}}$ is calculated as

$$H_{\text{in}} = H - H_d = H - N_d M,$$  \hspace{1cm} (4.9)$$
4.3 Vibrating Sample Magnetometer

where $N_d$ is the demagnetization factor, which depends on the shape of the sample. As shown in Fig. 4.18, the ferromagnetic thin films are in the form of a disk. The magnetic field is applied parallel to the sample surface. As the film thickness (less than 3 µm) is much smaller than the diameter of samples (10 mm), the demagnetization factor $N_d$ in (4.9) can be considered to be zero.

![Figure 4.18: The sample for VSM measurement. Ferromagnetic thin films (material Fe$_{49}$Co$_{49}$V$_2$ and Ni$_{81}$Fe$_{19}$) were produced by RF magnetron sputtering. Using the etching technology, the sputtered films were made in the form of a disk.](image)

The demagnetization field can be neglected here. Samples are assumed to be uniformly magnetized [66], the magnetization $M$ can be obtained with the sample volume $V$ as

$$M = \frac{1}{V} \sum m_i = \frac{m}{V}.$$  \hspace{1cm} (4.10)

### 4.3.1 Scalar Characterization with VSM

In the VSM setup, the axis of pickup coils is parallel to the magnetic field. As the magnetic field $H$ and magnetization $M$ are vector quantities, it is considered as scalar characterization. The magnetization component parallel to the magnetic field is determined.

In this work, DC measurement is carried out with a VSM system. Besides symmetric hysteresis loops, asymmetric hysteresis loops, such as FORCs and minor loops, can be determined. Fig. 4.19 shows the FORCs and minor loops measured on a 1.9 µm Fe$_{49}$Co$_{49}$V$_2$ thin film. As mentioned in Section 3.1, the FORCs can be applied for identification of Preisach hysteresis models.

### 4.3.2 Vector Vibrating Sample Magnetometer

Scalar VSM characterization presents only the magnetization component in the direction of the applied field. In the modern field of magnetic materials research, the full description of magnetization vectors is attracting more and more attention. Especially for anisotropic materials, which have different magnetic properties depending on the magnetization direction.

By applying another pickup coils system, which is perpendicular to the applied field, the VSM can be extended to a vector VSM (VVSM) for 2D vector characterization. This can be seen in Fig. 4.20(a), the two pickup coils systems are orthogonal to each other (biaxial pickup coils). In this work they are called longitudinal coils (LC) and transversal coils (TC). The parallel and perpendicular components of the magnetic moment are proportional to the
voltages induced in LC and TC. The relation described in (4.8) can be written as

\[ m_\parallel = K_{\text{long}} U_{\text{rms, long}}, \]
\[ m_\perp = K_{\text{tran}} U_{\text{rms, tran}}. \]  

(4.11)

As shown in Fig. 4.20(b), the angle between the two components is calculated by

\[ \alpha = \arctan \left( \frac{m_\perp}{m_\parallel} \right) = \arctan \left( \frac{K_{\text{tran}} U_{\text{rms, tran}}}{K_{\text{long}} U_{\text{rms, long}}} \right). \]  

(4.12)

### 4.3.3 Calibration Method

Accurate characterizations with VSM or VVSM require careful calibrations before measurement. Specifically, the factors \( K_{\text{long}} \) and \( K_{\text{tran}} \) in (4.11) need to be obtained using a reasonable calibration method.

The longitudinal pickup coils are calibrated by Mu-metal film (thickness of 25 µm) with known saturated magnetic polarization \( J_s = 0.8 \text{T} \). As a reference sample, the Mu-metal film is cut into a shape similar to the samples applied in measurements. When the reference sample is magnetized to saturation, the magnetic moment vector can be considered to align completely parallel to the field, which implies

\[ m_\parallel \approx m_s = VM_s = \frac{V J_s}{\mu_0}. \]

(4.13)
By combining (4.13) and (4.11), \( K_{long} \) can be obtained with \( J_s \) of the reference sample.

\[
K_{long} = \frac{V J_s}{\mu_0 U_{rms, long}}
\]  

(4.14)

The calibration of the transversal pickup coils is carried out by measuring the angular dependency of the pickup coils. A hard magnetic material with high coercivity and remanent magnetization is applied as the reference sample. The sample is magnetized to saturation then rotated in a zero field. The rotation is realized and controlled by means of a rotation stage. During rotation the induced voltages in LC and TC are measured in every angular increment. The remanent magnetization remains stable in the zero field, which is calculated by

\[
m_r = \sqrt{m_\parallel^2 + m_\perp^2}.
\]

(4.15)

After calibration, the resultant parallel and perpendicular components of magnetic moment should be orthogonal as shown in Fig. 4.21.

Figure 4.21: The parallel and perpendicular components of magnetic moment after calibration.
4.3.4 Vector Characterization with VSM

The vector characterization includes the magnetization component perpendicular to the applied field. This provides more information than the scalar characterization.

Fig. 4.22 presents the major hysteresis loops obtained by vector characterization. The isotropic samples were firstly magnetized to saturation then rotated in zero field. During rotation the magnitude of the remanent magnetization does not change but the direction rotates with the sample. After 90° rotation ($\theta = 90^\circ$), the remanent magnetization aligns close to the perpendicular direction. Major loops were measured both parallel and perpendicular to the applied field. As measurements start at the zero field, the start value of the perpendicular component $J_\perp$ is approximated to the remanent magnetic polarization $J_r$. When the field increases, the total magnetization increases and gradually aligns parallel to the field, while the perpendicular component decreases to a minimum.

![Hysteresis loops](image)

Figure 4.22: Major hysteresis loops measured both parallel and perpendicular to the applied fields. Samples: (a) 1.9 µm Fe$_{49}$Co$_{49}$V$_2$ thin film, (b) 1.7 µm Ni$_{81}$Fe$_{19}$ thin film.

The angular dependence of vector characterization is illustrated in Fig. 4.23. For comparison, the experiments described above were repeated with different rotation angles $\theta$. The corresponding primary magnetization curves start at different values. The start values of $J_\perp$ and $J_\parallel$ are approximated to the projections of the remanent magnetic polarization $J_r$ as

$$J_\perp \approx J_r \sin \theta, \quad J_\parallel \approx J_r \cos \theta. \quad (4.16)$$

Compared with isotropic materials, the anisotropic materials exhibit more obvious angular dependence of the magnetic properties. As shown in Fig. 4.24(a), an anisotropic sample\(^3\)

\(^3\)The sputtered thin film was magnetized at high temperature. The film is anisotropic along the film surface. Circular samples were made by applying laser cutting technology.
4.3 Vibrating Sample Magnetometer

Figure 4.23: Primary magnetization curves measured both perpendicular (a) and parallel (b) to the applied field. Before measurements the sample was rotated with \( \theta \) degree from original position. Sample: 1.9 \( \mu \)m Fe\(_{49}\)Co\(_{49}\)V\(_2\) thin film.

was magnetized along the sample surface in three different directions. Major loops were measured parallel to the applied field. It can be seen from the comparison, the sample can reach saturation with the lowest magnetic field along \( \theta = 0^\circ \), while with the highest magnetic field along \( \theta = 90^\circ \). It can be assumed that the easy axis of the material is close to the \( \theta = 0^\circ \) direction, and the hard axis is close to the \( \theta = 90^\circ \) direction.

Figure 4.24: Characterization of anisotropic sample. (a) scalar characterization along different directions. (b) vector characterization. Sample: 400nm Ni\(_{81}\)Fe\(_{19}\) thin film.
4 Experimental Techniques

Fig. 4.24(b) shows the vector characterization when the anisotropic sample was magnetized along the hard axis. The polarization component parallel to the applied field $J_\parallel$ is close to the hard axis, while the polarization component perpendicular to the applied field $J_\perp$ is close to the easy axis. When the magnetic field is zero, $J_\perp$ is much greater than $J_\parallel$. The reason can be analyzed according to the scalar measurements shown in Fig. 4.24(a). The anisotropic sample can get saturated with very low magnetic field along the easy axis. As a result, $J_\perp$ shows an obvious increase near the zero field.

4.3.5 Vector Characterization in Rotational Field

The VVSM can be applied for vector characterizations in a rotational field. As the electromagnet cannot be moved, the rotational field is realized by mechanically rotating the tested sample. [24, 65]. By adjusting the rotation angles and the magnitude of the field, the rotational field can be applied with different trajectories. In the characterization results shown here, both the magnetic polarization and magnetic field intensity are normalized for comparison.

The vector characterizations shown in Fig. 4.25 were carried out in a rotational field with circular trajectory. The sample was firstly magnetized in the $0^\circ$ direction and the magnetic field intensity was then rotated with a constant magnitude from $0^\circ$ to $360^\circ$. In the measurement shown in Fig. 4.25(a), the sample was firstly magnetized to negative saturation, then returned close to zero polarization by applying a positive coercive field (about 9kA/m). The measurement was made when the field started to rotate. It can be seen from the polar plot, during rotation the magnetic polarization vector rotates mostly in the negative direction. By increasing the rotation magnitude of the magnetic field to 25kA/m, the results become quite

![Figure 4.25: Vector characterizations of an isotropic sample in rotational field with circular trajectory. (a) the magnetic field rotates with a radium close to the coercive field 9kA/m. (b) the magnetic field rotates with radium of 25kA/m. Sample: 1.9 µm Fe$_{49}$Co$_{49}$V$_2$ thin film.](image-url)
different. As shown in Fig. 4.25(b), the magnetic polarization vector starts with a magnitude in the 0° direction, then rotates closely together with the magnetic field.

