Experimental and Numerical Investigations of Axisymmetric Turbulence

Der Technischen Fakultät der Universität Erlangen-Nürnberg

zur Erlangung des Grades

DOKTOR-INGENIEUR

vorgelegt von Özgür Ertunç

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Acknowledgments

During the Summer Academy 1997, I met with Prof. Dr. Dr. h.c. Franz Durst for the first time. At that time, we had a nice talk on the impact of fluid mechanics on our daily life and the research activities at LSTM-Erlangen, which find usually direct application in important aspects of our society, such as industry, environment and science. At that time, his approach to fluid mechanics encouraged me to pursue my PhD work at LSTM-Erlangen. However, this occasion would never be possible when Prof. Dr. Gürbüz Atagündüz encouraged me to join the Summer Academy.

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To Nevin, my beloved and companion, who endured the most hardest part of this effort, tolerated the obsession and the late nights that seemed necessary to bring it to completion. To her I owe more than I can tell.
Abstract

Axisymmetric turbulent flows were experimentally and numerically investigated. Different aspects of axisymmetric turbulent flows related to anisotropy-invariant model of turbulence (AI-model), which was constructed at LSTM-Erlangen by Jovanović, Otić & Bradshaw (2003), are highlighted. For this purpose, decay of nearly isotropic grid-generated turbulence, decay of anisotropic axisymmetric turbulence, axisymmetric contraction, axisymmetric expansion and successive axisymmetric strain were investigated for the turbulence Reynolds number range $15 < \text{Re}_\lambda < 60$. Measurements of velocity fluctuations were performed with hot-wire anemometry in two different flow facilities at LSTM-Erlangen: the closed-loop wind tunnel and the axisymmetric strain tunnel, which was designed and constructed in order to investigate axisymmetric strained grid-generated turbulence.

It was found that the increase in longitudinal Reynolds stress along axisymmetric contractions having high contraction ratios, which were reported in the literature, cannot be predicted with the rapid distortion theory and direct numerical simulations. This anomalous trend of measured longitudinal Reynolds stresses is referred to as the high contraction ratio anomaly in the present work. Experimental evidence is provided that the anomaly is due to inaccuracies in the measurements. The imperfect spatial resolution of X-wire probes, the mass flow rate fluctuations in the flow facility and the electronic noise of measurement instrumentation contaminate measurements of velocity fluctuations and become significant downstream in the contractions, especially in those having high contraction ratios. A measurement method employing two-point correlation measurements was developed to separate the turbulent velocity fluctuations from non-turbulent mass flow rate fluctuations and electronic noise. The imperfect wire resolution effect on the measurements was removed by a second measurement method, which was developed by adopting the correction methods proposed for isotropic turbulence, which are available in the literature, to the investigated anisotropic turbulent flows. After the application of the proposed methods for correcting measurements, not only was a continuous decrease in longitudinal Reynolds stress observed, but also an increase in transverse Reynolds stress was captured. These measurement methods were subsequently employed to all the selected axisymmetric turbulent flow cases.

As turbulence was generated with grids in the present study and homogeneity is a basic property of axisymmetric turbulence, the spatial inhomogeneity of turbulence at the wake of the grid was investigated. It is shown that the inhomogeneous turbulence
field generated in the vicinity of the grid is conserved at long distances away from the grid, even for grids having low solidity. The inhomogeneity level was found to be ±5% in Reynolds stresses and ±20% in the anisotropy of Reynolds stresses at locations more than 40 mesh away from the grid. Similar homogeneity measurements were performed in axisymmetric contraction and expansions. It is shown that the inhomogeneity of the grid-generated turbulence disappears after becoming isotropic in a contraction.

After resolving the aforementioned experimental ambiguities, measurements of Reynolds stresses, length scales, velocity correlation functions and velocity spectra were performed for unstrained and strained axisymmetric turbulent flow cases. It is shown that grid-generated turbulence is nearly isotropic in terms of the Reynolds stresses; however, as it decays, it has no tendency to reach a long-lasting isotropic state. Similar observations were made for grid-generated turbulence whose anisotropy is reduced after contracting it slightly. As regards the inhomogeneity of grid-generated turbulence, the constants of the power-law decay of grid-generated turbulence are shown to be dependent on the location of the measurements axis relative to the grid. One modeled term in the dissipation equation of the AI-model was checked against the measured unstrained turbulent flows in the present study and those in the literature, and also another model reported in the literature. No agreement could be detected among them.

All the measured turbulent flow cases were analyzed in terms of various statistical quantities. Special emphasis is given to the development of the Reynolds stress anisotropy as the flow is strained in the streamwise direction. Owing to the corrections applied in the measurements, the rapid distortion theory was experimentally validated, for the first time, for axisymmetric contractions with high contraction ratios. It is shown that rapid distortion theory predicts the anisotropy along rapid contractions very well and along moderately rapid contractions relatively well.

Reynolds stress predictions for the measured turbulent flow cases were performed with the AI-model, the rapid distortion theory and the \( k - \epsilon \) model. The AI-model yields good predictions for unstrained and slowly strained cases, but fails considerably for rapid axisymmetric strain. It is shown that the rapid pressure-strain term in the AI-model can be improved by calibrating it against the rapid distortion theory.
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Nomenclature

Roman letters

\( A \)
  anisotropy-invariant function, relating \( \epsilon_{ij} \) to \( a_{ij} \)
\( A \)
  cross-sectional area and calibration constant
  used in hot-wire measurements
\( A(x) \)
  cross-sectional area of straining duct as a function of streamwise distance from its inlet
\( A_{ij} \)
  anisotropy tensor of any second-order tensor \( Q_{ij} \), \( i \) and \( j = 1, 2, 3 \)
\( A_{\Pi} \)
  constant defining a power law decay process of \( \Pi \) at the wake of the grid
\( a \)
  value of \( \psi \) at two-component isotropic limiting state
\( a_{ij} \)
  anisotropy tensor of \( \overline{u_i u_j} \), \( i \) and \( j = 1, 2, 3 \)
\( B \)
  calibration constant used in hot-wire measurements
\( b \)
  value of \( \psi \) at one-component limiting state
\( C_{\mu} \)
  constant of \( k - \epsilon \) model
\( C_{\epsilon 1} \)
  constant of \( k - \epsilon \) model
\( C_{\epsilon 2} \)
  constant of \( k - \epsilon \) model
\( C \)
  anisotropy-invariant function, relating \( \Pi_{ij} \) to \( a_{ij} \)
\( C_{ij} \)
  convective transport of \( \overline{u_i u_j} \), appears in the transport equation of \( \overline{u_i u_j} \)
\( c \)
  contraction ratio of axisymmetric straining duct, \( c = A_{inlet}/A_{outlet} \)
\( c(x) \)
  local contraction ratio
\( c_{Lz}(x) \)
  elongation ratio of longitudinal integral length scale of \( v \) fluctuations in the longitudinal \( x \) direction (\( c_{L_u} \))
\( c_{Ly}(x) \)
  contraction ratio of transverse integral length scale of \( v \) fluctuations in the longitudinal \( x \) direction
\( c_{Lxz} \)
  elongation ratio of longitudinal integral length scale of \( u \) fluctuations in the longitudinal \( x \) direction
\( c_{Lyv} \)
  elongation ratio of longitudinal integral length scale of \( v \) fluctuations in the longitudinal \( x \) direction
\( c_{Luv} \)
  elongation ratio of Taylor’s dissipation length scale of \( u \) fluctuations in the longitudinal \( x \) direction
\( c_{Lvx} \)
  elongation ratio of Taylor’s dissipation length scale of \( v \) fluctuations in the longitudinal \( x \) direction
\( D \)
  sensitivity of the hot-wire measurement system to changes in effective cooling velocity around \( U_e = \overline{U}_e \)
<table>
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<th>Description</th>
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<td>$D_k$</td>
<td>turbulent diffusion of turbulent energy; appears in the transport equation of $k$</td>
</tr>
<tr>
<td>$D_\epsilon$</td>
<td>turbulent diffusion of dissipation; appears in the transport equation of $\epsilon$</td>
</tr>
<tr>
<td>$D_{v-k}$</td>
<td>viscous diffusion of turbulent energy; appears in the transport equation of $k$</td>
</tr>
<tr>
<td>$D_{v-\epsilon}$</td>
<td>viscous diffusion of dissipation; appears in the transport equation of $\epsilon$</td>
</tr>
<tr>
<td>$D_{ij}^t$</td>
<td>diffusive transport due to turbulent fluctuations; appears in the transport equation of $\bar{u}_i\bar{u}_j$</td>
</tr>
<tr>
<td>$D_{ij}^v$</td>
<td>viscous diffusive transport; appears in the transport equation of $\bar{u}_i\bar{u}_j$</td>
</tr>
<tr>
<td>$E_u$</td>
<td>one-dimensional power density spectra of $u$ fluctuations</td>
</tr>
<tr>
<td>$E_v$</td>
<td>one-dimensional power density spectra of $v$ fluctuations</td>
</tr>
<tr>
<td>$E(t)$</td>
<td>instantaneous signal potential read by the hot-wire anemometer $(\bar{E} + e)$</td>
</tr>
<tr>
<td>$\bar{E}$</td>
<td>mean signal potential</td>
</tr>
<tr>
<td>$e$</td>
<td>fluctuating component of the signal potential $E(t)$</td>
</tr>
<tr>
<td>$e^{mass}$</td>
<td>fluctuating component of $E(t)$ due to mass flow rate fluctuations</td>
</tr>
<tr>
<td>$e^{turb}$</td>
<td>fluctuating component of $E(t)$ due to turbulent velocity fluctuations</td>
</tr>
<tr>
<td>$e^{elec}$</td>
<td>fluctuating component of $E(t)$ due to electronic noise</td>
</tr>
<tr>
<td>$\epsilon_{ij}$</td>
<td>anisotropy tensor of $\epsilon_{ij}$, $i$ and $j = 1, 2, 3$</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>anisotropy-invariant function, relating $\Pi_{ij}$ to $a_{ij}$</td>
</tr>
<tr>
<td>$F_f$</td>
<td>flatness factor of the fluctuating quantity $f$</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency in Hz or any fluctuating quantity</td>
</tr>
<tr>
<td>$f(r)$</td>
<td>longitudinal two-point velocity correlation coefficient in isotropic turbulence</td>
</tr>
<tr>
<td>$g(r)$</td>
<td>transverse two-point velocity correlation coefficient in isotropic turbulence</td>
</tr>
<tr>
<td>$\Pi(x, y)$</td>
<td>any mean quantity</td>
</tr>
<tr>
<td>$h(r, t)$</td>
<td>two-point velocity correlation coefficient in isotropic turbulence based on third-order moments of velocity fluctuations</td>
</tr>
<tr>
<td>$I_{\Pi}(x, y)$</td>
<td>Inhomogeneity parameter of the arbitrary mean quantity $\Pi$</td>
</tr>
<tr>
<td>$II_a$</td>
<td>second invariant of $a_{ij}$</td>
</tr>
<tr>
<td>$III_a$</td>
<td>third invariant of $a_{ij}$</td>
</tr>
<tr>
<td>$J$</td>
<td>anisotropy-invariant weighting function</td>
</tr>
<tr>
<td>$K_1$</td>
<td>correction factor to correct the effect of finite wire length on Reynolds stresses measured by hot-wire probes</td>
</tr>
<tr>
<td>$K_2$</td>
<td>correction factor to correct the effect of finite wire length on integral length scales measured with hot-wire probes</td>
</tr>
<tr>
<td>$k$</td>
<td>turbulent kinetic energy $(q^2/2)$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>wavenumber in the streamwise ($x_1 = x$) direction $(2\pi f/U)$</td>
</tr>
<tr>
<td>$L_f$</td>
<td>integral length scale derived from $f(r)$</td>
</tr>
<tr>
<td>$L_g$</td>
<td>integral length scale derived from $g(r)$</td>
</tr>
<tr>
<td>$L_y$</td>
<td>integral length scale derived from $R_y$</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>local change of $\bar{u}_i\bar{u}_j$ in time; appears in the transport equation of $\bar{u}_i\bar{u}_j$</td>
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Nomenclature

\[ L_{ixj} \] integral length scale derived from \( R_{ii}(x_j) \), \( i \) and \( j = 1, 2, 3 \)

\[ L_{ux} \] longitudinal integral length scale of \( u \) fluctuations

\[ L_{vx} \] longitudinal integral length scale of \( v \) fluctuations

\( l_0 \) characteristic length scale in \( k - \epsilon \) model

\( l \) wire length

\( M \) mesh size defined as the length of the open edge grid

\( \dot{M}(t) \) instantaneous mass flow rate \( \left( \dot{M} + \dot{m}_i \right) \)

\( n \) calibration constant used in hot-wire measurements

\( n_\Pi \) constant defining a power law decay process of \( \Pi \) at the wake of the grid

\( P(t) \) instantaneous pressure \( \left( P + p \right) \)

\( \bar{P} \) mean pressure

\( p \) fluctuating component of pressure

\( P_{ij} \) production term in the transport equation of \( \bar{u}_i \bar{u}_j \)

\( P_k \) production term in the transport equation of \( k \)

\( P_\epsilon \) production of \( \epsilon \) appears in the transport equation of \( \epsilon \)

\( P^1_\epsilon \) production of \( \epsilon \) by the mean velocity gradients

\( P^2_\epsilon \) production of \( \epsilon \) by the mean velocity gradients

\( P_\tau \) production of \( \epsilon \) by the turbulent vortex stretching

\( Q_{ij}(r) \) any first-order correlation tensor in homogeneous turbulence

\( Q_{ij}(r) \) any second-order correlation tensor in homogeneous turbulence

\( Q_{ijk}(r) \) any third-order correlation tensor in homogeneous turbulence

\( Q_{ss} \) trace of \( Q_{ij} \)

\( q^2 \) trace of the \( \bar{u}_i \bar{u}_j \)

\( R \) ratio of the longitudinal Reynolds stress to the transverse stress \( \left( \bar{u}_1 \bar{u}_1 / \bar{u}_2 \bar{u}_3 \right) \)

\( R_0 \) \( R \) prior to rapid straining

\( R_{ij}(x_k) \) correlation coefficient of velocity fluctuation components \( \left( \bar{u}_i(0)\bar{u}_j(x_k) / \bar{u}_i(0)\bar{u}_j(0) \right) \), \( i, j \) and \( k = 1, 2, 3 \)

\( R_{y,u} \) correlation coefficient of \( u \) fluctuations in transverse \( y \) direction

\( R_{y,v} \) correlation coefficient of \( v \) fluctuations in transverse \( y \) direction

\( R_y \) correlation coefficient of \( u \) and \( v \) fluctuations in transverse \( y \) direction in an axisymmetric strained turbulent flow

\( R_x \) correlation coefficient of \( u \) and \( v \) fluctuations in longitudinal \( x \) direction in an axisymmetric strained turbulent flow

\( r \) position vector in homogeneous turbulence

\( \text{Re}_\lambda \) turbulent Reynolds number, \( (q \lambda_g / \nu) \)

\( \text{Re}_M \) mesh Reynolds number, \( \text{Re}_M = \bar{U}_{\text{grid}} M / \nu \)

\( \text{Re}_\lambda \) Taylor-scale Reynolds number, \( \text{Re}_\lambda = \bar{u}_{\text{avg}} \lambda_g / \nu \)

\( S_f \) skewness factor of the fluctuating quantity \( f \)

\( S \) measure of mean strain rate

\( S^* \) mean strain rate parameter

\( S_{ij} \) mean strain rate tensor

\( s \) coordinate parallel to the wire of hot-wire probe

\( t \) time

\( U(t) \) instantaneous streamwise velocity \( U(t) = U_1(t) \)

\( U_i(t) \) instantaneous velocity vector in cartesian coordinates, \( i = 1, 2, 3 \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{U}_i$</td>
<td>mean velocity vector in cartesian coordinates, $i = 1, 2, 3$</td>
</tr>
<tr>
<td>$U_e(t)$</td>
<td>instantaneous effective cooling velocity on the hot-wire ($\vec{U}_e + u_e$)</td>
</tr>
<tr>
<td>$\vec{U}_{\text{grid}}$</td>
<td>mean streamwise velocity on the grid</td>
</tr>
<tr>
<td>$\vec{U}_R$</td>
<td>reference mean velocity used to calibrate hot-wire probes</td>
</tr>
<tr>
<td>$u_i$</td>
<td>fluctuating velocity vector in cartesian coordinates, $i = 1, 2, 3$</td>
</tr>
<tr>
<td>$u_{\text{e}}$</td>
<td>Reynolds stress tensor, $i = 1, 2, 3$ and $j = 1, 2, 3$</td>
</tr>
<tr>
<td>$u_{\text{turb}}$</td>
<td>fluctuating component of the effective cooling velocity on the hot-wire</td>
</tr>
<tr>
<td>$u_{\text{mass}}$</td>
<td>longitudinal turbulent velocity fluctuations</td>
</tr>
<tr>
<td>$u_{\text{elec}}$</td>
<td>longitudinal Reynolds stress; $\overline{uu} = \overline{u_1u_1}$</td>
</tr>
<tr>
<td>$\bar{V}(t)$</td>
<td>instantaneous streamwise velocity $V(t) = U_2(t)$</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>transverse Reynolds stress; $\overline{ww} = \overline{u_2u_2}$</td>
</tr>
<tr>
<td>$\mathcal{W}$</td>
<td>weighting function for the $Re_{\lambda}^*$ of turbulence</td>
</tr>
<tr>
<td>$\overline{ww}$</td>
<td>lateral Reynolds stress; $\overline{ww} = \overline{u_3u_3}$</td>
</tr>
<tr>
<td>$X_i$</td>
<td>cartesian coordinates in fixed frame of reference, $i = 1, 2, 3$</td>
</tr>
<tr>
<td>$x_i$</td>
<td>cartesian coordinate in the streamwise direction, $x = x_1$</td>
</tr>
<tr>
<td>$x_{\text{cont}}$</td>
<td>distance between the grid and the test nozzle (contraction)</td>
</tr>
<tr>
<td>$x_{\text{exp}}$</td>
<td>distance between the grid and the test diffuser (expansion)</td>
</tr>
</tbody>
</table>

### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>angle between the hot-wire probe axis and the velocity vector ($\alpha = \alpha_{\text{eff}} + \theta$)</td>
</tr>
<tr>
<td>$\alpha_{\text{eff}}$</td>
<td>effective angle between the inclined wire and probe axis</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta, 1 for $i = j$ and 0 for $i \neq j$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>trace of the dissipation correlation $\epsilon_{ij} \left( \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right)$</td>
</tr>
<tr>
<td>$\epsilon^*$</td>
<td>trace of the dissipation correlation $\epsilon_{ij}$ calculated by using decaying $\overline{uu}$ data and assuming isotropy</td>
</tr>
<tr>
<td>$\tau$</td>
<td>total viscous dissipation of turbulent energy $\left( \epsilon + \nu \frac{\partial^2 \overline{uu}}{\partial x_i \partial x_i} \right)$</td>
</tr>
<tr>
<td>$\epsilon_{ij}$</td>
<td>dissipation tensor $\left( \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right)$</td>
</tr>
<tr>
<td>$\epsilon_{ijk}$</td>
<td>permutation symbol</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>destruction of dissipation; appears in the transport equation of $\epsilon$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>unit vector along the symmetry axis, $i = 1, 2, 3$</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>Taylor’s dissipation length scale derived from $f(r)$</td>
</tr>
<tr>
<td>$\lambda_g$</td>
<td>Taylor’s dissipation length scale derived from $g(r)$</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>Taylor’s dissipation length scale derived from $R_y$</td>
</tr>
<tr>
<td>$\lambda_{ij}$</td>
<td>Taylor’s dissipation length scale derived from $R_{ij}(x_j)$, $i$ and $j = 1, 2, 3$</td>
</tr>
<tr>
<td>$\lambda_{ux}$</td>
<td>longitudinal Taylor’s dissipation length scale of $u$ fluctuations</td>
</tr>
<tr>
<td>$\lambda_{vx}$</td>
<td>longitudinal Taylor’s dissipation length scale of $v$ fluctuations</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity of the fluid</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>ratio of the longitudinal Reynolds stress</td>
</tr>
</tbody>
</table>
after rapid axisymmetric contraction to the initial stress
\( \mu_2 \)

density of the fluid
\( \rho \)

denisty of the fluid
\( \rho \)

angle between the wire normal and the flow direction
\( \theta \)

characteristic time scale in \( k - \epsilon \) model
\( \tau_0 \)

constant of \( k - \epsilon \) model
\( \sigma_k \)

constant of \( k - \omega \) model
\( \sigma_k \)

specific dissipation rate in the \( k - \omega \) model
\( \omega_i \)

vorticity of velocity fluctuations
\( \omega_i \)

kinematic viscosity of the fluid
\( \nu \)

kinematic viscosity of the fluid
\( \nu_k \)

pressure-strain term; appears in the transport equation of \( \bar{u_i}u_j \)
\( \Pi_{ij} \)

fast pressure-strain term; appears in the transport equation of \( \bar{u_i}u_j \)
\( \Pi^f_{ij} \)

slow pressure-strain term; appears in the transport equation of \( \bar{u_i}u_j \)
\( \Pi^s_{ij} \)

term used to model the combination \( P^4_{\epsilon} - \gamma ; \) appears in the transport equation of \( \epsilon \) for homogeneous turbulence
\( \psi \)
Chapter 1

Introduction

1.1 General

Navier–Stokes equations, which govern the conservation of momentum for Newtonian fluids, have been known for more than 200 years. However, it has not been possible to solve them analytically, except for very special and also very simple flow cases with related boundary conditions. The non-linearity of this system of partial differential equations is the main barrier for its analytical solution. In principle direct numerical solutions of turbulent flow problem are possible, since the governing equations constitute a closed system. However, with increasing Reynolds number the ranges of length and temporal scales increase. Hence, more spatial and temporal resolution of the simulations are required, such that one rapidly reaches the limits of the available computer power. Therefore, it is impossible – for now and the foreseeable future – to apply direct numerical simulations for most of the turbulent flows of technological interest. Being aware of these difficulties, after Feynmann, it developed into folklore to say that turbulence is the last great unsolved problem of classical physics. In spite of the difficulties related to the nature of turbulence, numerous investigations aimed at understanding turbulence and, consequently, modeling and controlling turbulent flows have been conducted.

Since 1877, when Boussinesq pointed out that turbulence cannot be treated by deterministic laws due to its random nature, the use of the theory of probability was known to be necessary for the formulation of turbulent flows. Being aware of the randomness of turbulence, in his pioneering work Osborne Reynolds (1895) formulated turbulent flows with a statistical approach with which he decomposed the instantaneous values of variables into their mean and fluctuations about their mean value. Due to non-linear convective terms in Navier–Stokes equations, new unknown variables in the mean dynamic equations appeared, namely Reynolds stresses, and additional dynamic equations are necessary to solve these extra unknowns. However, the transport equations written for Reynolds stresses introduce more unknowns and as a result more equations have to be formulated. Each step of formulation multiplies the number of
unknown terms and, consequently, the number of dynamic equations. To overcome this closure problem, it is a common approach to stop formulating more equations for the unknowns at a preferred level of approximation and to model the unknown terms in terms of the known variables based on physical assumptions and mathematical constraints. Prandtl’s “Mischungsweg” model (1925) and Kolmogorov’s $k - \varepsilon$ model (1942) are examples of the first modeling attempts.

As far as the randomness in space and time is concerned, Taylor (1921) showed that there exist correlations between two points in space and time and between the velocity of a particle at one time and that of the same particle at a later time. After the first truly quantitative velocity fluctuation measurements with the hot-wire technique by Dryden & Kuethe (1929), Ziegler (1934) and Dryden, Schubauer, Mock & Skramstadt (1938) it became possible to show the correlations of turbulence in space and time and the spectrum of turbulence. In the second half of the 1930s, Taylor (1935a, 1937) and von Kármán (1937) simplified the governing equations systematically with the help of the isotropy assumption which originates from the space–time correlation considerations.

The works of G.I. Taylor (1935a, 1937, 1938) and von Kármán (1937) and von Kármán & Howarth (1938) gave a direction to systematic investigations of turbulent flows using a statistical approach involving spectrum of turbulence and correlations of fluctuating quantities in time and space. Considering the spatial distribution of various quantities, turbulent flows were classified into two main types: inhomogeneous and homogeneous. In a homogeneous turbulent flow field, all the statistical moments of fluctuations are invariant under translation. This property simplify the governing equations and reduce them to an initial value problem. Therefore, homogeneous turbulent flows constituted the framework of understanding and modeling of turbulent flow phenomena. Homogeneous flows were further classified into isotropic turbulence and axisymmetric turbulence; for the former, the governing dynamic and kinematic equations take their simplest form. Roughly based on the the classification mentioned above, each class of turbulent flows was intensively treated theoretically and experimentally up to the 1980s.

The dynamic equations of the second-order two-point velocity correlation functions for isotropic turbulence were first derived by von Kármán (1937) and von Kármán & Howarth (1938). Since this equation is not closed, there is no exact analytical solution. The kinematic basis of isotropic turbulence goes back to the work of Robertson (1940), who elaborated concisely and elegantly the methodology of the reduction of the forms of correlation tensors to meet the requirements of isotropic turbulence with the aid of invariant theory and the methodology for deriving the consequences of the equations of motion and continuity for the fundamental scalar functions defining the correlation tensors.

Kolmogorov (1941a, b) suggested that at sufficiently large Reynolds numbers, the motion of the large-scale eddies determined entirely by the amount of energy transferred to the smaller eddies and this energy is dissipated by the latter through the action of viscosity. Thus, an equilibrium between the transferred energy and the dissipated energy would be established. In other words, the motion of eddies less than a certain
size is determined uniquely by the local mean dissipation and the viscosity. Furthermore, in that state the motion of the small eddies are weakly related to the motion of the anisotropic and inhomogeneous larger eddies and their motion is nearly isotropic. The combination of these postulations is called *Kolmogorov's theory of local isotropy*. The importance of this theory was appreciated in the western turbulence community especially after the analysis and interpretation of the ideas of Kolmogorov made by Batchelor (1947). Later, the ideas of equilibrium and local isotropy were heavily used in many of the turbulence models.

Batchelor & Townsend (1948a, b) conducted experiments on decaying nearly isotropic grid-generated turbulence. They showed that the decay process is composed of initial, transition and final periods, in which the effect of the inertia forces on turbulence gradually decreases and the spectrum and the correlation functions develop into a self-similar nature. Batchelor & Townsend (1949) investigated the spectrum of decaying nearly isotropic grid turbulence to determine the ranges of wavenumbers in which the small-scale eddies are in equilibrium as proposed by Kolmogorov (1941). Comte-Bellot & Corrsin (1966, 1971) produced turbulence with improved isotropy by slightly contracting the grid-generated turbulence. Their investigations revealed that the turbulent stresses have a power-law decay in the initial and the transition periods. The coefficients of the power-law decay were later investigated by many researchers, e.g. Naudascher & Farell (1970) and Mohamed & Larue (1990). For an overview of the different aspects of isotropic turbulence, see Batchelor (1953), Hinze (1975, ch. 3) and Townsend (1976).

In non-idealized turbulent flows, turbulence is convected through regions of different sorts of distortions, for example in turbulent boundary layer. Hence, analysis of the influences of the mean velocity field distortions on turbulence quantities are of fundamental importance. This analysis can be most fruitful when the complex hierarchy of turbulence is considered. Therefore, the effects of the fundamental types of distortions in homogeneous turbulent flows, i.e. strain, shear and rotation, constituted a central area of turbulence research which was expected to reveal the interaction between the mean and the fluctuating motion.

The first investigations on the development of turbulence subjected to superimposed uniform distortion date back to Prandtl (1932, 1933) and Taylor (1935b), both of whom were interested in studying the effect of wind-tunnel contraction on the free stream turbulence. A formal analysis of the problem, assuming that the turbulence is initially isotropic, was made by Ribner & Tucker (1953) for axisymmetric contraction of turbulence and independently by Batchelor & Proudman (1954) for arbitrary irrotational distortions. They recognized that the turbulence has finite spatial and time correlation and therefore the effect of the local mean flow variations, which exist in a general turbulent flow, can be accepted as a superimposed distortion which locally creates a coordinated strain on turbulence. Only after assuming that the turbulence interacts strongly with the mean flow but only weakly with itself under certain conditions, can the effects of viscous dissipation and non-linear processes be neglected and the development of the velocity fluctuations and/or the vorticity fluctuations under superimposed distortions be calculated. This approach was later called the "rapid distortion
Fage (1934), Hall (1938), MacPhail (1944) and Dryden & Schubauer (1947) carried out the earliest experiments in order to investigate the effect of strain on grid-generated turbulence. Townsend (1954) systematically investigated the effect of irrotational uniform distortion without streamwise strain on homogeneous turbulence. These were the first investigations on distorted turbulence motivated and guided by a theory, namely RDT. Tucker & Reynolds (1968) and Reynolds & Tucker (1975) extended these investigations on irrotational distortion of homogeneous turbulence. They studied a range of distortion scenarios: plane strain, symmetric contraction and flattening contraction. Comparison of their experiments with the RDT revealed a good match. The studies of Uberoi (1956), Mills & Corrsin (1959), and Tan-atichat, Nagib, & Drubka (1980) were among the most remarkable experimental investigations on the effect of axisymmetric contraction applied to nearly isotropic grid-generated turbulence. Gence & Mathieu (1979) considered two successive plain strains without streamwise acceleration of the velocity. They investigated the development of anisotropic turbulence subjected to plain strain which does not match the principle axis of the Reynolds stress tensor. The first straining action was used to generate anisotropic turbulence with a known principle axis. During the second straining, they observed that turbulence tried to realign itself, and even transferred energy back to the mean flow, when the angle between the principle axis of the Reynolds stress tensor and the second strain tensor was excessive.

The relaxation of anisotropic homogeneous turbulence is another important sort of homogeneous turbulence. The turbulence was made to be anisotropic by either axisymmetric strain or plain strain. After straining, the mean velocity distortion was switched off, so that the effect of viscous dissipation and the slow part of the pressure-strain correlations of anisotropic turbulent fluctuations could be investigated isolated from the mean flow distortion. Uberoi (1956, 1957) and Mills & Corrsin (1959) were the first to observe that the anisotropic turbulence, which was generated by an axisymmetric contracting nozzle, tended to become isotropic while it was decaying. Therefore, this behavior of turbulence is called the “return-to-isotropy”. The anisotropic flows generated by plain strain were experimentally investigated by Tucker & Reynolds (1968) and more recently by Gence & Mathieu (1980). In general, these experimental studies showed that the decrease in anisotropy is essentially controlled by the type of anisotropy generated. Warhaft (1980) and Choi & Lumley (2001) also carried out remarkable investigations on the return-to-isotropy problem.

Homogeneous pure shear turbulence and pure rotation turbulence have also been studied by many researchers. Among the homogeneous flows, the pure shear flows are the most relevant basic turbulent flow to the turbulent boundary layer flows. Hinze (1975, ch. 4) gave a good insight into homogeneous shear flows. The review of Gence (1983) on homogeneous turbulence should be referred to for a more complete overview of homogeneous turbulence.

The second simplest type of turbulence, axisymmetric turbulence, is a subclass of homogeneous turbulent flows. Axisymmetric strained homogeneous turbulence and
1.2 Background on Numerical Simulations

After the availability of computation facilities around 1970s, modeling and numerical simulation of turbulent flows rapidly became the driving subjects of turbulence research. Although direct numerical simulations (DNS) of the Navier–Stokes equations are limited to low Reynolds numbers, they can deliver quantities which cannot be accurately measured or even cannot be measured at all, such as dissipation tensor and pressure-strain rate correlations. These capabilities of DNS allow one to test new ideas and theories on turbulence. Most importantly, the results of DNS have been utilized to construct and develop turbulence models. Besides that measurements were analyzed in comparison with the DNS data or even new measurement techniques were developed. There are increasing numbers of niche areas where DNS can contribute at Reynolds numbers that are already realizable, such as laminar-to-turbulent transition.

Similarly to the experimental investigations on turbulent flows, DNS simulations evolved in accordance with the complexity of the turbulent flows. Initial computations were performed for decaying isotropic turbulence by Orszag & Patterson (1972). Schumann & Herring (1976) and Schumann & Patterson (1978) studied the return-to-isotropy problem of axisymmetric turbulence. Rogallo (1981) extended the range of turbulent flows to strained and sheared homogeneous turbulence and was first to provide complete Reynolds stress budgets for a turbulent flow. Lee (1985) and Lee & Reynolds (1985) made a detailed simulation study of homogeneous turbulent flows. Initial attempt at the simulation of wall-bounded flows, which are inhomogeneous, were made by Schumann (1973). One of the prominent data sets of fully developed turbulent channel flow is due to Kim, Moin & Moser (1987). Coleman, Kim & Spalart (2000) performed the first strained wall-bounded simulations.

In parallel with the increasing availability of computational resources, the number of published DNS investigations has rapidly increased. However, utilization of the results in the development of turbulence models is not increasing at a comparable rate, probably owing to speed of education and the evolution of individuals who can understand and utilize that amount of information. However the efforts are not at all negligible. Lumley & Newmann (1977) were the first who extensively used DNS data for turbulence modeling. Initial systematic efforts at the application of DNS data in turbulence modeling were started at CTR NASA Ames in 1987 (Hunt, 1988). Hallbäck (1993), Hallbäck, Sjögren & Johansson (1993), Jovanović, Ye & Durst (1995) and Sjögren & Johansson (1998) are example studies in which DNS data were employed for modeling. More recently, Jovanović, Otić & Bradshaw (2003) constructed a second-order turbulence closure heavily based on the DNS databases. This closure is mainly utilized in this thesis as a theoretical framework.
Large eddy simulation (LES) is a reduced form of DNS, with which the energy containing large scales are solved directly and the small scales are modeled. Using this approach, the grid resolution requirements are reduced at the cost of accuracy, which can be compensated by better modeling of the effect of small scales on the large scales. Instead of the lower accuracy compared with DNS, with reduced requirements on computational resources, LES made it possible to study two-point correlations, length scale developments, vortex dynamics and transitions phenomena, etc. Therefore, it has been used in the development of turbulence closures, in addition to increasing application to technologically relevant flows.

1.3 Background on Axisymmetric Turbulence

A turbulent flow is called axisymmetric when the correlations of fluctuating quantities at any number of points are invariant under translation and, different to isotropic turbulence, under rotation and reflection only about the symmetry axis. These statistical properties make axisymmetric turbulence the second simplest type of turbulent flows. In spite of its greater complexity compared with isotropic turbulence, its analytical properties and easy experimental generation make it appealing for turbulence modeling studies.

Before going into the chronological summary of investigations in this field, brief comments should be made on the famous study of Lumley & Newmann (1977) in order to point out the significance of axisymmetric turbulence. When they analytically treated the anisotropy of the second-order correlation tensors of turbulence quantities, the importance of axisymmetric turbulence became more pronounced. When the second invariant of the anisotropy tensor of any second order correlation term in Reynolds stress transport equations is plotted as function of the third invariant, a triangle is formed, as depicted in Figure 1.1. This triangle bounds all states of the anisotropy of all correlation terms and called as the anisotropy-invariant map, hereafter abbreviated AI-map. The two side edges of this triangle correspond to the axisymmetric turbulence and their intersection point corresponds the isotropic turbulence. The left and right axisymmetric edges can be obtained by axisymmetric contraction or expansion of isotropic turbulence, respectively. The analysis of the DNS of two-dimensional channel turbulent boundary layers made by Fischer (2000) showed that at a considerable fraction of the height of the channel, the anisotropy invariant lies in the vicinity of the right axisymmetric edge of the AI-map, corresponding to axisymmetric expansion of isotropic turbulence. This implies that even more complex turbulent flows might attain simpler states of turbulence which are easy to handle theoretically and numerically.

It can easily be recognized from the AI-map in Figure 1.1 that there are five subclasses of axisymmetric turbulence: (1) axisymmetric strain (contraction and expansion), (2) anisotropic axisymmetric turbulence without strain (return-to-isotropy), (3) isotropic turbulence, (4) two-component isotropic turbulence and (5) one-component turbulence. The investigations on these axisymmetric turbulent flows form a valuable source
for understanding turbulence and the validation of turbulence models. Hence, in the following, a chronological overview of investigations on axisymmetric turbulence is provided by keeping to the classification given here.

The primary interests of the early investigations employing axisymmetric turbulence were mostly for practical purposes; for instance, the effect of contraction (strain) on grid-generated axisymmetric turbulent was investigated because of its application in wind tunnels and jet flows. As mentioned before, Prandtl (1932, 1933) and Taylor (1935b) made the first step towards the calculation of the effect of stream distortion on turbulence. They worked out the changes in a small disturbance which varies sinusoidally with respect to all three space coordinates, for a distortion which is sufficiently rapid. Later, this approach was fully developed by Ribner & Tucker (1953) and Batchelor & Proudman (1954) into the so-called rapid distortion theory. The original RDT predicts the effect of distortion on initially isotropic turbulence. Sreenivasan & Narasimha (1978) derived relations for an initially axisymmetric turbulence, which can be anisotropic. They showed that if the departure from isotropy is not large, the energy ratios can be expressed as the sum of their values given by the isotropic theory. RDT can handle axisymmetric distortions when the distortion time scale is much shorter than the turbulence time scales. However, this is a marginal state of turbulence. In many of the turbulent flows effect of dissipation and non-linear terms cannot be neglected. Because of the approximations and idealizations made, the RDT solution is of limited predictive value. However, it does describe well limiting situations where the approximations are satisfied and the rapid distortion analysis gives guidance for the
development of Reynolds stress closures. Reviews on RDT and its applications have been explained at length in reviews such as those by Hunt (1978), Savill (1987) and Hunt & Carruthers (1990).

Based on the Robertson’s (1940) methodology, Batchelor (1946) and Chandrasekhar (1950a) derived various forms of correlation tensors, which appear in the dynamic equations of turbulence, for axisymmetric turbulent flow. Their studies on the kinematics of axisymmetric turbulence revealed that there are four principle second-order velocity correlation functions and related length and time scales, whereas one correlation function is sufficient in isotropic turbulence. Lindborg (1995) extended that approach and made a thorough kinematic analysis of axisymmetric turbulence. He was able to derive measurable relations for the calculation of fast and slow turbulent pressure-strain rate correlations.