The magnetic field can also be adjusted to rotate with ellipse trajectories. The measurement shown in Fig. 4.26(a) started from zero polarization at positive coercive field 9kA/m. The magnitude of the magnetic field reaches maximum (15kA/m) at transversal directions (90° and 270°). In comparison with the result shown in Fig. 4.25(a), the magnetic polarization vector rotates closer to the magnetic field near transversal directions.

The measurement shown in Fig. 4.25(b) started at a positive remanent polarization in a zero field. The magnetic field component along 0° direction changes in a small range between zero and 10kA/m, the resultant polarization vector rotates close to the remanent polarization.

![Figure 4.26: Vector characterizations of an isotropic sample in rotational field with ellipse trajectory. (a) the magnetic polarization vector rotates from zero. (b) the magnetic polarization vector rotates from remanent polarization. Sample: 1.9 µm Fe$_{49}$Co$_{49}$V$_{2}$ thin film.](image)

The above mentioned characterizations are applied to an isotropic sample. If anisotropic material is magnetized in a rotational field, due to its direction dependent magnetic properties, the polarization vector behaves differently. Fig. 4.27 shows the vector characterizations of an anisotropic sample. The magnetic field intensity rotates with a constant magnitude.

In Fig. 4.27(a), the rotation magnitude of the magnetic field is 2kA/m. With reference to the major loops shown in Fig. 4.24(a), at 2kA/m, obvious difference of polarization exists along different directions. In the rotational field shown in Fig. 4.27(a), the magnetic field reach 2kA/m in the 0° direction then rotates from 0° to 360°. The resultant polarization rotates with an ellipse trajectory. From the shape of the ellipse, it can be assumed that the 0° direction is not completely parallel to the easy axis.

In Fig. 4.27(b), the rotation magnitude of the magnetic field is increased to 4kA/m. According to the measurement shown in Fig. 4.24(a), when the magnetic field reaches 4kA/m,
the sample is almost saturated along every direction. The polarizations display no obvious difference. Therefore, in the rotational field with radium of 4kA/m shown in Fig. 4.27(b), the rotation trajectory of the polarization vector is almost circular. During rotation the polarization vector rotates close to the magnetic field.

In this chapter three different characterization methods have been introduced. By means of the Epstein frame method, determination of the average magnetic properties can be carried out at different frequencies or temperatures. By applying the double C-yoke SST, the influence of plastic deformation can be evaluated using local magnetic property measurements. In order to characterize anisotropic materials, Epstein frame samples are cut from grain oriented steels along different directions. This is made by applying a VSM, because the sample can be mechanically rotated in the magnetic field. The VSM is widely utilized to detect the magnetic moment of small magnetic samples. By applying orthogonal pickup coils systems, vector characterization can be carried out in both undirectional and rotational magnetic field.

Reliable characterization methods provide accurate experimental data, which can be applied in identification of hysteresis models. The identification method and the numerical calculation procedure of the inverted Preisach model are introduced in the next chapter.
This chapter introduces the identification and numerical calculation method of the inverted Preisach model. The identification method plays an important role in the accuracy of the models. For comparison, the inverted Everett function, which is applied as the identification method for the pseudo-inverse of the Preisach model, is described in the first section. In order to calculate the inverted Everett matrix, a set of first order reversal curves (FORCs) are required as the experimental data for identification. By means of the inverted Everett function, the problem of determining weight function can be solved. However, this identification method requires a large amount of experimental data and careful work in measuring.

In comparison to the pseudo-inverse of the Preisach model, the proposed inverted hysteresis model is identified with less experimental data. Only major hysteresis loops are required to determine the parameters in the weight function. The determination of parameters is fulfilled by a numerical optimization method. A quasi-Newton method algorithm is applied to optimize the mean squared error (MSE) between the measured and simulated data. More details are introduced in Section 5.2.

In Section 5.3, the numerical calculation method of the inverted Preisach model is introduced. The double integral of the weight function is calculated by applying a numerical approximation method. The model is verified with hysteresis loops of both hard and soft materials. Simulations have been made on major loops, minor loops and reversal curves. The simulation results show good approximation to the measurement data.

In the last section, a short summary comparing the two identification methods is presented.

### 5.1 Identification with the Inverted Everett Function

As introduced in Section 3.1, with the pseudo-inverse of the Preisach model, the output is calculated by adding or subtracting the Everett function values at the vertices of the staircase interface. Therefore, to identify the pseudo-inverse of the Preisach model, the inverted Everett functions $E(\alpha, \beta)$ have to be calculated at all nodes of the Preisach plane. These values are arranged according to the coordinates of the nodes and form a so-called inverted Everett matrix. Normally, the Preisach plane is uniformly discretized by a $n \times n$ square mesh in the precalculation stage. The mesh values of $E(\alpha, \beta)$ form a triangular matrix $E_{n \times n}$ as the inverted Everett matrix.

$$
E = \begin{pmatrix}
E_{\alpha_1, \beta_1} & E_{\alpha_1, \beta_2} & \cdots & E_{\alpha_1, \beta_{n-1}} & E_{\alpha_1, \beta_n} \\
E_{\alpha_2, \beta_2} & E_{\alpha_2, \beta_3} & \cdots & E_{\alpha_2, \beta_{n-1}} & E_{\alpha_2, \beta_n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & E_{\alpha_{n-1}, \beta_{n-1}} & \cdots & E_{\alpha_{n-1}, \beta_n} & E_{\alpha_n, \beta_n}
\end{pmatrix}.
$$

(5.1)
5 Model Identification and Calculation

Figure 5.1: Identification of the pseudo-inverse of the Preisach model. On the left is the Preisach plane discretized by \( n \) (here \( n = 9 \)) mesh. In the middle is the input signal for measurement of FORCs. On the right are the FORCs for identification. All of the inverted Everett function results on these mesh nodes form the inverted Everett matrix.

The experimental data for identification is a discretized set of FORCs. The Everett function \( E(\alpha, \beta) \) is calculated according to (3.6). As the example illustrates in Fig. 5.1, the correspondence between the values of \( E(\alpha_i, \beta_j) \) and the points on the FORCs is described in the plot. E.g., \( E(\alpha_4, \beta_4) \) is calculated from half of the difference \( H_{\alpha_4} - H_{\alpha_4 \beta_4} \). To obtain \( H_{\alpha_4} \) and \( H_{\alpha_4 \beta_4} \), the magnetization \( M \) increases from negative saturation to \( \alpha_4 \), then decreases back to \( \beta_4 \). The calculated \( E(\alpha_4, \beta_4) \) can be indexed with \((\alpha_4, \beta_4)\) in the inverted Everett matrix.

Fig. 5.2(b) shows the inverted Everett matrix calculated from the FORCs shown in Fig. 5.2(a). During calculation, the Everett function values of vertices on the staircase interface are obtained from the inverted Everett matrix using an interpolation method. In this work the bilinear interpolation method is applied.

The measurement of FORCs is usually carried out point by point with the DC method. In order to obtain an abundant inverted Everett matrix for calculation, as shown in Fig. 5.2(a), FORCs distributed intensively inside the major loop should be measured. Therefore, an enormous amount of measuring points are required, which is difficult and time-consuming. Furthermore, measurement errors, e.g. the errors caused by noise in the measurement of the magnetic field, are carried over into the identification results [1, 5]. The simulation results are close to the experimental data, but the experimental errors may lead to the discontinuity and non-smoothness of the simulated curves.
5.2 Identification of the Inverted Preisach Model

The identification of the inverted weight function is accomplished with major loops. Compared with FORCs, it is easier to measure with both AC and DC methods. To decrease the experimental errors, mean experimental data can be obtained by repeating measurements. The advantage of identification with major loops becomes more obvious in the case of evaluating influence factors on hysteresis properties, such as frequency, temperature, and anisotropy. Measurements of identification data have to be carried out in different conditions. Therefore, the identification with major loops may attract more FE users.

During the identification process, the experimental data for identification is normalized for simplification as

\[ e(t) = \frac{M(t)}{M_{\text{max}}}, \quad f_{\text{meas}}(t) = \frac{H(t)}{H_{\text{max}}}. \] (5.2)

Figure 5.2: The calculated inverted Everett matrix. (a) illustrates the measured FORCs for identification (60 reversal curves). (b) is the surface of the inverted Everett matrix. (c) is the corresponding contour plot. Measurement of FORCs was carried out with VSM on 1.9 µm Fe\textsubscript{49}Co\textsubscript{49}V\textsubscript{2} thin film sample.
Consequently, the simulated magnetic field $H_{\text{simu}}(t)$ is calculated as

$$H_{\text{simu}}(t) = H_{\text{max}} f_{\text{simu}}(t),$$  \hspace{1cm} (5.3)

### 5.2.1 Simulation of the Forced Magnetization Effect

Some of the major loops are measured in intensive saturation. As shown in Fig. 5.3(a), the applied external magnetic field is further increased when the material is completely saturated. Theoretically, the magnetization retains an almost constant value and increases no further. This value is called the saturation magnetization. In practice, due to the forced magnetization effect, the magnetization (or polarization) keeps increasing with the applied magnetic field beyond actual saturation [42, 35]. The major loops with and without the forced magnetization effect are shown in Fig. 5.3(a).

![Figure 5.3: (a) The $M(H)$ loops with (black line) and without (red line) the forced magnetization effect. (b) The normalized $H(M)$ loops with (black line) and without (red line) the forced magnetization effect.](image)

For identification of the inverted Preisach model, the $M(H)$ loop is inverted and normalized to $f(e)$ loops. As shown in Fig. 5.3(b), the curves at $e = \pm 1$ (saturation magnetization) are almost vertical straight lines, which are especially obvious in the $f(e)$ loops obtained from the $M(H)$ loops measured in intensive saturation. It is difficult to model these curves and this may lead to numerical problems in simulation. Therefore, in the inverted modeling the vertical straight section should be removed from major loops as far as possible.