Axisymmetric turbulence can be generated experimentally by passing the flow through a grid (hereafter called grid-generated turbulent flow). Grid-generated turbulence is slightly anisotropic such that the axial fluctuations are larger than the transverse fluctuations. However, in experimental practice, it is accepted to be isotropic, or more accurately, nearly isotropic. This type of turbulent flow is supposed to retain its statistical symmetry properties, such as axisymmetry and homogeneity, when it is axisymmetrically strained through contraction and/or expansion. Generally, the development of the Reynolds stresses under strain is strongly dependent on the properties of the turbulent field exposed to strain, such as the energy and dissipation of turbulence, the homogeneity and anisotropy of turbulence, the spectral properties and the interaction of all of these with the applied strain.

There are numerous experimental studies on the axisymmetric contraction of nearly isotropic turbulence. Available experiments showed three distinct behaviors of the longitudinal and transverse normal Reynolds stresses for nearly isotropic grid-generated turbulence exposed to symmetric pure positive strain, that is, the contraction of turbulence through a symmetric nozzle. The first and most general behavior is that the positive symmetric strain promotes the anisotropies of the turbulent stresses such that the mean square of the axial (or streamwise, or longitudinal) velocity fluctuations $u^2$ decreases and that of the transverse (or lateral) fluctuations $v^2$ increases. Note that due to axisymmetry two transverse components should be equal, i.e. $v^2 = w^2$. RDT also predicts this kind of behavior. A second kind of behavior is initially like the first one but with an increasing $v^2$ further down stream of the nozzles. The contraction ratio of nozzles, which is the ratio of the inlet to the outlet cross-sectional area, helps to distinguish these three distinct behaviors. The first kind appears in nozzles with contraction ratios $\gamma < 9$ and the second behavior is typical for contraction ratios $\gamma > 9$. The third behavior is the continuous decay of both longitudinal and lateral components, with an increasing anisotropy caused by the slower decay of the $v^2$ component. This kind of behavior occurs in highly dissipative and energetic and weakly contracted flows.

The symmetric expansion of turbulence corresponds to pure symmetric negative strain. In this case, the two lateral components of Reynolds stress tensor decrease and the longitudinal component increases. In contrast to axisymmetric contraction, the number of
1.3 Background on Axisymmetric Turbulence

The effect of contraction on grid-generated turbulence was comprehensively investigated first by Uberoi (1956). In his experiments, the grid-generated turbulence was strained through square channel contractions. He measured Reynolds stresses on the axis of contractions and examined the spectrum of turbulence before and after the contraction to get an idea about the distortion of the turbulence structure. He tested contraction ratios of 4, 9 and 16. His results showed good quantitative agreement with RDT only for contraction ratios below 4. The development of stresses in his nozzles with contraction ratios 4 and 9 showed the first type of behavior mentioned above, whereas the contraction ratio 16 delivered the second type of behavior, i.e. $\bar{u}u$ increased at later stages of strain. He could not give any reason for this increase. Apart from the observations on the development of stresses, he found that turbulence in the converging nozzle is far from being locally isotropic. This finding suggests that the small-scale eddies in the dissipation range might not be nearly isotropic as suggested by Kolmogorov’s theory of locally isotropic turbulence, that is, the dissipation length scales were not isotropic at all. He remarked that it might also be possible that the turbulence Reynolds numbers in his experiments were not sufficiently high for local isotropy to occur.

Mills & Corrsin (1959) published one of the rare studies in which skewness factors of velocity and temperature fluctuations are presented along with the contraction. They tested only a contraction ratio of 4. The stresses showed the first kind of stress development. The skewness factor before the contraction was zero and increased through the contraction, which was not an expected behavior of homogeneous axisymmetric turbulence. The post- and pre-contraction correlation measurements of Mills & Corrsin clearly showed the anisotropic development of the four different correlation coefficient functions and related length scales. Among the correlation coefficients of velocity fluctuations, the anisotropy of the lateral velocity fluctuations was the most apparent. The anisotropy of the dissipative length scales confirmed the absence of local isotropy in the contracted flows investigated by Uberoi (1956).

In addition to the other type of uniform distortions, Reynolds & Tucker (1975) investigated the effect of axisymmetric contraction. A reasonable degree of agreement with RDT was observed when the RDT was modified to account for the dissipation. Relying on this good match with RDT, they commented that the vorticity amplification process, essentially like that of RDT, should be incorporated in the more comprehensive models of turbulence.

Hussain & Ramjee (1976) and Ramjee & Hussain (1976) examined the effect of the axisymmetric contraction ratio and the shape of the contraction on turbulence. Contraction ratios of 11, 22, 44.5, 64 and 100 were examined. Depending on the contraction ratio, they observed the first and second kinds of stress development. For increasing $\bar{u}u$, they mentioned that boundary layer- and fan blade-induced fluctuations could be responsible.

Tan-atichat, Nagib & Drubka (1980) made the most comprehensive study on the effects
of axisymmetric contraction on grid-generated turbulence. They tested a wide range
of initial conditions and contraction geometries. Contraction ratios of 1, 2, 4, 9, 16, 23.5
and 36 were examined. Different length scales of turbulence were generated by using
grids with different mesh sizes. The first and second kinds of behavior of velocity
fluctuations could be observed in this study. For almost all initial turbulent length
scales, nozzles with contraction ratios of 16, 23.5, and 36 always showed an increase in
\( u' \) at later stages of strain. Interestingly, for very small initial length scales generated by
wire gauze, an increase in \( u' \) in the nozzle with a contraction ratio of 4 was observed.

Warhaft (1980) studied the effect of axisymmetric contraction on turbulence and ther-
mal fluctuations. He has used a nozzle with a contraction ratio of 4. His experiments
were performed at a moderate mesh Reynolds number (based on mesh size and mean
velocity) compared with the experiments of Uberoi (1956). In his experiments, the first
kind of behavior of velocity fluctuations can be observed. He supplied also pre- and
post-contraction spectrum showing that the spectrum of streamwise velocity fluctua-
tions were effected at most, which is qualitatively in parallel to what is expected from
the RDT.

Han (1988) investigated the effect of a contraction on grid-generated turbulence. He
employed a nozzle with a contraction ratio of 10.56. He also observed the second type
of stress development, with a slight increase in \( u' \) which was accompanied by a nega-
tive dissipation calculated from the transport equation of turbulent kinetic energy for
axisymmetric turbulence. Since negative dissipation was not physically possible, he
concluded that the convective transport term, which was canceled owing to the homo-
genity assumption, should appear again to satisfy the balance equation. However,
inhomogeneity brings not only the effect of transport terms but also the inhomoge-
neous dissipation and the viscous dissipation terms. The most valuable part of his
work was the analysis of the error sources in the measurements of such flows.

A unique experiment, because of the applied constant strain rate, was reported by
Leuchter and Dupeuble (1993) . This was the only experiment in which the third type
of Reynolds stress development was observed. In contradiction to the previous exper-
iments, they measured a continuous decrease of longitudinal and transverse Reynolds
stresses. Since the contraction was located very close to the grid, 16.66 mesh lengths
downstream of the grid, the dissipation was made the dominant process in the contrac-
tion. Leuchter and Bertoglio (1995) also predicted these experiments with an excellent
degree of agreement.

The latest experimental investigation known to the author was performed by Sjögren
& Johansson (1998) with a wind tunnel even larger than that of Uberoi (1956). They
employed a nozzle with a contraction ratio of 9. The major objective of their work
was to generate data for the construction and validation of a Reynolds stress trans-
port model. In addition to the measurements of the turbulent stresses, they measured
components of the dissipation tensor and, for the first time, the fast and slow parts of
the pressure-strain rate correlations, by utilizing the method suggested by Lindborg
(1995). The strong anisotropy of the Taylor micro scales can also be observed in their
measurements. Their measurements of the pressure-strain rate showed that the slow
part of the pressure-strain term can be as much as the rapid part, even at locations where the flow experiences the maximum strain.

In addition to the above-mentioned axisymmetric strain studies, axisymmetric strain measurements with very low contraction ratios have been reported. Those of Uberoi & Wallis (1966), Comte-Bellot & Corrsin (1966) and Bennett & Corrsin (1978) are examples of this kind. The main objectives of these experiments were to improve the isotropy of the grid-generated turbulence and to obtain reliable decaying isotropic turbulence data for verifying already established theories on isotropic turbulence. Uberoi & Wallis (1966) observed that the post-contraction isotropic turbulence tends to its pre-contraction anisotropic state. However, this tendency was not observed by Comte-Bellot & Corrsin (1966), who modeled the initial decay period of isotropic turbulence with a power law for a wide range of Reynolds numbers. In the investigations of Bennett & Corrsin (1978), which dealt with grid-generated turbulence at low turbulent Reynolds numbers, isotropic turbulence returning to its prior anisotropy was also reported, but rather slower than that of Uberoi & Wallis (1966). Moreover, it was also not clear whether the increasing anisotropy of their post-contraction isotropic turbulence was the final period behavior of the decaying isotropic turbulence, which was known since the work of Batchelor & Townsend (1948b).

Some of the data from the work of Uberoi (1956) and Tan-atichat et al. (1980) were selected by Ferziger (1980) for the development and verification of turbulence models and validation of the simulations of homogeneous turbulence. The data of Leuchter & Dupeuble (1993) were chosen for the validation of large-eddy simulations of turbulent flows by Tavoularis, Jiménez & Leuchter (1998).

The most comprehensive DNS simulations of axisymmetric turbulence were provided by Lee (1985) and Lee & Reynolds (1985). Their study covered isotropic turbulence, simple distortion of isotropic turbulence with axisymmetric contraction, axisymmetric expansion and plain strain, relaxation of strained homogeneous turbulence and redistortion of already distorted turbulence. For the analysis of their data, they extensively employed the AI-map. They found that the anisotropy of the vorticity field always attains a higher value than the anisotropy of the Reynolds stresses. Interestingly, their simulations showed that the ratio between the second invariants of anisotropy tensor of the dissipation tensor and that of the Reynolds stress tensor is a function of the applied strain rate: for slow strain rates the anisotropy of the dissipation rate is smaller than that of the Reynolds stresses and for higher strain rates it becomes larger. It came out that the anisotropy in axisymmetric contraction is mainly controlled by the total strain rather than the mean strain rate, which is also one consequence derived from RDT. Therefore, the anisotropy generated by axisymmetric contraction can be well predicted with RDT. In contrast to contraction, the anisotropy of the expanded isotropic turbulence was found to be dependent on both the total strain and the strain rate, such that, for a given total strain, the greater is the applied strain, the more anisotropy the turbulence attains. The reason for this difference was explained by the dominance of the rapid pressure-strain rate term as a sink term during expansion. The authors noticed that with increasing total strain, distorted turbulence evolves to one of the limiting states shown in Figure 1.1. Unfortunately, these investigations are the
only modeling-relevant investigations on axisymmetric expansion reported in the literature.

In 1951, Rotta pointed out that the pressure-strain rate term in the transport equations of Reynolds stress is responsible for the energy transfer between different components of Reynolds stress tensor and it does not contribute to the total energy of turbulence. The first clear experimental confirmation came from Uberoi (1956), who observed that the anisotropy of the axisymmetric turbulence, which was made to be anisotropic with a converging nozzle, decreased after the contraction. Uberoi’s investigations showed that after symmetric contraction the longitudinal component gains more energy from the lateral components than it dissipates, whereas the lateral components lose more energy to the dissipation than they transfer to the longitudinal components. Moreover, he figured out that the anisotropic axisymmetric turbulence develop into a locally isotropic state faster than becoming isotropic through the action of the pressure-strain rate correlations. Later, this kind of homogeneous turbulence was investigated by other researchers, since it gives direct clues on the effect of the pressure-strain rate correlations. Uberoi (1957) rigorously analyzed this phenomenon further and projected his results to shear flows. The measurements of Mills & Corrsin (1959) confirmed those of Uberoi.

The return-to-isotropy phenomenon was also observed in the measurements of decaying grid turbulence of Comte-Bellot & Corrsin (1966), which was slightly anisotropic close to the grid. Tucker & Ali (1973) showed that the decay of anisotropy is independent of viscous dissipation and mainly the redistribution of vorticity in space which in effect means equalization of the turbulent velocity components. Similar observations were made in the measurements of Warhaft (1980). The relaxation simulations of Lee (1985) revealed that the turbulence kinetic energy after expansion decays much faster than those after the axisymmetric contraction and the plain strain. The investigations of Choi & Lumley (2001) involved experimental data on turbulence returning to isotropy after being symmetrically expanded. They found that the decay rate of anisotropy after symmetric contraction is faster than that after expansion.

Most of the experimental and numerical investigations on the return-to-isotropy problem were aimed at understand the nature of intercomponent energy transfer. Chou (1945) and Rotta (1951) found the analytical and modeling basis for the treatment of the pressure-strain rate term in the Reynolds stress transport equations. A significant attempt at modeling in homogeneous turbulent flows was made by Lumley & Newman (1977). This study was remarkable in several respects, such as the utilization of DNS results for modeling and the modeling strategy employed, which was based on the anisotropy of turbulent quantities. This attempt was followed again by Lumley (1978). Hallbäck, Sjögren & Johansson (1993), Johansson & Hallbäck (1994), Sjögren & Johansson (1998), Choi & Lumley (2001) and Jovanović et al. (2003) provided some of the important models of intercomponent energy transfer in axisymmetric turbulence, which were constructed on the framework established by Lumley (1978). To the author’s knowledge, there has been no systematic comparison study on the performance of these models.
1.4 Motivation of the Work and Structure of Thesis

Important aspects of isotropic turbulence, which is one of the limiting states of axisymmetric turbulence, have already been mentioned in Section 1.1. Nevertheless, one should note that the experimental and numerical generation of isotropic turbulence and its investigations over a wide range of turbulent Reynolds numbers are still challenging issues. Low Reynolds number effects in isotropic turbulence were investigated by Bennet & Corrsin (1978). For example, they reported velocity fluctuations with a finite skewness factor, monotonically changing from positive to negative values as turbulence decays. They also observed that the isotropic turbulence tends to be more anisotropic as the turbulence Reynolds number decreases. High Reynolds number isotropic turbulence was investigated in detail by Mydlarski & Warhaft (1996) with the utilization of an active grid for the generation of turbulence. They showed that the asymptotic state suggested by Kolmogorov (1941), in which the spectrum of turbulence becomes self-similar, can only be reached at turbulence Reynolds numbers of $10^4$, whereas many flows of technological interest are at a maximum $10^3$. This implies that turbulence closures should consider the effect of Reynolds numbers.

Another important issue is the applicability of the results of axisymmetric turbulence investigations to the treatment of turbulence in wall-bounded flows. Fischer (2000) and Jovanović (2004) showed that the dynamic equations for turbulent stresses for nearly parallel wall flows form, outside the very near-wall region, turbulence properties which nearly coincide with the system of equations which hold for the turbulence developing in axisymmetric expansions. This peculiarity permits straightforward application of nearly all closure approximations derived for axisymmetric turbulence to treat wall-bounded flows. Moreover, understanding different states of axisymmetric turbulence helps us to explain phenomenon, such as transition, and to apply our understanding for the control of turbulence, e.g. for drag reduction. Unique examples of such studies are due to Jovanović & Pashtrapanska (2004) who treated transition based on the anisotropy-invariants of turbulence, and Jovanović, Pashtrapanska, Frohnapfel, Durst, Koskinen, & Koskinen (2005) who investigated the drag reduction with polymer additives. These studies justify the efforts spent on understanding and modeling axisymmetric turbulence.

1.4 Motivation of the Work and Structure of Thesis

As has been made clear up to here, analytical, experimental and numerical investigations of homogeneous turbulence provide firm background information for understanding basic mechanisms involved in turbulent transport phenomena. Such studies can lead to significant improvements in the predictive tools used in engineering practice. With the appearance of the work of Lumley (1978), which strongly emphasizes the role of anisotropy in the dynamics of turbulence, the above-mentioned expectation is nowadays fully justified. The influence of anisotropy, probably accounts for $60−70\%$ of all basic mechanisms involved in turbulence dynamics and can be isolated and studied most efficiently in homogeneous turbulence. Nevertheless, the application of many of the turbulence models is limited to flows with moderate anisotropy of the turbulent
stresses. Advanced models of turbulence for anisotropic turbulent flows, which make use of all the advantages of the anisotropy invariant approach of Lumley, can be most efficiently tested by performing numerical predictions against corresponding DNS and measurements, provided that these are reliable, i.e. free from inhomogeneous effects and any kind of measurement and simulation artifacts.

Even for homogeneous turbulent flows, the construction of turbulence closure is complicated and, to a large extent, contains approximations. The major difficulty lies in the fact that constitutive relations for unknown turbulence correlations are generally non-linear, with a large number of unknown scalar functions, which cannot be exactly determined rationally. In the case of axisymmetric turbulence these correlations can be reduced to linear relationships with a finite number of independent functions, which can all be determined to satisfy limiting anisotropy states of turbulence with well-established behavior for high and low Reynolds numbers. For such turbulence it is possible to derive, with a certain degree of confidence, a fully closed set of model equations which can be used for numerical predictions and comparisons with corresponding experimental results. Only for this class of homogeneous turbulent flows, are comparisons of predictions and measurements are meaningful and can lead to firm conclusions about our capability to understand and predict turbulent flows, nevertheless under very restricted circumstances. For these reasons, reliable experiments on axisymmetric turbulence over wide range of turbulent Reynolds number and a wide range of anisotropies are necessary in order to fully test the capabilities and limitations of various closure schemes. Moreover, one should recognize that a reliable turbulence closure can be utilized inversely to control turbulence in fields of technological interest, such as transition, drag reduction and combustion.

The measured quantities in the axisymmetric turbulence experiments are strongly dependent on the properties of the generated turbulent flow, such as the homogeneity level, the isotropy level and the spectrum and the interaction of these with the applied distortion. As far as the experiments and the direct numerical simulations are concerned, the generation of well-defined states of turbulence and their investigations by means of measurements and simulations, at an acceptable accuracy for the turbulence closure development, are still challenging tasks. The first limitation, which applies to both approaches, is the achievable turbulence Reynolds number which can be obtained either with a large number of computational grids and long integration times or with large flow facilities running at high speeds. On the experimental side, only after Lindborg (1995) did the measurement of pressure-strain rate correlations indirectly become possible exceptionally for axisymmetric turbulence. Measurements at very low Reynolds numbers are problematic because of the spatial resolution problem of hot-wire probes. Moreover, in the available studies of axisymmetric turbulence, the data do not cover a wide range of turbulence Reynolds numbers and the quantities relevant for the turbulence closure development are lacking, for instance, the development of the velocity correlation coefficients, the length scales and the pressure-strain rate correlations.

In the author’s opinion, some axisymmetric turbulence-related issues, which will be discussed later, have not yet reached a concrete state and, therefore, further investi-
gations on them should be conducted. Before utilizing available experiments for the validation of homogeneous turbulence closures, the streamwise and transverse homogeneity of the investigated flows should be checked. Nevertheless, not all the investigated flows were analyzed for their homogeneity level. Similar immaturity persists in the experimental investigations of isotropic turbulence. Up to now, the available results do not give a complete description over a wide range of Reynolds numbers and most of the available experiments suffer from experimental uncertainties. The measurements of the axisymmetric contraction of isotropic turbulence suggest three different types of development of stresses, but that with increasing streamwise fluctuations occurring in nozzles with high contraction ratios cannot be observed in the DNS of Lee (1985) or confirmed by any theory. Hence, the results of these kinds of experiments are not accurate enough for the validation of the modeled equations in highly anisotropic axisymmetric flows. Even though the experiments on axisymmetric expansion are important owing to the dominant effect of the rapid part of the pressure-strain rate term in such flows, the number of investigations on this class of axisymmetric turbulence is even smaller than that of contraction. This situation hinders reliable construction and validation of turbulence closures. To the author's best knowledge, there has been no study on successive axisymmetric strain, such as contraction followed by expansion. In spite of the fact that the investigations on the return-to-isotropy problem were utilized to model the slow part of the pressure-strain rate term, there is no comprehensive comparison of existing models.

Axisymmetric turbulent flows were investigated experimentally and numerically in this work. The main objectives of the investigations were:

i. To provide reliable measurements of symmetrical distortions of isotropic and axisymmetric turbulence.

ii. To provide reliable measurements of decaying isotropic and anisotropic axisymmetric turbulence.

iii. To evaluate available models of axisymmetric turbulence.

These objectives were achieved mainly through experimental investigations of the following types of axisymmetric flows:

- Decay of nearly isotropic grid-generated turbulence.
- Decay of anisotropic axisymmetric turbulence.
- Axisymmetric contraction of nearly isotropic grid-generated turbulence.
- Axisymmetric expansion of nearly isotropic grid-generated turbulence.
- Axisymmetric contraction followed by an axisymmetric expansion (successive axisymmetric strain).

In order to achieve quantitatively reliable data for such flows, considerable theoretical and experimental efforts were initially undertaken to find out and eliminate possible
artifacts in the measurements. Thus, in addition to the investigations of the above mentioned types of flows, two subjects inevitably became important parts of the present study:

- To find out and eliminate the causes of contradictory results in the measurements of the axisymmetric contraction of nearly isotropic grid-generated turbulence with nozzles of high contraction ratio.

- To analyze the homogeneity of the grid-generated turbulence in unstrained and strained turbulence fields.

The anisotropy-invariant turbulence model (AI-model) of Jovanović et al. (2003) was chosen as the theoretical basis for the experimental investigations. Hot-wire measurements of Reynolds stresses, spectra and correlation coefficient functions in axisymmetric turbulent flows, at which the flow attained a maximum $Re_\lambda$ value around 60, were conducted to test this model for the investigated flow cases.

In the second chapter, the basic theoretical background for axisymmetric turbulent flows is provided for the discussions taking place in the subsequent chapters. In addition, necessary details of rapid distortion theory, the $k-\epsilon$ model and the AI-model are given in this chapter. Chapter 3 introduces the experimental facilities utilized in the course of this study and gives an overview of the measurements performed. In Chapter 4, the inhomogeneity of grid-generated turbulence is discussed based on the experimental investigations performed in this work. The effect of strain on the inhomogeneous grid-generated turbulence is also demonstrated. Isotropic and anisotropic decay of grid-generated turbulence are the subjects of Chapter 5.

In Chapter 6, a detailed literature survey of experimental and numerical work on the effect of axisymmetric contraction on grid-generated turbulence is provided. The high contraction ratio anomaly, which appears in the literature and in the present experiments, is pinpointed. Theoretical and experimental causes of this anomaly and experimental techniques developed for its removal are presented.

In Chapter 7, the present author’s experimental investigations on axisymmetric contraction, expansion and successive axisymmetric strain are analyzed looking at the developments of Reynolds stresses, corresponding anisotropies, turbulence length scales, correlation coefficient functions and spectra. In Chapter 8, data sets are selected from all the measured turbulent flow cases, and predictions are performed to them with AI-model, rapid distortion theory and the $k-\epsilon$ model. In the last chapter, the results and conclusions of the present work are summarized and a research approach for the improvement of the AI-model is presented.
Chapter 2

Theoretical Background

There are special types of turbulence which satisfy certain symmetry conditions in a statistical sense. Among these, isotropic turbulence is the simplest one and is defined from purely mathematical constraints such that all correlations, which are written relative to a coordinate system with respect to which the fluid has a zero mean motion, are invariant under arbitrary translation, rotation and reflection about this coordinate system. Axisymmetric turbulence is next to isotropic turbulence in order of simplicity in terms of its symmetry properties. In this type of turbulence, the correlations are invariant under arbitrary translations, rotations along the symmetry axis and reflections in planes through and normal to the symmetry axis. Owing to these purely mathematical constraints, the dynamic equations of turbulence and the forms of the tensors appearing in these equations gain simplicity. Thereby, these types of turbulence are usually investigated theoretically, experimentally and numerically in order to establish a concrete basis to improve our understanding of the more general types of turbulence. A detailed literature review of these investigations is provided in Chapter 1.

In the present chapter, first the governing transport equations for Reynolds stresses, turbulent kinetic energy and turbulent dissipation rate are introduced in their general form for incompressible flows. Later, the idea of homogeneous turbulence and the consequences (simplifications) of this kind of turbulence on the governing equations are presented. It is shown that axisymmetric turbulence is a subclass of homogeneous turbulence and isotropic turbulence is a subclass of the former. Subsequently, basic kinematic relations for axisymmetric turbulence are provided based on the publications of Robertson (1940), Batchelor (1946, 1953) and Chandrasekhar (1950). Kinematic relations involve the forms of the correlation tensors appearing in the transport equations of turbulence, basic two-point correlation functions and related length scales. As part of the kinematic considerations, the anisotropy-invariant map of Lumley & Newmann (1977) is introduced. It is shown in this map that axisymmetric turbulence represents two important limiting states of turbulence and, therefore, its modeling is treated as a key issue in the invariant Reynolds stress modeling. Finally, necessary details of the RDT, $k - \epsilon$ model and AI-model are provided in this chapter, since the predictions performed with them are compared with the measurements performed in this study.
2.1 Governing Transport Equations

The Navier–Stokes equations are generally applicable to all kinds of flows for which the continuum assumption is still valid. Defined by a set of partial differential equations, all turbulent flows to a greater extent are also determined by Navier–Stokes equations and by corresponding initial and boundary conditions. Following Reynolds (1895), one can split fluctuating flow quantities into an averaged and a fluctuating part. Introducing these in the Navier-Stokes equations and averaging them yield the Reynolds averaged Navier–Stokes equations and continuity equation for an incompressible, isothermal and constant viscosity flow which is not exposed to body forces:

\[\rho \left( \frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} \right) - \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right) - \mathcal{P} \delta_{ij} - \rho \bar{u}_i \bar{u}_j \right], \tag{2.1a}\]

\[\frac{\partial \bar{u}_i}{\partial x_i} = 0. \tag{2.1b}\]

In this set of equations, the unknown correlations \(\rho \bar{u}_i \bar{u}_j\) arising from the velocity fluctuations appear in the form of stresses. Even though \(\rho \bar{u}_i \bar{u}_j\) has the dimensions of stress, it is more conventional than convenient to refer to \(\bar{u}_i \bar{u}_j\) as the Reynolds stresses. Reynolds stresses cause the well-known closure problem for turbulent flow predictions, since they are not covered by the momentum and continuity equations given above. Transport (dynamic) equations for the Reynolds stresses (second-order moments of velocity fluctuations) and the higher order moments can be formulated by using the Reynolds averaging method and some algebraic manipulation (see, for instance, Hinze 1975, p. 323 and Jovanović 2004, p. 14). After such a derivation, the transport equations for the Reynolds stresses for incompressible and constant viscosity flow become

\[\frac{\partial \bar{u}_i \bar{u}_j}{\partial t} + \bar{U}_k \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_k} = - \left( \frac{u_j u_k}{\partial x_k} \frac{\partial \bar{U}_i}{\partial x_k} + \frac{u_i u_k}{\partial x_k} \frac{\partial \bar{U}_j}{\partial x_k} \right) - \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_k} - \frac{1}{\rho} \left( \frac{u_j}{\partial x_i} \frac{\partial p}{\partial x_i} + \frac{u_i}{\partial x_j} \frac{\partial p}{\partial x_j} \right) - 2 \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} \tag{2.2}\]

where the resultant correlations are: local change of \(\bar{u}_i \bar{u}_j\) in time \((L_{ij})\); convective transport of \(\bar{u}_i \bar{u}_j\) \((C_{ij})\); production of \(\bar{u}_i \bar{u}_j\) due to the deformation of mean velocity field \((P_{ij})\); diffusive transport due to turbulent fluctuations \((D_{ij}^t)\); velocity-pressure gradient correlations \((\Pi_{ij})\); viscous dissipation of Reynolds stresses \((\epsilon_{ij})\); and molecular (viscous) diffusive transport \((D_{ij}^\nu)\). Among the aforementioned correlations, \(D_{ij}^t\), \(\epsilon_{ij}\) and \(D_{ij}^\nu\) are new unknown correlations, again causing a closure problem. In order to overcome the closure problem, one should either model them as functions of the correlations closed by transport equations or write transport equations for them, which results in more unclosed terms. In the considerations given below, the former method is employed. The kinetic energy of turbulence:

\[k = \frac{1}{2} \bar{u}_i \bar{u}_i \tag{2.3a}\]
and the trace of the dissipation correlation \( \epsilon_{ij} \):

\[
\epsilon = \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k}
\]  

(2.3b)

are used in models for \( D_{ij} \), \( \epsilon_{ij} \) and \( D_{\nu} \). Therefore, transport equations of these terms should be modeled and solved.

The transport equation for the turbulent kinetic energy \( k \) can be obtained by contracting the dynamic equations for \( \overline{u_i u_j} \): (2.2)

\[
\frac{\partial k}{\partial t} + \overline{U_k} \frac{\partial k}{\partial x_k} = - \overline{u_i u_k} \frac{\partial \overline{U_i}}{\partial x_k} - \left( \frac{p}{\rho} + k \right) \frac{\partial u_k}{\partial x_k} - \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} + \nu \frac{\partial^2 k}{\partial x_k \partial x_k},
\]

(I)

which states that the change in the kinetic energy of turbulence per unit of mass and of time including the convective transport by the mean motion (I) is equal to (II) the work of deformation of the mean motion by the turbulence stresses (production of turbulence by the mean velocity gradients) plus (III) the convective diffusion by turbulence of the total turbulence mechanical energy, or the work by the total dynamic pressure of turbulence, plus (IV) the major contribution to the total viscous dissipation of turbulent energy (2.3b), plus (V) the viscous diffusion.

Note that in the way the transport equation 2.4 is written, the total viscous dissipation does not fully appear as an extra term. The total viscous dissipation of turbulent energy is

\[
\overline{\epsilon} = \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} + \nu \frac{\partial^2 u_i u_k}{\partial x_i \partial x_k}
\]  

(2.5)

However, based on order of magnitude analysis (Schenck 1999), in most of the flows the total viscous dissipation term \( \overline{\epsilon} \) can be approximated by the trace of the dissipation correlation \( \epsilon \) (2.3b). The dynamic equation for the viscous dissipation \( \epsilon \) is

\[
\frac{\partial \epsilon}{\partial t} + \overline{U_k} \frac{\partial \epsilon}{\partial x_k} = -2\nu \frac{\partial u_i}{\partial x_i} \frac{\partial u_i}{\partial x_k} \frac{\partial \overline{U_i}}{\partial x_k} - 2\nu \frac{\partial u_i}{\partial x_i} \frac{\partial u_i}{\partial x_k} \frac{\partial \overline{U_i}}{\partial x_k} - 2\nu \frac{\partial u_i}{\partial x_i} \frac{\partial^2 \overline{U_i}}{\partial x_i \partial x_k} - 2\nu \frac{\partial u_i}{\partial x_i} \frac{\partial u_i}{\partial x_k} \frac{\partial u_k}{\partial x_k}
\]

(II)

\[
- \nu \frac{\partial}{\partial x_k} \left( \frac{u_k}{\rho} \frac{\partial u_i}{\partial x_i} \frac{\partial u_i}{\partial x_i} \right) - 2\nu \frac{\partial u_i}{\partial x_i} \frac{\partial^2 p}{\partial x_i \partial x_i} \frac{\partial}{\partial x_i} + \nu \frac{\partial^2 \epsilon}{\partial x_k^2}
\]  

(III)

\[
D_{\epsilon} - \gamma + D_{\nu-\epsilon}
\]

(IV)

In general terms, \( P_\epsilon \) is the production of the dissipation, \( D_\epsilon \) is the turbulent diffusion of dissipation, \( -\gamma \) is the viscous destruction of dissipation and \( D_{\nu-\epsilon} \) is the viscous diffusion of dissipation.
2.2 Homogeneous Turbulence

In order to simplify the understanding of the physics of turbulence, analyses of certain states of turbulence were proposed, which simplify the above the governing equations. Among those proposed states, homogeneous turbulent flows are an important class. In a homogeneous turbulent flow field, all the statistical moments of fluctuations are invariant under translation. In addition to translational invariance of mean turbulent quantities, homogeneous turbulent flows should conform to the following:

1. They should be boundless, i.e. the mathematical problem associated with the evolution of this type of flow is of the initial-value type.

2. All correlations involving several points (structure functions) should depend only on the vectors joining one of them to any of the others.

3. If mean velocity gradients exist, they must be spatially uniform.

2.2.1 Transport Equations for Homogeneous Turbulence

In order to satisfy the first condition of homogeneity mentioned in the previous section, the lateral and transverse integral length scales ought to be much smaller than the corresponding dimensions of the duct.

By definition, the concept of axisymmetric turbulence includes the homogeneous turbulence assumption and transport equations for homogeneous turbulence govern axisymmetric turbulent flows. Grid-generated turbulence is an example of homogeneous turbulence. Nevertheless, when we look at the visualization of grid-generated turbulence in Figure 2.1, it is very hard to talk about a spatial invariance, especially in the flow direction. This assumption needs some explanations in order to avoid conceptual ambiguity. Assume that measurements of Reynolds stresses [or any statistical correlation \(Q(X_i, t)\)] are taken at different \(X_i\) coordinates, which are centers of the control volumes shown in Figure 2.1. Assuming that the flow is steady, the following dynamic equation should be valid for a stationary frame of reference \(X_1X_2X_3\):

\[
\overline{U_k} \frac{\partial u_i u_j}{\partial X_k} = \overline{u_i u_k} \frac{\partial U_i}{\partial X_k} - \frac{u_i u_k}{u_x} \frac{\partial U_j}{\partial X_k} - \frac{\partial u_i u_j u_k}{\partial X_k} - \frac{1}{\rho} \left( \frac{\partial p u_j}{\partial X_i} + \frac{\partial p u_i}{\partial X_j} \right) + \frac{1}{\rho} \left( \frac{\partial p}{\partial X_i} + p \frac{\partial u_i}{\partial X_j} \right) \\
- 2 \nu \frac{\partial u_i}{\partial X_k} \frac{\partial u_j}{\partial X_k} + \nu \frac{\partial^2 u_i u_j}{\partial X_k \partial X_k}.
\]

(2.7)

Now, if the control volumes are isolated and assumed to be the same control volume at different instants of time, one obtains the lower picture in Figure 2.1. In other words, the flow is observed from a frame of reference, \(x_1x_2x_3\) moving with the control volume. In this case, the observed mean velocity of the flow becomes zero, and the correlation
becomes a function of time and space, i.e. \( Q(x_i, t) \). The following equation governs the second moments of velocity fluctuations in the control volume:

\[
\frac{\partial u_i u_j}{\partial t} = -u_j u_k \frac{\partial U_i}{\partial x_k} - u_i u_k \frac{\partial U_j}{\partial x_k} - \frac{1}{\rho} \left( \frac{\partial p u_i}{\partial x_i} + \frac{\partial p u_j}{\partial x_j} \right) + \frac{1}{\rho} \left( \frac{p \partial u_j}{\partial x_i} + \frac{p \partial u_i}{\partial x_j} \right) - 2\nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k}. \tag{2.8}
\]

If the flow in this control volume is homogeneous, the spatial gradients of any mean quantity should drop. That is, for example, \( \overline{u_i u_j} \) is not anymore a function of \( x_i \) in the moving reference frame \( x_1, x_2, x_3 \). Then the transport equation for Reynolds stress (2.8) has the following form:

\[
\frac{\partial u_i u_j}{\partial t} = -u_j u_k \frac{\partial U_i}{\partial x_k} - u_i u_k \frac{\partial U_j}{\partial x_k} + \frac{1}{\rho} \left( \frac{\partial p u_i}{\partial x_i} + \frac{\partial p u_j}{\partial x_j} \right) - 2\nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k}. \tag{2.9}
\]

where the unknown terms in this equation system are the pressure-strain correlations (or pressure-velocity gradient correlations) \((\Pi_{ij})\) and the dissipation correlations \((\epsilon_{ij})\).

The left-hand side of the transport equations in homogeneous turbulence always have the time derivative in the moving control reference frame. Instead of measuring the change of any mean quantity \( \overline{\phi} \) in time, measurements are performed along a line parallel to the flow and are interpreted as the change in time with the help of Taylor’s frozen turbulence assumption:

\[
\frac{\partial \overline{\phi}}{\partial t} = \overline{U_1} \frac{\partial \overline{\phi}}{\partial X_1}. \tag{2.10}
\]

Note that, because of the spatial invariance of mean quantities in a homogeneous turbulent flow field, it is sufficient to measure only along one arbitrary line parallel to the flow. However, in order for this kind of treatment of streamwise inhomogeneity to be valid, the change of any mean turbulence quantity \( \overline{\phi} \) along \( X_1 \) should be limited as follows (Corrsin 1963):

\[
L \frac{d\overline{\phi}}{dX_1} << 1, \tag{2.11}
\]

where \( L \) is the integral length scale in the flow direction and \( \overline{\phi} \) can be also \( L \).

The equation of turbulent kinetic energy for homogeneous turbulence can be obtained by contracting (2.9), as follows:

\[
\frac{\partial}{\partial t} \left( \frac{q^2}{2} \right) = -u_j u_k \frac{\partial U_i}{\partial x_k} - \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k}. \tag{2.12}
\]

where \( q^2 = 2k = \overline{u_i u_i} \).

Note that, according to (2.5), for homogeneous turbulence \( \epsilon = \tau \). In the subsequent text \( \epsilon \) is called the total dissipation. Similarly to the above simplifications, the transport
Theoretical Background

Equation for $\epsilon$ (2.6) reduces to

$$\frac{\partial \epsilon}{\partial t} = -2\nu \frac{\partial u_i}{\partial x_l} \frac{\partial u_k}{\partial x_l} \frac{\partial \bar{U}_i}{\partial x_k} - 2\nu \frac{\partial u_i}{\partial x_l} \frac{\partial u_i}{\partial x_k} \frac{\partial \bar{U}_k}{\partial x_l} - 2\nu^2 \left( \frac{\partial^2 u_i}{\partial x_k \partial x_l} \right)^2 - \gamma,$$  

(2.13)

where $P_1^\epsilon$ and $P_2^\epsilon$ are the production of $\epsilon$ by the mean velocity gradients, $P_4^\epsilon$ is the term responsible for the production of $\epsilon$ due to vortex stretching and $\gamma$ is the viscous destruction of the dissipation rate.

2.2.2 Classification of Homogeneous Turbulence

For the sake of systematic analysis of the interactions between the mean and fluctuating motion, homogeneous turbulent flows can be classified according to the form of $\partial \bar{U}_i/\partial x_k$ as follows:

1. Homogeneous turbulent flows without mean velocity gradient (unstrained turbulence)
   
   (a) Homogeneous and isotropic turbulence
   
   (b) Homogeneous and anisotropic turbulence (return to isotropy)
2. Constant mean velocity gradient flows
   (a) Pure strain
   (b) Pure rotation
   (c) Uniform shear

A review of all kinds of homogeneous flows was given by Gence (1983). As regards to unstrained homogeneous flows, it will be shown in Section 2.3.2 that isotropic turbulence and some special anisotropic homogeneous turbulent flows are subclasses of axisymmetric turbulence. Strains of initially axisymmetric turbulence along axisymmetric contractions and expansions also deliver axisymmetric turbulences and are special subclasses of axisymmetric turbulence, which were experimentally investigated in this work.