To take the forced magnetization effect into account, an analytical tangent function was proposed with reference to the forward Preisach model proposed by Sutor in [35]. The function is expressed as

$$g[e](t) = b \tan \left[ a \frac{\pi}{2} e(t) \right],$$  \hspace{1cm} (5.4)
5.2 Identification of the Inverted Preisach Model

where the parameter \(a\) in (5.4) is limited to \([0, 1]\), because the range of the tangent function is limited to \((-1, 1)\). The reason for choosing the tangent function as a basis is due to the fact that most \(H(M)\) loops bear a similarity to the tangent function.

By adding the analytical tangent function in (5.4) to the inverted Preisach model, the simulation result \(f_{\text{simu}}(t)\) is calculated as

\[
f_{\text{simu}}(t) = \hat{\Upsilon}(e)(t) + g(e)(t). \tag{5.5}
\]

The graphic relation between \(\hat{\Upsilon}(e)(t)\) and \(f_{\text{simu}}\) is shown in Fig. 5.4(a). When the input \(e = 0\),

\[
f_{\text{simu}} = \hat{\Upsilon} = \frac{H_c}{H_{\text{max}}}, \tag{5.6}
\]

where \(H_c\) is the coercivity of the magnetic materials. This important parameter is usually presented in material catalogues. The difference between the measured and simulated coercivities will affect the accuracy of the electromagnetic simulation involving hysteresis effect. \(\hat{\Upsilon}(e)(t)\) can be considered as a hysteresis loop which shares the same coercivity with \(f_{\text{simu}}\). The hysteresis loop \(\hat{\Upsilon}(e)(t)\) is an ideal major loop without the forced magnetization effect. The constant part in saturation is shown with a red dashed line in Fig. 5.4(a).

Depending on the influence exerted by the forced magnetization effect, the slopes of the simulated \(H(M)\) loop can be controlled with the \(a\) factor in the analytical tangent function in (5.4). As shown in Fig. 5.4(b), with the increase of the \(a\) factor, the slope becomes steeper. To simulate the \(H(M)\) loop with less forced magnetization effect, which retains an almost constant value beyond saturation, the factor \(a\) can be set with a large value in the analytical tangent function in (5.4).
5.2.2 Parameter Determination

According to the weight function (see equ 3.21) introduced in section 3.2.2, the five parameters in the function, $A$, $\sigma_1$, $\sigma_2$, $\eta$ and $h$ are required to be determined. As the forced magnetization effect is taken into account in this inverted Preisach model, the parameters $a$ and $b$ in the tangent function (5.4) are also involved as the identification target.

The identification of the inverted model is fulfilled with measured major loops. The seven parameters are determined by a quasi-Newton method algorithm by optimization of the mean squared error (MSE) between the measured and simulated data. The MSE is calculated as

$$
\text{MSE}(f) = \frac{1}{N_T} \sum_{k=1}^{N_T} [f_{\text{meas}}(t_k) - f_{\text{simu}}(t_k)]^2,
$$

(5.7)

where $N_T$ is the size of the measured data $f_{\text{meas}}(t)$.

In this work a LabVIEW optimization VI based on the Broyden quasi-Newton method was applied to minimize the mean squared error (MSE). In the algorithm, unknown parameters are determined with an inverse method. As $\eta$ should always be an even number, it is manually set as a constant in the initialization of the algorithm. The MSE calculated by (5.7) is taken as the optimization target of the scheme. Relative change factors of the other six parameters are taken as a variable vector $X$. In the initialization part of the algorithm, as shown in (5.8), the six parameters are set with initial values, all the elements in variable vector $X$ are set to 1.

$$
P_{\text{ini}} = \begin{pmatrix}
\sigma_{1,\text{ini}} \\
\sigma_{2,\text{ini}} \\
A_{\text{ini}} \\
h_{\text{ini}} \\
a_{\text{ini}} \\
b_{\text{ini}}
\end{pmatrix}, 
X_{\text{ini}} = \begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
1
\end{pmatrix}.
$$

(5.8)

In the algorithm, as shown in (5.9), new parameters $P_k$ are calculated by the Hadamard product of $P_{\text{ini}}$ and the updated $X_k$ at every iteration.

$$
P_k = P_{\text{ini}} \odot X_k = \begin{pmatrix}
\sigma_{1,\text{ini}} \\
\sigma_{2,\text{ini}} \\
A_{\text{ini}} \\
h_{\text{ini}} \\
a_{\text{ini}} \\
b_{\text{ini}}
\end{pmatrix} \odot \begin{pmatrix}
x_{1,k} \\
x_{2,k} \\
x_{3,k} \\
x_{4,k} \\
x_{5,k} \\
x_{6,k}
\end{pmatrix} = \begin{pmatrix}
\sigma_{1,\text{ini}} \cdot x_{1,k} \\
\sigma_{2,\text{ini}} \cdot x_{2,k} \\
A_{\text{ini}} \cdot x_{3,k} \\
h_{\text{ini}} \cdot x_{4,k} \\
a_{\text{ini}} \cdot x_{5,k} \\
b_{\text{ini}} \cdot x_{6,k}
\end{pmatrix} = \begin{pmatrix}
\sigma_{1,k} \\
\sigma_{2,k} \\
A_k \\
h_k \\
a_k \\
b_k
\end{pmatrix},
$$

(5.9)

where $k$ is the index for the iterations. The new parameters $P_k$ are applied in the inverted Preisach model to calculate the MSE. The minimal MSE is iteratively determined until the stopping criteria is reached.

This quasi-Newton method algorithm is efficient (iteration steps $k < 1000$). However, in solving nonlinear problems, this method cannot guarantee convergence of solutions, especially in approximation of the $H(M)$ loops, which contain large vertical straight parts.
5.3 Calculation of the Inverted Preisach Model

The inverted Preisach model should be numerically implemented according to (3.14) for computation of the output. Depending on the value (+1 or −1) of the switch operators $\hat{\upsilon}_{\alpha\beta}$, the whole Preisach plane can be divided into two regions $S^+(t)$ and $S^-(t)$, where $\hat{\upsilon}_{\alpha\beta}$ are equal to +1 in $S^+(t)$ and they are equal to −1 in the $S^-(t)$ area. Therefore, (3.14) can be written as

$$\tilde{\chi}[e](t) = \int\int_{S^+(t)} \nu(\alpha, \beta) d\alpha d\beta - \int\int_{S^-(t)} \nu(\alpha, \beta) d\alpha d\beta. \quad (5.10)$$

The calculation in (5.10) involves double integrals of the weight function $\nu(\alpha, \beta)$ over the Preisach plane. Although a differentiable weight function $\nu(\alpha, \beta)$ has been proposed in 3.21, it is very difficult to find its antiderivative function. Furthermore, it is easier to apply a numerical method to compute a numerical approximation of the double integrals. In calculation of the inverted Preisach model, the approximation work is carried out in a precalculation procedure before the calculation stage.

5.3.1 Precalculation of V Matrix

In the precalculation stage, the Preisach plane is uniformly discretized by creating a $n \times n$ square mesh. As shown in Fig. 5.5(a), the square mesh covers the whole Preisach triangle. The middle points of every square are taken as the computation nodes. The black nodes $(\alpha_i, \beta_j)$ are located inside or on the diagonal of the Preisach triangle $P^' = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha_i \leq \beta \}$. The hollow nodes represent the nodes outside the Preisach triangle.

![Diagram of the Preisach plane discretization](image)

Figure 5.5: (a) The square mesh for discretization of the Preisach plane in the precalculation stage. The black nodes (●) are in the Preisach triangle, the hollow nodes (○) are outside the Preisach triangle. (b) shows the approximated $\nu(\alpha, \beta)$ surface.

As shown in Fig. 5.5(b), if $n$ is large enough, the integral of the weight function over the
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Square segment $\Delta s$ can be approximated by the volume of cubes as

$$\int \int_{\Delta s} v(\alpha, \beta) d\alpha d\beta \approx \Delta s v(\alpha_i, \beta_j), \quad (5.11)$$

where $(\alpha_i, \beta_j)$ is the center point of the square segment $\Delta s$. As shown in (5.12), the integral of the weight function over a specified area $S$ in the Preisach plane can be approximated by a sum of the cubic volume, which is included in $S$.

$$\int \int_{S} v(\alpha, \beta) d\alpha d\beta \approx \Delta s \sum_{(\alpha_i, \beta_j) \in S} v(\alpha_i, \beta_j), \quad (5.12)$$

Because the hollow nodes are outside the Preisach triangle, the integrals over these segments including hollow nodes are set to be zero. For the square segments including the nodes $(\alpha_i, \beta_j)$ on the diagonal $(\alpha_i = \beta_j)$, only half of the square segment is included in the Preisach plane. The integral over these segments $\Delta s(\alpha_i=\beta_j)$ is approximated as

$$\int \int_{\Delta s(\alpha_i=\beta_j)} v(\alpha, \beta) d\alpha d\beta \approx \frac{1}{2} \Delta s v(\alpha_i, \beta_j). \quad (5.13)$$

The approximated integrals over every unit segment of the mesh shown in Fig. 5.5(a) can be calculated as

$$v_{\alpha_i, \beta_j} = \begin{cases} 
\Delta s v(\alpha_i, \beta_j) & \text{if } \alpha_i < \beta_j, \\
\frac{1}{2} \Delta s v(\alpha_i, \beta_i) & \text{if } \alpha_i = \beta_j, \\
0 & \text{if } \alpha_i > \beta_j.
\end{cases} \quad (5.14)$$

All of $v_{\alpha_i, \beta_j}$ can form a $n \times n$ lower triangular matrix $V$ as

$$V = \begin{pmatrix}
v_{\alpha_1, \beta_1} & v_{\alpha_1, \beta_2} & \cdots & v_{\alpha_1, \beta_{n-1}} & v_{\alpha_1, \beta_n} \\
v_{\alpha_2, \beta_1} & v_{\alpha_2, \beta_2} & \cdots & v_{\alpha_2, \beta_{n-1}} & v_{\alpha_2, \beta_n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & v_{\alpha_{n-1}, \beta_{n-1}} & v_{\alpha_{n-1}, \beta_{n-1}} & v_{\alpha_{n-1}, \beta_n} & v_{\alpha_{n-1}, \beta_n} \\
\end{pmatrix}. \quad (5.15)$$

The precalculated weight function matrix $V$ is used as a constant in the calculation procedure. As an interpolation method is applied in updating the $S^+(t)$ and $S^-(t)$ regions in the Preisach plane, the mesh size, which is controlled by $n$ in discretization, plays an important role in the accuracy of the calculation results. In this work, $n$ is set to be greater than 100.