2.3 Axisymmetric Turbulence

2.3.1 Forms of Correlation Tensors

In addition to the translational invariance of mean turbulence quantities, in axisymmetric turbulence the correlations should also have rotational and reflectional symmetry about the symmetry axis of the flow. Based on the Robertson’s (1940) methodology, Batchelor (1946) and Chandrasekhar (1950) derived the forms of the correlation tensors which appear in the transport equations of turbulence for axisymmetric turbulent flow. The general form of axisymmetric tensors of the first, second and third order, which involve velocities at two points separated with the position vector \( r \) (see Figure 2.2) and obey the rotational and reflectional symmetry about the unit vector \( \lambda \) on the symmetry axis, are:

\[
Q_i(r) = A r_i + B \lambda_i \\
Q_{ij}(r) = A r_i r_j + B \lambda_i \lambda_j + C \delta_{ij} + D r_i \lambda_j + E r_j \lambda_i \\
Q_{ijk}(r) = A r_i r_j r_k + B \lambda_i \lambda_j \lambda_k + C r_i \delta_{jk} + D r_j \delta_{ki} + E r_k \delta_{ij} + F \lambda_i \delta_{jk} + G \lambda_j \delta_{ki} + H \lambda_k \delta_{ij} + I r_i r_j \lambda_k + J r_j r_k \lambda_i + K r_k r_i \lambda_j + L r_i r_j \lambda_k + M r_j \lambda_k \lambda_i + N r_k \lambda_i \lambda_j + \ldots
\]

where \( A, B, C, \ldots \) are scalar functions of \( r^2 \) and \( r_i \lambda_i \). As the distance between two points approaches 0, \( r \to 0 \), the forms of tensors in equation (2.14a) reduces to

\[
Q_i(r)_{r \to 0} = B \lambda_i \\
Q_{ij}(r)_{r \to 0} = B \lambda_i \lambda_j + C \delta_{ij} \\
Q_{ijk}(r)_{r \to 0} = B \lambda_i \lambda_j \lambda_k + F \lambda_i \delta_{jk} + G \lambda_j \delta_{ki} + H \lambda_k \delta_{ij}
\]
These are the forms of one-point correlation tensors for axisymmetric turbulence. These kinematic considerations for axisymmetric turbulence state that the off-diagonal components of the $u_iu_j$ tensor and all other second-order tensors should be equal to zero, and two diagonal components, which are the correlations in the directions perpendicular to the symmetry vector, should be equal to each other.

According to the relationship in equation (2.15b), following the notation of Jovanović & Otić (2000), the tensor for the second-order moments of velocity fluctuations, which is a second-order tensor, can be written as follows:

$$u_iu_j = A\delta_{ij} + B\lambda_i\lambda_j$$

(2.16)

For a turbulent flow, which flows in the $x_1$ direction and is axisymmetric in the same direction, i.e. $\lambda = (1,0,0)$, the Reynolds stress tensor is:

$$\overline{u_iu_j} = \begin{pmatrix} u_1u_1 & u_1u_2 & u_1u_3 \\ u_2u_1 & u_2u_2 & u_2u_3 \\ u_3u_1 & u_3u_2 & u_3u_3 \end{pmatrix} = \begin{pmatrix} A + B & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{pmatrix}.$$  

(2.17)

Similarly, the second-order tensors in equation (2.9) and the strain rate tensor $S_{ij}$ can be written as

$$\epsilon_{ij} = \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} = C\delta_{ij} + D\lambda_i\lambda_j,$$  

(2.18a)

$$\Pi_{ij} = \frac{p}{\rho} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = E\delta_{ij} + F\lambda_i\lambda_j,$$  

(2.18b)

$$P_{ij} = -u_ju_k \frac{\partial U_i}{\partial x_k} - u_iu_k \frac{\partial U_j}{\partial x_k} = G\delta_{ij} + H\lambda_i\lambda_j,$$  

(2.18c)

$$S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) = I\delta_{ij} + K\lambda_i\lambda_j,$$  

(2.18d)

where $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $I$ and $K$ are even scalar functions of $r^2$. It is shown in Section 2.6 that using these scalar functions one can interconnect the unknown correlations with each other and construct a turbulence closure.

### 2.3.2 Anisotropy-invariant Map and Limiting States of Turbulence

Any turbulence correlation represented by a second-order tensor $Q_{ij}$ has the functional form in equation (2.15b). Assuming that a turbulent flow, which flows in the $x_1$ direction and is axisymmetric in the same direction, i.e. $\lambda = (1,0,0)$, due to (2.15b) $Q_{ij}$ can be written as

$$Q_{ij} = B\lambda_i\lambda_j + C\delta_{ij} = \begin{pmatrix} B + C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{pmatrix},$$  

(2.19)
where \( B \) and \( C \) are any arbitrary functions. Depending on the values of \( B \) and \( C \), the following three states can be defined:

- isotropic turbulence for \( B = 0 \) \( \Rightarrow Q_{11} = Q_{22} = Q_{33} = C \), \( (2.20a) \)
- one-component turbulence for \( C = 0 \) \( \Rightarrow Q_{11} = B, \ Q_{22} = Q_{33} = 0 \), \( (2.20b) \)
- isotropic two-component turbulence for \( B = -C \) \( \Rightarrow Q_{11} = 0, \ Q_{22} = Q_{33} = C \). \( (2.20c) \)

It should be noted that these three states of turbulence are assumed to exist for all second-order correlations in the theory of axisymmetric turbulence.

Following Lumley (1978), an anisotropy tensor for \( Q_{ij} \) can be formulated as

\[
A_{ij} = \frac{Q_{ij}}{Q_{ss}} - \frac{1}{3} \delta_{ij} \quad (2.21)
\]

where \( Q_{ss} \) is the trace of \( Q_{ij} \) and \( \delta_{ij} \) is the Kronocker \( \delta \). It can be shown that the following relation between the diagonal terms of the \( A_{ij} \) should hold:

\[
A_{22} = A_{33} = -\frac{1}{2} A_{11} \quad (2.22)
\]

The first, second and third scalar invariants of \( A_{ij} \) are

\[
I = A_{ss}, \quad (2.23a)
II = A_{ij} A_{ji}, \quad (2.23b)
III = A_{ij} A_{jk} A_{ki}. \quad (2.23c)
\]

It was shown by Lumley (1978) that for an axisymmetric turbulence, where \( B \) and \( C \) takes arbitrary values, the following relation holds between second and third invariants of the anisotropy tensor:

\[
II = \frac{3}{2} \left( \frac{4}{3} III \right)^{\frac{2}{3}}. \quad (2.24)
\]
Figure 2.3: The anisotropy-invariant map for the second-order correlation terms in Reynolds stress transport equations: bounding and limiting states with corresponding experimental realizations. The labeling of the states of turbulence is adopted from Jovanović & Otić (2000).

He showed also that for two-component turbulence, which is only axisymmetric at the two limiting states, namely isotropic two-component and one-component turbulence, invariants are related by

$$II = \frac{2}{9} + 2III$$

(2.25)

When the relations (2.24, 2.25) are plotted in the $III - II$ plane, the so-called anisotropy-invariant map shown in Figure 2.3 is obtained. In this map, the relations (2.24, 2.25) are the bounding lines of the curvilinear triangle and the vertices of the triangle are the limiting states of axisymmetric turbulence defined in equations (2.20). Lumley (1978) showed that all physically realizable turbulence lies in this curvilinear triangle. Because of this fact, the invariant map is considered to be an important tool for modeling and understanding turbulent flows. Moreover, since axisymmetric turbulence forms the two bounding edges and, consequently, involves all limiting states of turbulence, correct modeling of axisymmetric turbulence is a very important issue for any Reynolds stress model.

Taking the anisotropy-invariant map as reference, five subclasses of axisymmetric turbulence can be defined:

1. Axisymmetric strain (contraction and expansion) of axisymmetric turbulence.
2. Isotropic turbulence.
3. Unstrained anisotropic axisymmetric turbulence.
4. Two-component isotropic turbulence.

5. One-component turbulence.

As illustrated in Figure 2.3, by straining initially isotropic turbulence one can move either along the left edge through a contraction and reach the two-component isotropic state, or along the right edge through an expansion and reach the one-component limit. Unstrained anisotropic axisymmetric turbulence can be obtained by stopping straining along either of the edges and letting turbulence decay (relax). In the present study, the first four subclasses are investigated. Generation of turbulence which is permanently at the one-component state is a challenging task, and might even be impossible.

2.3.3 Two-point Correlation Functions and Length Scales

Isotropic turbulence

Since von Kármán & Howarth (1938), it is well known that the isotropic turbulence field can fully be described by two two-point correlation functions

\[
\begin{align*}
    f(r) &= f(x_1) = \frac{u_1(0)u_1(x_1)}{u_1(0)u_1(0)}, \\
    f(r) &= f(x_2) = \frac{u_2(0)u_2(x_2)}{u_2(0)u_2(0)}, \\
    g(r) &= g(x_1) = \frac{u_2(0)u_2(x_1)}{u_2(0)u_2(0)}, \\
    g(r) &= g(x_2) = \frac{u_1(0)u_1(x_2)}{u_1(0)u_1(0)},
\end{align*}
\]

where \( f(r) \) and \( g(r) \) are the normalized correlations between the two velocity fluctuation components at two points a distance \( r \) apart and resolved parallel and perpendicular to the position vector \( r \), respectively. As illustrated in Figure 2.4a, \( r \) can be replaced by \( x_1 \) and \( x_2 \) such that \( r = x_1 = x_2 \). Even though \( f(x_1) = f(x_2) = f(r) \) and \( g(x_1) = g(x_2) = g(r) \), it is easier to measure \( f(x_1) \) and \( g(x_1) \) by using autocorrelation functions and such measurements were performed for the present work. Typical forms of \( f(x_1) \) and \( g(x_1) \) can be seen in Figure 5.14. Two-point correlation functions, as their names imply, show the degree of correlation of velocity fluctuations between two points. As \( r \) increases, the correlation decreases. The decrease in correlation in the vicinity of the vertex is mainly due to the dissipation of turbulence and the decrease far from the vertex is related to the randomization of the fluctuations as a result of nonlinear interaction of the different scales of turbulence and interaction of turbulence with the mean velocity field. Because of these properties, correlation functions are used to
derive dissipation and integral length scales of isotropic turbulence as follows:

\[-\frac{1}{\lambda_f^2} \left( \frac{\partial^2 f}{\partial r^2} \right)_{r=0} = 0, \quad (2.27a)\]

\[-\frac{1}{\lambda_g^2} \left( \frac{\partial^2 g}{\partial r^2} \right)_{r=0} = 0, \quad (2.27b)\]

\[L_f = \int_0^\infty f(r)dr, \quad (2.27c)\]

\[L_g = \int_0^\infty g(r)dr, \quad (2.27d)\]

where \(\lambda_f\) and \(\lambda_g\) are the longitudinal and transverse dissipation length scales and are measures for the radius of curvatures of \(f(r)\) and \(g(r)\) at their vertex, respectively. Note that, the reciprocals of the left-hand sides of equations (2.27a, b) are the radii of curvatures. \(L_f\) and \(L_g\) are the longitudinal and transverse integral length scales, which are defined as the area under the corresponding correlation functions.

One important Reynolds number in turbulence research is defined by \(\lambda_g\) and the energy of turbulence represented by \(\overline{u_1u_1}\) as follows

\[\text{Re}_\lambda = \frac{\sqrt{\overline{u_1u_1}\lambda_g}}{\nu}. \quad (2.28)\]

Mydlarski and Warhaft (1996) investigated a range of \(\text{Re}_\lambda\) at which Kolmogorov theories (1941a, b, 1942, 1962) are applicable. He showed that turbulence with \(\text{Re}_\lambda > 200\) (strong turbulence) is qualitatively different from turbulence with \(\text{Re}_\lambda < 100\) (weak turbulence). The effects of \(\text{Re}_\lambda\) on correlation curves and spectra of grid-generated turbulence are treated in Chapter 5.

**Anisotropic and Strained Axisymmetric Turbulence**

According to Batchelor (1946) and Chandrasekhar (1950), different from isotropic turbulence, four two-point correlation functions are needed to describe an axisymmetric turbulence field. As illustrated in Figure 2.4b, these are

\[R_{11}(x_1) = \frac{u_1(0)u_1(x_1)}{u_1(0)u_1(0)}, \quad (2.29a)\]

\[R_{22}(x_2) = \frac{u_2(0)u_2(x_2)}{u_2(0)u_2(0)}, \quad (2.29b)\]

\[R_{22}(x_1) = \frac{u_2(0)u_2(x_1)}{u_2(0)u_2(0)}, \quad (2.29c)\]

\[R_{11}(x_2) = \frac{u_1(0)u_1(x_2)}{u_1(0)u_1(0)}. \quad (2.29d)\]
2.3 Axisymmetric Turbulence

Figure 2.4: Illustration of basic two-point correlation functions for (a) isotropic turbulence and (b) axisymmetric turbulence. $\lambda$ is the vector about which turbulence has rotational and reflectional symmetry.

The corresponding dissipation and integral length scales are derived similarly to the isotropic turbulence as follows

\[
- \frac{1}{\lambda_{1,1}^2} = \left( \frac{\partial^2 R_{11}(x_1)}{\partial^2 x_1} \right)_{x_1=0}, \quad (2.30a)
\]
\[
- \frac{1}{\lambda_{2,2}^2} = \left( \frac{\partial^2 R_{22}(x_2)}{\partial^2 x_2} \right)_{x_2=0}, \quad (2.30b)
\]
\[
- \frac{1}{\lambda_{1,2}^2} = \left( \frac{\partial^2 R_{11}(x_1)}{\partial^2 x_1} \right)_{x_2=0}, \quad (2.30c)
\]
\[
- \frac{1}{\lambda_{1,1}^2} = \left( \frac{\partial^2 R_{11}(x_2)}{\partial^2 x_2} \right)_{x_1=0}. \quad (2.30d)
\]
\[ L_{1x_1} = \int_{0}^{\infty} R_{11}(x_1) \, dx_1, \quad (2.31a) \]
\[ L_{2x_2} = \int_{0}^{\infty} R_{22}(x_2) \, dx_2, \quad (2.31b) \]
\[ L_{2x_1} = \int_{0}^{\infty} R_{22}(x_1) \, dx_1, \quad (2.31c) \]
\[ L_{1x_2} = \int_{0}^{\infty} R_{11}(x_2) \, dx_2. \quad (2.31d) \]

In the present work, only \( R_{11}(x_1) \) and \( R_{22}(x_1) \) are measured and corresponding length scales are extracted from these curves in Chapter 7. For convenience, the index notation is abandoned in the following chapters and subscripts 1 and 2 are replaced by \( u \) and \( v \) for the velocity fluctuations and \( x_1 \) is replaced by \( x \). Accordingly, for example, \( R_{11}(x_1) \) becomes \( R_{uu}(x) \) and \( \lambda_{1x_1} \) becomes \( \lambda_{ux} \) in Chapter 7.

### 2.4 The Vortex Stretching and the Rapid Distortion Theory

The first investigations on the development of turbulence, subjected to superimposed uniform distortion, date back to Prandtl (1932, 1933) and Taylor (1935b), both of whom were interested in studying the effect of wind-tunnel contraction on the free stream turbulence. The semiquantitative vortex stretching theory of Prandtl is a simple and very illuminating description of the development of turbulence along straining ducts. Hence, this concept is briefly mentioned here. In this theory, it is accepted that \( u_2 \) and \( u_3 \) fluctuations are due to the vortex filaments aligned with the flow direction and \( u_1 \) fluctuations are due to those perpendicular to the flow direction, as illustrated in Figure 2.5. Turbulence can be imagined to be a collection of many of these vortex filaments. Thus, the effect of strain on turbulence can be, at least qualitatively, determined by considering its effect on vorticity. In an inviscid and incompressible flow, the total strength and the volume of the of the vortex filaments remain constant along the straining sections. According to Kelvin’s circulation theorem, the strength is the product of the angular velocity and the cross-sectional area of vortex filaments. Knowing that the peripheral velocities of the vortex filaments are measures of the velocity fluctuations \( u_1 \) and \( u_2 \), the ratio of the Reynolds stresses at the inlet and outlet of the axisymmetric straining ducts can be derived as

\[ \frac{u_1 u_1}_{\text{outlet}} = \frac{1}{c^2}, \quad (2.32a) \]
\[ \frac{u_2 u_2}_{\text{outlet}} = c, \quad (2.32b) \]

where

\[ c = \frac{A_{\text{inlet}}}{A_{\text{outlet}}} \quad (2.32c) \]
is the geometric contraction ratio of the straining ducts. These relations apply for both the contraction and the expansion. In addition to the development of the stresses, the development of the length scales can also be deduced from this simple treatment. For example, it is expected that \( L_{2,x} \) and \( L_{1,x} \) should be elongated along the contraction, whereas they become shorter along the expansion. Similar behavior should be expected from the dissipation scales derived from corresponding correlation functions. Examples of these trends can best be observed for the turbulence in Section 7.4, which is first contracted and then expanded.

After Prandtl and Taylor, a more detailed and formal analysis of the problem, assuming that the turbulence is initially isotropic, was made by Ribner & Tucker (1953) for axisymmetric contractions and independently by Batchelor & Proudman (1954) for arbitrary irrotational distortions. They recognized that the turbulence has finite spatial and time correlations and therefore the effect of the local mean flow variations, which exist in a general turbulent flow, can be accepted as a superimposed distortion which locally creates a coordinated strain on turbulence. Only after assuming that the turbulence interacts strongly with the mean flow but only weakly with itself under conditions explained below, the effects of viscous dissipation and nonlinear processes can be neglected and the development of the velocity fluctuations and/or the vorticity fluctuations under superimposed distortions could be calculated. This approach was called later the rapid distortion theory (RDT). Here, the discussion is limited to axisymmetric contractions.

Within the RDT, the dynamic equations for the fluctuating velocity and vorticity equations are taken as a basis. Utilizing the Reynolds averaging method, the random velocity, pressure and vorticity fields are accepted as the sum of the ensemble mean and fluctuating components, \( U_i(x,t) = \bar{U}_i + u_i(x,t) \), \( P(x,t) = \rho(\bar{T} + p(x,t)) \) and \( \omega(x,t) = \bar{\Omega} + \omega(x,t) \), where \( \bar{\Omega} = \nabla \times \bar{U} \). Then, restricting the flow to be incompressible with uniform density and having zero mean vorticity, \( \bar{\Omega} = 0 \), the dynamic equations for the vorticity fluctuation can be written as

\[
\frac{\partial \omega_i}{\partial t} + \frac{\partial u_i}{\partial x_j} \frac{\partial \bar{U}_j}{\partial x_i} = \nu \frac{\partial^2 \omega_i}{\partial x_k^2} - \frac{\partial u_k}{\partial x_i} \frac{\partial \omega_i}{\partial x_k} + \frac{\partial u_i}{\partial x_j} \frac{\partial \omega_j}{\partial x_k} - \omega_j \frac{\partial u_i}{\partial x_j} \tag{2.33a}
\]

where

\[
\frac{\partial u_i}{\partial x_i} = 0, \quad \omega_i = \epsilon_{ijk} \frac{\partial u_k}{\partial x_j} \tag{2.33b}
\]

so that \( \frac{\partial \omega_k}{\partial x_k} = 0 \). The terms (i) and (ii) are responsible for the advection and stretching of vorticity by the mean flow. The terms (iii) and (iv) are the viscous dissipation and the nonlinear contributions.

In the RDT formulation, the time-scale of the rate of distortion of turbulence is assumed to be much shorter than the characteristic time-scale of the large eddies. Because of this, the changes in the relative position of fluid particles are mainly due to the superimposed rapid distortion rather than the turbulence itself. Such an assumption allows the neglect of nonlinear and viscous effects, i.e. terms (iii) and (iv) in equation (2.33a). Hence, two main conditions have to be satisfied for the RDT approach to be valid in
Flow direction

(a) Contraction ($c > 1$)

(b) Expansion ($0 < c < 1$)

Figure 2.5: Illustration of the Prandtl’s semiquantitative vortex stretching theory applied along (a) contraction and (b) expansion.

distorted turbulent flows. First, the neglect of viscous effects requires that the duration of the distortion has to be so small that viscous dissipation does not produce a significant change in the turbulence energy, i.e.

$$ \int_{t'}^t \epsilon \, dt \ll \frac{1}{2} q^2 $$  \hspace{1cm} (2.34)

during the whole duration of distortion. Utilizing the well-known relation $\epsilon \propto (q^2)^{3/2} / L$, where $L$ is the average integral length scale of turbulence, an approximate version of this condition can be written as

$$ t - t' \ll \left( \frac{L}{q} \right)_{t=t'} $$  \hspace{1cm} (2.35)

For an axisymmetric distortion through a contraction, the duration of the distortion can be, in the worst case, accepted as being proportional to $D/U_1$, so that the above condition becomes

$$ \frac{q}{U_1} \ll \frac{L}{D} $$  \hspace{1cm} (2.36)
where $D$ is the inlet diameter of the contraction and $\bar{U}_1$ is the inlet mean stream velocity. It should be noted that the cases in which this condition is satisfied are not common, since both sides of the above inequality are usually small. An experimental approximation to this condition can be obtained when the distorting duct is placed sufficiently far from the turbulence-generating grid.

The second condition, which should be satisfied for the linearization of the governing equations of fluctuating velocity and vorticity components, requires that the contribution to the relative velocity of two neighboring particles from the turbulence should be small compared with that from the superimposed distortion. The reduced formulated form of this condition for axisymmetric irrotational distortion, i.e. contraction of turbulence through an axisymmetric nozzle of contraction ratio $c$, was given by Batchelor & Proudman (1954) as follows:

$$t - t' \ll \left[ \frac{L}{q} \left( \frac{\nu}{Lq} \right)^{1/2} \right]_{t=t} \left[ (c-1)^2 + 2(c^{-1/2} - 1)^2 \right]^{1/2},$$

which is more stringent than the first condition.

Hunt & Carruthers (1990) re-examined the conditions under which the viscous and nonlinear effects are negligible and found out that they are less stringent than Batchelor & Proudman (1954) suggested. For a turbulent flow with integral length and velocity scales $L$ and $u_0 = \sqrt{\frac{1}{3} q^2}$ and a length scale $l$ for the smallest eddies, he found the following criterion:

$$\frac{u_0}{L} \left( \frac{l}{L_x} \right)^{2/3} \ll \max \left( \frac{\Delta U}{L_D}, \frac{1}{T_D} \right),$$

where $T_D$ and $L_D$ are the period and the length of distortion, respectively, and $\Delta U$ is the difference in the mean velocity before and after the distortion. As stated by Hunt (1990), this criterion indicates that the linearization is justifiable either if the nondimensional strain rate, $\frac{\nu}{L_D} \frac{\Delta U}{T_D}$, is large enough or if the period of distortion is short enough, that is, $T_D \ll T_L = L/u_0$. Note that the latter condition is exactly the same as equation (2.36) suggested by Batchelor & Proudman (1954). Moreover, the criterion (2.38), suggested by Hunt & Carruthers (1990), shows that the neglect of nonlinear effects for the energy-containing eddies ($l \sim L$) is better justified than for smaller scale eddies ($l \ll L$).

If the turbulent flow and the irrotational distortion are chosen according to the criteria (2.37, 2.38), the linearized vorticity equation becomes

$$\frac{\partial \omega_i}{\partial t} + \bar{U}_j \frac{\partial \omega_i}{\partial x_j} = \omega_k \frac{\partial \bar{U}_i}{\partial x_k}.$$

In the RDT formulation, this equation was integrated after inserting the components of velocity fluctuations which are resolved into their Fourier wavenumber components. Thus, the derived relationships are show the effect of distortion on the spectrum and on the second-order moments of velocity components. According to the work of Batchelor
& Proudmann (1954), in which the turbulence is assumed to be isotropic before the distortion and the distortion is axisymmetric, the ratio of $\overline{u_1u_1}$ at time $t$ to that before the distortion is

$$\mu_1 = \frac{\overline{u_1u_1}}{\overline{u_1u_{10}}} = \frac{3}{4} c^{-2} \left[ 1 + \alpha^2 - \frac{1 + \alpha}{4\alpha^3} \ln \left( \frac{1 + \alpha}{1 - \alpha} \right) \right]$$

(2.40a)

and the ratios for lateral $\overline{u_2u_2}$ and transfer components $\overline{u_3u_3}$ are

$$\mu_2 = \mu_3 = \frac{3}{4} c + \frac{3}{4} c^{-2} \left( \frac{1}{2} \alpha^2 - \frac{1 - \alpha^2}{4\alpha^3} \ln \left( \frac{1 + \alpha}{1 - \alpha} \right) \right),$$

(2.40b)

where $\alpha^2 = 1 - c^{-3}$ and $c$ is the contraction ratio. This result of the linear analysis describes the growth of the Reynolds stresses with maximum magnitudes along contractions. Therefore, during the analysis of the measured data in the following chapters, it will be used for comparison as an extreme state.

Nevertheless, $\mu_1$ and $\mu_2$ were derived in their original form for initially isotropic turbulence. Sreenivasan & Narasimha (1978) derived relations for an initially axisymmetric turbulence, which can be anisotropic. They showed that if departure from isotropy is not large, the ratios of Reynolds stresses can be expressed as the sum of their value given by the isotropic theory and certain corrections as shown below:

$$\Delta \mu_1 = 0,$$

(2.42a)

$$\Delta \mu_2 = c - \mu_2^0,$$

(2.42b)

so that the Reynolds stress ratios read

$$\mu_1 = \mu_1^0,$$

(2.43a)

$$\mu_2 = \mu_2^0 - (R_0 - 1)(c - \mu_2^0).$$

(2.43b)

This formulation leads to less increase in the transverse energy component, in the case that the longitudinal component is initially larger than the transverse components, that is, for $R_0 > 0$. The $\mu_1$, $\mu_2$ and the local ratio of the longitudinal Reynolds stress to the transverse stress $R = \overline{u_1u_1}/\overline{u_2u_2}$ calculated by the above equations are given in Figure 2.6.

2.5 The Eddy Viscosity Concept and the $k - \epsilon$ Model

The modeling of the unknown Reynolds stresses $\rho \overline{u_iu_j}$ first started by relating them to the mean velocity field. In analogy with the viscous stresses in laminar flows, Bousson-
nesq (1877) proposed that the turbulent stresses are proportional to the mean-velocity gradients

\[ \rho \bar{u}_i \bar{u}_j = -\mu_t \left( \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right) + \rho \frac{2}{3} k \delta_{ij} \] (2.44)

with a constant \( \mu_t \) called turbulent eddy viscosity. This constant is not a fluid property but a local flow property which is always positive and greater than the dynamic viscosity of the fluid, \( \mu_t > \mu \). For incompressible flows the following relationship holds for the second-order moments of the velocity fluctuations,

\[ \bar{u}_i \bar{u}_j = -\nu_t \left( \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij} \] (2.45)

The second term on the right-hand side ensures that the sum of the diagonal components of \( \bar{u}_i \bar{u}_j \) results in \( 2k \).

The six unknown Reynolds stresses are reduced to only one unknown which is the eddy viscosity \( \nu_t \). In analogy with the kinetic theory of gases, where the viscosity is proportional to the average velocity and mean free path of the molecules, the turbulent viscosity may be expressed as

\[ \nu_t \propto u_0 l_0 \quad \text{or} \quad \nu_t \propto l_0^2 \tau_0 \] (2.46)

where \( l_0 \) is the characteristic length, \( \tau_0 \) is the characteristic time and \( u_0 = l_0 / \tau_0 \) is the characteristic velocity of the turbulence. In other words, \( l_0 \) and \( \tau_0 \) represent the finite spatial and time correlation of turbulence. The energy of turbulent motion is mainly drained from the mean flow by the large-scale turbulent motion, although the dissipation of this energy is affected by the small-scale turbulent motions, i.e. smallest eddies. The large-scale eddies can be accepted as being the controlling agents of the turbulent motion. Thus, the time and length scales of energy-containing eddies, which are
integral time and length scales of turbulence, are selected as the most common characteristic scales in the eddy viscosity concept.

For the determination of $\nu_t$, a considerable number of models have been proposed with different levels of complexity determined by the number of differential equations utilized to find the characteristic scales in (2.46). For the sake of completeness, a list of them is given below.

- **Zero-equation models**: The length and time scales are specified algebraically using empirical information from experiments. Prandtl's mixing length model (1925) and the Cebeci-Smith model (1974) are the most prominent.

- **One-equation models**: The velocity scale is assumed to be the turbulent kinetic energy, $u_0 = \sqrt{k}$, and is calculated by solving the transport equation for $k$; $l_0$ is specified empirically. This type of model was first proposed by Kolmogorov (1942) and Prandtl (1945) in the following form:

\[
\nu_t = C_\mu k^{1/2} l_0
\]  
(2.47)

where $C_\mu$ is a constant determined empirically.

- **Two-equation models**: Two separate transport equations are solved to determine the velocity and length scales in (2.46). The $k - \epsilon$ and $k - \omega$ models, which are the most applied models in many applications, belong to the class of two-equation models. The eddy viscosity $\nu_t$ is given by the relation

\[
\nu_t = C_\mu l^2
\]  
(2.48)

for the $k - \epsilon$ model and

\[
\nu_t = \frac{k}{\epsilon}
\]  
(2.49)

for the original $k - \omega$ model, where $\omega$ is the specific dissipation rate, i.e. dissipation rate of kinetic energy $k$ per unit $k$.

For comprehensive information on these models, the review by Speziale (1991), the book by Mohammadi and Pironneau (1994), which is devoted to the $k - \epsilon$ model and the book by Wilcox (1998) which is focused on the $k - \omega$ model, can be consulted.

Due to its popularity, a fairly detailed account of the $k - \epsilon$ model of Spalding & Launder (1971) and Jones & Launder (1972) is provided below. Later, this model will be used to predict the effect of irrotational distortion on turbulence and its performance will be compared with those of RDT and the invariant model of Jovanović et al. (2003) explained in the next section.

In the $k - \epsilon$ model, the length and time scales of turbulence are calculated from the turbulent kinetic energy, $k$, and the turbulent dissipation rate, $\epsilon$:

\[
l_0 \propto \frac{k \sqrt{k}}{\epsilon} \quad \text{and} \quad \tau_0 = \frac{k}{\epsilon}
\]  
(2.50)
Thus, using equation (2.46), the relation for $\nu_t$ (2.48) is derived. The transport equations for the kinetic energy (2.4) and the dissipation rate (2.6) are solved to obtain $k$ and $\epsilon$. However, only after modeling the unknown terms in these transport equations, can the system of equations (one continuity and three momentum transport equations for the mean flow quantities and two transport equations for $k$ and $\epsilon$) be closed and solved. The transport equation for $k$ (2.4) requires the modeling of the diffusive transport term, whereas in the equation for $\epsilon$ (2.6), the production, the dissipation and the destruction of the dissipation rate should be modeled. Using gradient approximations for the diffusion terms and dimensional arguments for the others, these unknown terms were modeled. The modeled transport equations are

\[
\frac{\partial k}{\partial t} + \mathbf{U} \cdot \nabla k = \nu_t \left( \frac{\partial \mathbf{U}_i}{\partial x_k} + \frac{\partial \mathbf{U}_k}{\partial x_i} \right) \frac{\partial \mathbf{U}_i}{\partial x_k} + \frac{\partial}{\partial x_k} \left( \frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial x_k} \right) - \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} + \nu \frac{\partial^2 k}{\partial x_k^2} \tag{2.51}\]

for $k$ and

\[
\frac{\partial \epsilon}{\partial t} + \mathbf{U} \cdot \nabla \epsilon = C_{\epsilon 1} \frac{\epsilon}{k} \nu_t \left( \frac{\partial \mathbf{U}_i}{\partial x_k} + \frac{\partial \mathbf{U}_k}{\partial x_i} \right) \frac{\partial \mathbf{U}_i}{\partial x_k} + \frac{\partial}{\partial x_k} \left( \frac{\nu_t}{\sigma_k} \frac{\partial \epsilon}{\partial x_k} \right) - C_{\epsilon 2} \frac{\epsilon^2}{k} + \nu \frac{\partial^2 \epsilon}{\partial x_k^2} \tag{2.52}\]

for $\epsilon$, where the $C_{\mu}, \sigma_k, \sigma_\epsilon, C_{\epsilon 1}$ and $C_{\epsilon 2}$ are empirical constants. Launder & Sharma (1974) reported standard values for the constants:

\[
C_{\mu} = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3, \quad C_{\epsilon 1} = 1.44, \quad C_{\epsilon 2} = 1.92 \tag{2.53}\]

Nevertheless, the five constants in the $k-\epsilon$ model are not universal for every turbulent flow, therefore this model always requires fine tuning to obtain correct results, when, of course, the results are known. Rodi (1980) reported cases, where this model has been successfully applied with the above values of the constants.

The modeled equations for $k$ and $\epsilon$ simplify for homogeneous turbulence as follows:

\[
\frac{\partial k}{\partial t} = \nu_t \left( \frac{\partial \mathbf{U}_i}{\partial x_k} + \frac{\partial \mathbf{U}_k}{\partial x_i} \right) \frac{\partial \mathbf{U}_i}{\partial x_k} - \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} \tag{2.54a}\]

\[
\frac{\partial \epsilon}{\partial t} = C_{\epsilon 1} \frac{\epsilon}{k} \nu_t \left( \frac{\partial \mathbf{U}_i}{\partial x_k} + \frac{\partial \mathbf{U}_k}{\partial x_i} \right) \frac{\partial \mathbf{U}_i}{\partial x_k} - C_{\epsilon 2} \frac{\epsilon^2}{k} \tag{2.54b}\]

The major limitation of the eddy viscosity concept originates from the assumption of a single length and time scale for a turbulent flow, which contradicts the nature of turbulence. Moreover, since the eddy viscosity is a scalar quantity and it behaves like an isotropic property, the eddy viscosity concept cannot distinguish the anisotropy of length and velocity scales appearing in most turbulent flows. Pope (1975) has already suggested a more general tensorial representation of the eddy viscosity. The drawbacks of this model will be presented for anisotropic axisymmetric turbulence in Chapter 8. Despite known limitations and drawbacks of the $k-\epsilon$ model, and also the $k-\omega$ model, they have found widespread use in engineering applications, due to their robustness.
2.6 Anisotropy-invariant Modeling of Axisymmetric Turbulence

One of the main motivations of the present study was to generate experimental data for the validation and further development of the turbulence closure of Dr. Jovanović (Jovanović et al. 2003, Jovanović 2004), therefore a summary of the construction of this model and conclusions related to the present study are given below. However, details of derivations and reasonings for many physical assumptions are not described unless they are necessary in the context of this thesis.

Based on the principles of the invariant theory of isotropic turbulence, which was founded by Robertson (1940), Jovanović & Otić (2000) constructed a constitutive relationship between the tensor of second-order moments of velocity fluctuations and the strain rate tensor for axisymmetric turbulence. Because of the functional form of any second-order correlation given in equation (2.15b), similar constitutive relations hold between any two second-order tensors in axisymmetric turbulence. This property of the tensors was utilized deliberately by Jovanović, Otić & Bradshaw (2003) for the construction of their new turbulence closure. Jovanović et al. (2003) derived constitutive relations between unknown and known tensors. These relations are scalar functions of second and third invariants of the anisotropy tensor of second-order velocity fluctuations. These scalar functions, or so-called invariant functions, are interpolation functions between limiting states of turbulence defined on the anisotropy-invariant map at very high and very low $Re_\lambda$ (see Figure 2.3). Jovanović and his coworkers were inspired by the work of Lumley (1978), who tried to use the scalar invariants of the anisotropy tensors for the construction of a turbulence closure. In his book, Jovanović (2004) explained all constructive details and discussions on basic physical assumptions of this turbulence closure.

2.6.1 Relations Between Unclosed Correlations

The unclosed correlations in the dynamic equations for $\overline{u_iu_j}$ (2.9) are the pressure-strain correlations (or pressure-velocity gradient correlations) ($\Pi_{ij}$) and the dissipation correlations ($\epsilon_{ij}$). These terms should be related to $\overline{u_iu_j}$ to close the equation system. Jovanović & Otić (2002) showed that in axisymmetric turbulence any two second-order correlation tensors can be related to each other by utilizing the forms of the tensors given in (2.18). For example, between $\overline{u_iu_j}$ (2.16) and $\epsilon_{ij}$ (2.18a) the following relation can be derived by eliminating the $\lambda_i\lambda_j$ term:

$$\frac{\epsilon_{ij}}{\epsilon} - \frac{1}{3} \delta_{ij} = \frac{q^2}{\epsilon} \frac{\epsilon - 3C}{q^2 - 3A} \left( \frac{\overline{u_iu_j}}{q^2} - \frac{1}{3} \delta_{ij} \right) a_{ij},$$

(2.55)
where $a_{ij}$ and $e_{ij}$ are the anisotropy tensors of $\overline{u_i u_j}$ and $\epsilon_{ij}$, respectively. Hence, in a more compact form,

$$e_{ij} = A a_{ij},$$

(2.56)

where

$$A = \frac{q^2 \epsilon - 3C}{q^2 - 3A^2}$$

(2.57)

In the next section, it is shown that $A$ is constructed as a scalar function of the invariants of the Reynolds stress anisotropy tensor $a_{ij}$, i.e. indirectly as a function of $\overline{u_i u_j}$. Here these functions are called scalar-invariant functions. Furthermore, taking the square of equation (2.56) and the definition of the second invariants of the tensor in (2.23), the following relation can be derived:

$$A = \left( \frac{I_{IIe}}{I_{IIa}} \right)^{1/2}$$

(2.58)

Ye (1996) showed that the resulting relation

$$e_{ij} = \left( \frac{I_{IIe}}{I_{IIa}} \right)^{1/2} a_{ij}$$

(2.59)

and the DNS of Rogallo (1981) agree well. Thus, the dissipation tensor becomes

$$e_{ij} = A^2 a_{ij} + \frac{1}{3} \epsilon \delta_{ij}.$$ 

(2.60)

Jovanović et al. were able to write $\Pi_{ij}$ in terms of $P_{ij}$ and $a_{ij}$:

$$\Pi_{ij} = C a_{ij} + a_{ij} P_{ss} + \mathcal{F} \left( \frac{1}{3} P_{ss} \delta_{ij} - P_{ij} \right),$$

(2.61)

in which the slow pressure strain term ($\Pi_{ij}^s$) is modeled by the scalar-invariant function $C$ and rapid (fast) pressure-strain term is modeled by the scalar-invariant function $\mathcal{F}$. Hence, the modeled transport equation for $\overline{u_i u_j}$ reads

$$\frac{\partial \overline{u_i u_j}}{\partial t} = P_{ij} + C a_{ij} + a_{ij} P_{ss} + \mathcal{F} \left( \frac{1}{3} P_{ss} \delta_{ij} - P_{ij} \right) - 2A^2 a_{ij} - \frac{2}{3} \epsilon \delta_{ij}.$$ 

(2.62)

Assuming that the mean velocity field is known, $\epsilon$ in (2.60) is the last unclosed term. Jovanović et al. enlarged the system of equations by the dynamic equation for $\epsilon$ (2.6) and modeled $P_\epsilon^1$, $P_\epsilon^2$, $P_\epsilon^3$ and $\gamma$ as follows:

$$P_\epsilon^1 + P_\epsilon^2 = -2A^2 \overline{\epsilon u_i u_j} \frac{\partial \overline{U_i}}{\partial x_k},$$

(2.63)

$$P_\epsilon^4 - \gamma \simeq \psi \epsilon^2 k,$$

(2.64)
so that the dynamic equation for $\epsilon$ reads

$$\frac{\partial \epsilon}{\partial t} = 2A \frac{\epsilon}{k} \frac{\partial U_i}{\partial x_k} - \psi \frac{\epsilon^2}{k},$$

(2.65)

where $\psi$ is the fourth scalar-invariant function in this closure. Details of the above derivations and underlying physical assumptions can be found in Jovanović (2004).

### 2.6.2 Scalar-invariant Functions

Relation (2.24) between second and third invariants of second-order tensor correlations is exact for axisymmetric turbulence. Thus, knowing the values of $A, C, F$ and $\psi$ at the limiting states of turbulence, it is possible to formulate them as interpolation functions between two limiting states along axisymmetric branches of the anisotropy-invariant map with the help of a weighting function, say $J$. For example, for $A$ the interpolation equation can be written based on the invariants of the anisotropy tensor $a_{ij}$ of the second-order moments of velocity fluctuations $u_iu_j$ as follows:

$$A \simeq (1 - J)(A)_{1C} + J(A)_{iso}, \quad III_a > 0,$$

(2.66a)

$$A \simeq (1 - J)(A)_{2C-iso} + J(A)_{iso}, \quad III_a < 0,$$

(2.66b)

such that

$$J = 1, \quad \text{for isotropic turbulence,} \quad II_a = III_a = 0,$$

(2.67a)

$$J = 0, \quad \text{for two-component turbulence,} \quad II_a = \frac{2}{9} + 2III_a,$$

(2.67b)

where $II_a$ and $III_a$ are second and third invariants of the anisotropy tensor $a_{ij}$ and $(A)_{iso}$, $(A)_{1C}$ and $(A)_{2C-iso}$ are the limiting state values of the function $A$. According to relations (2.21,2.23), $a_{ij}$ and invariants of it are defined as

$$a_{ij} = \frac{u_iu_j}{q^2} - \frac{1}{3} \delta_{ij},$$

(2.68)

$$I_a = a_{ss} = 0,$$

(2.69a)

$$II_a = a_{ij}a_{ji},$$

(2.69b)

$$III_a = a_{ij}a_{ik}a_{jk}.$$

(2.69c)

Finally, the invariant weighting function $J$, which satisfies the conditions in (2.67), is

$$J = 1 - 9 \left( \frac{1}{2} II_a - III_a \right)$$

(2.70)

This approach was suggested by Lumley (1978) and exploited by Jovanović et al. (2003) for the construction of all the scalar functions by assuming that the invariants of the
anisotropy tensors of other correlations follow exactly the change of those of \( \overline{u_i u_j} \). Note that the weighting function \( J \) was necessarily formulated in terms of the invariants of \( a_{ij} \) for the sake of closure.

Scalar functions might take different values at the same limiting state depending on the ratio of the dissipation length scale \( \lambda_g \) to the integral length \( L_g \) in the turbulent flow. This ratio was found to be a function of the turbulent Reynolds number

\[
\text{Re}_\lambda^* = q\lambda_g/\nu
\]

as follows:

\[
\frac{\lambda_g}{L_g} = -0.049\text{Re}_\lambda^* + \frac{1}{2} \left( 0.009604\text{Re}_\lambda^{*2} + 10.208 \right)^{1/2},
\]

which is valid as \( \text{Re}_\lambda^* \to 0 \) and \( \text{Re}_\lambda^* \to \infty \). Note that \( \text{Re}_\lambda^* \) is different to \( \text{Re}_\lambda \) (2.28). Thus, in addition to the limiting states dictated by the kinematic relations, one should consider \( \text{Re}_\lambda^* \to 0 \) and \( \text{Re}_\lambda^* \to \infty \) as two extra limits at each kinematic limiting state. A normalized form of relation (2.72)

\[
\mathcal{W} = 0.626 \frac{\lambda_g}{L_g}
\]

can then be utilized to interpolate the scalar-invariant functions between their values at very high and very low \( \text{Re}_\lambda^* \). For example, according to Jovanović et al. (2003), for vanishing anisotropy, \( A \) has two different values and its value at any intermediate \( \text{Re}_\lambda^* \) can be approximated as

\[
A \to (1 - \mathcal{W})(A)_{\text{Re}_\lambda^* \to \infty} + \mathcal{W}(A)_{\text{Re}_\lambda^* \to 0}, \quad III_a \to 0
\]

Figure 2.7 shows the scalar-invariant functions at all limiting states, which were proposed by Jovanović et al. (2003) based on DNS and theoretical considerations. Making use of these values together with equations (2.24, 2.70, 2.74), the scalar-invariant functions \( A, C, F \) and \( \psi \) take the following forms:

\[
A \approx \left\{ \begin{array}{ll}
1 + \left( 1 - 9 \left[ \frac{1}{2} I_{II_a} - \frac{3}{4} \left( \frac{2}{3} I_{II_a} \right)^{\frac{3}{2}} \right] \right) (\mathcal{W} - 1), & III_a > 0, \\
1 + \left( 1 - 9 \left[ \frac{1}{2} I_{II_a} + \frac{3}{4} \left( \frac{2}{3} I_{II_a} \right)^{\frac{3}{2}} \right] \right) (\mathcal{W} - 1), & III_a < 0,
\end{array} \right.
\]

\[
C \approx \left\{ \begin{array}{ll}
1 - 9 \left[ \frac{1}{2} I_{II_a} - \frac{3}{4} \left( \frac{2}{3} I_{II_a} \right)^{\frac{3}{2}} \right] (2.5\mathcal{W} - 2.5) \epsilon, & III_a > 0, \\
1 - 9 \left[ \frac{1}{2} I_{II_a} + \frac{3}{4} \left( \frac{2}{3} I_{II_a} \right)^{\frac{3}{2}} \right] (2.5\mathcal{W} - 2.5) \epsilon, & III_a < 0,
\end{array} \right.
\]

\[
\psi \approx \left\{ \begin{array}{ll}
1 - 9 \left[ \frac{1}{2} I_{II_a} - \frac{3}{4} \left( \frac{2}{3} I_{II_a} \right)^{\frac{3}{2}} \right] (2.5\mathcal{W} - 2.5) \epsilon, & III_a > 0, \\
1 - 9 \left[ \frac{1}{2} I_{II_a} + \frac{3}{4} \left( \frac{2}{3} I_{II_a} \right)^{\frac{3}{2}} \right] (2.5\mathcal{W} - 2.5) \epsilon, & III_a < 0,
\end{array} \right.
\]
Theoretical Background

\[ F \approx \begin{cases} 
0.6 + 3.6 \left[ \frac{1}{2} II_a - \frac{3}{4} \left( \frac{2}{3} II_a \right)^{\frac{3}{2}} \right], & III_a > 0, \\
0.6 + 3.6 \left[ \frac{1}{2} II_a + \frac{3}{4} \left( \frac{2}{3} II_a \right)^{\frac{3}{2}} \right], & III_a < 0,
\end{cases} \]

(2.75c)

\[ \psi \approx \begin{cases} 
9b \left[ \frac{1}{2} II_a - \frac{3}{4} \left( \frac{2}{3} II_a \right)^{\frac{3}{2}} \right] + \left\{ 1 - 9 \left[ \frac{1}{2} II_a - \frac{3}{4} \left( \frac{2}{3} II_a \right)^{\frac{3}{2}} \right] \right\} (1.8 - 0.4W), & III_a > 0, \\
9a \left[ \frac{1}{2} II_a + \frac{3}{4} \left( \frac{2}{3} II_a \right)^{\frac{3}{2}} \right] + \left\{ 1 - 9 \left[ \frac{1}{2} II_a + \frac{3}{4} \left( \frac{2}{3} II_a \right)^{\frac{3}{2}} \right] \right\} (1.8 - 0.4W), & III_a < 0,
\end{cases} \]

(2.75d)

The limiting states of \( \psi \) (2.75d) at the two-component isotropic limit and one-component limit, which are represented by \( a \) and \( b \), respectively, are suggested to be

\[ a = 2.5, \]

(2.76a)

\[ b = 0. \]

(2.76b)

With this last step, the turbulence closure for axisymmetric turbulence proposed by Jovanović et al. (2003) is finalized.

2.6.3 Overview of the Modeled System of Equations

Finally, a closed system of ordinary differential equations (2.62 and 2.65) are obtained in the form of an initial value problem:

\[ \frac{\partial u_i u_j}{\partial t} \approx P_{ij} + C a_{ij} + a_{ij} P_{ss} + F \left( \frac{1}{3} P_{ss} \delta_{ij} - P_{ij} \right) - 2 A \epsilon a_{ij} - \frac{2}{3} \epsilon \delta_{ij}, \]

\[ \frac{\partial \epsilon}{\partial t} \approx 2 A \epsilon \frac{\partial U_i}{\partial x_k} \frac{\partial U_i}{\partial x_k} - \psi \frac{\epsilon^2}{k}, \]

in which the scalar-invariant functions are formulated as functions of the second invariant \( (II_a) \) of the anisotropy tensor \( a_{ij}, \) \( Re^*_\lambda \) and \( \epsilon \) (2.75a-d):

\[ A \approx A(II_a, Re^*_\lambda), \]

\[ C \approx C(II_a, Re^*_\lambda, \epsilon), \]

\[ F \approx F(II_a), \]

\[ \psi \approx \psi(II_a, Re^*_\lambda). \]

Thereby, this system of equations can be integrated if the mean velocity field and the initial values for \( \overline{u_i u_j} \) and \( \epsilon \) are known. This model is used in Chapter 8 to predict Reynolds stress development of the conducted experiments. Only the mean velocity
field and the initial values of Reynolds stresses are taken from the measurements and $\epsilon$ is calculated from the turbulent kinetic energy equation (2.12).

One can use this set of equations in order to design ducts in which turbulence is tailored according to the needs of the application. Designing of burners, for which anisotropy of turbulence plays an important role at the flame front, breaking down of agglomerates within straining ducts and turbulence control in general are a few examples of application areas of this anisotropy-invariant model.

### 2.6.4 Expanded Forms of Modeled Equations

The development of Reynolds stresses during axisymmetric contraction and expansion, according to the invariant turbulence closure, can be best understood by expanding each term in the modeled dynamic equations (2.62, 2.65).

As can be seen in equation (2.9), the time derivative of Reynolds stresses is composed of a production term, pressure velocity-gradient correlation term and dissipation term. For axisymmetric turbulence, the kinematics of turbulence (2.17) dictate that $u_2\overline{u_2}$ and
\( \bar{u}_3 \bar{u}_3 \) should be equal to each other:

\[
\bar{u}_2 \bar{u}_2 = \bar{u}_3 \bar{u}_3 \quad (2.77a)
\]

and

\[
\bar{u}_1 \bar{u}_2 = \bar{u}_1 \bar{u}_3 = \bar{u}_2 \bar{u}_3 = 0 \quad (2.77b)
\]
at all locations. Hence, it is sufficient to consider the dynamic equations for \( \bar{u}_1 \bar{u}_1 \) and \( \bar{u}_2 \bar{u}_2 \):

\[
\frac{\partial \bar{u}_1 \bar{u}_1}{\partial t} = P_{11} + \Pi_{11} - 2\epsilon_{11}, \quad (2.78a)
\]

\[
\frac{\partial \bar{u}_2 \bar{u}_2}{\partial t} = P_{22} + \Pi_{22} - 2\epsilon_{22}, \quad (2.78b)
\]

and the transport equation for \( \epsilon \) (2.13).

Moreover, the continuity equation for the mean velocity field delivers the following equality for a steady flow:

\[
\frac{\partial \bar{U}_1}{\partial x_1} = -2 \frac{\partial \bar{U}_2}{\partial x_2}. \quad (2.79)
\]

**Production correlations**

Expansion of the production term \( P_{ij} \) in equation (2.9) to obtain \( P_{11} \) gives

\[
P_{11} = -2\bar{u}_1 \bar{u}_1 \frac{\partial \bar{U}_1}{\partial x_1} \quad (2.80)
\]

This equation states that the production of \( \bar{u}_1 \bar{u}_1 \) is always negative if the flow is contracted, i.e. \( \partial \bar{U}_1 / \partial x_1 < 0 \).

Similarly, the production term for \( \bar{u}_2 \bar{u}_2 \) is

\[
P_{22} = -2\bar{u}_2 \bar{u}_2 \frac{\partial \bar{U}_2}{\partial x_2}.
\]

Due to the continuity equation (2.79), this term reduces to

\[
P_{22} = \bar{u}_2 \bar{u}_2 \frac{\partial \bar{U}_1}{\partial x_1}, \quad (2.81)
\]

which implies that \( \bar{u}_2 \bar{u}_2 \) will always be produced for \( \partial \bar{U}_1 / \partial x_1 > 0 \) in homogeneous axisymmetric turbulent flow.

Due to the equality of the transverse lateral stress components, the trace of the \( P_{ij} \) term becomes

\[
P_{ss} = (\bar{u}_2 \bar{u}_2 - \bar{u}_1 \bar{u}_1) 2 \frac{\partial \bar{U}_1}{\partial x_1}. \quad (2.82)
\]
Similarly, the production term in the total kinetic energy of turbulence equation (2.12) becomes half of $P_{ss}$:

$$ P_k = (\bar{u}_2\bar{u}_2 - \bar{u}_1\bar{u}_1) \frac{\partial \bar{U}_1}{\partial x_1} $$  \hspace{1cm} (2.83)

The message from this equation is that for $\partial \bar{U}_1/\partial x_1 > 0$, production starts as soon as the transverse stress component becomes larger than the longitudinal one.

**Pressure-strain correlations**

The expanded form of the modeled pressure-strain correlation in equation (2.61) for $i = 1$ and $j = 1$ is

$$ \Pi_{11} = C a_{11} + a_{11} P_{ss} + F \left( \frac{1}{3} P_{ss} - P_{11} \right) . $$

Note that the multiplier of $F$ in the above equation represents the anisotropy of the production tensor $P_{ij}$. Further expansion of the above equation gives

$$ \Pi_{11} = C a_{11} + \frac{\partial \bar{U}_1}{\partial x_1} \left[ -2 a_{11} (\bar{u}_1\bar{u}_1 - \bar{u}_2\bar{u}_2) + F \frac{1}{3} (4 \bar{u}_1\bar{u}_1 + 2 \bar{u}_2\bar{u}_2) \right] $$ \hspace{1cm} (2.84a)

Similarly, utilizing the relation between the diagonal components of the anisotropy tensor $a_{ij}$ (2.22), $\Pi_{22}$ and $\Pi_{33}$, which are equal to each other, the following form is obtained:

$$ \Pi_{22} = -\frac{1}{2} C a_{11} + \frac{\partial \bar{U}_1}{\partial x_1} \left[ a_{11} (\bar{u}_1\bar{u}_1 - \bar{u}_2\bar{u}_2) - F \frac{1}{3} (2 \bar{u}_1\bar{u}_1 + \bar{u}_2\bar{u}_2) \right] . $$ \hspace{1cm} (2.84b)

The terms including $C$ represent the effect of the slow pressure-strain term and the rest represent that of the fast pressure-strain term. Thus, along the left edge of the AI-map, that is, along the contraction, where $a_{11} < 0$ and $\partial \bar{U}_1/\partial x_1 > 0$, the slow pressure-strain term acts like a sink for $\bar{u}_1\bar{u}_1$ and like a source for $\bar{u}_2\bar{u}_2$. Whether rapid pressure-strain acts like a source or a sink is mainly determined by the balance in the parenthesis in front of $\partial \bar{U}_1/\partial x_1$. However, since the rapid pressure-strain term redistributes the turbulent energy among components, at the left edge of AI-map where $\bar{u}_2\bar{u}_2 > \bar{u}_1\bar{u}_1$, $\Pi_{11}$ and $\Pi_{22}$ should be source and sink, respectively.

**Dissipation correlations**

According to partitioning of the total dissipation rate (2.60) into its components, $\epsilon_{11}$ and $\epsilon_{22}$ are

$$ \epsilon_{11} = A \epsilon a_{11} + \frac{1}{3} \epsilon, $$ \hspace{1cm} (2.85a)

$$ \epsilon_{22} = -\frac{1}{2} A \epsilon a_{11} + \frac{1}{3} \epsilon. $$ \hspace{1cm} (2.85b)
According to this kind of partitioning, the dissipation of $\overline{u_1u_1}$ is weakened and that of $\overline{u_2u_2}$ is augmented for $a_{11} < 0$. In other words, the smaller the Reynolds stress, the less it dissipates.

The modeled dynamic equation for the total dissipation rate $\epsilon$, in its expanded form, is

$$\frac{\partial \epsilon}{\partial t} = -6A\epsilon \frac{\partial U_1}{\partial x_1} a_{11} - \psi \frac{\epsilon^2}{k},$$

where the first term represents production of the dissipation due to the mean velocity gradient and the difference between longitudinal and transverse Reynolds stresses. For instance, strain of initially isotropic turbulence through a contraction would have $\frac{\partial U_1}{\partial x_1} > 0$ and $a_{11} < 0$ and the first term becomes a source for the dissipation and its contribution increases with increasing anisotropy. This behavior is the same for straining of initially isotropic turbulence through an expansion. On the other hand, the second term always reduces the dissipation and the amount of reduction is proportional to the second power of the dissipation rate. Negative production of dissipation can be obtained when $a_{11}$ and $\frac{\partial U_1}{\partial x_1}$ have the same sign, for instance when initially turbulence is first contracted and then expanded.
Chapter 3

Experimental Facilities and Overview of Experimental Investigations

3.1 Flow Facilities

3.1.1 Axisymmetric Strain Tunnel (AST)

To study axisymmetric turbulence, the experimental facility sketched in Figure 3.1 was designed and set up. The design of the wind tunnel was carried out with the intention of performing the study of axisymmetric strained turbulence. The actual straining was obtained by axisymmetric ducts, like contractions (converging nozzles) and expansions (diffusers) having different contraction ratios. Turbulent flow was produced with turbulence-generating grids possessing different mesh sizes.

The main components of the experimental facility were an open-circuit sucking-type wind tunnel driven downstream by a radial blower, a hot-wire measurement system together with signal conditioning and data acquisition instruments and a specially built mass flow rate control system. The latter is described in Durst et al. (2003). Hereafter this experimental facility is called an axisymmetric strain tunnel (AST).

The air was sucked into the flow facility through an inlet nozzle which had a smooth wall curvature converging to a duct having a diameter of 600 mm. In this constant-diameter duct, the flow was exposed to honeycomb-type flow straighteners (1) and a set of screens (2) which condition the flow in such a way that a uniform, low-turbulence level flow approached the turbulence-generating grid. The honeycomb flow straighteners embraced 3 mm diameter tubes of length 30 mm. The following three sets of wire screens were made up of 0.25 mm thick wires providing a mesh size of 0.8 mm. The duct led to a settling chamber (3) having a length of 300 mm after the last wire screen.
Experimental Facilities and Overview of Experimental Investigations

After this, the flow was contracted with a 9.77:1 contraction (4), corresponding to a reduction in the diameter to 192 mm. With this contraction, the flow was accelerated not only to attain higher flow velocities but also to damp the remaining background disturbances. The first 9.77:1 contraction (4) was followed by a 192 mm diameter duct in which turbulence developed before entering the straining-ducts (8). The geometric details of the straining ducts are provided in Section 3.1.2.

All turbulence-generating grids (5) were located in the 192 mm diameter duct downstream of the exit of the first nozzle. Metal grids with punched square holes were used to produce the investigated grid turbulence. Three grids with mesh sizes of M = 5, 7 and 10 mm were used. In the present investigations, mesh size is defined as the side length of the open square of the turbulence-generating grid. The widths of the grid rods were 1.1, 1.6 and 2.2 mm and the thicknesses were 1.2, 1.6 and 2.2 mm, respectively. The corresponding porosities were 0.826, 0.815 and 0.826, respectively. The grids were chosen such that their porosities were over 0.57, so that, as suggested by Corrsin (1963), the instabilities and the inhomogeneities due to blockage effects of the grids were not expected to be present in the generated turbulent flow fields.

After the straining ducts (8), the flow expanded to a chamber which was always kept under a small subpressure with respect to the surrounding atmospheric pressure. The inner wall of this chamber was carefully covered with pyramid-shaped foam (10) for damping acoustic waves. Before the flow reached the outlet of this chamber, it passed between vertical sound-damping profiles (11). These profiles were placed so as to damp the acoustic disturbances emerging from the blower. This minimized the effect of possible acoustic disturbances on the investigated turbulent flows by damping the acoustic resonance of the wind tunnel, especially within the straining ducts. The whole tunnel was connected to the blower by flexible pipe to avoid mechanical vibrations caused by the blower and the resulting flow disturbances.

In order to perform hot-wire measurements at various locations of the flow, the hot-wire probes (7) were mounted on a traversing unit (9), enabling movements to be made in three orthogonal directions, with positioning errors of ±0.1 mm in all three directions. To permit undisturbed measurements, the traversing unit was located inside the acoustically damped chamber, as sketched in Figure 3.1.

The Reynolds number of the flow was that of the grid-generated turbulence and set for each experiment by changing the mesh size and the flow speed. The maximum attainable flow speed on the grid was dependent on the straining duct employed, because of the different pressure drops caused by these ducts. For the highest contraction ratio, 14.75, the AST could afford 2.5 m/s and, for the measurements with rest of the straining sections, 5 m/s was chosen to be the maximum velocity (see Table 3.2).
3.1 Flow Facilities

3.1.2 Straining Ducts

Four basic strain configurations were investigated: zero strain (decay of isotropic and anisotropic turbulence), axisymmetric contraction, axisymmetric expansion and axisymmetric contraction followed by an axisymmetric expansion. The dimensions of the test sections employed are given in Table 3.1. As for the zero-strain configuration, three ducts of constant diameter were employed: one for the investigations of nearly isotropic grid-generated turbulence, one for the investigations of grid-generated turbulence with improved isotropy and one for investigations of the decay of anisotropic turbulence. After all the converging nozzles, ducts of length 10 cm with constant diameter were added to prevent sudden expansion effects on the flow in the nozzle.

<table>
<thead>
<tr>
<th>Straining section</th>
<th>Contraction ratio</th>
<th>Total expansion angle [°]</th>
<th>Inlet diameter [m]</th>
<th>Length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst. Diameter duct-1</td>
<td>-</td>
<td>-</td>
<td>0.192</td>
<td>0.525</td>
</tr>
<tr>
<td>Cst. Diameter duct-2</td>
<td>-</td>
<td>-</td>
<td>0.170</td>
<td>0.200</td>
</tr>
<tr>
<td>Cst. Diameter duct-3</td>
<td>-</td>
<td>-</td>
<td>0.100</td>
<td>0.350</td>
</tr>
<tr>
<td>Contraction-1</td>
<td>1.270</td>
<td>-</td>
<td>0.192</td>
<td>0.100</td>
</tr>
<tr>
<td>Contraction-2</td>
<td>3.690</td>
<td>-</td>
<td>0.192</td>
<td>0.200</td>
</tr>
<tr>
<td>Contraction-3</td>
<td>14.750</td>
<td>-</td>
<td>0.192</td>
<td>0.200</td>
</tr>
<tr>
<td>Expansion-1</td>
<td>0.720</td>
<td>7.000</td>
<td>0.192</td>
<td>0.600</td>
</tr>
<tr>
<td>Expansion-2</td>
<td>0.700</td>
<td>7.000</td>
<td>0.100</td>
<td>0.350</td>
</tr>
</tbody>
</table>

Table 3.1: Dimensions of the straining sections employed.
3. Experimental Facilities and Overview of Experimental Investigations

3.1.3 Wind Tunnel of LSTM-Erlangen

The preliminary investigations in the AST showed transverse inhomogeneity of measured quantities persisting even after 40 mesh size away from the grid, even though the grids had low solidity. It was suspected that either the maximum attainable speed in the AST, 5 m/s, was not sufficient to have good mixing in the wake of the grid (low Re effect), or the duct diameter to mesh size ratio was not sufficiently small. Hence, in order to investigate the effect of the grid Reynolds number and the duct size on the homogeneity of grid-generated turbulence, measurements in the closed-loop wind tunnel of LSTM-Erlangen were conducted. A side view of the wind tunnel is shown in Figure 3.2. This was a Göttinger-type low-speed tunnel with an open test section. The open section of the wind tunnel was closed to prevent the generation of free shear turbulence and, consequently, its influence on the measurements. A grid with 10 mm mesh size was installed after the exit of the contraction. The closed test section was 1.87 m in width, 1.4 m in height and 2 m in length. In Figure 3.3, the test section and the hot-wire probe configuration employed are shown. As can be seen, the hot-wire probes were installed on a 3-D traverse system, which allowed the probes to be traversed to 100 mesh size away from the grid of largest mesh size, whereas in the AST the probes could be traversed only 50 mesh size for the same grid (see Table 3.2). A Pitot-static tube was fixed on the same traversing system close to the hot-wire probes for the calibration of hot wires.

The wind tunnel was equipped with a temperature regulator, which kept the flow at a given temperature within ±0.1 K. The maximum operating speed at the test section could be set up to 60 m/s. Because of the large drag force applied on the grid at high velocities, in the present investigations at the maximum only 12 m/s was achieved. Note that the flow in AST could reach only 5 m/s on the turbulence-generating grid, i.e. the maximum grid Reynolds number was almost doubled by the utilization of the wind tunnel.

3.2 Instrumentation for Turbulence Measurements

To conduct velocity measurements, a DISA 56C01 hot-wire anemometer unit with four DISA CT56C17 constant-temperature hot-wire bridges was employed. Two single normal wire (SN-wire) and one X-wire probes were used. This probe configuration was selected to account for the irrotational velocity fluctuations appearing as flow disturbances. The theoretical background of the design of this probe system and the necessary measurement and data processing methods are given in detail in Chapter 6.

The normal wires were 0.77 and 1 mm in length and the two inclined wires of the X-wire probe were 1.15 mm in length. The distance between the inclined wires was 1 mm and their geometric inclinations were 43° and 44° with respect to the probe axis. All the hot-wires employed were 5 μm in diameter. In order to minimize the electronic
Figure 3.2: Dimensions (mm) of the closed-loop wind tunnel of LSTM-Erlangen.

Figure 3.3: The test section of the wind tunnel and the hot-wire probe configuration employed.
noise, the gain and high-frequency filter settings of the hot-wire bridge amplifier were chosen to be 1. With these settings, the anemometer had a cut-off frequency of 22 kHz. The signals from the anemometers were filtered through low-pass filters of back-up amplifiers with their upper frequency limit set at 11 kHz. Hence the measurement system could sense and record all turbulence fluctuations up to this frequency limit of 11 kHz. This upper frequency limit was much higher than the expected highest turbulent velocity fluctuation frequencies. The latter was estimated to be 8 kHz at the maximum.

Velocity calibrations of the hot-wire probes employed in the AST were performed at the exit of the test contractions (8 in Figure 3.1). For the purpose of calibration, the static pressure difference between the entrance and the exit of the contraction was measured with a SETRA differential-pressure transducer and, by using the Bernoulli equation and the law of conservation of mass, the velocity at the exit of the nozzle was determined. The calibration in the wind tunnel was performed with the reference velocity obtained from a Pitot-static tube installed on the traverse in the vicinity of the hot-wire probes. During the calibration and the measurements, the temperature of the flow was measured with a PT100 temperature sensor in order to correct the measured data for temperature drifts.

The filtered hot-wire signals, pressure transducer signal and temperature signal were acquired by a 16-bit A/D converter (NI 6052E DAQ card) installed in a personal computer. The data was recorded on hard disks. All the signals were grounded only over the computer in order to prevent ground loop disturbances on the measured signal. During the measurements, a resolution of 0.076 mV was achieved with the A/D converter. The minimum r.m.s. voltage reading of an analog r.m.s. meter at turbulence-free flow is around 0.2 mV. Hence the resolution of the data acquisition system permitted low-turbulence intensity measurements, with high digitalization accuracy.

The sampling rate for each set of measurements was chosen such that only statistically uncorrelated data were sampled, unless the spectra and the two-point correlation functions were measured. For this purpose, the integral time scales, \( \tau \), were calculated via autocorrelation measurements in the test section for every measurement case and the sampling frequencies was chosen such that they were less that \( 1/2\tau \) to ensure zero eddy turbulence correlation between two consecutive data. The number of data for each measurement point was chosen to be 50000, which resulted in less than 1% statistical uncertainty with 99% confidence for the mean velocity and around 1.2% statistical uncertainty with 95% confidence for the mean square of the turbulent velocity fluctuations.

In order to investigate the effect of mass flow rate fluctuations inherent in the flow facilities, artificial mass flow rate fluctuations were introduced into the flow with a mass flow rate control valve. Thus, it was possible to check the validity of the suggested measurement method accounting for such non-turbulent velocity fluctuations in a controlled manner. This valve was developed by LSTM-Erlangen and Drägerwerke. It was driven by pressurized air and could deliver precise mass flow rates as any function of time with the help of a coupled control unit. Details of this equipment and examples
of its application in fluid mechanics research can be found in Durst et al. (2003) and Ertunc et al. (2003). In the present investigations, only sinusoidal mass flow rate pulsations were generated by providing the input sinusoidal signal to the control unit by a signal generator. The mass flow rate control valve was placed at the end of the wind tunnel between the blower and the sound-damped room to prevent vortical structures being produced by the interaction between the flow from the control valve and the surrounding flow which could have approached the hot-wire probes.

3.2.1 Calibration of the Hot-wire Probes

The generalized forms of SN-wire and X-wire probes are shown in Figure 6.5. For a single hot-wire (normal or inclined) and for a velocity field fully correlated over the entire length of the wire, the measured signal potential, \( E(t) \), can be related to the effective cooling velocity, \( U_e(t) \), which is the velocity perpendicular to the wire, in the following way (Bruun, 1995):

\[
E(t)^2 = A + BU_e(t)^n. \tag{3.1}
\]

This equation represents a linear relation, where \( A \), \( B \) and \( n \) are calibration parameters which are individually valid for each calibrated wire. For calibration of the wires, the mean value of the effective cooling velocity, \( U_e \), should be known from an independent measurement, so that \( A \), \( B \) and \( n \) can be calculated by using the equation

\[
E^2 = A + BU_e^n. \tag{3.2}
\]

For the calibration in the AST, measurements of pressure difference between the inlet and the outlet of the nozzles were utilized as the independent reference velocity \( U_R \). The relation between the reference velocity and the measured pressure difference is

\[
U_R = \sqrt{\frac{(P_{\text{inlet}} - P_{\text{outlet}})}{\rho(1 - 1/c^2)}}, \tag{3.3}
\]

where \( c \) is the contraction ratio. In the wind tunnel, the reference velocity was obtained from Pitot-static tube measurements.

The velocity calibration data for all four wires (two normal and two inclined) were acquired before each measurement. Depending on the contraction ratio, the mean velocity along the nozzles axis might cover a wide range of velocity for which the velocity calibration should be performed. However, if the calibration constants are determined for the whole velocity range, as shown in Figure 3.4 a, then equation (3.2) cannot represent the measured calibration data and the calibration errors might be a few percent. In order to reduce the calibration error, equation (3.2) was assumed to be valid for narrower ranges of velocities; therefore, calibration was made only around the mean signal potential measured at each measurement location. The calibration constants \( A \) and \( B \) were determined for all the wires by keeping \( n = 0.45 \). Shown in Figure 3.4 b,
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Figure 3.4: (a) A typical velocity calibration curve obtained for a velocity range 1-14 m/s. (b) The errors introduced by different calibration methods.

This method reduced the calibration error to below 1%. The calibration error was calculated by using the reference velocity and the velocity found from equation (3.2). It was concluded that dynamic determinations of calibration constants $A$ and $B$ reduced the error such that any optimization of the value of $n$ did not make any significant difference to the calibration error.

During the velocity calibration and the measurements, the temperature of the flow was always recorded, so that changes in the temperature of the flow during measurements could be corrected. For correcting the influence of temperature drifts on the measurements, the method suggested by Kanevč & Oka (1973) was applied. The temperature drift remained at ± 1 K during almost all measurements.

For the angle calibration of the X-wire probe, the effective angle ($\alpha_e$) method suggested by Bradshaw (1971) was applied. In this method, it is assumed that the directional sensitivity of the hot-wire is dependent only on the effective cooling velocity $U_e$, for which the following equation is valid:

$$U_e = |U| \cos(\alpha) = |U| \cos(\alpha_{eff} + \theta), \quad (3.4)$$

where $|U|$ is the absolute value of the velocity vector, $\theta$ is the angle between the probe axis and the velocity vector and $\alpha_{eff}$ is the effective wire angle (see Figure 6.5). Note that the effective angle and the geometric angle need not necessarily be equal to each other, because of the prongs effect. Moreover, the yaw and pitch coefficients drop out, since they become part of the effective angle.

Replacing the effective velocity (3.4) for $\theta = 0$ in equation (3.2), one obtains

$$E^2 = A + B(|U| \cos(\alpha))^n = A + \hat{B}(\alpha)|U|^n, \quad (3.5)$$

where

$$\hat{B}(\alpha) = B \cos^n(\alpha). \quad (3.6)$$
3.2 Instrumentation for Turbulence Measurements

Hence the angle calibration of the X-wire can be performed in two steps. First, the aforementioned velocity calibration is performed at $\theta = 0^\circ$, from which $A$ and $\hat{B}(\alpha_{\theta=0^\circ}) = \hat{B}(\alpha_{eff})$ can be found. In order to find the unknowns $B$ and $\alpha_{eff}$, at a constant velocity, the angle $\theta$ of the hot-wire probe is changed and the output signal magnitude is read. In the present study, the probes were mounted on an arc-shaped traversing system enabling $\theta$ to be changed between $25^\circ$ to $-25^\circ$ with $5^\circ$ steps (see Figure 3.3). Thus, for each $\theta$, the corresponding mean of the signal potential is

$$\langle E^2(\theta) \rangle = A + \hat{B}(\alpha)|\bar{U}|^n.$$

(3.7)

For small changes in the angle $\theta$, the quantities $B$ and $\alpha_{eff}$ can be treated independently from $\theta$, so that the following relation can be used to find out the unknowns:

$$\left( \frac{E^2(\theta) - A}{E^2(\theta = 0) - A} \right)^{\frac{1}{n}} = \cos(\alpha_{eff} + \theta) \cos \alpha_{eff}.$$  

(3.8)

By algebraic manipulation of the above equation, one can find a linear form of this equation as

$$\cos \theta - \left( \frac{E^2(\theta) - A}{E^2(\theta = 0) - A} \right)^{\frac{1}{n}} = \tan \alpha_{eff} \sin \theta.$$  

(3.9)

Plotting $\cos \theta - \hat{E}_\theta$ versus $\sin \theta$, one obtains two straight lines essentially crossing at $0^\circ$, as shown in Figure 3.5 a. The slopes of these lines yield the effective angles $\alpha_{eff}$ of the two inclined wires of the X-wire probe. Knowing $\alpha_{eff}$, one can calculate $B$ from equation (3.6), which is the last of the angle calibration. The error introduced by this angle calibration is shown in Figure 3.5 b.

In the present investigations, it was seen that the effective angle was not dependent on the reference velocity $\bar{U}_R$, which was kept constant during the angle calibration.
3. Experimental Facilities and Overview of Experimental Investigations

Figure 3.5: (a) Typical angle calibration curves of an X-wire probe. (b) The error introduced by the applied angle calibration method at different angles between the probe and the flow direction, $\theta$.

3.3 Flow Quality of the Flow Facilities

3.3.1 The AST

The background turbulence in duct-1 was investigated to determine the quality of the flow without any turbulence-generating grid before it entered any of the straining ducts. For this purpose, the intensity and the power spectrum density of the velocity fluctuation components $u$ and $v$ were measured in duct-1.

The turbulence intensity is presented in Figure 3.6a; $u'$ and $v'$ are the root mean square of the axial and transverse fluctuations, respectively. To show only the flow-induced fluctuations, the contributions of the electronic noise were excluded in Figure 3.6a. The turbulence intensities of both velocity components increased along the constant cross-section duct-1, which is a clear indication of the developing boundary layer.

The power spectrum density of velocity fluctuations $u$ and $v$ at the inlet and exit of duct-1 are shown in Figure 3.6b. The first peculiarity is that the energy-containing fluctuations were in a very low frequency range. This means that the background disturbances did not possess a turbulent nature, hense, they should be irrotational. The increase towards the exit of the duct in the low-frequency values of the $v$ spectra is greater more than those of the $u$ spectra. This can be related to the unsteady changes in the thickness of the layer which induce irrotational fluctuations of $v$ at the center of the duct. The spikes at high frequencies are the remaining electronic disturbances. As demonstrated in Figure 3.11 for the wind-tunnel measurements, the amplitudes of spectra for a grid-generated turbulence are an order of magnitude higher than the background disturbances in the measured signal, hence they do not constitute a serious problem.
Extensive investigations on the disturbances induced by the flow and the measurement equipment were performed. In Chapter 6, all these investigations are presented in detail. Concerning the disturbances caused by the developing boundary layer, it was found that the boundary layer induces irrotational flow fluctuations at the center of the duct. No measures were taken to reduce the effect of the developing boundary layer; instead, a measurement method was developed to extract its contribution from the measurements. With the application of this method, not only is the effect of the boundary layer on the measurements avoided, but also the acoustic disturbances and disturbances induced by the radial blower. This method was successfully applied in order to extract turbulence information from the measurements. The treatment of electronic noise and irrotational flow fluctuations is extensively explained in Chapter 6.