5.3.2 Calculation Procedure

According to (5.10), to calculate the output, it is critical to update the $S^+(t)$ and $S^-(t)$ regions of the Preisach plane. As introduced in (3.15) in Section 3.2.1, in the memory area of the
Preisach plane, the modified switch operator \( \hat{\vartheta}_{\alpha, \beta}[M](t) \) is calculated from the history value of the relay switch operator \( \hat{\gamma}_{\alpha, \beta}[M](t) \). Therefore, the history value of \( \hat{\gamma}_{\alpha, \beta} \) in area \( P = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \geq \beta \} \) should also be saved in calculation. All of the switch values of \( \hat{v} \) and \( \hat{\gamma} \) are saved in a \( n \times n \) matrix \( S \), which has the same size as the precalculated matrix \( V \).

\[
S = \begin{pmatrix}
\hat{v}_{\alpha_1, \beta_1} & \hat{v}_{\alpha_1, \beta_2} & \ldots & \hat{v}_{\alpha_1, \beta_{n-1}} & \hat{v}_{\alpha_1, \beta_n} \\
\hat{v}_{\alpha_2, \beta_1} & \hat{v}_{\alpha_2, \beta_2} & \ldots & \hat{v}_{\alpha_2, \beta_{n-1}} & \hat{v}_{\alpha_2, \beta_n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\hat{v}_{\alpha_{n-1}, \beta_1} & \hat{v}_{\alpha_{n-1}, \beta_2} & \ldots & \hat{v}_{\alpha_{n-1}, \beta_{n-1}} & \hat{v}_{\alpha_{n-1}, \beta_n} \\
\hat{v}_{\alpha_n, \beta_1} & \hat{v}_{\alpha_n, \beta_2} & \ldots & \hat{v}_{\alpha_n, \beta_{n-1}} & \hat{v}_{\alpha_n, \beta_n}
\end{pmatrix}
\tag{5.16}
\]

The matrix \( S \) is called the switch matrix here. As shown in (5.16), the value of \( \hat{\gamma}_{\alpha, \beta} \) is saved in the lower triangular part of matrix \( S \), while the value of \( \hat{\vartheta}_{\alpha, \beta} \) is stored in the upper triangular part. During calculation, according to the current input \( e(t) \), a linear interpolation method is applied to update the interface between \( S^+(t) \) and \( S^-(t) \) regions in the Preisach plane, which are the +1 and −1 regions in the matrix \( S \).

The production of the weight function and the corresponding switch value is fulfilled by applying the Hadamard product of the \( S \) matrix and the \( V \) matrix as

\[
S \circ V = \begin{pmatrix}
\hat{v}_{\alpha_1, \beta_1} \cdot v_{\alpha_1, \beta_1} & \hat{v}_{\alpha_1, \beta_2} \cdot v_{\alpha_1, \beta_2} & \ldots & \hat{v}_{\alpha_1, \beta_{n-1}} \cdot v_{\alpha_1, \beta_{n-1}} & \hat{v}_{\alpha_1, \beta_n} \cdot v_{\alpha_1, \beta_n} \\
\hat{v}_{\alpha_2, \beta_1} \cdot v_{\alpha_2, \beta_1} & \hat{v}_{\alpha_2, \beta_2} \cdot v_{\alpha_2, \beta_2} & \ldots & \hat{v}_{\alpha_2, \beta_{n-1}} \cdot v_{\alpha_2, \beta_{n-1}} & \hat{v}_{\alpha_2, \beta_n} \cdot v_{\alpha_2, \beta_n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\hat{v}_{\alpha_{n-1}, \beta_1} \cdot v_{\alpha_{n-1}, \beta_1} & \hat{v}_{\alpha_{n-1}, \beta_2} \cdot v_{\alpha_{n-1}, \beta_2} & \ldots & \hat{v}_{\alpha_{n-1}, \beta_{n-1}} \cdot v_{\alpha_{n-1}, \beta_{n-1}} & \hat{v}_{\alpha_{n-1}, \beta_n} \cdot v_{\alpha_{n-1}, \beta_n} \\
0 & 0 & \ldots & 0 & \hat{v}_{\alpha_n, \beta_n} \cdot v_{\alpha_n, \beta_n}
\end{pmatrix}
\]

The integral of the weight function over the whole Preisach plane is numerically realized by the sum of all elements in the Hadamard product matrix as

\[
\hat{\Gamma}[e](t) = \int_{\beta \geq \alpha} v(\alpha, \beta) \hat{\vartheta}_{\alpha, \beta}[e](t) \, d\alpha \, d\beta
\]

\[
\approx \sum_{i=1}^{n} \sum_{j=1}^{n} \hat{v}_{\alpha_i, \beta_j} \cdot v_{\alpha_i, \beta_j}
\]

\[
= \sum (S \circ V). \tag{5.17}
\]

As the weight function matrix \( V \) is a lower triangular matrix, by applying the Hadamard product to the \( S \) matrix and \( V \) matrix, the resultant matrix in (5.3.2) is also a lower triangular matrix. The value of the relay switch operator \( \hat{\gamma}_{\alpha, \beta} \) becomes zero by the Hadamard product. In this way, the values of relay switch operators \( \hat{\gamma} \) in the \( S \) matrix are saved and updated for calculation of \( \hat{v} \), but they are not involved in the integral approximation.

Finally, the resultant \( \hat{\Gamma}[e](t) \) is applied in (5.6) and (5.3) for computation of the output magnetic field \( H_{\text{simu}}(t) \).
5.4 Validation of the Inverted Preisach Model

To evaluate the accuracy of the inverted model, relative errors between the measured and simulated magnetic field are calculated and expressed as

$$\delta[k] = \frac{|H_{\text{meas}}[k] - H_{\text{simu}}[k]|}{H_{\text{max}}}.$$  \hspace{1cm} (5.18)

For an input with a size of $N$, the accuracy of the model is evaluated with the mean relative error $\overline{\delta}$.

In this work, simulations were performed on hysteresis curves of several different magnetic materials, which have been mentioned in Chapter 4. The inverted model was verified for both soft and hard magnetic materials. Besides major hysteresis loops, minor loops and FORCs can also be simulated. By comparison, the simulation results show good approximation to the measurement data, which can be demonstrated and evaluated with relative errors.

5.4.1 Simulation of Major Loops

The proposed inverted hysteresis model is identified with major loops. The agreement between simulated and measured major loops demonstrates the accuracy of the identification method.

The following presents the simulated major loops of six materials. Table 5.1 lists the resultant parameters determined by applying the quasi-Newton method algorithm. It can be seen that the parameter $\eta$ for the soft material M330-35A ($\eta = 10$) is much greater than $\eta$ applied for the other materials. The electrical steel M330-35A can easily be magnetized with a low magnetic field. The magnetic permeability increases rapidly near the coercivity ($53.73 \text{ A/m}$) and decreases when the material reaches saturation. In the case of the inverted hysteresis model, the change of the output relative to input is analyzed with the reluctivity, which increases rapidly when the input reaches the maximum (saturation magnetization). As parameter $\eta$ is the exponent in the weight function expression 3.21, the branch slope of major loops can be controlled with the value of $\eta$. The major loops with a sharp slope near the maximum of the input can be approximated with a large value of $\eta$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$A$</th>
<th>$h$</th>
<th>$\eta$</th>
<th>$b$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M330-35A</td>
<td>1.22</td>
<td>2.15</td>
<td>2.05</td>
<td>0.438</td>
<td>10</td>
<td>0.981</td>
<td>0.0068</td>
</tr>
<tr>
<td>M1300-100A</td>
<td>1.07</td>
<td>2.12</td>
<td>4.43</td>
<td>0.399</td>
<td>4</td>
<td>0.977</td>
<td>0.0118</td>
</tr>
<tr>
<td>Fe$<em>{49}$Co$</em>{49}$V$_2$</td>
<td>0.596</td>
<td>0.823</td>
<td>2.02</td>
<td>-0.178</td>
<td>2</td>
<td>0.87</td>
<td>0.0873</td>
</tr>
<tr>
<td>Ni$<em>{81}$Fe$</em>{19}$</td>
<td>0.903</td>
<td>0.542</td>
<td>1.83</td>
<td>-0.608</td>
<td>2</td>
<td>0.77</td>
<td>0.175</td>
</tr>
<tr>
<td>35Z155-RD</td>
<td>0.996</td>
<td>1.89</td>
<td>32</td>
<td>0.405</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35Z155-TD</td>
<td>0.828</td>
<td>1.33</td>
<td>40.5</td>
<td>0.464</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
5.4 Validation of the Inverted Preisach Model