The turbulence background intensity plot in Figure 3.6a shows more isotropic fluctuations, whereas in Figure 3.6b, the power spectrum values in the low-frequency range are remarkably anisotropic. Remember that the integration of the power spectrum densities of $u$ and $v$ in whole frequency range, which is 1–40000 Hz in the present measurements, delivers the mean squares $\overline{u^2}$ and $\overline{v^2}$. Thus, although the amplitudes of high-frequency components are lower than those of the low-frequency components, they contribute more to the integral value. In order to demonstrate the effect of the high-frequency noise, the background turbulence measurements in Figure 3.6 were reprocessed with a digital low-pass filter whose cut-off frequency was selected to be 49 Hz. As can be seen in Figure 3.7a, the streamwise turbulence intensity was almost unaffected, whereas the transverse intensity decreased drastically close to the entrance of duct-1. When the power spectrum density in Figures 3.6b and 3.7b are compared, no change in the low-frequency range can be observed. Although the selected cut-off frequency of the low-pass filter was somewhat arbitrary, comparison revealed that the energy of $u$ fluctuations is composed mainly of low-frequency fluctuations and $v$ fluctuations were influenced by the high-frequency noise, especially close to the inlet when they are extremely low. It is fair to argue that it is a formidable task to distinguish with accuracy the flow-induced fluctuations and the instrument noise.

In order to visualize the extent of the boundary layer in the core of the flow, the profiles of mean velocity and Reynolds stresses along the radius of a cross-section perpendicular to the flow direction were measured. Since the boundary layer would be tripped by the introduced grids, measurements with and without grids were performed. The measurement cross-section was 0.33 m away from the grid location, which corresponds to 66M, 47.1M and 33M mesh distance for grids of 5, 7 and 10 mm mesh sizes, respectively.

The measured profiles were normalized by the values at the center of the channel and plotted for half of the cross-section in Fig. 3.8. Large-scale inhomogeneity in mean velocity and in Reynolds stresses was not observed in the core of the flow. Near the wall, flow became inhomogeneous due to the wall. The profiles in grid-generated turbulence showed that the thickness of the boundary layer was around 20% of the radius of the duct in mean velocity, and around 15% in Reynolds stresses; 15% of boundary layer thickness means that only 72% of the total area was homogeneous at this cross-section. When the development of the turbulent boundary layer is considered, one
3. Experimental Facilities and Overview of Experimental Investigations

Figure 3.6: The background turbulence in duct-1: (a) The streamwise and transverse turbulence intensities without the contributions of electronic noise and (b) the power spectrum density of $u$ and $v$ fluctuations at the inlet and exit of duct-1.

Figure 3.7: The background turbulence in duct-1 after filtering with a low-pass filter which was active above 49 Hz: (a) The streamwise and transverse turbulence intensities and (b) the power spectrum density of $u$ and $v$ fluctuations at the inlet and exit of duct-1.
can easily conclude that, as the straining sections are located far away from the grid, the homogeneous core would be limited to a smaller portion of the cross-section before the flow is strained. Moreover, it was interesting to observe that stresses in the boundary layer were multiple times the stresses in the core of the grid-generated turbulent flow. For these two reasons, it was most likely that the on-axis measurements, especially inside the contraction, were contaminated by boundary layer effects in the present experiments and in the experiments available in the literature.

Being aware of the above-mentioned states of the flow in the AST, the plots in Figure 3.6 are accepted to assess the quality of the flow facility. The AST was further utilized since

- the intensities were less than 0.1%;

- the existing low-frequency fluctuations was not expected to interact with the grid-generated turbulence, which, in general, possesses fluctuations with higher frequency;

- significant portion of the core flow was not affected by the boundary layer before the straining sections;

- a measurement method was developed and utilized to account for irrotational streamwise velocity fluctuations (see Chapter 6).

![Figure 3.8: The extent of boundary layer in the core flow and the transverse homogeneity in duct-1 at x=0.33 m: (a) Normalized mean velocity $\overline{U}$; (b) normalized $\overline{uu}$; (c) normalized $\overline{vv}$.](image-url)
3.3.2 The Wind Tunnel

The background turbulence intensity in the test section of the wind tunnel is shown in Figure 3.9. The zero \( x \)-coordinate corresponds to the inlet of the closed test section where the grid was installed at later stages of the investigations. The contributions of the electronic noise were excluded in Figure 3.9 to show only the flow-induced fluctuations. The measurements revealed that the transverse fluctuations were always higher than the streamwise fluctuations except for the lowest velocity 5 m/s. The \( v \) fluctuations clearly increased with increasing mean velocity, whereas the \( u \) fluctuations did not have this tendency at low mean velocities. The measurements were performed with mean velocities less than 12 m/s, at which the background intensities were low enough for reliable measurements of grid-generated turbulence.

The power spectrum densities of \( u \) and \( v \) fluctuations in Figure 3.10 show a turbulence-like spectrum with spectral energies distributed in a wide frequency range, whereas the spectrum in the AST did not possess energies at high frequencies. Moreover, the anisotropy of turbulence can also be realized from the power spectrum density plots. As far as the spikes in the power density spectrum are concerned, their energies are an order of magnitude lower than those measured for grid-generated turbulent case (Figure 3.11) at the same flow speed and the same location in the wind-tunnel. These spikes are mainly electronic noise and contribution to the Reynolds stresses can be subtracted as discussed in Chapter 6.
3.4 Description of the Test Cases

In order to investigate different forms of axisymmetric turbulence, several sets of measurements were performed. The measurements differed from each other in terms of the effects investigated and the quantities measured. The effects investigated were mentioned in Section 1.4. Three kinds of measurements were mainly performed:

- Stress measurements in which only statistically independent samples were acquired along the symmetry axis of the test sections.
- Spectrum and autocorrelation measurements for which signals were acquired in packages with very high sampling rates.
- Homogeneity measurements in which the flow fields were spatially fine resolved in axial and transverse directions.

In Table 3.2, an overview of the measurements together with experimental conditions are given. For all on-axis measurements, spectrum and correlations measurements were performed. Only a few homogeneity measurements are available with spectrum information. An overview of the transverse homogeneity investigations is given in this table.

As will also be made clear in the Chapter 6, hot-wire measurements in converging nozzles with high-contraction ratios should be corrected for its insufficient spatial resolution and for the irrotational fluctuations, which exist in the flow facility and amplified in the contraction. In the framework of this thesis, two correction methods were developed to account for these effects. Hence, in addition to the experiments investigating different aspects of axisymmetric turbulence, some measurements were made for the
Figure 3.11: Comparison of the power spectrum density of the background turbulence and grid-generated turbulence at the same flow speed and location in the wind tunnel.

development of measurement techniques and checking the flow quality in the flow facilities.

In all the measurements, the distances between the straining sections employed and the turbulence-generating grid were selected such that the flow had reached a reasonable isotropic homogeneous state prior to entering the straining sections.
### Measurements on the symmetry axis

<table>
<thead>
<tr>
<th>Straining-section</th>
<th>Mesh sizes [mm]</th>
<th>Start of test-section [m]</th>
<th>Start of measurements [m]</th>
<th>$U_{ref}$ [m/s]</th>
<th>$Re_{st}$ range</th>
<th>$Re_{st}$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decay of grid-generated turbulence (wind tunnel)</td>
<td>-</td>
<td>8.0</td>
<td>-</td>
<td>0.04</td>
<td>3, 5, 6, 8, 10, 12</td>
<td>1584-6336</td>
</tr>
<tr>
<td>Decay of grid-generated turbulence (AST)</td>
<td>duct-1</td>
<td>5.0, 7.0, 10.0</td>
<td>-</td>
<td>0.1</td>
<td>2, 5</td>
<td>666-4000</td>
</tr>
<tr>
<td>Decay of grid-generated turbulence with improved isotropy (AST)</td>
<td>after contraction-1, in duct-2</td>
<td>5.0, 7.0, 10.0</td>
<td>0.225, 0.425 (start of cont.)</td>
<td>0.1, 0.325</td>
<td>2, 5, 6</td>
<td>1333-2666</td>
</tr>
<tr>
<td>Decay of anisotropic turbulence (AST)</td>
<td>after contraction-2, in duct-3</td>
<td>5.0, 7.0, 10.0</td>
<td>0.425 (start of cont.)</td>
<td>0.525</td>
<td>4</td>
<td>833-3333</td>
</tr>
<tr>
<td>Effect of contraction (AST)</td>
<td>contraction-1 (c=1.27)</td>
<td>5.0, 7.0, 10.0</td>
<td>0.225, 0.525 (start of cont.)</td>
<td>0.2</td>
<td>2.5, 5</td>
<td>833-3333</td>
</tr>
<tr>
<td></td>
<td>contraction-2 (c=3.69)</td>
<td>5.0, 7.0, 10.0</td>
<td>0.225, 0.525 (start of cont.)</td>
<td>0.125, 0.45</td>
<td>2.5, 5</td>
<td>833-3333</td>
</tr>
<tr>
<td></td>
<td>contraction-3 (c=14.75)</td>
<td>5.0, 7.0, 10.0</td>
<td>0.225, 0.525 (start of cont.)</td>
<td>0.125, 0.45</td>
<td>2.5</td>
<td>833</td>
</tr>
<tr>
<td>Effect of expansion (AST)</td>
<td>expansion-1 (c=0.72)</td>
<td>5.0, 7.0, 10.0</td>
<td>0.325 (start of exp.)</td>
<td>0.3</td>
<td>2.5, 5</td>
<td>833-3333</td>
</tr>
<tr>
<td>Effect of successive strain (AST)</td>
<td>contraction-2, expansion-2</td>
<td>5.0, 7.0, 10.0</td>
<td>0.425 (start of cont.)</td>
<td>0.325</td>
<td>2.5, 5</td>
<td>833-3333</td>
</tr>
</tbody>
</table>

### Measurements of transverse homogeneity

| Grid-generated turbulence (wind tunnel) | - | 8.0 | - | 0.04 | 3, 5, 6, 8, 10, 12 | 1584-6336 | 15-60 |
| Grid-generated turbulence (AST) | duct-1 | 10.0 | - | 0.02 | 5 | 3333 | 15-35 |
| Effect of contraction (AST) | duct-1, contraction-1, duct-2 | 10.0 | 0.225 (start of cont.) | 0.1, 0.4 | 5 | 3333 | 15-35 |
| Effect of expansion (AST) | expansion-1 (c=0.72) | 10.0 | 0.325 (start of exp.) | 0.3 | 5 | 3333 | 15-35 |
Chapter 4

Inhomogeneity of Grid-generated Turbulence Under Zero Strain and Finite Strain

In order to ensure the homogeneity of the turbulent field after the grid, Corrsin (1963) suggested that three conditions should be satisfied. First, the grid should have a porosity larger than 57% to prevent coalescing jets. Second, the diameter or the height of the flow duct, say $D$, must be much larger than the length scale of the energy-containing eddies, which is of the same order as the mesh size of the grid ($M$), i.e. $D/M >> 1$. Since the boundary layer scales with the mesh size, the larger the ratio $D/M$ the less is the effect of the walls on the measured data expected. Third, the turbulence becomes homogeneous after at least 40 mesh size downstream of the grid. This means that measurements made after that distance reflect the properties of homogeneous turbulence. Similarly, any kind of straining duct such as a contraction or an expansion should start at a downstream location at least $x/M > 40$ away from the grid in order to have only the effect of strain on the turbulence and not the inhomogeneity and the strain. Lack of one or more of these conditions might lead to an inhomogeneous turbulence field.

To the author’s knowledge, there have been just a few studies which investigate the homogeneity of the grid-generated turbulence. Corrsin in 1942 studied grid turbulence and derived the above-mentioned rules which can be found in his later publication (Corrsin, 1963). Batchelor & Townsend (1948) and Batchelor & Stewart (1950) recognized the lack of inhomogeneity for very fine grids (grids of $1/4$ in mesh size). Grant & Nisbet (1957) measured transverse profile measurements at the wake of the grids showing wavy r.m.s. values of turbulent velocity fluctuations. In their measurements, for a grid having 70% porosity, the inhomogeneity of Reynolds stresses can reach up to $\pm 15\%$ at $x/M = 80$ for a mesh size of $1/4$ in and $\pm 6\%$ at $x/M = 30$ for a mesh size of 2 in. Their investigations reveal that with increasing mesh size the inhomogeneity drops. Moreover, they showed that the location of the measurement axis, which is parallel to the flow direction, has a drastic effect on the curve of decaying grid turbulence. In a detailed study on turbulence manipulators, Loehrke & Nagip (1972) measured stand-
ing wavy structures. However, they measured transverse profiles located at the most 10 mesh size away from the grid. In a study by Liu, Ting & Rankin (2004), the inhomogeneity of the r.m.s. of velocity fluctuations increased in the streamwise direction and reached up to 30% at \( x/M = 40 \) for perforated plates of 65% porosity. They showed that the greater the solidity the greater ist the inhomogeneity of turbulence and, for a constant porosity, the larger the mesh diameter the less is the inhomogeneity of the r.m.s. of velocity fluctuations.

When measurements were being performed along the symmetry axis of contractions, it was realized that for the same grid and grid Reynolds number Reynolds stresses deviated from measurement to measurement, especially close to the grid. After detailed investigations, it was found that the relative location of the grid with respect to the measurement axis was the main influencing factor for the observed deviations. As part of experimental procedure used in the present work, the grid was removed for the calibration of the hot-wires and placed back for the measurements, which changed the relative location of the rods of grid with respect to the measurement axis from one experiment to the other. Transverse profile measurements at the wake of the grid revealed wavy transverse profiles of Reynolds stresses similar to those of Grant & Nisbet (1957), despite the fact that the 67% porosity of the grids used was higher than the 57% porosity suggested by Corrsin (1963).

Although grid-generated turbulence has been utilized in many experimental turbulence investigations, and despite the aforementioned observations, the homogeneity of grid-generated turbulence did not attract enough academic interest and, consequently, remained as an ad hoc assumption. Since axisymmetric turbulence is based on an assumption of homogeneity, it was found necessary to establish the extent of homogeneity of grid-generated turbulence in the present thesis work. For this purpose, planes perpendicular to the grid and 50 mm in the transverse direction were scanned with 1 mm spatial resolution in the transverse direction. Two test rigs were utilized for these investigations: the wind-tunnel and the AST. Detailed investigations on the inhomogeneity of grid-generated turbulence were conducted in the wind tunnel because of large test section dimensions, providing \( D/M >> 1 \) and higher achievable velocities. Hence, the effects of grid Reynolds number on the inhomogeneity of measured turbulence quantities in the wake of the grid could be checked in the absence of wall effects. While known sources of artifacts were kept to a minimum in these measurements, inhomogeneity data obtained from wind tunnel investigations were used as reference for the evaluation of the inhomogeneity of the turbulence measurements in the AST.

The effect of axisymmetric strain on turbulence is the most important aspect of the present work. Therefore, homogeneity measurements were also performed along two contractions and one expansion in AST in order to observe the effect of strain field on a spatially inhomogeneous turbulence field.

The scanned planes are composed of \( N_x \) measurement points in the flow direction and \( N_y \) measurement points in the transverse direction, such that the spatial resolution of the scan in both directions are \( \Delta x \) and \( \Delta y \). In order to visualize the homogeneity of a measured mean quantity \( \overline{H} \) in the scanned plane, the inhomogeneity parameter \( I_\pi \) is
defined as
\[ I_{H}(x,y) = \frac{H(i\Delta x, j\Delta y) - \frac{1}{N_y} \sum_{j=1}^{N_y} H(i\Delta x, j\Delta y)}{\frac{1}{N_y} \sum_{j=1}^{N_y} H(i\Delta x, j\Delta y)} \cdot 100, \tag{4.1} \]
which is the percentage deviation of the variable \( H(x, y) \) from the absolute value of its average value calculated along an \( x = \) constant line. The inhomogeneity of the mean streamwise velocity, Reynolds stresses and anisotropy of Reynolds stresses \( a_{11} \), which are designated \( I_U, I_{uu}, I_{vv}, I_{uv} \), and \( I_{a_{11}} \), respectively, are analyzed below.

Anisotropies of Reynolds stresses are of vital importance for modeling and predictions. The fields of stress anisotropy are quantified by the parameter \( a_{11} \), which is defined by equation (2.68) as
\[ a_{11} = \frac{u_1u_1}{q^2} - \frac{1}{3} = \frac{uu}{q^2} - \frac{1}{3}. \tag{4.2} \]

In a homogeneous turbulence field, it is necessary that the off-diagonal components of the Reynolds stress tensor \( \bar{uu}, \bar{uw}, \text{and} \bar{vw} \) are zero. In order to verify this condition, the ratio \( \bar{uu}/u'v' \) is plotted for each scanned field, where the prime indicates the root mean square. Moreover, in an ideal homogeneous turbulence, distribution functions of any fluctuating quantity \( f \) should show a symmetry around their mean and should have a normal distribution. The symmetry is monitored through the skewness factor:
\[ S_f = \bar{f}^3/f'^3. \tag{4.3} \]

For a perfect symmetric distribution, the skewness factor should read be zero. The flatness factor:
\[ F_f = \bar{f}^4/f'^4 \tag{4.4} \]
is the measure of the normal distribution and it should be 3 for a perfect normal distribution. During the analysis of each case, fields of the skewness and flatness factors of velocity fluctuations \( S_u, S_v, F_u \), and \( F_v \) are evaluated.

A detailed analysis of all the measured cases and evaluated variables are documented in a report by Erkan & Lienhart (2006). In this chapter, only selected data from this report are presented.

### 4.1 Inhomogeneity of Grid-generated Turbulence Under Zero Strain

The mesh size and the rod thickness of the square punched grid used for turbulence generation in the wind tunnel experiments, respectively were 8 and 2 mm. The corresponding porosity of the grid was 64%. The grid Reynolds number was controlled by setting the mean flow rate. Thus, in the following analysis each graph is labeled with the mean flow rate. The measurements in the wind tunnel were performed in a plane...
Inhomogeneity of Grid-generated Turbulence Under Zero Strain and Finite Strain

which extends 1060 mm in the streamwise direction and 50 mm in transverse direction. Since such a measurement field corresponds to a narrow, long strip, in the subsequent analysis the measured field could only be visualized by reducing the aspect ratio in the plots.

The inhomogeneity of the mean velocity field and its change with flow rate are shown in Figure 4.1. Red indicates regions of higher velocity and blue indicates regions of lower mean velocity than the average of the mean velocity along \( x = \text{constant} \) line. The locations of grid rods are depicted on the \( y \)-axis of these plots. Hence, mean velocity decreases behind the rods and increases in the open area between two rods. When plots for different flow rates are compared, it is remarkable that the inhomogeneity field shows only a small change. In the vicinity of the grid \( x/M < 15 \), inhomogeneity of the streamwise velocity is over \( \pm 10\% \) but it ceases below \( \pm 2\% \) for \( x/M > 20 \). \( I_{\tau} \) levels off between \( \pm 1\% \) for \( x/M > 60 \).

The inhomogeneity of the normal stresses \( \overline{uu} = \overline{u_1u_1} \) and the off-diagonal stress \( \overline{uv} = \overline{u_1u_2} \) for different flow rates are shown in Figures 4.2 and 4.3. Broadly seen are the regions having either positive or negative deviations extending along the whole plane. These regions repeat in a cyclic manner in the transverse direction. Similar plots can be found in Ertunc & Lienhart (2006) for \( \overline{vv} = \overline{u_2u_2} \). The total measurement time of one of these cases was around 3 days. Hence, it is really impressive to observe first, standing regions of inhomogeneity behind the grid even after 120 mesh size, and secondly, the persistence of these regions at almost the same locations for all flow rates.

In the inhomogeneity fields of stresses, traces of each grid rod can clearly be seen up to \( x/M \approx 15 \). Fluctuations show very high values behind the rods for mean flow rates \( U_{\text{grid}} = 8 \) m/s and 12 m/s. These regions correspond to the separated wake flow of the rods. However, the origin of the grid-generated turbulence is the free shear region between the low- and high-speed regions in the vicinity of the grid shown in Figure 4.1. For \( x/M > 15 \), positive regions coalesce with positive regions and vice versa, so that wider strips of positive and negative regions occur which further coalesce with each other downstream of the grid. In three dimensions the inhomogeneity field can be imagined to be composed of continuously braiding positive and negative strips. Far downstream of the grid, the level of the inhomogeneities of normal stresses does not drop but shows spatial fluctuations between \( \pm 5\% \). \( I_{\tau} \) fluctuates between \( \pm 50\% \). It can be concluded that measurements of normal stresses along any two \( y = \text{constant} \) lines might deviate \( \pm 5\% \) from each other. Moreover, depending on the selected \( y = \text{constant} \) lines, the deviation of measured \( \overline{ww} \) values would be much larger than the normal stresses. In contrast to the \textit{ad hoc} assumption that \( \overline{ww} \) in grid-generated turbulence is close to zero, \( \overline{ww} \) might reach even up to 30\% of the normal stresses (see Figure 4.4).

The anisotropy field of \( a_{11} \) and its inhomogeneity field are shown in Figures 4.5 and 4.6. In the vicinity of the grid, regions with higher values of \( a_{11} \) become larger with increasing grid Reynolds number. Similarly, the inhomogeneity of \( a_{11} \) also becomes stronger close to the grid. Moreover, long-lasting negative and positive inhomogeneity strips can also be seen in Figure 4.6, with deviations approaching \( \pm 20\% \). \( a_{11} \) has a
4.1 Inhomogeneity of Grid-generated Turbulence
Under Zero Strain

Inhomogeneity of Grid-generated Turbulence

Under Zero Strain

tendency to drop in the flow direction. It is shown in Chapter 5 that $a_{11}$ decreases faster close to the grid and further downstream it levels off to some value depending on the grid Reynolds number and measurement position, as already shown in Figure 4.5.

The skewness and flatness factors of $u$ and $v$ fluctuations of these experiments were documented in detail by Ertunc & Lienhart (2006). They showed that the skewness factors take values $\pm 0.1$ for $x/M > 20$ and the inhomogeneity of the turbulence field also becomes visible within the $S_v$ field. It was also found that the $F_u$ field is spatially more homogeneous than the $F_v$ field. It was observed that the spatial inhomogeneity of $F_v$ close to the grid grows with increasing Reynolds number, as shown in Figure 4.7. Further downstream of the grid, both flatness factors showed a tendency to be less than 3 and take values between 2.85 and 3.0. This shows that large-amplitude fluctuations are less than expected for a normal distribution. Note that when the properties of the velocity fluctuation distribution are considered to be the whole criterion of homogeneity, the measured skewness and flatness values downstream of the grid give an impression that the flow fields are homogeneous, although spatial inhomogeneity of the measured mean quantities exists.

The present investigations on unstrained grid-generated turbulence clearly show that obeying all the rules given by Corrsin (1963), which are accepted and followed by many researchers in the past, is not consistent with the generation of a homogeneous turbulent field. The lateral profiles of $\overline{uu}$ and $\overline{vv}$ components show that an inhomogeneous field consists of elongated positive and negative regions standing in space and coalescing with each other. The inhomogeneity fields do not show any dependence on the grid Reynolds number, which was adjusted by the flow rate. Only the $a_{11}$ and $F_v$ values in the vicinity of the grid increase with increasing grid Reynolds number.

These investigations on the inhomogeneity of decaying grid-generated turbulence not only support those of Grant & Nisbet (1957), but also reveal the development of the inhomogeneity in detail. These experiments clearly show that certain amount of inhomogeneity persists to exit in grid-generated flow far downstream of the grid, and grid turbulence investigations which do not take this fact into consideration can be misleading. Mohammed & LaRue (1990) and Camussi, Barbagallo, Guj & Stella (1996) reported rare studies in which the homogeneity of the generated turbulence field was provided. However, in these investigations a few planes perpendicular to the flow direction were coarsely scanned. Despite the observed inhomogeneity, the turbulence field was accepted to be sufficiently homogeneous in the work of Mohammed & LaRue (1990). Camussi et al. could not even detect inhomogeneity far from the grid, most probably because of the low spatial resolution of the scanned planes. In the present work, no measures were undertaken to generate a more homogeneous turbulence; instead, measured data were analyzed by taking the presented spatial inhomogeneity into consideration.
Figure 4.1: $I_{TR}$ fields in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
4.1 Inhomogeneity of Grid-generated Turbulence
Under Zero Strain

Figure 4.2: $I_{uu}$ fields in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
Figure 4.3: $I_{uv}$ fields in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
4.1 Inhomogeneity of Grid-generated Turbulence Under Zero Strain

Figure 4.4: Level of the off-diagonal stress components with respect to normal stress components $\overline{uv}/u'u'$ in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
4. Inhomogeneity of Grid-generated Turbulence Under Zero Strain and Finite Strain

Figure 4.5: Level of the anisotropy $a_{11}$ in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
Figure 4.6: $I_{a_{11}}$ fields in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
Figure 4.7: $F_v$ fields in the wake of the grid in the wind tunnel for different grid velocities. Corresponding $Re_M$ are (a) 3200, (b) 4267 and (c) 6400.
4.2 Comparison of Unstrained and Strained Grid-generated Turbulence Fields

Homogeneity measurements of unstrained grid-generated turbulence fields were also performed in the AST in order to determine the effect of the duct size on the spatial homogeneity of the generated grid turbulence. The test duct has a diameter of 192 mm, which is almost 10% of the width and height of the wind tunnel test section. Homogeneity measurements were performed with a grid having a 10 mm mesh size and 2.2 mm rod width and a corresponding porosity of 67%. The grid Reynolds number \( \text{Re}_M \) was 3000.

The homogeneity measurements of strained grid-generated turbulence fields were performed along contractions having \( c = 1.27 \) and 3.69 and an expansion having \( c = 0.72 \). Turbulence was generated with a grid having a 10 mm mesh size and 2.2 mm rod width and a corresponding porosity of 67%. It is shown in Section 4.1 that inhomogeneity fields qualitatively change with increasing distance from the grid. Thus, locating each contraction at \( x/M = 32.5 \) and 52.5, the effects of contractions on different inhomogeneous fields were tested.

The most remarkable effect of contraction with \( c = 3.69 \) can be seen with \( I_{a11} \) in Figure 4.8b and c: as turbulence approaches an isotropic state, i.e. \( a_{11} \to 0 \) (see Figure 4.8a and d), the amount of deviation increases and, after passing the state \( a_{11} = 0, I_{a11} \) ceases down to a range \( \pm 5 \). Nevertheless, in contrast to \( I_{a11} \), the \( I_{\tau\tau} \) (Figure 4.10c) and \( I_{v\tau} \) fields are still inhomogeneous. Interestingly, the isotropic states reached in the contraction seem to function like a barrier, after which the inhomogeneity of the Reynolds stresses drop drastically. The homogenization effect of the isotropic state should be investigated in more detail, since it might be used to generate more homogeneous turbulence fields.

Grid-generated turbulence fields both in the wind tunnel and in the AST and turbulence fields in contraction and in expansion are compared in Figures 4.9-4.14. Of the investigated cases and evaluated data reported in detail in Ertunc & Lienhart (2006), only the most important results are presented there. From the measured turbulence fields in the wind tunnel, the data set for \( U_{grid} = 6 \) m/s and, out of four contraction experiments, the contraction with \( c = 3.69 \) located at \( x/M = 32.5 \) and the expansion experiment are selected for comparison.

When the unstrained grid-generated turbulence measurements in the wind tunnel and in the AST are compared in Figures 4.9-4.14, no remarkable difference can be observed. Hence, the turbulence investigations in the AST can reasonably be considered to be free from the effect of flow duct size as regards the homogeneity of the turbulence field.

Earlier coalescence of positive and negative regions of inhomogeneity is expected along the contractions when compared with the unstrained turbulent field. Although the expected kind of coalescence cannot be observed in the fields presented in Figures 4.9c
and 4.10c, the positive and negative strips are not as distinct as the unstrained turbulent fields shown in the same figures. This can be accepted as an indirect implication of early coalescence.

The $a_{11}$ plots in Figure 4.11 show the general character of different types of flows. The anisotropy levels of decaying turbulence cases decrease fast in the vicinity of the grid and level to some value further downstream, whereas they decrease below 0 during contraction and increase during the expansion. The inhomogeneity plot of $I_{a_{11}}$ in Figure 4.12 shows the distinct homogenization effect of the contraction, which has already been discussed.

No difference can be observed between the $S_u$ plots in Figure 4.13, whereas a clear increase in $F_u$ during the contraction can be seen in Figure 4.14. It was already shown in Ertunç & Lienhart (2006) that skewness and flatness factors of $v$ fluctuations have this kind of behavior.

In the report of Ertunç & Lienhart (2006), it was shown that the contraction with $c = 1.27$ does not show any considerable effect on the inhomogeneity field of mean velocity, stresses, skewness and flatness factors. Similar to these results, as can be seen in Figures 4.9-4.14, there is little effect of the expansion on the oncoming turbulence field except the expected increase in anisotropy shown in Figure 4.11d. Hence, it can be concluded that the inhomogeneity field is influenced when the strain is high enough as in the case of contraction with $c = 3.69$. 
4.2 Comparison of Unstrained and Strained Grid-generated Turbulence Fields

Figure 4.8: Level of the anisotropy and its inhomogeneity in the contraction with $c = 3.69$: (a) $a_{11}$ field at $x_{cont}/M = 32.5$; (b) $I_{a_{11}}$ field at $x_{cont}/M = 32.5$; (c) $a_{11}$ field at $x_{cont}/M = 52.5$; (d) $I_{a_{11}}$ field at $x_{cont}/M = 52.5$. 
Figure 4.9: Comparison $I_T$ fields: (a) decay in the wind tunnel, (b) in the AST; (c) in the axisymmetric contraction; (d) in the axisymmetric expansion.
4.2 Comparison of Unstrained and Strained Grid-generated Turbulence Fields

Figure 4.10: Comparison $I_{uu}$ fields: (a) decay in the wind tunnel, (b) in the AST; (c) in the axisymmetric contraction; (d) in the axisymmetric expansion.
Figure 4.11: Comparison of the $a_{11}$ fields: (a) decay in the wind tunnel, (b) in the AST; (c) in the axisymmetric contraction; (d) in the axisymmetric expansion.
Figure 4.12: Comparison of $I_{a11}$ fields: (a) decay in the wind tunnel; (b) in the AST; (c) in the axisymmetric contraction; (d) in the axisymmetric expansion.
Figure 4.13: Comparison of $S_u$ fields: (a) decay in the wind tunnel, (b) in the AST; (c) in the axisymmetric contraction; (d) in the axisymmetric expansion.
4.2 Comparison of Unstrained and Strained Grid-generated Turbulence Fields

Figure 4.14: Comparison of $F_v$ fields: (a) decay in the wind tunnel; (b) in the AST; (c) in the axisymmetric contraction; (d) in the axisymmetric expansion.
Decay of Isotropic and Anisotropic Axisymmetric Turbulence

In this chapter, investigations of unstrained axisymmetric turbulent flows are described. For these kinds of turbulent flows, the dynamic equations of the Reynolds stresses $u_iu_j$, the kinetic energy $k = q^2/2 = u_iu_i/2$ and the dissipation rate of turbulence $\epsilon$ are obtained from equations (2.9, 2.12 and 2.13) by setting the mean velocity gradients to zero:

$$\frac{\partial u_iu_j}{\partial t} = \frac{1}{\rho} \left( p \frac{\partial u_j}{\partial x_i} + p \frac{\partial u_i}{\partial x_j} \right) - 2\nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k},$$

$$\frac{\partial k}{\partial t} = -\epsilon,$$

$$\frac{\partial \epsilon}{\partial t} = -2\nu \frac{\partial u_i}{\partial x_i} \frac{\partial u_i}{\partial x_l} - 2\nu^2 \left( \frac{\partial^2 u_i}{\partial x_k x_l} \right)^2.$$  

Since the investigated flow cases involves only one independent mean velocity $\bar{U}_1$ and two independent components of Reynolds stress tensor $u_ju_j$, the index notation for the velocity components is omitted in the subsequent text and are denoted as

$$\bar{U}_1 = \bar{U}, \quad \bar{u}_1u_1 = \bar{u}, \quad \bar{u}_2u_2 = \bar{v}, \quad \bar{u}_3u_3 = \bar{w}.$$  

However, subscripts denoting direction will be used when necessary.

The following three different kinds turbulent flows, which are governed by equations (5.1a-c), were experimentally investigated:

**Grid-generated, nearly isotropic turbulence:** The decaying grid-generated turbulent flows measured in the wind tunnel and in the AST are representatives of this kind of turbulent flow. Details of the measurements are given in Table 3.2. In grid-generated turbulent flow measurements, here and in the literature, it is always observed that the streamwise turbulent stress $\bar{u}$ is slightly higher than the transverse stresses $\bar{v} = \bar{w}$ and both streamwise and transverse stresses decay.
5. Decay of Isotropic and Anisotropic Axisymmetric Turbulence

Besides the decay of the stresses, the anisotropy of the stresses $a_{11}$ decrease and levels off some value before reaching to isotropy (Figure 5.1a).

**Grid-generated turbulence with improved isotropy:** The grid-generated turbulence was strained through the converging nozzle with a contraction ratio 1.27, so that the $\overline{u^2}$ component decreased faster than the transverse stress components in the contraction and, consequently, a more isotropic state was obtained downstream of the nozzle as shown in Figure 5.1b. Uberoi & Wallis (1966), Comte-Bellot & Corrsin (1966) and later Bennett & Corrsin (1978) applied this technique with the expectation of obtaining a better experimental approximation of isotropic turbulence and its decaying characteristics.

**Anisotropic axisymmetric turbulence:** The anisotropic axisymmetric turbulence was generated by straining the grid-generated turbulence through the nozzle having $c = 3.69$, so that $\overline{v^2} = \overline{w^2} > \overline{u^2}$. The turbulence decayed in the constant-diameter duct connected at the exit of the contraction. Hence, this kind of turbulent flow is called relaxation from axisymmetric contraction in the DNS investigations of Ro-
5.1 How Isotropic is the Grid-generated Turbulence?

Isotropic turbulence and grid-generated turbulence have been studied extensively by various researchers. Some of the important studies have already been mentioned in Chapter 1 and more comprehensive accounts can be found in Batchelor (1953) and Hinze (1975). In the present work, first, data on the simple grid-generated turbulence and turbulence with improved isotropy are analyzed in comparison with certain relationships which should hold for ideally isotropic turbulence. In addition, the von Kármán & Howarth equation is used to evaluate the isotropy level of the measurements. These analyses were aimed at establishing whether only decreasing the anisotropy satisfies other theoretical criteria of ideally isotropic turbulence.

Despite the extensive investigations of grid-generated turbulence, constants of the decay laws of turbulence, extracted from measurements originating from different wind tunnels and researchers, might deviate from each other because of the several reasons. In this chapter, it is shown that the inhomogeneity of grid-generated turbulence presented in Chapter 4 is one of these reasons. In addition to the above analysis, developments of length scales, correlation curves and the one-dimensional energy spectrum of the grid-generated turbulence are provided in order to show the effects of the distance and the grid Reynolds number on them.

At the isotropic limit (vanishing anisotropy), Jovanović et al. (2003) suggested a relationship for the length scale ratio $\lambda_g/L_g$ (2.72) which is used to weight the invariant functions $A$, $C$, $F$ and $\psi$ in the $Re_\lambda$ range $0 - \infty$ [see equations (2.75a-d)]. Measured length scales of grid-generated turbulence are used to check the validity of equation (2.72). Moreover, the values of $\psi$ attained both at the isotropic limit and during anisotropic decay along the left edge of the AI-map are compared with the $\psi$ models of suggested by Jovanović et al. (2003) (2.75d) and Lumley & Newman (1977).

5.1. How Isotropic is the Grid-generated Turbulence?

5.1.1 Anisotropy of Grid-generated Turbulence

The development of $a_{11}$ in cases of grid-generated turbulence and turbulence with improved isotropy are shown in Figure 5.2. The mesh Reynolds number $Re_M$ is controlled by the flow speed in the wind tunnel and by the mesh size and the flow speed in the AST. The size of the wind tunnel allowed the furthest measurements from the grid. Figure 5.2a shows that $a_{11}$ increases for $x/M < 40$, decreases until $x/m \approx 100$ and levels off, for $x/M > 100$. The initial anisotropies shown in Figure 5.2b are remarkably damped during the contraction, as shown in Figure 5.2c. The leveling off of $a_{11}$ cannot be observed in either of the AST measurements.
5. Decay of Isotropic and Anisotropic
Axisymmetric Turbulence

Figure 5.2: Development of \(a_{11}\): (a) Decay of grid-generated turbulence in the wind tunnel; (b) decay of grid-generated turbulence in the AST; (c) decay of grid-generated turbulence with improved isotropy in the AST.

No dependence of \(a_{11}\) on \(Re_M\) can be observed in wind tunnel measurements. However, both measurements in the AST (Figures 5.2b, c) show that if \(Re_M\) is controlled by the mesh size, the smaller is the mesh size, the higher the anisotropy becomes. Note that although \(M=10\ mm\), the \(U_{\text{grid}}=2\ m/s\) case has the lowest \(Re_M\) and the case with \(M=5\ mm\) and \(U_{\text{grid}}=5\ m/s\) delivers the highest anisotropy values. This observation was also made in the other experiments. Since all the grids have the same porosity, it is hard to understand this behavior of turbulence for the \(M=5\ mm\) case.

By using Taylor’s "frozen turbulence" hypothesis, the energy balance equation (5.1b) can be rewritten as

\[
\epsilon = -U \frac{d\kappa}{dx},
\]

so that the total dissipation rate can be extracted from the measurements of \(U\), \(\overline{uu}\) and
For isotropic turbulence, it is expected that $k = 3/2\pi u$. Hence the total dissipation rate for isotropic turbulence can be written as

$$\epsilon^* = -\frac{U^3}{2} \frac{d\pi u}{dx}. \tag{5.4}$$

This means that, if grid-generated turbulence is fully isotropic, measurements of only $U(t)$ would be sufficient to find the total dissipation rate. Although grid-generated turbulence has never been shown to be fully isotropic in terms of $a_{11}$, there are respected investigations that use this kind of assumption, e.g. Mohamed & LaRue (1990). Thereby, it is interesting to look at the development of $\epsilon/\epsilon^*$, which should give 1.0 for fully isotropic turbulence. As can be seen in Figures 5.3a-c, all the measured cases are away from 1.0 and do not show a significant tendency towards 1.0. It is interesting to observe in Figure 5.3a that the actual dissipation rate $\epsilon$ increases more than the expected isotropic dissipation with increased $Re_M$. Moreover, after $x = 0.4$ m (50M) the $\epsilon/\epsilon^*$ ratio drops with a constant slope and take values below 1.0 far downstream of the grid. The presented $\epsilon/\epsilon^*$ ratios clearly show how far it is reasonable to assume $k = 3/2\pi u$ in grid-generated flows.

Another way of checking the isotropy is based on the length scale relations. In a fully isotropic turbulence, the ratio between the streamwise and transverse dissipation length scales and the integral length scales should take following values:

$$\lambda_f/\lambda_g = \sqrt{2}, \tag{5.5a}$$
$$L_f/L_g = 2. \tag{5.5b}$$

These ratios are extracted from the measured correlation curves $f(r)$ and $g(r)$ using equations (2.27), and are shown in Figure 5.4 for grid-generated turbulence and turbulence with improved isotropy. The $\lambda_f/\lambda_g$ ratio is accepted as the signature of local isotropy, since it reflects the isotropy about the dissipation scale. Since the work of Kolmogorov (1941a, b, 1942), it is known that small-scale turbulence becomes more isotropic and homogeneous with increasing Reynolds number. This can also be observed in grid-generated turbulence measurements (Figure 5.4a): after a rapid increase $\lambda_f/\lambda_g$ levels off at a value which is higher than $\sqrt{2}$ and approaches to $\sqrt{2}$ with increasing $Re_M$. As shown in Figure 5.4c, $L_f/L_g$ also experiences a leveling off around 1.95 far from the grid. The measurements of turbulence with improved isotropy deliver a constant level of $\lambda_f/\lambda_g$ lower than $\sqrt{2}$. This occurs simply because of the vortex stretching along the contraction. In other words, the transverse length scales are squeezed and the streamwise length scales are elongated along the contraction. The $L_f/L_g$ ratio in Figure 5.4d also reflects this effect.

### 5.1.2 Decay Law of Isotropic Turbulence

The first two comprehensive theoretical and experimental studies on the decay of isotropic turbulence are due to Taylor (1935) and von Kármán & Howarth (1938). By utilizing the two-point correlation technique and continuity equation, von Kármán &
Howarth (1938) were able to show that, in an isotropic turbulent flow of an incompressible fluid, where $u = v = w$ and $\rho = \text{constant}$, the following relationship holds between the longitudinal and transverse velocity correlation coefficient functions, $f(r,t)$ and $g(r,t)$ (2.29):

$$f(r,t) - g(r,t) = -\frac{1}{2} \frac{\partial f}{\partial r}$$

This equation implies that despite all the normal Reynolds stresses at a point being equal to each other in an isotropic turbulent flow, $f(r,t)$ and $g(r,t)$ are not equal to each other. Equation 5.6 was verified experimentally for grid-generated turbulence (see Taylor 1937). Using the equation of motion and equation (5.6) von Kármán &
Howarth (1938) derived the dynamic equation for \( f(r, t) \):

\[
\frac{\partial}{\partial t} \left( \bar{u} f \right) + \bar{u} \frac{3/2}{r^4} \frac{\partial}{\partial r} \left( r^4 h \right) = 2\nu \bar{u} \frac{1}{r^4} \frac{\partial}{\partial r} \left( r^4 \frac{\partial f}{\partial r} \right),
\]

where \( h(r, t) \) is the third-order velocity correlation coefficient function, defined as

\[
h(r, t) = \frac{w(0, t) v(0, t) u(r, t)}{v(0, t) v(0, t) u(0, t)} = \frac{w(0, t) w(0, t) u(0, t)}{w(0, t) w(0, t) u(0, t)}.
\]

The terms with \( h \) and \( \nu \) represent inertial and viscous processes, respectively. Close to the grid, in the so-called initial period of decay, both of these processes are active. Far from the grid, inertial effects ceases compared with the viscous effects and when the
Decay of Isotropic and Anisotropic Axisymmetric Turbulence

Reynolds number is sufficiently small, the viscous term dominates the decay of turbulence and enters into the final period of decay. The governing equation (5.7) becomes linear at this state of turbulence, for which Batchelor & Townsend (1948b) found the following self-similar solution:

\[ f(r, t) = \exp \left[-r^2/(8\nu t)\right]. \]  

(5.9)

Although this solution matches the experiments very well, it can be obtained 600 M after the grid and the grid Reynolds number should be so low that it is not of general interest. \( f(r, t) \) does not preserve its shape in the initial period of decay, therefore a self-similar solution does not exist. However, self-preservation is not a necessary condition for the validity of equation (5.7). Moreover, as the distance between two points approaches zero, i.e. \( r \to 0 \), equation (5.7) gives an exact relation for the decay of the viscous stress in time:

\[ \frac{\partial}{\partial t} (\overline{uu}) = 10\nu \left( \frac{\partial^2 f}{\partial^2 r} \right)_{r=0} \overline{uu} = -20\nu \frac{\overline{uu}}{\lambda_f^2}; \]  

(5.10)

where \( \lambda_f \) is Taylor’s microscale derived from the longitudinal correlation coefficient \( f(r, t) \) (2.27). Since \( \lambda_f = \sqrt{2} \lambda_g \) should hold for isotropic turbulence, the decay relation (5.10) can be written in its most common form:

\[ \frac{\partial}{\partial t} (\overline{uu}) = 10\nu \left( \frac{\partial^2 f}{\partial^2 r} \right)_{r=0} \overline{uu} = -10\nu \frac{\overline{uu}}{\lambda_g^2}. \]  

(5.11)

Finally, with the utilization of Taylor’s hypothesis and the definition of Taylor’s dissipation-scale Reynolds number \( \text{Re}_\lambda \) (2.28), equation (5.11) can be written as

\[ \frac{\overline{U}}{\overline{uu}^2} \nu \frac{d\overline{uu}}{dx} = -10 \frac{1}{\text{Re}_\lambda^2}; \]  

(5.12a)

or

\[ -\text{Re}_\lambda^2 \frac{\overline{U}}{\overline{uu}^2} \nu \frac{d\overline{uu}}{dx} = 10. \]  

(5.12b)

These equations are valid for the decay of fully isotropic turbulence without exception for the initial and final periods of decay. Since equation (5.12a or b) involves energy, dissipation and dissipation scale information, either can be used to check the isotropy level of the decaying turbulence experiments. Moreover, since decay of isotropic turbulence is the simplest turbulent flow problem, any turbulence model should truncate to equation (5.12a or b). In Chapter 8, several turbulence models are tested using this relation. In Figure 5.5, data for grid-generated turbulence and turbulence with improved isotropy are plotted in comparison with equations (5.12a, b). Not surprisingly, none of the measured cases obeys the decay law.

At this point, it is interesting to reconsider the effect of slight inhomogeneity of the grid-generated turbulence fields presented in Chapter 4. Hence, \( \overline{U}_{grid} = 6 \text{ m/s} \) case in the wind tunnel and one homogeneity measurement the downstream of the nozzle having \( c = 1.27 \) are compared with the decay laws (5.12a and b). The inhomogeneity
5.1 How Isotropic is the Grid-generated Turbulence?

Figure 5.5: Comparison of measured decay with the theoretical decay laws: (a) Equation 5.12a; (b) equation 5.12b.

fields $I_{q^2}$ are shown in Figure 5.6a and d. In order to compare the experimental data with the decay laws, least-squares curve fits were performed along $y = \text{constant}$ lines for $\overline{uu}$, $\overline{vv}$, $q^2$, $\lambda_g$ and $\epsilon$.

The following commonly accepted simple power law forms (see, for example, Hinze 1975, p. 259-277, or Mohamed & LaRue 1990) are used in order to capture the decaying Reynolds stresses, turbulence energy and dissipation rate:

\[
\frac{\overline{uu}}{U^2} = A_{uu} \left( \frac{x}{M} \right)^{n_{uu}},
\]

\[
\frac{\overline{vv}}{U^2} = A_{vv} \left( \frac{x}{M} \right)^{n_{vv}},
\]

\[
\frac{q^2}{U^2} = A_{q^2} \left( \frac{x}{M} \right)^{n_{q^2}},
\]

\[
\frac{\epsilon M}{U^3} = A_{\epsilon} \left( \frac{x}{M} \right)^{n_{\epsilon}}.
\]

The effect of inhomogeneity on the decay constants can clearly be seen in Figure 5.6. For instance, $A_{uu} n_{uu}$ can vary $\pm 55\%$ and $\pm 7\%$, respectively, along the transverse $y$-axis in a grid-generated turbulence field. This finding is in agreement with those of Grant & Nisbet (1957). Since the inhomogeneity for the turbulence with improved isotropy is less than the simple grid-generated turbulence, the scatter of decay constants of the former along $y$-axis is also less.

The terms in equations (5.12a and b) are evaluated for the measured case by using the decay constants found for $y = \text{constant}$ lines, and are compared with the theoretical values in Figure 5.7a and b.

Anisotropy, total dissipation rate, length scale and decay law considerations show that
Figure 5.6: Inhomogeneity of turbulent energy $I_{q^2}$ and corresponding decay constants $n$ and $A$: (a, b, c) Grid-generated turbulence in the wind tunnel; (d, e, f) turbulence with improved isotropy.
none of the generated turbulent flows delivers a fully isotropic state for both the large- and the small-scale turbulences. The author is convinced that a good approximation of isotropic turbulence with static grids is more an exception than the rule. Therefore, here and in most of the experimental literature, data on grid-generated turbulence and turbulence with improved isotropy should be accepted as a rough estimate of isotropic turbulence; moreover, each experimental data set should be considered on its own. Furthermore, if the homogeneity problem of grid-generated turbulence is not solved, investigations on finding universal decay constants would remain ambiguous.

5.2 Length Scale and $\psi$ Considerations for Vanishing Anisotropy

The development of the length scales is best observed in the wind tunnel measurements, since measurements could be extended up to 140 mesh size. As can be seen in Figure 5.8, as expected, both $\lambda_g$ and $L_g$ decrease with increasing $Re_M$ and increase with increasing $x/M$. The $\lambda_f$ and $L_f$ scales show a similar tendency, but they are not shown here. In Figure 5.9a, the local $Re_\lambda$ is shown to be function of both streamwise location and $Re_M$. For the experimentalist, it is important to know the change of $Re_\lambda$ with $Re_M$ at constant $x/M$ locations: it is shown in Figure 5.9b that the relation is linear. Hence, for a specific grid, measurements of $Re_\lambda$ at two velocities are sufficient to estimate the $Re_\lambda$ at other velocities. For the design of strained homogeneous turbulence experiments, the change of $\lambda_g$ with $\overline{U_{grid}}$ must be known (see Section 7.1). In Figure 5.10, it is...
shown that both $\lambda_g$ and $L_g$ are proportional to $U_{grid}^{-1}$ for a fixed distance from the grid.

It is shown in Section 2.6.2 that the length scale ratio $\lambda_g/L_g$ is used to weight the scalar invariant functions $A$, $C$, $F$ and $\psi$ in a $Re^*$ range $0 - \infty$ and therefore is an important term in the anisotropy-invariant model of Jovanović et al. (2003). Since the grid-generated turbulence measurements performed in the wind tunnel have the widest range of $Re_{\lambda}$, length scale ratio data in Figure 5.11 are used to check the validity of the suggested relation (2.72) for $\lambda_g/L_g$ at vanishing anisotropy. Although it is shown in Figures 5.9a and 5.11b that $Re_{\lambda}$ and $\lambda_g/L_g$ are not constant in the streamwise direction, data starting from $x/M = 60$ are accepted to be sufficiently constant and averaged for the comparison. As can be seen in Figure 5.12, the proposed $\lambda_g/L_g$ model underestimates $\lambda_g/L_g$ but reproduces the general behavior of the measurements. It should be noted that the $\lambda_g/L_g$ model is constructed for purely isotropic turbulence, which could not be obtained within the present investigations.

According to equation (2.65), the modeled form of the dynamic equation of total dissipation rate in the absence of strain (5.1c) is

$$\frac{\partial \epsilon}{\partial t} = -\psi \frac{\epsilon^2}{k},$$

where $\psi$ models the difference between the turbulence production of the dissipation rate and the viscous destruction. Using equation (5.1b) and Taylor’s frozen turbulence hypothesis, $\psi$ can be written in the following form

$$\psi = -\frac{k}{\epsilon} \frac{d\epsilon}{dx} \left( \frac{dk}{dx} \right)^{-1},$$

which can easily be extracted from the Reynolds stress measurements. The $\psi$ values of grid-generated turbulence and turbulence with improved isotropy are compared.
5.3 Correlation Functions and
One-dimensional Power Density Spectra

The normalized correlation functions \( f(x) \) and \( g(x) \) are defined by equation \((2.29)\). Correlation coefficients are derived from autocorrelation functions with the help of Taylor’s frozen turbulence hypothesis, i.e. the measured change in time converted to the spatial change in the streamwise direction \( x \). Therefore, \( x \) is selected to denote the independent variable rather than \( r \). As can be seen in Figure 5.14a and b, \( f(x) \) and \( g(x) \) drop faster close to the vertex with increasing \( Re_M \), as expected. Although \( f(x) \) (Figure 5.14a) shows negative values, with increasing \( Re_M \) the negative region of the correlation function disappears and \( f(x) \) takes a form which can be represented by an exponential decay function. The overlapping correlation functions at different
5. Decay of Isotropic and Anisotropic Axisymmetric Turbulence

Figure 5.10: Development of length scales during the decay of grid-generated turbulence in wind tunnel with respect to $U_{grid}$: (a)$\lambda_g$; (b)$L_g$.

Figure 5.11: Development of length scale ratios during the decay of grid-generated turbulence in wind tunnel at different Re$_M$: (a)$\lambda_f/L_f$; (b)$\lambda_g/L_g$.

$x/M$ (Figure 5.14c and d) suggest that they scale with the integral length scales for a specific Re$_M$. This finding is in agreement with many of the other studies, which are summarized in Hinze (1975, p. 59).

Before presenting the spectra, the background of formulation for the one-dimensional power density spectra is briefly introduced. The Fourier transform pair of a finite
5.3 Correlation Functions and
One-dimensional Power Density Spectra

Figure 5.12: Comparison of the $\lambda_g/L_g$ model (2.72) with those of measured grid-generated turbulence in the wind tunnel.

A record of $u(t)$ where $0 < t < T$ can be written as

$$u(t) = \int_{-\infty}^{\infty} a_u(f)e^{i2\pi ft} df$$  \hspace{1cm} (5.16a)

$$a_u(f) = \int_{0}^{T} u(t)e^{-i2\pi ft} dt$$  \hspace{1cm} (5.16b)

from which the one-dimensional energy spectra of $u$ fluctuations can be calculated by

$$E_u(f) = \frac{2}{T}|a(f)|^2.$$  \hspace{1cm} (5.17)

Note that $\overline{uu}$ can be calculated from $E_u(f)$ as follows:

$$\overline{uu} = \int_{0}^{\infty} E_u(f) df.$$  \hspace{1cm} (5.18)

The above equations are used to evaluate $E_v(f)$ $v$ fluctuations by using the $v(t)$ signal instead of $u(t)$. In connection with isotropic correlation functions $f$ and $g$, $E_{11}$ and $E_{22}$ are commonly used for $E_u$ and $E_v$ in the literature (for instance, see Pope 2000, p. 224-229). However, as already explained in Section 2.3.3, in strained axisymmetric turbulence there are four basic correlation functions, therefore $E_{11}$ and $E_{22}$ are replaced by $E_u$ and $E_v$ to avoid misinterpretation.
5. Decay of Isotropic and Anisotropic Axisymmetric Turbulence

For better interpretation of the spectra, they can be written in terms of the wavenumber component in the streamwise direction $k_1$:

\begin{align}
    k_1 &= \frac{2\pi f}{U}, \\
    E_u(k_1) &= \frac{U}{2\pi} E_u(f), \\
    E_v(k_1) &= \frac{U}{2\pi} E_v(f).
\end{align}

Figure 5.13: Comparison of the measured $\psi$ with several models for isotropic turbulence.

The developments of the spectra in the grid-generated turbulence in the wind tunnel are analyzed here. The spectra presented here are the averages of 100 spectra, which are evaluated from 100 independent records of 1 second. The change in the form of spectra due to $Re_M$ and $x/M$ can be assessed by normalizing it with the corresponding Reynolds stress, so that according to (5.18) the areas under the spectrum curves become 1. Increasing $Re_M$ (Figure 5.15a) results in an increase in energy at high frequencies where dissipation takes place. Since energy at high frequencies dissipates first, this range of frequencies vanishes first with increasing $x/M$ (Figure 5.15b). This behavior explains also the development of the dissipation length scale $\lambda_g$ in Figures 5.8a and 5.10a.

Kolmogorov (1941a, b) suggested that for sufficiently large Reynolds number, there should be a wavenumber range, between the energy containing and dissipation range,
Figure 5.14: Development of correlation functions $f(x)$ and $g(x)$: (a, b) with increasing $Re_M$ at $x/M = 80$ and (c, d) with increasing streamwise location $x/M$ at $Re_M = 3168$.

in which the spectra scale with $k_{1}^{-5/3}$. This range is called the inertial subrange. The $Re_\lambda$ achieved within the present experiments, let us say at $x/M = 80$, varies from 15 to 40. According to Mydlarski & Warhaft (1996), the turbulence fields generated in our wind tunnel fall into class of weak turbulence. According to their work, one reaches the so-called strong turbulence regime for $Re_\lambda > 200$ at which the isotropy and local isotropy considerations of Kolmogorov hold. In order to reach $Re_\lambda = 200$ at $x/M = 80$ in our wind tunnel, according to Figure 5.9b, we should have run the wind tunnel with a mean speed of 60 m/s, which was impossible. However, it is still interesting to observe the development of the inertial subrange with increasing $Re_M$. For this purpose, the spectra are converted into a wavenumber representation by using the equations (5.19a-c). The $E_u(k_1)$ spectra and fitted lines having a $k_{1}^{-5/3}$ dependency
5. Decay of Isotropic and Anisotropic Axisymmetric Turbulence

Figure 5.15: Development of normalized spectra $E_u(f)$: (a, b) with increasing $Re_M$ at $x/M = 80$; (c, d) with increasing streamwise location $x/M$ at $Re_M = 3168$.

Figure 5.16: $E_u(k_1)$ spectra at $x/M = 80$ showing the development of the inertial subrange with increasing $Re_M$.

are shown in Figure 5.16. It can be clearly seen that inertial subrange grows starting from $Re_M = 2112$. In the inertial subrange of isotropic turbulence having $k_1^{-5/3}$ scaling, the ratio of the transverse to longitudinal spectra is expected to be $4/3$ (see Pope 2000, p. 229). $E_v(k_1)/E_u(k_1)$ ratios (Figure 5.17) do not represent a developed inertial subrange.

The spectrum which has delivered successful approximations of measured grid-generated turbulence at energy-containing scales and at sufficiently high $Re_M$ is (see Hinze 1975,
5.4 Decay of Anisotropic Axisymmetric Turbulence

The \( \psi \) term (2.75d) makes the right-hand side of (5.1c) a function of second invariant \( II_a \) (2.69) of anisotropy tensor \( a_{ij} \) and also \( \text{Re}^*_\lambda \). Experiments and the simulations of the decay of axisymmetric anisotropic turbulence would deliver \( \psi(II_a, \text{Re}^*_\lambda) \) data along the left and right edges of the AI-map (Figure 2.3), which can be used to build a model for the \( \psi \) term. This kind of turbulent flow can be generated simply by straining the grid-generated turbulent flow either with an axisymmetric nozzle or diffuser and letting the

Hence, \( E_u(k_1) \) is normalized by dividing it by \( \overline{\mu u L_f} \) and compared with equation (5.20) in Figure 5.18, so that the scaling of the spectra with the total energy and the energy-containing scales becomes visible. The increase in energy at high wavenumbers with increasing \( \text{Re}_M \) is also visible in Figure 5.18a. The theoretical spectrum (5.20) matches the measured spectra in the energy-containing range and, as expected, deviates at high wavenumbers (dissipation range). The changes in the dissipation range with \( x/M \) (Figure 5.18b) are not as large as those with \( \text{Re}_M \). Hence, both considerations suggest that the properties of the dissipation range should be included in the normalization. However, finding a model spectrum which can successfully model the whole wavenumber range is a research field on its own and outside the scope of this thesis. For a good review on such spectral considerations, the interested reader should refer to the book of Pope (2000).

Figure 5.17: Development of the \( E_v(k_1)/E_u(k_1) \) ratio: (a, b) with increasing \( \text{Re}_M \) at \( x/M = 80 \) and (c, d) with increasing streamwise location \( x/M \) at \( \text{Re}_M = 3168 \).

\[ E_u(k_1) = \frac{2}{\pi} \frac{\overline{\mu u L_f}}{1 + k_1^2 L_f^2}. \]  

(5.20)
turbulence decay (relax) downstream of the straining duct. Such measurements can partly be found in the work of Uberoi (1956, 1957), Mills & Corrsin (1959), Tucker & Ali (1973), Warhaft (1980), Sjögren & Johansson (1998) and Choi & Lumley (2001). The DNS simulations of Rogallo (1981) and Lee (1985) covered this kind of turbulent flows. However, the measured and simulated data either are not delivered detailed enough for extracting $\psi$ information or are inaccurate. Hence, in order to validate $\psi$ (2.75d) as suggested by Jovanović et al. (2003) and Lumley & Newman (1977), measurements were performed here in a circular duct connected to the exit of the nozzle having $c = 3.69$.

As can be seen in Figures 5.19a-c, $\overline{uu}$ is frozen, $\overline{vv}$ is slowly decaying and, consequently, the generated anisotropic turbulence has a very slow tendency to reduce its anisotropy. Similar behavior of $\overline{uu}$ can also be observed in the measurements of Uberoi (1956). The anisotropies plotted on the AI-map (Figure 5.20) confirm that the anisotropy changes are very small within the measurable length.

The $\psi$ values are extracted from the present measurements and from the data of Sjögren & Johansson (1998) by using equation (5.15). The experiments of Sjögren & Johansson (1998) were selected for two-reasons: first, the printed data are sufficient to extract $\psi$; second, the size of their flow facility is the largest, i.e. reliable data are expected. Two models of $\psi$, that (2.75d) suggested by Jovanović et al. (2003) and that suggested by Lumley & Newman (1977) are compared with the experimental data in Figure 5.21. The first irritating observation is the total disagreement of the $\psi$ models. The present experiments overshoot both $\psi$ models, but consistently deliver $\psi$ values around 2.55, which is very close to the two-component isotropic limit which is set in the model of Jovanović et al. The data of Sjögren & Johansson (1998) also show very interesting behavior: one set of their data roughly matches the model of Jovanović et al., whereas
the other two sets match the model of Lumley & Newman. The disagreements within experimental data and within the models of $\psi$ suggest that there are unresolved problems on both the experimental and modeling sides.

As far as the present measurements are concerned, it was observed that far downstream of the contractions, irrotational fluctuations due to the boundary layer contribute to the velocity fluctuations measured at the core of the flow. The data presented here are corrected for many experimental artifacts including the irrotational velocity fluctuations by using several experimental techniques extensively explained in Chapter 6.
Figure 5.19: Development of (a) \( \bar{U} \), (b) \( \bar{u} \bar{u}, \bar{v} \bar{v} \) and (c) \( a_{11} \) during the decay of anisotropic turbulence downstream of contraction having \( c = 3.69 \).
Figure 5.20: Courses of measured Reynolds stresses on the AI-map at different $Re_M$.

Figure 5.21: Comparison of $\psi(I_{IIa}, Re^*_\lambda)$ models and the available experimental data.
Chapter 6

High-Contraction Ratio Anomaly and Its Removal

6.1 Effect of Contracting Nozzles

The effects of contracting nozzles on grid-generated turbulence have been investigated by many researchers. The available experimental investigations were mainly carried out to obtain a deeper understanding of favorable pressure gradients on grid-generated turbulence and/or to supply data for turbulence model developments. As already mentioned in Section 1.3, there are three distinct behaviors of the mean square of the axial (longitudinal) velocity fluctuation $u^2$ and the transverse fluctuation $v^2$.

These behaviors are listed below once more:

**Type 1:** The most general behavior was the continuous decrease in $u^2$ and the continuous increase in $v^2$. Hence the anisotropy tensor component $a_{11}$ increased in absolute value towards the two-component isotropic state $-1/3$, such that turbulence moves on the left axisymmetric border of the AI-map from the nearly isotropic state towards the two-component isotropic state. RDT also predicts this kind of behavior.

**Type 2:** In almost all measurements with nozzles having contraction ratios higher than 9, initially $u^2$ decreased and $v^2$ increased, but, after a while, $u^2$ started to increase in the contraction. This means that turbulence initially moved towards the two-component isotropic limiting state and changed its direction towards the isotropic state.

**Type 3:** When the nozzle is kept close to the grid and the strain values are kept low, it was observed that $u^2$ and $v^2$ decreased continuously, but $v^2$ decreased more slowly than $u^2$, so that $a_{11}$ attained states on the left axisymmetric border of the AI-map.

In Table 6.1, measurements in the literature are grouped according to the above classification. All the experiments performed with nozzles with contraction ratios higher than
6. High-Contraction Ratio Anomaly and Its Removal

<table>
<thead>
<tr>
<th>Stress behavior</th>
<th>Authors</th>
<th>Contraction ratio, c</th>
<th>Inlet section size [m]</th>
<th>Re_M</th>
<th>Re_L</th>
</tr>
</thead>
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<tr>
<td>Type 1</td>
<td>Uberoi (1956)</td>
<td>4, 9</td>
<td>0.6096 x 0.6096</td>
<td>3710, 6150, 12300</td>
<td>35, 44, 87</td>
</tr>
<tr>
<td></td>
<td>Mills &amp; Corrsin (1959)</td>
<td>4</td>
<td>0.6096 x 0.6096</td>
<td>7420 -</td>
<td>-</td>
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<tr>
<td></td>
<td>Reynolds &amp; Tucker (1975)</td>
<td>not provided</td>
<td>0.762 x 0.432</td>
<td>6780 -</td>
<td>-</td>
</tr>
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<td></td>
<td>Tan-atichat et al. (1980)</td>
<td>2, 4, 9</td>
<td>0.154</td>
<td>100-8000</td>
<td>2.3-31</td>
</tr>
<tr>
<td></td>
<td>Warhall (1980)</td>
<td>4</td>
<td>0.4 x 0.4</td>
<td>9700 -</td>
<td>-</td>
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<tr>
<td></td>
<td>Sjögren &amp; Johansson (1998)</td>
<td>9</td>
<td>0.8 x 1.2</td>
<td>not provided</td>
<td>≈ 32</td>
</tr>
<tr>
<td>Type 2</td>
<td>Uberoi (1956)</td>
<td>16</td>
<td>0.6096 x 0.6096</td>
<td>3710</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Hussain &amp; Ramjee (1976)</td>
<td>11</td>
<td>0.254</td>
<td>234 -</td>
<td>-</td>
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<td></td>
<td>Ramjee &amp; Hussain (1976)</td>
<td>11, 22, 44.5, 64, 100</td>
<td>0.254</td>
<td>141-2210</td>
<td>2.3-31</td>
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<td></td>
<td>Tan-atichat et al. (1980)</td>
<td>4, 9, 16, 23.5, 36</td>
<td>0.154</td>
<td>100-8000</td>
<td>2.3-31</td>
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<td></td>
<td>Han (1988)</td>
<td>10.56</td>
<td>0.489</td>
<td>3400</td>
<td>≈ 20</td>
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<tr>
<td>Type 3</td>
<td>Leuchter and Dupeuble (1993)</td>
<td>4 (cut.strain)</td>
<td>0.3</td>
<td>7950</td>
<td>80,60</td>
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Table 6.1: Overview of the available measurements in the literature in which effects of symmetric contractions on turbulence were investigated.

9 showed an increase in \( \overline{u} \) downstream of the nozzles, i.e. Type 2 behavior. In other measurements, in which nozzles with contraction ratios \( \leq 9 \) were employed, Type 1 behavior was observed.

6.2 Anomalous Behavior of Reynolds Stresses

For some of the experimental studies given in Table 6.1, the developments of the mean square of the longitudinal (axial) velocity fluctuation \( \overline{u}^2 \) along contracting nozzles are presented in Figure 6.1. For each case presented in this figure, the data were normalized with the \( \overline{u} \) value at the outlet of the nozzles and the nozzle length \( L \). Hence,
data that drop below the value 1 represent cases in which \( \overline{u'u'} \) starts to increase in the contraction. As can be seen in Figure 6.1, the mean square of longitudinal velocity fluctuation data decreased monotonically along the axis of the converging nozzles employed with contraction ratios 4 and 9 in the experiments of Uberoi (1956) and 2 and 4 in the experiments of Tan-atichat et al. (1980), whereas Tan-atichat et al. also measured an initial decrease in \( \overline{u'u'} \) followed by an increase downstream of the nozzles with \( c = 4 \) and 9. Note that one of the measurements of Tan-atichat et al. (1980) for \( c = 4 \) was performed with small-scale turbulence at the inlet of the nozzle and showed an increase in \( \overline{u'u'} \), whereas the other measurement with large-scale turbulence at the inlet of the nozzle did not show an increase along the same nozzle. These measurements of Tan-atichat et al. (1980) for \( c = 4 \) reveal a qualitative inconsistency but, at the same time, show the effect of the conditions in such experiments. A similar qualitative inconsistency exists between the \( c = 9 \) data of Uberoi (1956) and of Tan-atichat et al. (1980).

In contrast to the experimental findings with increasing \( \overline{u'u'} \) downstream of nozzles (Type 2), according to the RDT results in Figure 2.6, the mean square of the longitudinal velocity fluctuations should always decrease monotonically. Moreover, the development of \( \overline{u'u'} \) in nozzles is only a function of the contraction ratio. Hence, for the same contraction ratio, at least qualitatively, the same behavior of axial stress in different experiments is expected, which was not the case in the reported studies.

It is also wrong to expect that all these experiments satisfy the conditions under which results of RDT become valid. However, direct numerical simulation studies of Rogallo (1981) and Lee (1985) on the effect of strain on homogeneous turbulence, have always shown a continuous decrease in \( \overline{u'u'} \) irrespective of the applied strain, i.e. Type 1 behavior.

Launder, Reece & Rodi (1975) employed the \( c = 16 \) data of Uberoi (1956) to calibrate their isotropization model. At the Stanford Conference in 1980, Ferziger selected homogeneous turbulent flow test cases for the validation of turbulence models from the measurements of Tan-atichat et al. (1980), which had increasing trends for \( \overline{u'u'} \) downstream in the contracting nozzle employed.

For the reasons mentioned above, the increase in \( \overline{u'u'} \) during positive axisymmetric strain of grid-generated turbulence is accepted as anomalous. In the subsequent text, this behavior is called ‘high contraction ratio anomaly’, since it manifested itself commonly in measurements performed in nozzles with high contraction ratios.

Tsugé (1984) also observed the anomaly of strained grid turbulence in nozzles of high contraction ratio. He studied the evolution of turbulence in a contracted flow. He found analytically that only small eddies decay through the contraction, in accord with the conventional prediction, and the large eddies are amplified with an increase in the mean flow velocity. He managed to validate his results by reproducing the measurements of Uberoi (1956) and Hussain & Ramjee (1976) for contraction ratios of 16 and 11, respectively, with his analytical model. Nevertheless, he assumed that these measurements reflected reality.
6. High-Contraction Ratio Anomaly and Its Removal

During the course of turbulence modeling studies at LSTM-Erlangen, this anomalous development of turbulence was realized by Jovanović et al. (2003) during the validation of their turbulence model, when they were trying to use the homogeneous turbulent flow test cases selected by Ferziger (1980). The predictions of Jovanović et al. (2003) showed excellent agreement with experiments for \( c < 9 \). Nevertheless, as shown in Figure 6.2, they could not predict the increase in \( \overline{uu} \) reported in the above-mentioned high contraction ratio measurements.

When the dynamic equations of \( \overline{uu} \) and \( \overline{vv} \) (2.78) for homogeneous flows are considered, it can be seen that only the pressure-strain rate correlation term in the \( \overline{uu} \) equation can act as a source term when the turbulence is positively strained and \( \overline{uu} \) becomes smaller than \( \overline{vv} \). Since measurements of pressure-strain rate correlations in axisymmetric turbulence are very cumbersome, the turbulence model of Jovanović et al. (2003) can be utilized for the analysis of this term. According to the modeled pressure strain rate correlations (2.84a) and the dissipation correlations (2.85a), when turbulence is nearly isotropic before entering the contraction, \( \alpha_{11} \approx 0 \), the time derivative of \( \overline{uu} \) should always be negative. That means that \( \overline{uu} \) should decay. As the axisymmetric turbulent flow approaches the two-component isotropic limit, the pressure-strain rate correlations (2.84a) and the dissipation correlations (2.85a) tend to zero. Since \( \overline{uu} \) ceases completely at the two-component isotropic state, production cannot be effective any longer, so the time derivative of \( \overline{uu} \) becomes zero. Note that the zero time-derivative at the two-component isotropic state is one consequence of the realizability conditions of Reynolds stress models discussed by Schumann (1977). Hence, under continuous strain, it is expected that turbulence at a nearly isotropic state continuously develops into a more anisotropic state and attains a zero gradient in time at the two-component isotropic state. Nevertheless, dynamic equations set no constraint that the time deriva-

Figure 6.2: Predictions of the mean square of the longitudinal velocity fluctuation, \( \overline{uu} \), and transverse velocity fluctuation, \( \overline{vv} \), along the symmetry axis of the nozzles. Test cases are selected from Tan-atichat et al. (1980) in the Proceedings of the Stanford Conference by Ferziger (1980): test case numbers (a) 0375B and (b) 0375E.
6.2 Anomalous Behavior of Reynolds Stresses

Figure 6.3: Measurements of $\overline{uu}$ and $\overline{v^2}$ (a) at low contraction ratio ($c = 3.69$) and (b) at high contraction ratio ($c = 14.75$). The measurements are corrected for the background turbulence. Lines are predictions.

The authors note that such a state of turbulence is not probable since it cannot sustain owing to the immediate sinking action of the production and the dissipation term.

Similar axisymmetric strained turbulence measurements were performed by the present in a wind tunnel of very low background turbulence with the help of hot-wire anemometry, using nozzles of low ($c = 3.69$) and high ($c = 14.75$) contraction ratios. Example results, which are corrected for the background disturbances in the flow, are shown in Figure 6.3. These experiments confirm that high contraction ratio measurements suffer from the increase in $\overline{uu}$ downstream of the contraction. As can be seen in the same figure, the predictions are in very good agreement for the investigated contraction ratio of 3.69, whereas for the nozzle with high contraction ratio the prediction of $\overline{uu}$ falls below the measured values as latter start to increase.

In addition to the discrepancy between the theory and the experiments, the hot-wire measurements performed with a single normal wire (SN-wire) probe and an X-wire probe showed markedly different increases in the $\overline{uu}$ downstream of the contraction. As shown in Figure 6.4, this difference is only substantial in the nozzle with a high contraction ratio.

The above results, and especially the discrepancy between experiments and theory, triggered the author’s interest in the high contraction ratio anomaly observed in axisymmetric strained turbulence. Investigations were started to find its cause and these are described in this chapter. Due to the difference in measurements of longitudinal velocity fluctuations performed with SN-wires and X-wires, experimental contaminations were analyzed in two steps: first for the SN-wire measurements and later for the X-wire measurements.
6. High-Contraction Ratio Anomaly and Its Removal

Figure 6.4: Comparison of \( \overline{uu} \) measurements performed with SN-wire and X-wire (a) at low contraction ratio \( (c = 3.69) \) and (b) at high contraction ratio \( (c = 14.75) \). The data are not corrected for the background turbulence.

In the analysis of the SN-wire contaminations, it is shown analytically that one important cause of the high contraction ratio anomaly lies in measurements contaminated by mass flow rate fluctuations, which are caused by the flow facility and amplified as the flow is strained by a contracting nozzle. It is also shown that another reason is the electronic noise of the measurement system. It is postulated that these contaminations appear in the measurements, which are performed with both SN-wire and X-wire, as part of the longitudinal velocity fluctuations for strained turbulent flows through nozzles with high contraction ratios.

In the second step, the response equations of the X-wire probe, the conventional measurement and data processing methods are analyzed. It is shown that the finite wire length and the distance between the two wires of the X-wire probe may not be sufficiently small to resolve turbulent structures undergoing a thinning in the transverse direction as they flow through the contractions which were employed in many of the studies presented in the literature. These findings are experimentally confirmed in this chapter.

The spatial resolution problems of hot-wire probes have been known since Dryden, Schubauer, Mock & Skramstadt (1938). Researchers such as Dryden et al. (1938), Uberoi & Kovazsnay (1953), Frenkiel (1954), Wyngaard (1968) and Bremhorst (1972) established a sound theoretical basis for the effect of the spatial resolution of single normal and X-wire probes on the measured statistical quantities of turbulence. These studies are extensively utilized in the hot-wire measurements of turbulent boundary layers in the vicinity of the walls where thin streaky turbulent structures are expected to exist according to the pioneering work of Kline (1967). Johansson & Alfredsson (1983), Klewicki & Falco (1990) and Zhu & Antonia (1995) are important examples of such investigations dealing with the consequences and the corrections of the spatial resolu-
tion problem of hot-wire measurements in boundary layers. Unfortunately, the same emphasis was not given to the measurements of turbulence in nozzles with high contraction ratios and, according to the author’s knowledge none of the measurements in the literature were corrected for the contaminations owing to the limited spatial resolution of the hot-wire measuring probes.

The main goal of this chapter is to verify all of the above-mentioned arguments and achieve anomaly-free measurements of longitudinal and transverse velocity fluctuations in nozzles with high contraction ratio which can be used for the validation of the turbulence models. This goal is again achieved in two steps. First, a measurement method based on two-point velocity correlation measurements was developed for the measurements of longitudinal velocity fluctuations and realized with two SN-wire probes. This technique allowed turbulence data to be extracted from the measured signals, which were contaminated with flow rate fluctuations and the electronic noise. The proposed method is shown to be more reliable than a family of commonly used correction methods in such investigations. In the second step, the spatial resolution problems of X-wire probes are treated by separating the spatial resolution problem due to the finite separation between the wires of the X-wire probe and to the finite length of the wires of the X-wire probe, noting that the latter affects also the SN-wire measurements. One X-wire probe was added to the two SN-wire configurations. A new measurement and data processing technique was developed which directly eliminated the wire separation effect and corrected the measurements only for the finite wire length. Experimental evaluations of the two suggested measurement methods were successfully conducted. The author showed that both $\overline{u'\overline{u}}$ and $\overline{v'\overline{v}}$ can be measured accurately with the combination of the two methods.