The simulated major loops of electrical steels M330-35A and M1300-100A are shown in Fig. 5.6(a) and (b). $\hat{\Upsilon}(e) \cdot H_{\text{max}}$ is presented with the measured and simulated major loops for comparison. $\hat{\Upsilon}(e) \cdot H_{\text{max}}$ can be considered as a major loop, which shares the same coercivity and saturation magnetic polarization with the measured major loop. It can be observed in Fig. 5.6(c) and (d) that the relative errors $\delta$ of both simulation results are under 13%. However, large errors are distributed near the maximum of the input magnitude (12.75% for M330-35A and 8.89% for M1300-100A). This may be due to the difficulty in defining the saturation magnetization and the asymmetry of the measured major loops. But the simulated coercivity, which is usually taken as an important material parameter, has a reasonably small relative error (less than 1% for both results).
Figure 5.7: Simulated major loops of Fe\textsubscript{49}Co\textsubscript{49}V\textsubscript{2} and Ni\textsubscript{81}Fe\textsubscript{19} thin film samples. Measurements were carried out with VSM at room temperature. (a) presents the measured and simulated major loops of a 1.9 µm Fe\textsubscript{49}Co\textsubscript{49}V\textsubscript{2} thin film sample. (c) shows the relative errors between the measured and simulated major loops in (a). The measured and simulated major loops of a 1.7 µm Ni\textsubscript{81}Fe\textsubscript{19} thin film sample are presented in (b). The relative errors are shown in (d).

The simulated major loops of Fe\textsubscript{49}Co\textsubscript{49}V\textsubscript{2} and Ni\textsubscript{81}Fe\textsubscript{19} thin film samples are shown in Fig. 5.7(a) and (b). Compared with the results of Fe\textsubscript{49}Co\textsubscript{49}V\textsubscript{2}, the simulated major loop of Ni\textsubscript{81}Fe\textsubscript{19} is not as close to the measurement data. As shown in Fig. 5.7(b), the magnetic field increases sharply near the maximal polarization 0.8T. This loop can hardly be approximated. It has a maximal relative error 10.56% near the remanent polarization 0.32T.

The oriented electrical steel 35Z155 has excellent magnetic property in the rolling direction (RD) compared with the transverse direction (TD). As shown in Fig. 5.8(a) and (b), the 35Z155-RD has a very low coercivity (24.83 A/m), it can be saturated by applying a magnetic field of 50 A/m. The measured major loop of 35Z155-TD has an abnormal shape. Both of the
5.4 Validation of the Inverted Preisach Model

Figure 5.8: Simulated major loops of oriented electrical steels 35Z155 in rolling (RD) and transverse directions (TD). Measurements were carried out with the Epstein frame method at 50Hz at room temperature. (a) presents the measured and simulated major loops of 35Z155-RD, (c) shows the relative errors between the measured and simulated major loops in (a). The measured and simulated major loops of 35Z155-TD are presented in (b). The relative errors are shown in (d).

major loops are difficult to approximate. By applying the inverted Preisach model, the fitted loops have acceptable relative errors. The mean relative error $\overline{\delta}$ is 6.43% for 35Z155-RD and 3.57% for 35Z155-TD.

The results presented above demonstrate that the simulated major loops have good agreement with measurement data, especially for the major loops of non-oriented steels. The asymmetry of the measured major loops and the improper definition of the saturation magnetization can lead to large relative errors. In order to decrease relative errors, the major loops for identification should contain as little of the constant part as possible. Aside from the errors caused by the experimental data, the discretization size of the Preisach triangle $n$ plays another important role in the accuracy of the simulated results.
Figure 5.9: Relative errors of the simulations performed with different discretization size $n$. (a) shows the mean relative errors $\delta$ to different discretization size $n$. (b) shows the maximal relative errors $\delta_{\text{max}}$ to different discretization size $n$.

As shown in Fig. 5.9(a) and (b), relative errors decrease obviously when the discretization size $n$ increases from 10 to 100. This change is not as significant when $n$ is larger than 100. Therefore, most simulations in this work were performed with a discretization size of 100.

5.4.2 Simulation of Asymmetric Hysteresis Loops

The inverted Preisach model keeps the wiping out and congruency properties, which enables it to be applied in the simulation of asymmetric hysteresis loops, such as FORCs (see Fig. 5.10), hysteresis curves with minor loops (see Fig. 5.11) and the magnetizing hysteresis loops (see Fig. 5.12). The simulated reversal curves, asymmetric and symmetric minor loops are guaranteed to be located inside the major loops. Before calculation started, the inverted model was identified with the outer major loops. Parameters of the weight function were determined by applying the quasi-Newton method algorithm. In calculation the measured magnetic polarization was taken as the input. The magnetic field was computed and compared with the experimental data.

As shown in Fig. 5.10, Fig. 5.11 and Fig. 5.12, the simulated asymmetric hysteresis loops are quite close to the experimental data. The mean relative error $\overline{\delta}$ is 1.42% for the FORCs shown in Fig. 5.10. $\overline{\delta}$ is about 2.19% for the hysteresis curves with minor loops shown in Fig. 5.11. $\overline{\delta}$ is about 1.61% for the magnetizing hysteresis loops shown in Fig. 5.12.
5.4 Validation of the Inverted Preisach Model

Figure 5.10: Simulation results of FORCs. Measurement was carried out with VSM on a 1.9 µm Fe$_{49}$Co$_{49}$V$_2$ thin film sample at room temperature. (a) presents the measured and simulated FORCs. (b) shows the input magnetic polarization. (c) presents the measured and simulated magnetic field to the data index. (d) shows the relative errors. Reversal points on the FORCs are at 0kA/m, 5kA/m, 10kA/m, 15kA/m, 20kA/m, 30kA/m, 40kA/m and 60kA/m.
Figure 5.11: Simulation results of minor loops. Measurement was carried out with VSM on a 1.9 µm Fe_{49}Co_{49}V_{2} thin film sample at room temperature. (a) presents the measured and simulated hysteresis curves with minor loops. (b) shows the input magnetic polarization. (c) presents the measured and simulated magnetic field to the data index. (d) shows the relative errors. Minor loops are located at 0 kA/m, 10 kA/m, 20 kA/m and 30 kA/m.
Figure 5.12: Simulation results of magnetizing hysteresis loops. Measurement was carried out with the Epstein frame method on M330-35A steel samples at 10Hz at room temperature. (a) presents the measured and simulated magnetizing hysteresis loops. (b) shows the input magnetic polarization. (c) presents the measured and simulated magnetic field to the data index. (d) shows the relative errors. The discretization size $n = 200$. 

(a)

(b)

(c)

(d)
5.5 Comparison of Identification Methods

The identification method of a hysteresis model plays an important role in enhancing the modeling accuracy. In this section, the identification method of the inverted Preisach model is compared with the Everett function method applied in the pseudo-inverse approach. Table 5.2 shows the comparison details. When the inverted Preisach model is applied, only a major loop is required to determine the 7 parameters in the weight function. With the pseudo-inverse approach, a set of FORCs are measured to calculate the inverted Everett matrix $E_{n \times n}$. The measurement of major loops is easier and faster than the measurement of FORCs. Moreover, the experimental errors in measuring major loops can be reduced by averaging the testing values of repeated measurements. But this is difficult in the measurement of FORCs. As the inverted Everett matrix is directly calculated from the FORCs, the experimental errors may be carried over into the simulation results.

To compare the identification accuracy, simulations of FORCs were performed both with the inverted Preisach model and the pseudo-inverse approach. Simulation results are shown in Fig. 5.13 and Fig.5.14. The inverted Everett matrix $E_{n \times n}$ is calculated by applying the bilinear interpolation method. The discretization size of the inverted Preisach triangle is 100. As shown in Table. 5.2, by applying the pseudo-inverse approach, the mean relative error is 2.67% for 40 FORCs and 1.97% for 60 FORCs. Adding 20 more reversal curves for identification decreases the mean relative error by about 26%. However, this gain is offset by the considerable measurement effort required. Compared with the pseudo-inverse approach, the inverted Preisach model requires only a major loop for identification. The identification process is simpler and faster than the pseudo-inverse approach. The simulation results display good agreement with the experimental data. These will be great advantages in attracting FE users.

<table>
<thead>
<tr>
<th>Models</th>
<th>Inverted Preisach Model</th>
<th>Pseudo-inverse Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental data</td>
<td>Less</td>
<td>More</td>
</tr>
<tr>
<td>A major loop</td>
<td>A set of FORCs</td>
<td></td>
</tr>
<tr>
<td>Determined parameters</td>
<td>7 parameters in weight function</td>
<td>Inverted Everett matrix $E_{n \times n}$</td>
</tr>
<tr>
<td>Measurement methods</td>
<td>Easy and fast Both AC or DC methods</td>
<td>Hard and slow DC method</td>
</tr>
<tr>
<td>Mean relative error (40 FORCs)</td>
<td>2.37%</td>
<td>2.67%</td>
</tr>
<tr>
<td>Mean relative error (60 FORCs)</td>
<td>2.09%</td>
<td>1.97%</td>
</tr>
</tbody>
</table>
5.5 Comparison of Identification Methods

Figure 5.13: Simulation results of FORCs with 40 reversal curves. The FORCs were measured with VSM on a 1.9 µm Fe$_{49}$Co$_{49}$V$_2$ thin film sample. (a) shows the input magnetic polarization. (b) presents the simulation results computed by applying the inverted Preisach model. (d) shows the relative errors of the simulation results in (b). The simulation results computed by applying the pseudo-inverse approach are shown in (c). The relative errors are presented in (e).
Figure 5.14: Simulation results of FORCs with 60 reversal curves. The FORCs were measured with VSM on a 1.9 µm Fe\textsubscript{49}Co\textsubscript{49}V\textsubscript{2} thin film sample. (a) shows the input magnetic polarization. (b) presents the simulation results computed by applying the inverted Preisach model. (d) shows the relative errors of the simulation results in (b). The simulation results computed by applying the pseudo-inverse approach are shown in (c). The relative errors are presented in (e).
6 Calculation Optimization

The inverted Preisach model introduced in this work can be identified with less experimental data. With small relative errors it can be applied to describe the hysteresis behavior of both hard and soft magnetic materials. To include the inverted model into FE codes, the calculation efficiency is another important factor.