### 6.3 Measurement Contaminations of Strained Axisymmetric Turbulence

Most of the available extended fluid flow investigations in turbulence research have been carried out by hot-wire anemometry. This measuring technique can be considered to be fully developed and it is common knowledge how to apply the technique also to low-level turbulence measurements. The invaluable work of Commte-Bellot (1976) and Bruun (1995) can be referred to for an overview of this technique. Nevertheless, raw hot-wire measurements are always contaminated to some degree with different kinds of artifacts which originate from the instrumentation and/or from the measured flow itself. Discussions on types of contamination and methods of corrections can also be found in the aforementioned studies and additionally one can refer to Hinze (1975) for discussions on several of these effects. In the following theoretical considerations on hot-wire measurements with SN-wire and X-wire probes, special emphasis is given to the contaminations appearing in measurements of turbulence which are undergoing high convective acceleration, as in the case of turbulent flow through a symmetric converging nozzle with high contraction ratio.
Consider the generalized forms of SN- and X-wire probes given in Figure 6.5. For a single hot-wire (normal or inclined) and for a velocity field fully correlated over the entire length of the wire, the measured signal potential, \( E(t) \), can be related to the effective cooling velocity, \( U_e(t) \), which is the velocity perpendicular to the wire by equation (3.2) which is repeated here as

\[
E(t)^2 = A + BU_e(t)^n. \tag{6.1}
\]

For a stationary turbulent flow, the velocity and the signal potential can be decomposed into their mean components \( \bar{U}_e \) and \( \bar{E} \) and the fluctuating components in time \( u_e \) and \( e \) respectively. The linearized response equation (6.1) can be used directly to obtain the mean and the instantaneous cooling velocity so that further statistical values, such as Reynolds stresses, can be computed.

The relation between the measured voltage fluctuations and the corresponding velocity fluctuations is

\[
e = \left( \frac{\partial E}{\partial U_e} \right)_{U_e=\bar{U}_e} u_e, \tag{6.2}
\]

which can be derived by linearizing the Taylor series expansion of the voltage output of the hot-wire around the mean cooling velocity \( \bar{U}_e \). The term \( \frac{\partial E}{\partial U_e} \) is the velocity sensitivity of the whole measurement system together with the wire, which is calculated at the mean effective cooling velocity \( \bar{U}_e \). In the subsequent text, velocity sensitivity is denoted \( D \). The value of the velocity sensitivity around the mean effective cooling velocity can be obtained by using the linearized cosine law of cooling (6.1):

\[
D = \left( \frac{\partial E}{\partial U_e} \right)_{U_e=\bar{U}_e} = \frac{nB \bar{U}_e^{n-1}}{2E}. \tag{6.3}
\]
Ideally, the measured and the actual velocity data should be equal to each other, but this is not always the case because of the artifacts. To differentiate the measured quantities of the velocity field from the actual quantities, an asterisk is assigned to the measured quantities. Hence, the measured fluctuation of the cooling velocity becomes

\[ u^*_e = \frac{1}{D} e^* \]  

(6.4)

In a test facility, it cannot be avoided that the flow equipment employed produces mass flow rate fluctuations. When the wire is placed perpendicular to the flow, the effective cooling velocity component would be the instantaneous local streamwise velocity of the flow \( U(t) \). In a turbulent flow, this velocity component can be decomposed into the mean component, \( \bar{U} \), the velocity fluctuations due to turbulence, \( u_{turb} \), and additional velocity fluctuations mainly caused by mass flow rate fluctuations, \( u_{mass} \), inherent in most wind tunnels. Note that \( t \) in parentheses is dropped from the fluctuating variables since they are time dependent by definition. The total instantaneous velocity at a point in the nozzle is therefore

\[ U(t) = \bar{U} + u_{mass} + u_{turb}. \]  

(6.5)

This velocity causes an output signal of the hot-wire anemometer which has fluctuating components with contributions from the turbulent velocity fluctuations, the mass flow rate fluctuations and additionally its own electronic noise:

\[ E^*(t) = \bar{E} + e_{mass} + e_{turb} + e_{elec}. \]  

(6.6)

There might be additional contributions such as acoustic noise arising from the blower or from the turbulent boundary layers in the flow facility. Uberoi (1956), Bennet & Corrsin (1978), Tan-atichat et al. (1980) and Han (1988) discussed acoustic contaminations together with non-vortical flow contaminations at the core of the flow induced by boundary layer fluctuations and the blowers. Tan-atichat & Harandi (1987) showed that the acoustic disturbances are one of the reasons for the high contraction ratio anomaly. However, will not be discussed separately the effect of acoustic noise and boundary layer-induced non-vortical fluctuations, since they can be treated as part of the mass flow rate fluctuations of the whole flow facility.

In the following sections, the effects of the mass flow rate fluctuations in the flow, the electronic noise of the measurement instruments and the spatial resolution of hot-wire probes will be analyzed by using the basic relations given above.

### 6.3.1 Effect of Mass Flow-rate Fluctuations

Concerning a flow only with mass flow rate fluctuations, the instantaneous total mass flow rate can be written as the sum of the mean and the fluctuating mass flow rates:

\[ \dot{M}(t) = \bar{M} + \dot{m}. \]  

(6.7)
This corresponds to the following cross-sectional overall velocity at a certain measuring location:

\[ U(t) = \bar{U} + u^{\text{mass}}. \]  

For the flow through a nozzle, the velocity can be assumed to be uniform in a transverse plane and, hence, the local mean velocity and the cross-sectional average velocities are identical, so that the connection between equations (6.7) and (6.8) is

\[ \bar{M} + \dot{m} = (\bar{U}_0 + u_0^{\text{mass}}) \rho A_0 = (\bar{U}_1 + u_1^{\text{mass}}) \rho A_1(x), \]  

where the subscript 0 indicates the inlet plane of a nozzle and 1 the measuring plane. From equation (6.9), we obtain

\[ \frac{u_1^{\text{mass}}}{u_0^{\text{mass}}} = \frac{A_0}{A_1(x)} = c(x), \]  

where \( c(x) = A_0/A_1(x) \) is the area ratio between the nozzle inlet area and the cross-sectional area at any intermediate measurement location \( x \) along the nozzle axis; in other words, \( c(x) \) is the local contraction ratio. It is called the contraction ratio of the nozzle if the inlet and outlet area of the nozzle are considered. Equation (6.10) shows that the mass flow rate fluctuation-caused velocity fluctuations, \( u^{\text{mass}} \), amplify along the nozzle axis due to the area ratio \( c \) so that the ratio of the mean square values of the longitudinal velocity fluctuations is

\[ \frac{u_1^2}{u_0^2} = \frac{u_1^{\text{mass}}}{u_0^{\text{mass}}} = c(x)^2. \]  

Hence the mass flow rate fluctuations cause velocity fluctuations in a nozzle and these amplify along the axis of a converging nozzle and, therefore, should be higher for nozzles with high contraction ratios in comparison with nozzles with low \( c \) values.

### 6.3.2 Effect of Electronic Noise

In a flow free of turbulence and of mass flow rate fluctuations, the measured voltage value is due only to the mean flow and the electronic noise:

\[ E(t) = \overline{E} + e^{\text{elec}}. \]  

If the electronic noise is not eliminated properly, according to equation (6.4) it will manifest as a velocity fluctuation as follows:

\[ u^* = \frac{1}{D} e. \]
Although $e_{elec}^2$ takes a constant value at all points along the nozzle symmetry axis, when it is converted to $\overline{uu^*}$ using the above relation, the inlet and the outlet values become

\begin{align}
\frac{u_{0}^{elec}u_{0}^{elec}}{D_0^2} &= 1 \\
\frac{u_{1}^{elec}u_{1}^{elec}}{D_1^2} &= 1
\end{align}

(6.14a)

(6.14b)

so that the ratio of the two measured pseudo-velocity fluctuation mean squares is given by

\begin{align}
\frac{u_1 u_1}{u_0 u_0} &= \frac{u_{0}^{elec}u_{0}^{elec}}{u_{1}^{elec}u_{1}^{elec}} = D_0^2/D_1^2 = c(x)^2(1-n) \frac{E_1^2}{E_0^2},
\end{align}

(6.15)

where $0.4 < n < 0.5$ in general, which means an amplification of the measured $\overline{uu^*}$ value proportional to $c(x)^2(1-n)$ multiplied by the ratio of the mean hot-wire voltage readings, which is always higher than 1. This result suggests that the effect of electronic noise can be properly corrected if its contribution to the signal is subtracted from the voltage readings. In general investigations of turbulent flow with modern hot-wire equipment, this noise contribution can be neglected. However, this issue becomes critical if the mean velocity variation in the flow is large and the turbulence level becomes very low at the same time, which is the case when the grid-generated turbulence experiences elongational strain imposed by a nozzle with a high contraction ratio.

### 6.3.3 Contaminated Strained Turbulence Measurements

Assume that the measurements deliver only the turbulent velocity fluctuations:

\begin{align}
U(t) = \overline{U} + u^{turb}.
\end{align}

(6.16)

According to the simplest theory, namely Prandtl’s (1932) vortex stretching theory, the streamwise component of the velocity fluctuation should decay according to the following relation:

\begin{align}
\frac{u_1 u_1}{u_0 u_0} = \frac{u_1^{turb} u_1^{turb}}{u_0^{turb} u_0^{turb}} = \frac{1}{c(x)^2}.
\end{align}

(6.17)

This equation implies that if only the turbulence fluctuations are measured, a decrease in the mean square of the longitudinal velocity fluctuation with $1/c(x)^2$ would be observed. However, as explained above, the mass flow rate fluctuations and the contributions of electronic noise become amplified as the ratio $c(x)$ increases. Hence all of these effects add up in the hot-wire measurements and yield

\begin{align}
\overline{uu^*} &= u^{turb} u^{turb} + u^{mass} u^{mass} + u^{elec} u^{elec}.
\end{align}

(6.18)

The decrease in turbulence and amplification of the above-mentioned artifacts along the nozzle are demonstrated in Figure 6.6 for a particular measurement in the high...
contraction ratio nozzle used in this study. As can be seen, the mass flow rate fluctuations are amplified even more drastically than the electronic noise and, therefore, it should be expected that this effect will appear in the measured longitudinal energy component $\overline{u'^2}$ given by equation (6.18).

In order to demonstrate the combined effect of turbulence and the contaminations due to flow rate fluctuations and electronic noise along a nozzle with a high contraction ratio, an example flow was constructed by assuming the following conditions at the inlet of the nozzle: 99.7% of the measured $\overline{u'^2}$ is due to turbulence and 0.25% and 0.05% to mass flow rate fluctuations and electronic noise, respectively. These values were commonly met in the author’s measurements where the background turbulence intensity level was 0.05% and the grid turbulence intensity was around 4% at the inlet of the nozzle, whose contraction ratio and shape were identical with those of the nozzle used in this study. The inlet conditions of known experiments in the literature are not very different from those in the author’s experiments.

By using equations (6.11), (6.15) and (6.17), all these effects were calculated relative to the measured total $\overline{u'^2}$ given by equation (6.18). The calculated curves in Figure 6.7 demonstrate that the total $\overline{u'^2}$ level decreases initially and, as the local contraction ratio $c(x)$ increases along the flow direction, the contaminations dominate the turbulence fluctuations and are measured as part of the longitudinal velocity fluctuation, if not accounted for.

In this example calculation, the decay of turbulence was not taken into account. However, involving a decay process in the calculations will lead to the occurrence of the anomaly much earlier in the contraction and with a higher relative level. Therefore, according to the above considerations, it is safe to argue that the claimed high contraction ratio anomaly for the mean square of the longitudinal velocity fluctuations obtained by SN-wire measurements is caused by the mass flow rate fluctuations in the flow and by not properly corrected electronic noise.
Figure 6.7: Simulation of a flow case in which 99.7% of the measured $\overline{uu}$ at the inlet of the contraction is due to turbulence, 0.05% to mass flow rate fluctuations and 0.25% to electronic noise.

It is worth pointing out that the above calculation is specific for a particular measurement and a particular experimental facility. This suggests that the development of $\overline{uu^*}$ along the nozzle axis is dependent on the following points: turbulence properties at the inlet of the nozzle, instrumentation noise, flow quality and the nozzle shape. As the case may be, the anomaly becomes visible earlier or later along the nozzle with an absolute level depending on the properties of the flow and the experimental arrangement.

6.3.4 Effect of Wire Length and Separation

In a turbulent flow, which is undergoing convective acceleration in a symmetrical nozzle, the length scales also become highly anisotropic in addition to the increase in the anisotropy of second order moments of velocity fluctuations (Reynolds stresses). The length scales in the acceleration direction become longer and those in the transverse direction become shorter. When the transverse length scales decrease to values comparable to the wire length, at the same instant the velocities at different positions on the wires and between the wires no longer fully correlate. This state is called incomplete correlation.

Dryden et al. (1938) were the first not only to investigate the spatial resolution problem of hot-wire measurements but also to give correction equations for the measured moments of velocity fluctuations and the correlation functions which are measured by two hot-wire probes. A short account of their work was given by Hinze (1975). Their method and correction factors were specific to SN-Wires. The other prominent studies on the wire-length corrections for SN-wire probes are due to Uberoi & Ko-
vasznay (1953) and Frenkiel (1954). Wynagard (1968) was the first to investigate the spatial resolution problem of X-wire probes and showed that the $u$ and $v$ fluctuations measured with X-wire probes might involve contributions from $v$ and $u$ fluctuations respectively. These counter-contributions were called the cross-talk terms. Bremhorst (1972) considered both the wire length and the wire separation effects on measurements performed with X-wire probes. However, in his formulation Bremhorst partially neglected the effect of the cross-talk terms which start to dominate the error made in the measurements when turbulence is highly anisotropic. In the next few paragraphs, the importance of cross-talk terms is clarified for homogeneous turbulence undergoing convective acceleration.

The cosine law of cooling for two identical inclined wires of an X-wire probe given in Figure 6.5 are

$$E_1^2 = A + BU_{e1}^n,$$  \hspace{1cm} (6.19a)
$$E_2^2 = A + BU_{e2}^n.$$  \hspace{1cm} (6.19b)

Thus, for a complete correlation on the wires of an X-wire probe in a flow with $\overline{W} = 0$, the measured effective cooling velocities should be composed of the longitudinal and transverse velocity components:

$$U_{e1} = U_1(t)\cos(\alpha) + V_1(t)\sin(\alpha),$$  \hspace{1cm} (6.20a)
$$U_{e2} = U_2(t)\cos(\alpha) - V_2(t)\sin(\alpha),$$  \hspace{1cm} (6.20b)

where the subscripts 1 and 2 denotes the velocities measured with inclined wires 1 and 2, respectively. The classical calculation based on the assumption that $U_1(t) = U_2(t) = U(t)$ and $V_1(t) = V_2(t) = V(t)$. This assumption results in the following instantaneous longitudinal and transverse velocities:

$$U(t) = \frac{U_{e1} + U_{e2}}{2\cos(\alpha)},$$  \hspace{1cm} (6.21a)
$$V(t) = \frac{U_{e1} - U_{e2}}{2\sin(\alpha)}.$$  \hspace{1cm} (6.21b)

The fluctuating components are calculated by subtracting the mean of the velocity components from $U(t)$ and $V(t)$. For a full correlation over the wire, from equation (6.4), the fluctuating potentials read from the two identical inclined hot-wires are

$$e_1 = D_{e1}u_{e1},$$  \hspace{1cm} (6.22a)
$$e_2 = D_{e2}u_{e2},$$  \hspace{1cm} (6.22b)

where $D_{e1}$ and $D_{e2}$ are the velocity sensitivity of given in equation (6.3) around the mean effective velocities $U_{e1}$ and $U_{e2}$, respectively. Assuming that $\overline{V} = 0$, which is the case on the symmetry axis of a converging nozzle, $D_{e1}$ becomes equal to $D_{e2}$.

Knowing that the effective fluctuating velocities are

$$u_{e1} = u\cos(\alpha) + v\sin(\alpha),$$  \hspace{1cm} (6.23a)
$$u_{e2} = u\cos(\alpha) - v\sin(\alpha),$$  \hspace{1cm} (6.23b)
and using the fluctuating potentials (6.22), the Reynolds stresses \( \overline{uu} \), \( \overline{vv} \) and \( \overline{uv} \) can be found as:

\[
\begin{align*}
\overline{uu} & = \frac{e_1^2 + 2e_1e_2 + e_2^2}{4D^2\cos(\alpha)^2}, \\
\overline{vv} & = \frac{e_1^2 - 2e_1e_2 + e_2^2}{4D^2\sin(\alpha)^2}, \\
\overline{uv} & = \frac{e_1^2 - e_2^2}{4D^2\cos(\alpha)\sin(\alpha)},
\end{align*}
\]

(6.24a) \( \text{to } (6.24c)

where the mean squares of the fluctuating signal potentials are

\[
\begin{align*}
\overline{e_1^2} & = D^2u_1^e, \\
\overline{e_2^2} & = D^2u_2^e, \\
\overline{e_1e_2} & = D^2u_1u_2.
\end{align*}
\]

(6.25a) \( \text{to } (6.25c)

The measured velocity fluctuation moments given in equations (6.24) should reflect the actual values if there is complete correlation over the entire length of the wires. In the case of incomplete correlation over the wire system depicted in Figure 6.5, the fluctuating components of the anemometer signals are proportional to the average cooling of the wire:

\[
\frac{1}{l_X} \int_0^{l_X} u_e(s) ds,
\]

(6.26)

therefore the measured actual mean squares of the fluctuating signal potentials are

\[
\begin{align*}
\overline{e_1^*} & = D^2 \frac{1}{l_X} \left( \int_0^{l_X} u_{e1}(s) ds \right)^2, \\
\overline{e_2^*} & = D^2 \frac{1}{l_X} \left( \int_0^{l_X} u_{e2}(s) ds \right)^2, \\
\overline{e_1e_2^*} & = D^2 \frac{1}{l_X} \int_0^{l_X} u_{e1}(s) ds \int_0^{l_X} u_{e2}(s) ds.
\end{align*}
\]

(6.27a) \( \text{to } (6.27c)

As a consequence of zero transverse mean velocity, the velocity sensitivities of both wires are taken to be equal and designated \( D \). Moreover, \( D \) is also assumed to be constant at all positions of the wires.

The square of the integrals in equations (6.27) involve the effect of the incomplete correlation on the measurements, but their calculations are not trivial. For the calculation of these equations, the method used by Dryden et al. (1938) and Bremhorst (1972) is followed. The wires of length \( l \) are treated to be composed of \( n \) segments of length \( \Delta s \) as shown in Figure 6.5, such that each of these segments is cooled with a different effective fluctuating velocity:

\[
\begin{align*}
{u_{e1,i}} & = u_{1,i}\cos(\alpha) + v_{1,i}\sin(\alpha), \\
{u_{e2,i}} & = u_{2,i}\cos(\alpha) - v_{2,i}\sin(\alpha), \text{ where } i = 1 \ldots n.
\end{align*}
\]

(6.28a) \( \text{to } (6.28b)\)
Hence the integral terms in equations (6.27) can be approximated by the discrete summations as

\[
\bar{e}_1^2 = D^2 \frac{1}{l^2} \Delta s^2 \left[ \cos(\alpha)^2 \left( \sum_{i=1}^{n} u_{1,i} \right)^2 + 2 \cos(\alpha) \sin(\alpha) \left( \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{1,i} \right) \right. \\
\left. \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \sin(\alpha)^2 \left( \sum_{i=1}^{n} v_{1,i} \right)^2 \right], \quad (6.29a)
\]

\[
\bar{e}_2^2 = D^2 \frac{1}{l^2} \Delta s^2 \left[ \cos(\alpha)^2 \left( \sum_{i=1}^{n} u_{2,i} \right)^2 - 2 \cos(\alpha) \sin(\alpha) \left( \sum_{i=1}^{n} u_{2,i} \sum_{i=1}^{n} v_{2,i} \right) \right. \\
\left. \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \sin(\alpha)^2 \left( \sum_{i=1}^{n} v_{2,i} \right)^2 \right], \quad (6.29b)
\]

\[
\bar{e}_1 \bar{e}_2 = D^2 \frac{1}{l^2} \Delta s^2 \left[ \cos(\alpha)^2 \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} u_{2,i} - \cos(\alpha) \sin(\alpha) \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{2,i} \right. \\
\left. \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \cos(\alpha) \sin(\alpha) \sum_{i=1}^{n} v_{1,i} \sum_{i=1}^{n} u_{2,i} - \sin(\alpha)^2 \sum_{i=1}^{n} v_{1,i} \sum_{i=1}^{n} v_{2,i} \right], \quad (6.29c)
\]

In a homogeneous flow, the following equalities hold:

\[
\left( \sum_{i=1}^{n} u_{1,i} \right)^2 = \left( \sum_{i=1}^{n} u_{2,i} \right)^2, \quad (6.30a)
\]

\[
\left( \sum_{i=1}^{n} v_{1,i} \right)^2 = \left( \sum_{i=1}^{n} v_{2,i} \right)^2, \quad (6.30b)
\]

\[
\sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{1,i} = \sum_{i=1}^{n} u_{2,i} \sum_{i=1}^{n} v_{2,i}, \quad (6.30c)
\]

\[
\sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{2,i} = \sum_{i=1}^{n} u_{2,i} \sum_{i=1}^{n} v_{1,i}, \quad (6.30d)
\]

so that, using only the left-hand side of the above equalities, the simplified forms of
the $e_1^*, e_2^*$ and $e_1 e_2^*$ in equations (6.29a, b, c) become

$$e_1^* = D^2 \frac{1}{l_X} \Delta s^2 \left[ \cos(\alpha)^2 \left( \sum_{i=1}^{n} u_{1,i} \right)^2 + 2 \cos(\alpha) \sin(\alpha) \left( \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{1,i} \right) \right. \right.$$ 

$$+ \sin(\alpha)^2 \left( \sum_{i=1}^{n} v_{1,i} \right)^2 \right],$$

(6.31a)

$$e_2^* = D^2 \frac{1}{l_X} \Delta s^2 \left[ \cos(\alpha)^2 \left( \sum_{i=1}^{n} u_{1,i} \right)^2 - 2 \cos(\alpha) \sin(\alpha) \left( \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{1,i} \right) \right. \right.$$ 

$$+ \sin(\alpha)^2 \left( \sum_{i=1}^{n} v_{1,i} \right)^2 \right],$$

(6.31b)

$$e_1 e_2^* = D^2 \frac{1}{l_X} \Delta s^2 \left[ \cos(\alpha)^2 \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} u_{2,i} - \sin(\alpha)^2 \sum_{i=1}^{n} v_{1,i} \sum_{i=1}^{n} v_{2,i} \right].$$

(6.31c)

The above set of equations implies that all three quantities $e_1^*, e_2^*$ and $e_1 e_2^*$ are affected by the incomplete correlation due to the finite wire length and $e_1 e_2^*$ is additionally affected by the separation between wires. Note that $e_1 e_2^*$ involves the cross-talk terms. Inserting the above relations in (6.24) and labeling the velocity fluctuation moments with an asterisk denoting that they are just the quantities reduced from the measured signal potentials in (6.31), we obtain

$$\bar{u} u^* = \frac{\Delta s^2}{2l_X^2} \left\{ \left( \sum_{i=1}^{n} u_{1,i} \right)^2 + \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} u_{2,i} \right. \right.$$ 

$$+ \tan(\alpha)^2 \left[ \left( \sum_{i=1}^{n} v_{1,i} \right)^2 - \sum_{i=1}^{n} v_{1,i} \sum_{i=1}^{n} v_{2,i} \right] \right\},$$

(6.32a)

$$\bar{v} v^* = \frac{\Delta s^2}{2l_X^2} \left\{ \left( \sum_{i=1}^{n} v_{1,i} \right)^2 + \sum_{i=1}^{n} v_{1,i} \sum_{i=1}^{n} v_{2,i} \right. \right.$$ 

$$+ \cot(\alpha)^2 \left[ \left( \sum_{i=1}^{n} u_{1,i} \right)^2 - \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} u_{2,i} \right] \right\},$$

(6.32b)

$$\bar{u} v^* = \frac{\Delta s^2}{l_X^2} \sum_{i=1}^{n} u_{1,i} \sum_{i=1}^{n} v_{1,i}.$$ 

(6.32c)

These equations show that the measured $\bar{u} u^*$, for instance, contains all contributions from the $u$ fluctuations and contributions due to the $v$ fluctuations. For an ideal X-wire measurement, half of the $\bar{u} u^*$ is contributed by the cross-correlation terms, i.e. the sec-
ond term in equation (6.32a), and the contributions from the $v$ fluctuations are compensated by the cross-correlation term in the third term; $\overline{uv}^*$ can be similarly interpreted. Now, the consequences of incomplete correlation together with high convective acceleration can be better explained. As the transverse scales become shorter, all the terms in equations (6.32a, b) start to show lower values than the real ones and especially the cross-correlation terms drops more than the others since loss of correlation in these occurs both due to the finite wire length and the separation between the wires. Hence, for example, contributions of the $v$ fluctuations in $\overline{uv}^*$ cannot be compensated any longer. Consequently, $\overline{uv}^*$ starts to increase, if the actual value of $\overline{uv}$ increases, which is the case when turbulence is strained in a nozzle. Similarly, in the case of a flow through a converging nozzle, the measured $\overline{uv}^*$ do have a lower value than the real one unless $\overline{uv}$ increases.

In the present measurements in nozzles with high contraction ratio, the SN-wire measurements, which are corrected for the electronic noise and flow rate fluctuations, do not show any increase in the $\overline{uv}^*$, whereas X-wire measurements with the same corrections show a remarkable increase. An example of such a measurement is shown in Figure 6.8a. As can be seen in Figure 6.8b, $\overline{e_1e_2}$ crosses the zero line when the anisotropy becomes zero and starts to increase at negative values as $\overline{vv}^*$ increases. After a while, it starts to decrease in its absolute value and at the same time $\overline{uv}^*$ determined by by X-wire measurements starts to increase and deviate from $\overline{uv}^*$ determined by SN-wire measurements. The development of the integral length scale and the dissipation scale of turbulence in the transverse direction were estimated by using the rapid distortion theory and Prandtl’s vortex stretching analogy (see Sections 6.4.2 and 6.6 for details).

As shown in Figure 6.8c, at the locations where the increase in $\overline{uv}^*$ starts, the transverse length scales of turbulence dropped to the wire lengths of the SN-wire and X-wire probes, $l_{SN}$ and $l_x \cos(\alpha)$, respectively, and the distance between the wires of the X-wire probe $\Delta z_X$. Since SN-wire measurements of $\overline{uv}^*$ do not show any increase, based on the derived equations (6.32) and the development of transverse length scales in the nozzle, it is fair to argue that the increase in $\overline{uv}^*$ in the present X-wire measurements is due to the limitation dictated by the spatial resolution of the X-wire probe.

Similarly to the X-wire probe, an SN-wire probe suffers from the spatial resolution problem because of its finite wire length. Following the same approach as above, the relation between the measured mean square of streamwise fluctuations and the actual fluctuations can be found as

$$\overline{uu}_{SN}^* = \frac{\Delta s^2}{l_{SN}^2} \left\{ \sum_{i=1}^{n} u_{SN,i} \right\}^2$$ \hspace{1cm} (6.33)

Dryden et al. (1938) evaluated the above equation to find a correction factor as a function of ratio of the wire length to the integral length scale, and showed that the measured moments of streamwise velocity fluctuations can be, around 10%, less than the actual values. In Section 6.4.2, his derivations will be briefly cited and, based on his
Figure 6.8: Measured data showing the signs of incomplete correlation on an X-wire probe. The values of $\overline{uu}$ and $\overline{vv}$ are corrected for the electronic noise and the flow rate fluctuations. (a) Normal Reynolds stresses obtained by X-wire and SN-wire measurements. (b) Mean values of signal correlations for X-wire measurements. (c) Estimated dissipation and integral length scales in the transverse direction and the hot-wire dimensions.
method, a new measurement and data processing method will be presented with correction factors different from those given by Dryden et al. (1938) for SN-wire probes and by Bremhorst (1972) for X-wire probes.

6.4 Proposed Measurement and Data Processing Methods

The considerations in Section 6.3 show that reliable hot-wire measurements of turbulent velocity fluctuations in nozzles require considerable and different types of corrections to yield the turbulent velocity fluctuations in the longitudinal direction, and also in the transverse direction. Although this fact is generally known, it has not been taken into account in the past when the effects of elongational straining of grid turbulence were studied. In this section, two measurement and data processing methods are constructed: one for the elimination of flow rate fluctuations and electronic noise effects in $\overline{uu}$ measurements performed with SN-wire probes, and the other to correct for the effect of the imperfect spatial resolution.

6.4.1 Removal of Mass Flow-rate Fluctuations and Electronic Noise

It is a common exercise to accept the background turbulence of a flow facility as the total non-turbulent contribution in turbulent fluctuation measurements. This kind of interpretation readily suggests the most common correction method, which is the subtraction of background $\overline{uu}$ measurements (measurements without a grid) from the grid-generated turbulence measurements as follows:

$$\overline{uu}_{\text{grid}} - \overline{uu}_{\text{no-grid}} = \overline{u^\text{turb}u^\text{turb}}_{\text{grid}} + \overline{u^\text{mass}u^\text{mass}}_{\text{grid}} + \overline{u^\text{elec}u^\text{elec}}_{\text{grid}}$$

$$- \overline{u^\text{mass}u^\text{mass}}_{\text{no-grid}} - \overline{u^\text{elec}u^\text{elec}}_{\text{no-grid}}$$

$$= \overline{u^\text{turb}u^\text{turb}}_{\text{grid}}$$

This kind of correction was applied by Uberoi (1956), Comte-Bellot & Corrsin (1966), Bennettt & Corrsin (1978) and Han (1988). However, correcting only for the background turbulence failed to remove the anomaly in those experiments with high contraction ratio nozzles. There are several reasons for this. First, this correction method does not take into account the spatial resolution problem of hot-wire probes. Second, the assumption that the contributions of mass flow rate fluctuations of the flow facility to the background $\overline{uu}$ measurements, i.e. the free-stream turbulence, keep their value at the same level in flows with and without the grid may not be correct. One simple reason for this expectation of a difference is the change in the pressure-drop flow-rate characteristic of the whole facility, which will consequently influence the resultant mass flow rate fluctuation level of the flow facility. Another important reason,
which was discussed also by Bennett & Corrsin (1978), is that the development of the boundary layer along the nozzle wall is influenced by the outer turbulent field. That means that the induced grid-turbulence would strongly affect the boundary layer in the flow facility after the grid and especially in the nozzle so that boundary layer fluctuations in the nozzle cause non-turbulent fluctuations of velocity within the whole cross-section, which are then recorded as part of measured turbulence quantities.

An alternative hot-wire measurement method based on the two-point correlation measurement technique was therefore developed to eliminate flow rate fluctuations and electronic noise in the measurements of turbulent $u\overline{u}$ performed with SN-wire probes. Uberoi (1956) mentioned such a method but he did not realize it. The main assumption of the method is that any non-vortical velocity fluctuation, irrespective of its cause, should correlate across the whole cross-section perpendicular to the flow. Therefore, if two measurement points are chosen far away from each other such that turbulence quantities are not correlated, the correlation between the velocity measurements at these two points will yield the contribution of non-vortical velocity fluctuations in the measured $u\overline{u}$. However, it is essential that the two measurement points should be in a homogeneous flow field so that the assumption of uniformity of all statistical quantities of turbulence at this transverse plane can be assumed. That is, none of these points should be selected in the vicinity of the wall where the assumption of homogeneity is not valid.

To explain the strategy for correct measurements, necessary derivations are provided below. These form the basis of the final measuring method applied in these investigations.

If one performs simultaneous measurements at two points, say A and B, the measured mean square values of longitudinal velocity fluctuations, $u_A^2$ and $u_B^2$, and the correlation between the two points $u_A u_B$ are given by

$$
\overline{u_A u_A^*} = \overline{u_A^{turb} u_A^{turb}} + \overline{u_A^{mass} u_A^{mass}} + \overline{u_A^{elec} u_A^{elec}},
$$

$$
\overline{u_B u_B^*} = \overline{u_B^{turb} u_B^{turb}} + \overline{u_B^{mass} u_B^{mass}} + \overline{u_B^{elec} u_B^{elec}},
$$

$$
\overline{u_A u_B} = \overline{u_A^{turb} u_B^{turb}} + \overline{u_A^{mass} u_B^{mass}} + \overline{u_A^{elec} u_B^{elec}}.
$$

If the distance between two measurement points is selected such that the turbulent fluctuations in the longitudinal direction will no longer be correlated, one can write

$$
\overline{u_A^{turb} u_B^{turb}} = 0.
$$

The assumption of homogeneity at the measurement cross-section implies

$$
\overline{u_A^{turb} u_A^{turb}} = \overline{u_B^{turb} u_B^{turb}} = \overline{u_A^{turb} u_B^{turb}}.
$$

Moreover, based on the main assumption of the proposed method stating that the non-vortical fluctuations at one cross-section should be correlated and should have the same value at all points on that cross-section, one can write

$$
\overline{u_A^{mass} u_B^{mass}} = \overline{u_A^{mass} u_A^{mass}} = \overline{u_A^{mass} u_B^{mass}} = \overline{u_B^{mass} u_B^{mass}}.
$$
As a result of the above assumptions, the measured correlations in equation (6.35) reduce to

\[
\begin{align*}
\bar{u}_A u_A^* &= u_{\text{turb}} u_{\text{turb}} + u_{\text{mass}} u_{\text{mass}} + u_{\text{elec}} u_{\text{elec}} \\
\text{with } \quad u_{\text{elec}} u_{\text{elec}} &= D_A e_{\text{elec}}^2 e_{\text{elec}}^A, \\
\end{align*}
\]

(6.37a)

\[
\begin{align*}
\bar{u}_B u_B^* &= u_{\text{turb}} u_{\text{turb}} + u_{\text{mass}} u_{\text{mass}} + u_{\text{elec}} u_{\text{elec}} \\
\text{with } \quad u_{\text{elec}} u_{\text{elec}} &= D_B e_{\text{elec}}^2 e_{\text{elec}}^B, \\
\end{align*}
\]

(6.37b)

\[
\begin{align*}
\bar{u}_A u_B^* &= u_{\text{mass}} u_{\text{mass}} + u_{\text{elec}}^A u_{\text{elec}}^B \\
\text{with } \quad u_{\text{elec}}^A u_{\text{elec}}^B &= D_A D_B e_{\text{elec}}^A e_{\text{elec}}^B. \\
\end{align*}
\]

(6.37c)

To remove the contribution of the mass flow rate fluctuations, the two-point correlation measurement can be subtracted from the one-point correlation measurements as follows:

\[
\begin{align*}
\bar{u}_A u_A^* - \bar{u}_A u_B^* &= u_{\text{turb}} u_{\text{turb}} + u_{\text{elec}}^A u_{\text{elec}}^A + u_{\text{elec}}^A u_{\text{elec}}^B, \\
\bar{u}_B u_B^* - \bar{u}_A u_B^* &= u_{\text{turb}} u_{\text{turb}} + u_{\text{elec}}^B u_{\text{elec}}^B + u_{\text{elec}}^A u_{\text{elec}}^B. \\
\end{align*}
\]

(6.38a)

The equations for the contributions of electronic noise in equation (6.37) are derived from equation (6.4). The correlations \(e_{\text{elec}}^A e_{\text{elec}}^A\), \(e_{\text{elec}}^B e_{\text{elec}}^B\) and \(e_{\text{elec}}^A e_{\text{elec}}^B\) in equation (6.37) can be measured when the air around the wires is absolutely still. This condition can be achieved by isolating the wires from the environment with small boxes. Since these quantities are velocity independent, the contribution of electronic noise in the measurements can be calculated for each wire by calculating the local values of velocity sensitivity parameters \(D_A\) and \(D_B\) using equation (6.3). Finally, the correct turbulent \(v v\) can be evaluated by subtracting all electronic noise correlations from equation (6.38).

The realization of this method and its application in such investigations will be presented in Section 6.5. Further, an experimental comparison will be provided between the method proposed here and the conventional method of measurement.

### 6.4.2 Removal of Wire Length and Wire Separation Effect

The correction method suggested by Dryden et al. (1938) is for SN-wire probes and the method suggested by Bremhorst (1972) partially neglects the effect of cross-talk terms in equations (6.32), which become important in the case of highly anisotropic flows, as shown in Section 6.3.4.

In the subsequent text, the formulation of Dryden et al. (1938) and Bremhorst (1972) is followed to construct a correction method which can be also applied in measurements of highly anisotropic turbulent flows. The proposed method requires three separate measurements: all performed at the same locations by one SN-wire probe, one inclined wire probe (SY-wire) and one with the same SY-wire probe rotated 180°. By performing
the measurements with one single normal and one inclined wire at the same location, the effect of the wire separation is avoided. The suggested method is constructed in such a way that the wire length effect on SN- and SY-wire measurements can be treated with the method of Dryden et al. to obtain real values of $\bar{uu}$ and $\bar{vv}$.

The mean square of fluctuating signals recorded by one inclined wire at two orientations, differing by $180^\circ$ axial rotation, corresponds to equations (6.31a, b). Dropping the subscript for wire index in these equations, the sum of $e^2_1$ and $e^2_2$ gives

$$e^2_1 + e^2_2 = 2D^2 \frac{1}{l_X} \Delta s^2 \left[ \cos(\alpha)^2 \left( \sum_{i=1}^{n} u_i \right)^2 + \sin(\alpha)^2 \left( \sum_{i=1}^{n} v_i \right)^2 \right],$$

(6.39)

where subscripts 1 and 2 denote the first and second measurements. As a result of this operation, the contributions of the second terms in equations (6.31a, b) are eliminated.