In comparison to the inverted Preisach model, the pseudo-inverse approach needs much more experimental data for identification. However, the integral expression of the pseudo-inverse approach can be rewritten in a discrete form (see equations in 3.12 or 3.13). By means of the discrete form, the output is calculated by adding or subtracting the inverted Everett function values, which are computed on the vertex of the staircase line. In the precalculation stage, the inverted Everett function values are calculated on each node of the mesh covering the whole Preisach triangle. All these Everett function values are saved in the inverted Everett matrix as constant. During calculation, the Everett function values on the vertices of the staircase line are obtained by interpolation in this inverted Everett matrix. In this way, it costs little memory space and it is very efficient in calculation.

As introduced in Section 5.3.2, the inverted Preisach model is computed by applying a numerical approximation method. A square mesh is generated to cover the whole Preisach triangle. In each mesh cell the switch value and weight function value are calculated and saved in two matrices. During the calculation process, all the elements in the switch matrix have to be updated according to the current input. This costs both much computation time and considerable memory usage.

To improve the calculation efficiency, research was conducted on the discrete form of the inverted Preisach model. In this chapter the new algorithm utilizing the discrete form is introduced. This algorithm was evaluated by comparing it with the previous algorithm and the pseudo-inverse approach.

6.1 Discrete Form of the Inverted Preisach Model

In this work, the discrete form of the inverted Preisach model is proposed with reference to the pseudo-inverse approach. As shown in Fig. 6.1, the inverted Preisach triangle $T$ is separated into two subsets $S^+(t)$ and $S^-(t)$ by a staircase interface $G(t)$. The modified switch operators $\hat{\upsilon}_{\alpha\beta}$ in the $S^+(t)$ area are equal to +1, while the ones in the $S^-(t)$ area are equal to −1. The

---

1This discrete form and the algorithm evaluation introduced in this chapter has been published in [67, 68]
inverted Preisach model in (5.10) can be written as

\[
\hat{\Upsilon}[e](t) = -\int_I \nu(\alpha, \beta) d\alpha d\beta + 2\int_{S^+(t)} \nu(\alpha, \beta) d\alpha d\beta.
\]  

(6.1)

Once the model has been identified, the integral of the weight function over the whole Preisach triangle \( T \) can be considered as a constant. Therefore, the output \( \hat{\Upsilon}[e](t) \) in (6.1) depends only on the integral of the weight function over the positive region \( S^+(t) \).

To find the discrete form of the inverted Preisach model in (3.14), geometric analysis of the Preisach plane was performed and this is illustrated in Fig. 6.1. The Preisach plane of the inverted Preisach model is in \( P' = \{ (\alpha, \beta) \in \mathbb{R}^2 : \alpha \leq \beta \} \). The staircase line \( G(t) \) separates the \( S^+(t) \) and \( S^-(t) \) subsets in the Preisach plane. The points \( (M_k, m_k) \) are the vertices of \( G(t) \). \( M_k \) and \( m_k \) are the \( \alpha \) and \( \beta \) coordinates of these vertices. The values of \( M_k \) and \( m_k \) can be obtained from the extrema of input history.

Figure 6.1: Geometric interpretation of the rectangular areas in the \( S^+(t) \) subset of the inverted Preisach plane. \( G(t) \) is the staircase interface dividing \( S^+ \) and \( S^- \) subsets. (a) The magnetization process starts from the positive saturation state. (b) The magnetization process starts from the negative saturation state.

As shown in Fig. 6.1, the \( S^+(t) \) subset can be divided into several rectangles and a triangle. The rectangle \( R \) in Fig. 6.1 can be the difference between two trapezoids, e.g., the rectangle \( R \) in Fig. 6.1(a). In another case, the rectangle can be the difference between a trapezoid and a
6.1 Discrete Form of the Inverted Preisach Model

triangle, such as the rectangle \( R \) in 6.1(b). The points \((M_k, m_k)\) are the upper right vertices of these trapezoids or triangles.

In this work, an area \( S(\alpha_p, \beta_q) \) is defined as \( S(\alpha_p, \beta_q) = \{(\alpha, \beta) \in P', \alpha \leq \alpha_p, \beta \leq \beta_q\} \). As shown in Fig. 6.2, the area \( S(\alpha_p, \beta_q) \) always starts from the bottom left vertex of the Preisach triangle \((-1, -1)\) and takes the point \((\alpha_p, \beta_q)\) as the upper right vertex. When the point \((\alpha_p, \beta_q)\) is located on the line \( \alpha = \beta \), as shown in Fig. 6.2(a), the area \( S(\alpha_p, \beta_q) \) is a triangle. Otherwise the area \( S(\alpha_p, \beta_q) \) is a trapezoid [see Fig. 6.2 (b)].

Here a function \( F(\alpha_p, \beta_q) \) is defined as the integral of the weight function over the area \( S(\alpha_p, \beta_q) \). It is expressed as

\[
F(\alpha_p, \beta_q) = \iint_{S(\alpha_p, \beta_q)} \nu(\alpha, \beta) \, d\alpha d\beta. \tag{6.2}
\]

A rectangle \( R_k \) shown in Fig. 6.1 can be considered as the difference between \( S(M_k, m_k) \) and \( S(M_k, m_{k-1}) \). The integral of the weight function over this rectangle \( R_k \) is calculated as

\[
\iint_{R_k(t)} \nu(\alpha, \beta) \, d\alpha d\beta = \iint_{S(M_k, m_k)} \nu(\alpha, \beta) \, d\alpha d\beta - \iint_{S(M_k, m_{k-1})} \nu(\alpha, \beta) \, d\alpha d\beta
= F(M_k, m_k) - F(M_k, m_{k-1}). \tag{6.3}
\]

As shown in Fig. 6.1, besides these rectangles in the \( S^+(t) \) subset, there is always an extra triangle. The upper right vertex of this triangle is on the point \((e(t), e(t))\). The integral of the weight function over this triangle can be calculated by \( F(e(t), e(t)) \). Similarly, the integral of the weight function over the whole Preisach triangle \( T \) can be calculated by \( F(1, 1) \). According to the geometric interpretation shown in Fig. 6.1, the integral expression in (6.1)
6 Calculation Optimization

can be rewritten in a discrete form as

\[ \hat{Y}[e](t) = -F(1,1) + 2F(e(t),e(t)) + 2 \sum_{k=1}^{n(t)} [F(M_k,m_k) - F(M_k,m_{k-1})], \]  

(6.4)

where \( n(t) \) is the number of rectangles that \( S^+ \) can be divided into.

In this work, an optimized algorithm is designed to calculate the output with this discrete form in (6.4). Similarly to the pseudo-inverse approach, the integral computation can be replaced by adding or subtracting the F function values on the vertices of staircase interface.

6.2 Precalculation of the F Matrix

In the optimized algorithm, a precalculation procedure is required to calculate F function values in the mesh cells. As introduced in Section 5.3.1, a \( n \times n \) square mesh is generated to uniformly discretize the whole Preisach triangle. The integral of the weight function over each mesh cell is numerically approximated as \( a_{\alpha,\beta} \). As expressed in (5.15), all \( a_{\alpha,\beta} \) values calculated in the mesh can form a \( n \times n \) lower triangular matrix, which is called the weight function matrix and labeled with \( V \).

As introduced in (6.2), \( F(\alpha_p,\beta_q) \) is the integral of the weight function over area \( S(\alpha_p,\beta_q) \). In calculation, it can be numerically approximated by summing all the unit integrals \( a_{\alpha,\beta} \) covered inside the area \( S(\alpha_p,\beta_q) \). Mathematically \( F(\alpha_p,\beta_q) \) is calculated by selecting a submatrix of \( V \) and summing all the \( a_{\alpha,\beta} \) elements in this submatrix. This can be expressed as

\[ F(\alpha_p,\beta_q) = \int \int_{S(\alpha_p,\beta_q)} a(\alpha,\beta) \, d\alpha \, d\beta \approx \sum_{i=1}^{p} \sum_{j=1}^{q} a_{\alpha,\beta} = F_{\alpha_p,\beta_q}. \]  

(6.5)

All \( F_{\alpha_p,\beta_q} \) calculated on nodes \((\alpha_p,\beta_q)\) can form a \( n \times n \) lower triangular matrix \( F \) where

\[
F = \begin{pmatrix}
F_{\alpha_1,\beta_1} & F_{\alpha_1,\beta_2} & \ldots & F_{\alpha_1,\beta_n} & F_{\alpha_2,\beta_n} \\
F_{\alpha_2,\beta_2} & F_{\alpha_2,\beta_3} & \ldots & F_{\alpha_2,\beta_n} & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & F_{\alpha_{n-1},\beta_{n-1}} & \ldots & F_{\alpha_{n-1},\beta_n} & F_{\alpha_n,\beta_n}
\end{pmatrix}.
\]

(6.6)

Fig. 6.3 presents an example to explain how \( F_{\alpha_p,\beta_q} \) is calculated from \( a_{\alpha,\beta} \). The nodes of the mesh \(^2\) represent the nonzero elements in the \( V \) and \( F \) matrices. The elements \( a_{\alpha,\beta} \) with \((i > j)\) are zero as they are out of the Preisach plane. As shown in Fig. 6.3, the element \( F_{\alpha_4,\beta_4} \) is an approximated integral over a triangle, the \( a_{\alpha,\beta} \) included inside the dashed square are selected as a submatrix. \( F_{\alpha_4,\beta_4} \) is calculated by summing all the elements in this submatrix. In the same way, \((\alpha_4,\beta_7)\) is the upper right vertex of a trapezoid, \( F_{\alpha_4,\beta_7} \) is calculated by summing

\(^2\)The black nodes in Fig. 6.3 represent the selected elements in \( V \) and \( F \) matrices. The zero elements in the triangular part of \( V \) and \( F \) matrices are not shown here.
6.2 Precalculation of the $F$ Matrix

Figure 6.3: Calculation of $F_{\alpha p, \beta q}$ from $v_{\alpha i, \beta j}$. The Preisach plane is discretized by a $n \times n$ (here $n = 9$) square mesh.

all the elements included inside the dashed rectangle.

In the precalculation stage of the optimized algorithm, parameters of the weight function are determined from the measured major loops. Integrals of the weight function over mesh cells are numerically approximated and saved in a weight function matrix $V$. According to (6.5) and (6.6), F function values can be calculated from the elements in $V$ matrix. These F function values form a F matrix.

Fig. 6.4 presents the $V$ matrix and the $F$ matrix for simulating the hysteresis curves of an Fe$_{49}$Co$_{49}$V$_2$ thin film sample and M330-35A steel samples. The $V$ matrix shown in Fig. 6.4(c) has a distinct increase near the vertices of the Preisach triangle. This can be deduced from the shape of major loop measured on M330-35A steel samples. With a very low coercivity the magnetic field changes sharply relative to the magnetic polarization near saturation.

The matrix $F$ is precalculated and applied as a constant matrix in the calculation procedure. According to the discrete form of the inverted model in (6.4), the output is calculated by adding or subtracting the F function values. During calculation, vertices of the staircase interface $G(t)$ are updated according to the current input. The values of $F(\alpha, \beta)$ on the vertices are obtained by applying the interpolation method in the $F$ matrix.
Figure 6.4: The weight function matrix $V$ and the calculated $F$ matrix. All of these matrices are normalized. (a) is the weight function matrix $V$ identified from the major loop of an Fe$_{49}$Co$_{49}$V$_2$ thin film sample. (b) is the $F$ matrix calculated from the $V$ matrix in (a). (c) is the weight function matrix $V$ identified from the major loop of M330-35A steel samples at 10Hz at room temperature. (d) is the $F$ matrix calculated from the $V$ matrix in (c).

### 6.3 Updating of the Staircase Interface $G(t)$

In the calculation procedure of the optimized algorithm, it is not necessary to save or update the entire switch matrix $S$ according to the current input. Instead, only the vertices of the staircase interface $G(t)$ are required to be saved and updated with a matrix $G$ where

$$G = \begin{bmatrix} M_0 & M_1 & \cdots & M_k \\ m_0 & m_1 & \cdots & m_k \end{bmatrix}. \quad (6.7)$$

The $\alpha$ and $\beta$ coordinates of vertices are saved in the two rows of this matrix. $k$ represents the number of vertices on the staircase interface $G(t)$.

As introduced in Section 3.2.1, by applying different switch operators $\tilde{\gamma}_{\alpha\beta}$ and $\tilde{\upsilon}_{\alpha\beta}$, the resultant staircase interfaces $L(t)$ and $G(t)$ have geometrical correlation in the Preisach plane (see Fig. 3.7). The coordinates of vertices on the staircase interface $G(t)$ can be obtained by
means of its geometrical relation with \( L(t) \). In calculation, a matrix \( L \) is applied to save the coordinates of vertices on the staircase interface \( L(t) \) as

\[
L = \begin{bmatrix}
M_0' & M_1' & \cdots & M_k' \\
m_0' & m_1' & \cdots & m_k'
\end{bmatrix}.
\]

(6.8)

The coordinates of vertices on the staircase interface \( L(t) \) directly refer to the extrema of input history. According to the geometrical correlation between \( L(t) \) and \( G(t) \), the vertices of \( G(t) \) can be obtained from the vertices \( L(t) \) by a coordinate conversion

\[
m_i = 1 - M_i' + e(t), \\
M_i = e(t) - m_i' - 1,
\]

(6.9)

where \( i = 1, 2, \ldots, k \). \( e(t) \) is the normalized input and 1 is the maximal input after normalization.

Based on the wiping out property of the Preisach model, the vertices in the staircase interface \( L(t) \) are updated according to the local input extrema. As shown in Fig. 6.5, in the increasing case, the precious local input maxima which are not greater than the current input \( e \) will be wiped out, such as the local maximum \( M_2 \leq e \). In the staircase interface \( L(t) \) of the Preisach plane, the vertices whose \( \alpha \) coordinate are equal to these wiped maxima will be deleted.

This occurs in a similar manner for monotonically decreasing input. The precious local input minima which are not smaller than the current input \( e \) will be wiped out, such as the local minimum \( m_3 \geq e \). In the staircase interface \( L(t) \) of the Preisach plane, the vertices whose \( \beta \) coordinate are equal to these wiped minima will be deleted.
In calculation, the corresponding columns including these wiped out extrema will be deleted in the $L$ matrix. The updated $L$ matrix is converted to the $G$ matrix by the coordinate conversion described in (6.9). According to the vertices coordinates saved in the $G$ matrix, the corresponding F function values are obtained by bilinear interpolation in the $F$ matrix. These F function values are applied to calculate the output with the discrete form in (6.4).

### 6.4 Evaluation of the Optimized Algorithm

To evaluate the efficiency of the optimized algorithm, the average memory usage and execution time of the program are calculated by applying the profile performance and memory tool in LabVIEW. The two evaluation factors are compared with the previous algorithm and the pseudo-inverse approach.

As shown in Fig. 6.6(a), the average memory usage is roughly linear, proportional to the size of input. It clearly shows that a large amount of memory is in use when the previous algorithm is applied for calculation. The average memory used in the previous algorithm is about 1.5 times the average memory usage required in the pseudo-inverse approach. The optimized algorithm and the pseudo-inverse approach uses nearly the same amount of memory in calculation. In the previous algorithm a $n \times n$ switch matrix $S$ is applied to save the state value of switch operators $\hat{\upsilon}_{\alpha\beta}$ in the whole Preisach plane.

According to the definition of $\hat{\upsilon}_{\alpha\beta}$ in (3.15), in the memory area of the Preisach plane the value of switch operators depend upon their history value. Therefore, the switch matrix $S$ cal-

![Figure 6.6: Efficiency comparison of the three algorithms with average memory usage (a) and average execution time (b). $N$ is the size of the input $M[k]$. The discretization size $n$ is set to be 100 in the simulation.](image)
culated in the last step has to be saved in a register to calculate the current $S$ matrix. However, for the pseudo-inverse approach and the optimized algorithm, only the vertices coordinates of the staircase interface are updated in calculation. In this way, lots of memory usage is economized in the calculation procedure.

As shown in Fig. 6.6(b), the previous algorithm costs the most execution time. The optimized algorithm is about 28 times faster than the previous algorithm. Comparing the pseudo-inverse approach and the new algorithm, there is no significant difference in the calculation efficiency between the two algorithms. This demonstrates that the coordinate conversion from the $L$ matrix to the $G$ matrix costs little execution time. Compared with the previous algorithm, which needs to calculate the $n^2$ switch operators in every step, the optimized algorithm performs more efficiently in calculation.

The comparisons in Fig. 6.6 demonstrate that the calculation efficiency of the inverted Preisach model has clearly been improved by applying the discrete form in computation. The efficiency of the optimized algorithm is very close to the pseudo-inverse approach. In this way, the inverted Preisach model proposed in this thesis not only has an advantage in identification, but it is also efficient in calculation.
7 Conclusions and Outlook

7.1 Conclusions

In this thesis an inverted hysteresis model is proposed to include ferromagnetic hysteresis in the electromagnetic field calculations. The model is based on the classic Preisach model, which has been widely applied within FE applications, since it approaches the requirements of accuracy, simplicity, and efficiency. The proposed inverted model applies a modified switch operator, which makes it possible to approximate the weight function distribution with a differentiable function. This inverted Preisach model with analytic weight function has advantages in identification. A limited number of parameters in the weight function can be identified from a major hysteresis loop. Compared with a set of FORCs required in the pseudo-inverse approach, a major hysteresis loop contains less experimental data and the measurement is more accessible. In order to evaluate the accuracy of this model, simulations were made on both symmetric and asymmetric hysteresis curves of magnetic materials. The mean relative errors of the simulation results are less than 7%. From an application point of view, accurate modeling with less experimental data may attract more FE users.

Besides the advantage in identification, the efficiency in calculation plays another important role in the implementation work. As this inverted model keeps the basic properties of the Preisach model, its integral expression can be rewritten in a discrete form. The application of this discrete form has obviously improved the calculation efficiency. The execution time of the optimized algorithm is about 28 times faster than the previous algorithm. Comparing the optimized algorithm with the pseudo-inverse approach, there is no significant difference in the execution time and memory usage.

• Inverted Preisach Model with Differentiable Weight Function

The inverted Preisach model proposed here introduces the magnetization \( M \) (or magnetic flux \( B \)) as input and takes the magnetic field strength \( H \) as output. Without an iterative procedure, the inverted model can be directly implemented into the FE formulations, in which the magnetic vector potential is taken as an unknown.

In this thesis analysis concentrated firstly on the pseudo-inverse of the Preisach model, which is most commonly applied to realize the Preisach inversion. The pseudo-inverse keeps the switch function of the forward Preisach model and simply exchanges the roles of the input and output. The advantage of the pseudo-inverse approach is the possibility of applying the Everett function method. In this way, it is not necessary to find a suitable weight function for the inverted model. However, the inverted Everett functions have to be identified from a set of FORCs, which contain a large amount of experimental data. The measurement of FORCs is
tedious and difficult. Furthermore, the experimental errors may be taken into the simulation results. To solve this problem, an attempt was made to find a suitable analytic weight function, which could be identified with less experimental data. With the help of FORCs, the distribution of the weight function was calculated by applying numerical approximation. This demonstrates that the weight function is partly positive and partly negative in the Preisach plane (see Fig. 3.2). It is difficult to find a suitable function to approximate this distribution.

In the Preisach integral, the weight function is multiplied by the switch operator $\hat{\gamma}_{\alpha\beta}$, which can be represented by rectangular hysteresis loops. The Preisach model is constructed as a superposition of an infinite number of the operators $\hat{\gamma}_{\alpha\beta}$ [16]. Fig. 7.1(a) and (c) present the $M(H)$ loop (forward major loop) and the elementary rectangular loop of $\hat{\gamma}_{\alpha\beta}$. As shown in Fig. 7.1(a), the magnetization output increases along the lower branch and decreases along the upper branch. The elementary rectangular loop in Fig. 7.1(c) has corresponding ascending and descending paths. In contrast, in the $H(M)$ loop (inverted major loop) shown in Fig. 7.1(b), the magnetic field increases along the upper branch and decreases along the lower branch. The pseudo-inverse of the Preisach model still keeps the switch operator $\hat{\gamma}_{\alpha\beta}$, whose rectangular loop has opposite paths to the inverted major loop. This may be the leading cause of the complicated weight function distribution.

According to this analysis, modification was made on the switch operator $\hat{\gamma}_{\alpha\beta}$. A modified switch operator $\hat{\upsilon}_{\alpha\beta}$ was proposed and applied in the new inverted Preisach model. As shown in Fig. 7.1(d), the rectangular hysteresis loop of $\hat{\upsilon}_{\alpha\beta}$ has similar ascending and descending paths to the inverted major loop shown in Fig. 7.1(b). By means of the modified switch operator $\hat{\upsilon}_{\alpha\beta}$, the approximated weight function distribution is almost completely positive in the Preisach plane (see Fig. 3.9). According to the shape of the approximated distribution,
7.1 Conclusions

A differentiable analytic expression was proposed as the weight function $\nu(\alpha, \beta)$ of the inverted Preisach model. The application of analytic weight function can improve the flexibility of modeling. Furthermore, the new inverted model retains the wiping out and congruency properties, which can offer great advantage in calculation.

• **Simple Identification from Major Hysteresis Loops**

To identify the inverted Preisach model, only major loops are required as experimental data. It is also possible to use minor loops to improve the simulation accuracy. Seven parameters are determined from measured major loops. A quasi-Newton method algorithm is applied to optimize the mean squared error (MSE) between the measured and simulated data.

The inverted model was verified for both soft and hard magnetic materials. Simulations were made on hysteresis loops of magnetic thin films, oriented and non-oriented electrical steels sheets. Besides symmetric hysteresis loops, asymmetric hysteresis curves, such as minor loops or FORCs can also be simulated. By comparison, the simulation results produced by the inverted hysteresis model show good approximation to the measurement data. The accuracy was evaluated with relative errors. The maximal relative errors mainly distributed near the saturation magnetization input. The main possible reasons are

- It is difficult to define an exact magnetization value in saturation. Due to the forced magnetization effect, the magnetization may keep increasing with the magnetic field after technical saturation is reached.

- It is hard to choose the maximal magnetic field in major loop. In the inverted modeling, the vertical straight part should be removed from $H(M)$ loops as far as possible. The vertical straight part will increase the difficulty and errors in the identification with major loops.

- The measured major loop is asymmetric. According to the definition of the Preisach model, the major loops are supposed to be symmetric.

The discretization size of the Preisach triangle also affects the relative errors of simulation. Both the maximal and the mean relative errors are clearly reduced by increasing the discretization size $n$. Most simulations in this work were performed with a discretization size of 100. The mean relative errors of these simulation results are less than 7%.

• **Efficient Calculation with Numerical Discrete Formulation**

In the numerical computation of the inverted Preisach model, the double integral of weight function over the whole Preisach plane is calculated by applying numerical approximation:

1. A $n \times n$ square mesh is generated to cover the whole Preisach triangle.

2. The elementary square area $\Delta s$ is multiplied to the $\nu(\alpha_i, \beta_j)$ value calculated at the middle points of each mesh square. The product $\Delta s \cdot \nu(\alpha_i, \beta_j)$ is taken as the approximated double integral of the weight function over mesh square.
7 Conclusions and Outlook

3. All the resulted products \( \Delta s \nu(\alpha_i, \beta_j) \) can be saved in a weight function matrix \( \mathbf{V}_{n \times n} \).

4. A switch matrix \( \mathbf{S}_{n \times n} \) is applied to save the current switch operator values at the middle points of each mesh square.

5. Multiplication of the weight function and the switch value is fulfilled by applying the Hadamard product of the \( \mathbf{S} \) matrix and the \( \mathbf{V} \) matrix.

6. The integral of the weight function over the whole Preisach plane is approximated by the sum of all elements in the Hadamard product matrix \( \mathbf{S} \circ \mathbf{V} \).

This method requires the \( \mathbf{S} \) matrix to be constantly updated according to the current input. It costs much memory usage and increases the computation time.

To make the calculation procedure comparable to the pseudo-inverse approach, the Everett function method is taken for reference. A function \( F(\alpha_p, \beta_q) \) is defined to be equal to the integral of the weight function over area \( S(\alpha_p, \beta_q) \). The area \( S(\alpha_p, \beta_q) \) always starts from the bottom left vertex of the Preisach triangle and takes the point \( (\alpha_p, \beta_q) \) as the upper right vertex. The positive set \( S^+ \) of the Preisach plane can be subdivided into segments. Each segment can be calculated with \( F(\alpha, \beta) \) functions. Hence, the integral expression of the inverted model has been rewritten in a numerical discrete form. During calculation, vertices of the staircase interface are updated according to the current input. The output is calculated by adding or subtracting the function values of \( F(\alpha, \beta) \). In comparison to updating the \( \mathbf{S} \) matrix, much memory space is saved and the calculation time decreases.

To evaluate the calculation efficiency, a comparison of memory usage and execution time was made between the previous algorithm, the pseudo-inverse approach and optimized algorithm. This demonstrates that: the previous algorithm utilizing the \( \mathbf{S} \) matrix costs the most memory usage; the new algorithm utilizing the numerical discrete formulation is about 28 times faster than the previous algorithm; comparing the pseudo-inverse approach and the new algorithm, there is no significant difference in the execution time and memory usage of the two algorithms.

• Three Different Characterization Methods for Various Experimental Data

In this work three different characterization methods were applied: the Epstein frame method, the double C-yoke SST and the vibrating sample magnetometer.

• By means of the Epstein frame method, the average magnetic properties of electrical steel samples were characterized. The obtained results show good consistency with the standard data. Measurements can be carried out at different frequencies (10Hz – 400Hz) and at different temperatures (-40° C – 140°).

• By applying the double C-yoke SST, the local magnetic properties of electrical steel samples were characterized. The change of the local magnetic properties can be applied to evaluate the influence of plastic deformation.
• By utilizing a VSM, the magnetic moment of small magnetic samples, such as the magnetic thin films, can be detected. By adding an orthogonal pickup coils systems, vector characterization can be carried out in both undirectional and rotational magnetic field.

These measurement methods provide experimental data for identification of the inverted Preisach model. The measurement results in different conditions, such as frequency, temperature, orientation, can be applied for model expansion in further research.

7.2 Outlook

The inverted model proposed in this thesis is based on the classic Preisach model, which is mostly considered as a mathematical approach rather than a truly physical model. Hence, this model can be applied mostly in the computation including hysteresis effect, especially in the FE analysis.

A FE computation kernel has been developed in the Chair of Sensor Technology, University of Erlangen-Nuremberg. It is possible to include this model in the FE code and apply it to solve electromagnetic problems. In this way, the accuracy and calculation efficiency can be compared with the iteration methods and other inverted hysteresis models.

The proposed inverted Preisach model is a rate-independent scalar model. As ferromagnetic hysteresis could be influenced by many factors, this inverted model can be extended to take the following effects into account.

• Dynamic effect
• Thermal effect
• Stress effect
• Effect caused by plastic deformation

The expansion could be accomplished with reference to the expansion of the forward Preisach models [35, 55, 69]. Moreover, by introduction of an additional rotation operator, the inverted hysteresis model can be applied to solve numerical problems where rotational electromagnetic fields are involved [23, 70, 71, 72, 73, 74, 75].
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3. 2013 June, IEEE Magnetics Society Summer School Assisi (poster), “A new inverted hysteresis model with modified switch operator and differentiable weight function”.

4. 2010 November, 55th MMM Conference Atlanta (poster), “A measurement setup for acquiring the local magnetic properties of plastically deformed soft magnetic materials”.


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