In the present investigations, the Reynolds stresses together with the length scales become anisotropic due to straining of the flow, especially in nozzles with high contraction ratios. Utilizing the vortex stretching explanation of Prandtl (1932), it can visually be understood that the vortex filaments aligned parallel to the symmetry axis of the contraction become thinner and longer so the length scales in the transverse $y$-direction become shorter and in the longitudinal $x$-direction become longer. Therefore, the integration over the wires can be accepted to be dominantly influenced by the correlation in the transverse $y$-direction. Assuming homogeneity of the turbulence over the wires in Figure 6.5, the transverse correlation coefficients between the velocity fluctuations at any two segments of the wire, e.g. $i^{th}$ and $j^{th}$ segments, can be written as

$$R_{y,u}(|i - j| \Delta \cos(\alpha)) = \frac{u^i u^j}{\bar{uu}},$$

(6.40a)

$$R_{y,v}(|i - j| \Delta \cos(\alpha)) = \frac{v^i v^j}{\bar{vv}}.$$

(6.40b)

After expanding equation (6.39), following Dryden et al. (1938), it can be shown that

$$e^2_1 + e^2_2 = 2D^2 \frac{1}{l_X} \Delta s^2 \left[ 2\bar{u}u \cos(\alpha)^2 \int_{0}^{l_X} (l_X - s) R_{y,u}(s \cos(\alpha)) \, ds \right. + 2\bar{v}v \sin(\alpha)^2 \int_{0}^{l_X} (l_X - s) R_{y,v}(s \cos(\alpha)) \, ds \right]$$

(6.41)

In the measurements of Mills & Corrsin (1959), the transverse correlation coefficients after the contraction were close to each other. Moreover, all transverse length scales in a contraction provided by Sjögren & Johansson (1998) were of the same order of magnitude. Hence it is reasonable to assume that the transverse correlation coefficients are equal:

$$R_{y,u} \approx R_{y,v} = R_y,$$

(6.42)
and equation (6.41) can be written as
\[
e^2_1 + e^2_2 = D^2 \frac{2}{l_X^2} \left[ 2\overline{u}\cos(\alpha)^2 + 2\overline{v}\sin(\alpha)^2 \right] \int_0^{l_X} (l_X - s) R_y(s \cos(\alpha)) ds, \tag{6.43}
\]
whereas measurements assuming a complete correlation over the wire, i.e. equations (6.28b, 6.25), would give
\[
e^2_1 + e^2_2 = D^2 \left[ 2\overline{u}^* \cos(\alpha)^2 + 2\overline{v}^* \sin(\alpha)^2 \right]. \tag{6.44}
\]
A comparison of equation (6.43) with equation (6.44) gives the following relation between the real sum of the fluctuating effective cooling velocities and the measured value:
\[
\frac{2\overline{u}\cos(\alpha)^2 + 2\overline{v}\sin(\alpha)^2}{\left( u^2_{e1} + u^2_{e2} \right)} = \frac{l_X^2}{2 \int_0^{l_X} (l_X - s) R_y(s \cos(\alpha)) ds}. \tag{6.45}
\]
Changing the variables of integration from \(s\) to \(y\) such that
\[
y = s \cos(\alpha), \tag{6.46}
\]
equation (6.45) becomes
\[
\frac{2\overline{u}\cos(\alpha)^2 + 2\overline{v}\sin(\alpha)^2}{\left( u^2_{e1} + u^2_{e2} \right)} = \frac{l_X^2 \cos(\alpha)^2}{2 \int_0^{l_X \cos(\alpha)} (l_X \cos(\alpha) - y) R_y(y) dy}. \tag{6.47}
\]
For \(\alpha = 0^\circ\), the above equation will be valid for the measurement of \(\overline{uu}\) with SN-wire, such that the right hand side of the above equation becomes exactly the same correction factor \(K^2_1\) evaluated by Dryden et al. (1938),
\[
K^2_1 = \frac{l^2}{2 \int_0^l (l - y) R_y(y) dy}. \tag{6.48}
\]
Thus \(K^2_1\) can be used to correct SN-wire measurements when \(l = l_{SN}\) and to correct the sum of two slanted wire measurements (6.47) when \(l = l_X \cos(\alpha)\) as follows:
\[
\overline{uu} = K^2_1 \overline{uu}^*, \tag{6.49a}
\]
\[
2\overline{u}\cos(\alpha)^2 + 2\overline{v}\sin(\alpha)^2 = K^2_1 \left( u^2_{e1} + u^2_{e2} \right). \tag{6.49b}
\]
The correct value of \(\overline{uu}\) can be obtained from the SN-wire measurements after correcting for the wire length by using the single normal wire correction (6.49a). Hence, the last step of correction is the replacement \(\overline{uu}\) in (6.49b) with that measured by SN-wire (6.49a) in order to obtain the correct value of \(\overline{uu}\).
However, $K_1^2$ (6.48) has to be evaluated for each hot-wire probe at each measurement location. It is necessary to know the correlation coefficient function $R_y$ for the calculation of the correction factor $K_1$. Based on their measurements, Dryden et al. (1938) utilized

$$R_y = e^{-|y/L_y|}, \quad (6.50)$$

where $L_y$ is the integral length scale of turbulence along the axis parallel to the wire and defined as

$$L_y = \int_0^\infty R_y dy. \quad (6.51)$$

Hence, $K_1$ in equation (6.48) becomes

$$K_1 = \frac{1}{\sqrt{2}} \left( \frac{l}{L_y} \right) \left[ \frac{1}{(\frac{l}{L_y}) - 1 + e^{-|\frac{l}{L_y}|}} \right]^{1/2} \quad (6.52)$$

The proposed form of $R_y$ by Dryden et al. (1938) does not have a zero gradient in its vertex and it gives essentially positive values for all $r$, whereas the correct curve should have zero gradient at $r = 0$, and it attains negative values at large distances of $r$. Moreover, even in isotropic turbulent flow, where $R_y$ equals the transverse correlation coefficient $g(r)$, $g(r)$ takes different forms in the initial and final decay periods. Therefore, in the present study, a more general function of the following form was assumed:

$$R_y = e^{-|y/L_y|^m} \quad (6.53)$$

For different states of turbulence between the grid and contraction, where turbulence is accepted to be isotropic, the correlation curves were analyzed and the value of the coefficient $m = \sqrt{2}$ was found to deliver the best fitting correlation curves to the measured curves. A comparison of the measured correlation curves and the assumed correlation curves is shown in Figure 6.9. Notwithstanding the fact that

$$R_y = e^{-|y/L_y|^2} \quad (6.54)$$

cannot also have negative values, this form of the function will be employed here, since it describes the general form of the turbulent flow generated in this work. For a more detailed discussion on the form of $g(r)$, readers should refer to the work of Hinze (1975).

Hence, utilizing $R_y$ (6.54), $K_1$ was evaluated numerically and is compared with that suggested by Dryden et al. (1938) in Figure 6.10. The latter has a linear form and attains higher values than the former up to values $l/L_y \approx 4$. This happens since the form suggested by Dryden et al. (1938) underestimates the real correlation curve close to its vertex (see Figure 6.9).

Since the calculation of $K_1$ is cumbersome for each flow case, the following fourth-order polynomial fit to the numerically evaluated $K_1$ is used

$$K_1 = 1 + 0.05953 \left( \frac{l}{L_y} \right) + 0.06884 \left( \frac{l}{L_y} \right)^2 - 0.01621 \left( \frac{l}{L_y} \right)^3 + 0.00127 \left( \frac{l}{L_y} \right)^4. \quad (6.55)$$
6. High-Contraction Ratio Anomaly and Its Removal

Figure 6.9: Measured correlation coefficients in comparison with the correlation coefficient functions employed for the wire length correction.

In the above suggested method, it is necessary to know the integral length scale $L_y^*$, which is calculated from the correlation coefficient function defined by equation (6.51). However, if the measurements are contaminated owing to the wire length and wire separation effect, the measured correlation and, consequently, the integral length scale would not reflect the actual ones. Again, Dryden et al. (1938) showed that the measured correlation curve is related to the real one as follows:

$$R_y^*(y) = \frac{\int_0^1 (l-s) R_y(\sqrt{s^2+y^2}) \, ds}{\int_0^1 (l-s) R_y(s) \, ds}.$$  \hfill (6.56)

Using the definition of the integral length scale (6.51), another correction factor for the measured length scale can be derived as

$$K_2 = \frac{L_y^*}{L_y} = \frac{1}{L} \int_0^\infty R_y^*(y) \, dy.$$  \hfill (6.57)

In the present study, $K_2$ which was evaluated by using $R_y$ (6.54) was used. A comparison of the $K_2$ factors calculated as suggested by Dryden et al. (1938) is given in Figure 6.10. The selected functional form of $R_y$ delivers a moderate correction to the length scale compared with that suggested by Dryden et al. (1938) It is more useful for the correction that the evaluated $K_2$ factor is represented as a function of the measured ratio $l/L_y^*$ with a fourth-order polynomial curve:

$$K_2 = 0.994296 + 0.121488 \left( \frac{l}{L_y^*} \right) - 0.028164 \left( \frac{l}{L_y^*} \right)^2$$

$$+ 0.001604 \left( \frac{l}{L_y^*} \right)^3 + 0.000136 \left( \frac{l}{L_y^*} \right)^4.$$  \hfill (6.58)

Hence the measured value of the integral length scale would be sufficient to calculate the real value and then calculate the correction factor of measured velocity moments $K_1$ given by the fourth-order polynomial (6.55).
One of the aims of the present study was to provide experimental data on axisymmetric strained turbulence for the validation of turbulence models; it is also necessary to provide the longitudinal and transverse integral length scales, \( L_x \) and \( L_y \), and the dissipation scales, \( \lambda_x \) and \( \lambda_y \), at least, before the contraction where turbulence can be accepted to be isotropic so that length scales are

\[
L_x \approx L_f, \quad L_y \approx L_g, \quad \lambda_x \approx \lambda_f, \quad \lambda_y \approx \lambda_g, \tag{6.59}
\]

where subscripts \( f \) and \( g \) refer to the basic longitudinal and transverse correlation coefficients \( f(r) \) and \( g(r) \), respectively, in isotropic turbulence. Similarly to the correction of integral length scales, dissipation scales measured before contraction should also be corrected. Frenkiel (1954) suggested the following correction:

\[
\frac{\lambda_y^2}{\lambda_y^2} = \frac{1}{2} \left[ \left( \frac{1}{6} \frac{l^2}{\lambda_y^2} + 1 \right) + \sqrt{\left( \frac{1}{6} \frac{l^2}{\lambda_y^2} + 1 \right)^2 - \frac{2}{9} G \frac{l^2}{\lambda_y^2}} \right]. \tag{6.60}
\]

The dissipation length scale \( \lambda_y \) and \( G \) originate from the following approximation of \( R_y(y) \) for small distances of \( y \) in isotropic turbulence

\[
R_y(y) = 1 + \frac{y^2}{2} \frac{d^2 R_y(0)}{dy^2} + \frac{y^4}{24} \frac{d^4 R_y(0)}{dy^4} + \ldots \\
= 1 + \frac{y^2}{2} \frac{d^2 R_x(0)}{dx^2} + \frac{y^4}{8} \frac{d^4 R_x(0)}{dx^4} + \ldots, \tag{6.61}
\]

from which the dissipation scale and \( G \) are defined as

\[
\frac{\lambda_y^2}{\lambda_y^2} = -\frac{2}{\frac{d^2 R_y(0)}{dy^2}} = -\frac{1}{\frac{d^2 R_x(0)}{dx^4}}, \tag{6.62}
\]

\[
G = \lambda_y^4 \frac{d^4 R_x(0)}{dx^4}. \tag{6.63}
\]
6. High-Contraction Ratio Anomaly and Its Removal

6.5 Experimental Realization of Two-point Correlation Measurements

The experimental investigations necessary for the development of the proposed measurement methods were conducted in the AST at LSTM-Erlangen, which was described in Chapter 3. In Section 6.4.1, a new measurement approach based on two-point correlation measurements is suggested to obtain measurements of the mean square of the longitudinal velocity fluctuations, $\overline{uu}$, with an SN-wire probe, which are free from contaminations due to mass flow rate fluctuations and electronic noise. To carry out such measurements, two SN-wire probes were employed. This is readily seen in Figure 6.11, showing the two probe holders connected by a streamlined probe holder, such that the relative distance between the two hot-wires was 14 mm. This distance was selected to be larger than the maximum mesh size of 10 mm, so that the two hot-wires did not read correlated turbulent velocity fluctuations, which was one of the conditions required by the proposed correction method for error-free turbulent velocity fluctuation measurements.

In order to validate the applicability of the proposed measurement method, the mass flow rate control valve mentioned in in Chapter 3 was utilized. The following two control measurements were performed for the validation. First, a turbulent flow was generated with a grid of mesh size of 10 mm and it was contracted through the test
nozzle with a contraction ratio of 14.75. The velocity was measured along the symmetry axis of the test duct and the test nozzle. Later, under the same conditions, the flow produced by the wind tunnel was pulsated with an additional sinusoidal flow induced by the installed mass flow rate control valve and velocity measurements were performed again. The wind tunnel had a volume flow rate of 2900 l/min and the Reynolds number in the test section based on the diameter of the test section was 22900. The additional sinusoidal flow rate pulsation had a frequency of 57 Hz, an amplitude of 82.5 l/min and a mean value of 87.5 l/min.

As can be seen in Figure 6.12, the uncorrected data show an increase in \( \bar{u}u \) due to the contracting nozzle, even without the artificial pulsations, but \( \bar{u}u \) increased more for the pulsed flow case. This increase can be accepted as verification of the previously postulated mass flow rate fluctuation effect on the \( \bar{u}u \) measurements. Another conclusion which can be derived from the uncorrected \( \bar{u}u \) data is its dependence on the flow facility and the measurement instrumentation. Therefore, the demonstrated influence of these dependences on the published data is rather arbitrary and should be considered only qualitative.

After the data from the two measurements had been processed with the method proposed in Section 6.4.1, the anomalies were removed and both measurements resulted in the same \( \bar{u}u \) curve. The removal of the anomaly and the agreement of the two curves in Figure 6.12 prove that the precautions taken during the realization of the correlation measurement method were correct. Moreover, they suggest that the proposed method can be used safely even for small-amplitude flow rate fluctuations, which are not supposed to influence turbulent properties to a large extent, under non-contracted flow conditions.
6. High-Contraction Ratio Anomaly and Its Removal

6.5.1 Contributions of Mass Flow-rate Fluctuations and Electronic Noise

In order to demonstrate the contributions of the electronic noise and flow rate fluctuations to the $\overline{uu}$ measurements, these contaminations were determined with the help of the proposed measurement method for one of the flow cases generated with a mesh size of 5 mm and a contraction ratio of 14.75. The results depicted in Figure 6.13 clearly show that as the contraction ratio increases, i.e. downstream of the test nozzle, the contributions of electronic noise and the flow rate fluctuations became the main part of the measured longitudinal stresses. These results confirm the results of the analytical considerations in Sections 6.3.1 and 6.3.2. Therefore, both types of measurement contamination can be accepted as the sources of the observed anomalous increase in $\overline{uu}$ in SN-wire measurements and partially in X-wire measurements seen in the literature for grid-generated turbulence strained through high contraction ratio nozzles. There are, of course, different degrees of influence on the measurements, depending on the experimental facility.

6.5.2 Comparison of Correction Methods

In order to appreciate fully the superiority of the proposed method over conventional correction methods, further experiments were conducted with grids of different mesh sizes, but at the same Reynolds number calculated on the basis of the test section diameter and center line velocity. Two methods of correction were applied to the measurements. As can be seen in Figure 6.14a, the uncorrected $\overline{uu}$ data scatter considerably and there is a clear increase in the measured $\overline{uu}$ downstream of the contraction because of...
the amplified effects of random disturbances in the whole flow facility, inherent mass flow rate fluctuations and electronic noise. The application of the conventional correction method devised for single-point measurements reduces this increase, as shown in Figure 6.14b. Nevertheless, this method cannot account for the kinks in the data, whereas the application of the proposed method, depicted in Figure 6.14c, resulted in smoother measurements of $\overline{uu}$. The kinks in the uncorrected measurements can be random mass flow rate bursts, which might occur due to separation on the blower blades or sudden changes of the environmental conditions. However, whatever the reason, the proposed method was proved to be capable of removing its effect from the measurements.

Uberoi (1956) and Han (1988) employed the conventional correction method for their single-point velocity measurements, but they could not correct for the random mass flow rate fluctuations or, moreover, for the mass flow rate fluctuations induced by the grid in the whole flow facility and in the boundary layer approaching the nozzle. Hussain & Ramjee (1976), Ramjee & Hussain (1976) and Tan-atichat et al. (1980) did not apply even the conventional methods of correction. Therefore, their results for nozzles with high contraction ratios should be treated with caution.

### 6.6 Experimental Realization of Measurements Free from Spatial Resolution Problems

The method suggested in Section 6.4.2 requires one SN-wire measurement and two SY-wire measurements at the same locations along the contraction. In order to perform these measurements, an X-wire probe was added in the middle of the two SN-wire probes as shown in Figure 6.11b. One of the SN-wire probes and one of the inclined wires of X-wire probe were utilized to perform the suggested measurements. The second SN-wire probe was used to detect the flow rate fluctuations and the X-wire probe was used in the conventional manner to obtain data to demonstrate the effect of the proposed measurement method.

For correcting for the spatial resolution effect, the ratio of the the wire length to the transverse integral length scale, i.e. $l/L_y$, should be known at all measurement locations. In the contraction, the length scales also become anisotropic in addition to the Reynolds stresses: the longitudinal scales are elongated and the transverse scales are shortened (see for instance Mills & Corrsin 1959). Therefore, the investigated turbulent flow should be divided into two regions in terms of the development of the length scales: the nearly isotropic region before contraction and the axisymmetric anisotropic region in the contraction.

Measurements of transverse correlation measurements with two wires located in a plane perpendicular to the symmetry axis were not performed in the present investigations. Instead, the SN-wire measurements were performed with high sampling frequency, so that autocorrelation of the transverse and longitudinal fluctuations could
Figure 6.14: $\overline{uu}$ component measurements along a converging nozzle of contraction ratio 14.75: (a) Uncorrected measured data; (b) data corrected by using the conventional correction method for single-point measurements; (c) data corrected by using the proposed correction method utilizing two-point correlation measurements. The dashed lines are shown only to aid the eye in observing the trend of the data.
be evaluated and by using Taylor’s frozen turbulence assumption the correlation coefficient functions \( R^*_x \) were calculated. In isotropic turbulence, the following relation between \( R^*_x \) and \( R^*_y \) holds:

\[
R^*_y(r) = R^*_x(r) + \frac{1}{2} r \frac{dR^*_x(r)}{dr},
\]

hence \( R^*_y \) can also be calculated. Using \( R^*_x \) and \( R^*_y \), one can evaluate the measured integral and the dissipation length scales \( \lambda^*_y \) and \( L^*_y \) by using relations (6.51) and (6.62). Knowing the length of the SN-wire, \( l \), the real integral length scales were found by using the correction factor \( K_2 \) (6.58) in equation (6.57). The dissipation scales were also corrected by using equation (6.60).

Different from the isotropic turbulence, as formulated by Batchelor (1946) and Chandrasekhar (1950), in axisymmetric turbulence there are two transverse and two longitudinal elementary correlations and corresponding integral and dissipation length scales. Using RDT, Townsend (1976) showed analytically that the elongation of the longitudinal integral scale of the \( v \) fluctuations is a function of the contraction ratio \( c(x) \):

\[
c_{L_v}(x) = \frac{L_v(x)}{L_v(0)} = c(x)^2 \frac{1}{\mu_2},
\]

where \( \mu_2 \) is the amplification ratio of \( \overline{v^2} \) given by equation (2.40) and it is also a function of the contraction ratio. According to the rapid distortion theory, the longitudinal length scales of \( v \) fluctuations experience the most elongation and this can also be recognized in the measurements of Sjögren & Johansson (1998).

Notwithstanding the fact that the measurements are contaminated, recognizing that the \( v \) fluctuations are the dominant fluctuations in the nozzle, the elongation of the longitudinal integral length scales of \( v \) fluctuations were compared with those estimated with the help of rapid distortion theory (6.65) in Figure 6.15a. The good agreement between the measurements and estimated elongation ratio assures that the rapid distortion theory can be utilized for the evaluation of transverse length scales which are necessary for the wire length corrections. Hence, although the transverse scales were not directly measured in the present investigations, using the measured length scales in the nearly isotropic region before contraction, the development of the length scales can approximately be estimated with the help of RDT.

In order to account for the development of transverse length scales, first it was assumed that two transverse correlation coefficients of \( u \) and \( v \) fluctuations are approximately equal to each other, as already stated by equation (6.42). Hence, it is also assumed that the corresponding dissipation and integral length scales are also approximately equal and they are represented by \( \lambda_y \) and \( L_y \), respectively. In the measurements of Sjögren & Johansson (1998), it can be seen that the transverse length scales of \( u \)- and \( v \) fluctuations are of the same order of magnitude.

For the development of the transverse length scales, the semiquantitative vortex stretching theory of Prandtl (1932) was partially employed. According to this theory, as the
Figure 6.15: Development of the velocity, integral length scales in terms of contraction and elongation ratios: (a) Measured longitudinal integral length scales of $v$ fluctuations compared with length scale development calculated by RDT-theory; (b) development of transverse and longitudinal length scales with mean velocity contraction ratio.

As shown in Figure 6.15a, the measured $L_x$ immediately follows the elongation ratio calculated by equation (6.65) but not the local contraction ratio $c(x)$ calculated from the measured streamwise velocity. Therefore, it is more convenient to use the elongation ratio to calculate the contraction (thinning) of the diameter of longitudinal vortex filaments.

Finally, the local values of transverse dissipation and integral length scales were obtained by multiplying the initial values of those measured before contraction, which were already corrected for the spatial resolution by using $K_2$ in equations (6.57) and (6.58), with the inverse of the square root of the local elongation factor estimated by using equation (6.65):

$$L_y(x) = L_y(0) \frac{1}{\sqrt{c_{L_x}}} \quad (6.66a)$$

$$\lambda_y(x) = \lambda_y(0) \frac{1}{\sqrt{c_{L_x}}} \quad (6.66b)$$

The development of the length scales in Figures 6.8 and 6.15b were calculated as explained above. The anisotropy of the length scales can be seen in Figure 6.15 for the turbulent flow contracted through a nozzle with contraction ratio 14.75. At the exit of the nozzle, the ratio between the longitudinal and transverse integral length scales of $v$ fluctuations approaches almost 100, which indicates at very long and thin turbulent eddies.
6.6 Experimental Realization of Measurements

Free from Spatial Resolution Problems

Figure 6.16: Influence of the correction for the wire separation and the wire length effect: (a) for $c = 14.75$ and (b) for $c = 3.69$. The dashed lines are shown only to aid the eye in observing the trend of the data.

Only after knowing the transverse length scale development, the correction factor $K_1$ (6.55) for SN-wire and inclined-wire measurements can be calculated and consequently the readings can be corrected by using the correction equation (6.49a) for $u^*$ and the correction equation (6.49b) for $v^*$.

Note that the influence of the wire separation is eliminated by three separate measurements at the same locations and the influence of the wire length is corrected. In order to demonstrate the relative contribution of the correction for the influence of wire length, in Figure 6.16 the correction for the wire length was made ineffective by setting $K_1 = 1$, so that only the influence of the wire separation became visible. When Figures 6.16a and b are compared, it is clear that the influence of the correction is almost negligible for the nozzle with the contraction ratio $c = 3.69$. In general, as expected, the correction for the spatial resolution is remarkable in the contraction where the transverse length scales become smaller than the wire length and the separation between the two wires of the X-wire probe. The reduction in $u^*$ in Figure 6.16a is due to the decoupled measurement with the SN-wire probe, which eliminates the ill-correlated cross-talk terms. In the investigated flow cases, the correction for wire length was more visible for $v^*$.

6.6.1 Final Results After Corrections

One set of measurements with a nozzle having a high contraction ratio $c = 14.75$ is provided in this chapter and the rest of the measurements are presented in Chapter 7. Three grids with different mesh sizes, $M = 5, 7,$ and $10$ mm, but with the same soli-dity, were employed to generate turbulence. The data which are corrected for all the artifacts mentioned in this work and the data measured with the X-wire probe and
corrected only for the flow rate fluctuations and electronic noise are presented in Figure 6.17.

The error in the $\overline{\nu}$ measurements performed with the X-wire probe approaches 35% at the exit of the nozzle and the measurements of $\overline{\nu}$ are absolutely wrong in a substantial portion of the nozzle.

After the proposed measurement and correction methods, in the investigated turbulent flows $\overline{\nu}$ did not show any sign of increase in the nozzle, which is in agreement with the theoretical expectations. The differences in the measured and corrected $\overline{\nu}$ with the SN-wire and X-wire probes support the fact that the anomaly of $\overline{\nu}$ is induced by the insufficient spatial resolution of the X-wire probe, which starts to dominate the measurements as the turbulence undergoes a convective acceleration and, consequently, the turbulent stresses and the turbulent length scales become highly anisotropic.

The large correction to $\overline{\nu}$ unfortunately suggests that most of the experiments performed, especially with high contraction ratio nozzles, involve strong deviations from the actual values not only in $\overline{\nu}$ but also $\overline{\nu}$. However, the data of Uberoi (1956) for a nozzle with $c = 16$, which shows signs of the high contraction ratio anomaly, are exceptional. To the author’s knowledge, his experimental set-up was the largest in size and for the mentioned data set the length scale close the exit of the contraction was estimated to be around 5 mm in diameter. The size of the X-wire probe was not reported, nevertheless, if the wire length and the wire separation of his X-wire probe were around 5 mm, one can fairly argue that his data were also contaminated.

As mentioned in Section 6.2, the dynamic equations for $\overline{\nu}$ and $\overline{\nu}$ imply that an increase in $\overline{\nu}$ is rather an exception when turbulence is exposed to positive strain. Hence, based on the analysis and results presented here, it is right to argue that the data in Figure 6.17 represent the actual turbulence behavior of $\overline{\nu}$ and $\overline{\nu}$ when grid-generated turbulence passes through a nozzle with high contraction ratio.

When the work of Tsugé (1984) is reconsidered in the light of the present study, one can see that he reproduced data of Uberoi (1956) and Hussain & Ramjee (1976) for contraction ratios of 16 and 11, respectively, with his model. It is not clear whether the $c = 16$ measurement of Uberoi was contaminated; however, when the size of the flow facility employed by Hussain & Ramjee (1976) is considered, it can fairly be accepted that their data were contaminated. Hence the outcome of the work of Tsugé (1984) is questionable and even misleading.
6.6 Experimental Realization of Measurements
Free from Spatial Resolution Problems

Figure 6.17: Data measured as proposed in the present work in comparison with conventional X-wire measurements. All data presented are already corrected for flow rate fluctuations and electronic noise. See Table 3.2 for other details of the corrected data.
Chapter 7

Effect of Axisymmetric Strain on Grid-generated Turbulence

The effects of axisymmetric strains generated by axisymmetric contraction, expansion and contraction followed by an expansion were investigated experimentally in the AST. An overview of the investigations has already been given in Table 3.2. The inlet $Re_\lambda$ of the experiments varied between 15 and 50 depending on the mesh size and the mean streamwise velocity on the grid, $\bar{U}_{grid}$. The main objective of the analysis presented in this chapter was to investigate the effects of strain rate, $Re_\lambda$ on the developments of anisotropy, length scale, correlation coefficients and spectra. Based on these analyses, important aspects related to turbulence modeling are pointed out.

Another important objective of the measurements was to generate reference data for the validation of turbulence models. However, as already shown in Chapter 6, in order to acquire reliable turbulence quantities, raw hot-wire measurements have to be corrected for various types of contamination. The measured stresses in nozzles with contraction ratios 14.75 and 3.69 were corrected for the influence of flow rate fluctuations, electronic noise and the hot-wire spatial resolution. Measurements performed in other straining sections were corrected only for the flow rate fluctuations and electronic noise. The presented length scales, correlation coefficients and spectra were not corrected for the discussed artifacts, since corrections of these quantities require other methods to be developed which are outside the scope of the present work.

7.1 Non-dimensional Parameters

The non-dimensional parameters which are used hereafter for the analysis of the measured data can be classified into two: those defining the strain field and parameters defining the changes in the flow field as a function of the former parameters. These parameters are discussed below.
The mean strain rate tensor is defined as

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) . \]  

(7.1)

For axisymmetric strain, the mean strain rate tensor is

\[ S_{ij} = \frac{\partial U_i}{\partial x_j} . \]  

(7.2)

The strain rate tensors for axisymmetric contraction and expansion can be written as

\[
S_{ij} = \frac{2}{\sqrt{3}} S \begin{pmatrix}
+1 & 0 & 0 \\
0 & -1/2 & 0 \\
0 & 0 & -1/2
\end{pmatrix},
\]

(7.3a)

\[
S_{ij} = \frac{2}{\sqrt{3}} S \begin{pmatrix}
-1 & 0 & 0 \\
0 & +1/2 & 0 \\
0 & 0 & +1/2
\end{pmatrix},
\]

(7.3b)

respectively, where

\[ S = \left( \frac{1}{2} S_{ij} S_{ij} \right)^{1/2} \]  

(7.4)

is a parameter which can be used as a measure of the mean strain rate. For axisymmetric strain, it reads

\[ S = \frac{\sqrt{3}}{2} \left| \frac{\partial U_1}{\partial x_1} \right| . \]  

(7.5)

An important non-dimensional parameter is the ratio of the time scale of the distortion of the turbulence to that of the mean flow field:

\[ S^* = \frac{S q^2}{\epsilon} . \]  

(7.6)

This parameter is called the mean strain rate parameter and \( q^2/\epsilon \) is called the eddy turnover time. \( S^* \) can be used to judge the rapidity of the applied strain: it is accepted that for \( S^* >> 1 \) the RDT applies to the flow; see, for example, Lee (1985), Lee, Piomelli & Reynolds (1986) and Lee (1990). Hence this parameter will be used to check the relative rapidity of the applied strain.

Another important parameter related to the applied mean strain is the contraction ratio of the nozzles \( c \), i.e. the ratio of the inlet area of the nozzle to the outlet area of the nozzle:

\[ c = \frac{A_{\text{inlet}}}{A_{\text{outlet}}} . \]  

(7.7)

Ideally, \( c \) is equal to the total strain in the flow direction, which can also be written as

\[ c = \frac{A_{\text{inlet}}}{A_{\text{outlet}}} = \exp \int_0^t \frac{\partial U_1}{\partial x_1}(t')dt' \]  

(7.8)
where \( t' \) is the time of flight of the fluid element during strain:

\[
t' = \int_0^x \frac{1}{U_1(x')} \, dx'.
\] (7.9)

Note that for a nozzle with a contraction ratio (it can be called also expansion ratio) \( c \), the average mean velocity gradient along the straining section can be approximated as

\[
\frac{\partial U_1}{\partial x_1} = \frac{U_{1, \text{inlet}}(c - 1)}{L},
\] (7.10)

where \( L \) is the length of the straining section. This equation shows proportionality of the mean strain on the initial velocity and the total strain \( c \). Hence, for straining initially isotropic turbulence, the mean strain rate parameter \( S^* \) can be written as

\[
S^* = \sqrt{\frac{3}{2}} \left| \frac{U_{1, \text{inlet}}(c - 1)}{L} \right| \frac{\lambda_g}{\nu}.\] (7.11)

It is shown in Chapter 5 that \( \lambda_g \propto U_{1, \text{inlet}}^{-1.0} \). Hence, the rapidity of the applied strain in the experiments can be set to higher values by either decreasing \( L \), decreasing \( U_{1, \text{inlet}} \) or increasing the total strain \( c \).

Another set of non-dimensional parameters are related to the anisotropy of turbulence. For simplicity, the ratio between the two normal stresses can be used as a rate of anisotropy during the evaluation of experimental results:

\[
R = \frac{\mu u}{\mu v}.
\] (7.12)

Nevertheless, it is not common to utilize \( R \) for detailed analysis of the stresses. However, the non-dimensional anisotropy parameters (2.69), which are of importance, can be written in terms of \( R \) as follows:

\[
a_{11} = \frac{2(R - 1)}{3(R + 2)},
\] (7.13a)

\[
II_a = \frac{2(R - 1)^2}{3(R + 2)^2},
\] (7.13b)

\[
III_a = \frac{2(R - 1)^3}{9(R + 2)^3}.
\] (7.13c)

Hence, in the subsequent analysis the effect of strain on the anisotropy of turbulence is visualized by using the indicators of the anisotropy (7.13a) and the contraction ratio \( c \).
7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

Data presented in this section were measured in nozzles having contraction ratios 1.27, 3.69 and 14.75. After analyzing the rapidity of the applied strains, the development of stresses and their anisotropies are analyzed. The developments of length scales, correlation coefficients and spectra are presented. The effects of total strain $c$ and $Re_\lambda$ on all the investigated quantities are discussed.

7.2.1 Rapidity of the Applied Strains

The rapidity of the applied strains in the present study and the important strain experiments in the literature are quantified by using the mean strain rate parameter (7.6). For the calculation of $S^*$, the average strain rate along the whole nozzle was taken and the turbulent energy and the dissipation were chosen to be those at the inlet of the contractions. As can be seen in Figure 7.1, except for the experiments of Leuchter (1993), in all the experiments available in the literature $S^*$ values were larger than 1. In the experiments of Uberoi (1956) with $c = 16$, $S^*$ reaches values close to 200. In the present study, measurements in a wide range of $S^*$ were performed by applying nozzles with different contraction ratios and different grid Reynolds numbers. Among the present measurements, $c = 14.75$ measurements can fairly be accepted to be rapid enough such that RDT applies to these measurements. Experiments with nozzles having $c = 3.69$ nozzles delivered $S^*$ values slightly above 1; therefore, only after being compared with RDT can their rapidity be judged. The experiments performed with the $c = 1.27$ nozzle are far from being rapid.

7.2.2 Development of Reynolds Stresses and Their Anisotropies

In Chapter 6, it was shown that the mean square of the axial fluctuations $\overline{uu}$ should decrease and the transverse one $\overline{vv}$ should develop to higher levels than $\overline{uu}$. Hence, when turbulence is distorted with symmetric nozzles, stress developments in Group-1 and Group-3 are possible. In order to give a picture of the dimensional data, example measurements of Reynolds stresses in three different contractions are provided in Figure 7.2. An immediate observation is the increasing anisotropy with increasing contraction ratio.

Examples of mean velocity gradients in the tested nozzles are provided in Figure 7.3. It can be seen that in the present experiments the velocity gradient along the contractions was not kept constant. Note that according to equation (7.10), the amplitude of $d\overline{U}_1/dx_1$ along one nozzle changes with the inlet mean velocity. This change is demonstrated for $c = 3.69$ in Figure 7.3.
7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

The effect of contraction ratio $c$ on the anisotropy of Reynolds stresses is shown on the AI-map in Figure 7.4. Broadly speaking, turbulence comes close to the two-component anisotropic state with increasing $c$. When the anisotropy parameters $a_{11}, II_a$ and $III_a$ of the nine experiments (involving three contractions) are plotted with respect to the local contraction ratio $c(x)$ as in Figure 7.5, it can be said that the anisotropy of turbulence is dominantly controlled by the contraction ratio, i.e. total strain. Moreover, the comparison of the measurements with the modified RDT theory of Sreenivasan & Narasimha (1978), which was discussed in Section 2.4, shows very good agreement.

In the measurements presented so far, the nozzles were located 0.225 m away from the grid. This distance corresponds to 45, 32 and 22.5 mesh sizes for grids with 5, 7 and 10 mm mesh sizes, respectively. According to Corrsin (1963), these distances were just around the homogenization length, 30 mesh size. In order to check the validity of the already presented results, another set of measurements were made with contractions located 0.525 m away from the grid, i.e. 105, 75 and 52.5 mesh sizes away from the grid. The comparison of these with the former measurements for $c = 14.75$ is provided in Figure 7.6. These measurements delivered data almost identical with those from the former measurements. Moreover, as can be seen in Figure 7.1, although $S^*$ for $c = 14.75$ experiments varies from 7 to 70, the anisotropy developments of those experiments compared in Figure 7.6 do not show any remarkable difference from each other.

In order to check the effect of grid Reynolds numbers, $c = 3.69$ experiments were performed at two different speeds and three mesh sizes. The comparison given in Figure 7.7 shows almost no difference in the anisotropies of the generated turbulence. The good agreement anisotropy of turbulence generated by a moderate contraction
Figure 7.2: Stress development in nozzles with different contraction ratios and grid Reynolds numbers.
7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

Figure 7.3: Typical forms of mean velocity gradients in the tested nozzles.

ratio $c = 3.69$ with RDT becomes more visible in Figure 7.7.

A close look to the $c = 1.27$ case in Figure 7.8 show a strong dependence of anisotropy development on initial anisotropy values. Such an effect is not easy to observe in experiments with $c = 14.75$ and 3.69, because the large change in the anisotropy due to rapid straining through these nozzles suppresses the small differences in the initial anisotropy.

Concerning the dependence of the anisotropy on $S^*$ and $Re_\lambda$, it is worth noting that DNS of axisymmetric strained turbulence of Johansson and Hallbäck (1994) showed that the rapid pressure-strain term $\Pi_{ij}$ had a very weak dependence on $Re_\lambda$ in the range 8.6–17 and a strong $S^*$ dependence in the range 1–9. They found that the dependence of $\alpha_{ij}$ on $S^*$ decreases very fast as $S^*$ increases; in other words, the effect of rapidity is small where rapid effects dominate. Hence, observations made in the present investigations parallel those of Johansson and Hallbäck (1994).

The present measurements show that axisymmetric contraction experiments having $S^*$ values slightly above unity can be well approximated with RDT. Experimental results show that for $c = 3.69$ and especially for $c = 14.78$, the total strain in the flow direction $c(x)$ is the only relevant parameter describing the anisotropy of the turbulent stresses, as expected from RDT. To the author’s knowledge, for the first time in the literature, the measurements in a nozzle having a high contraction ratio confirm the RDT. Moreover, contrary to the widely held belief that RDT holds only under extreme flow conditions, measurements show that the conditions under which RDT applies are not
that marginal as was accepted by some turbulence modelers. Because RDT delivers good results for a wide range of $S^*$ values which are common to many practical flows, the present data together with RDT constitute an important class of verification source for turbulence models.

### 7.2.3 Axisymmetry of the Turbulent Flow in Contracting Nozzles

It is necessary to check the axisymmetry of the turbulent flow field which is generated. Three non-dimensional parameters can be used as indicators of the axisymmetry: the development of the off-diagonal Reynolds stress components relative to the normal stress components, skewness factor and flatness factor. In Chapter 4, these were already investigated in detail for grid-generated turbulence and turbulence in straining sections. Different to the data provided in Chapter 4, the effect of the distance between the grid and the contraction is presented in Figure 7.9. For the nozzle with $c = 14.75$ there is a remarkable reduction in $\overline{uv}$ with increased distance between the nozzle and grid. However, it should be noted that in order that off-diagonal stress components become effective in total energy balance, the off-diagonal components of the strain rate tensor $S_{ij}$ should exist, which was not the case for axisymmetric strain. Hence, although the experiments with $c = 14.75$ nozzles show higher relative $\overline{uv}$ values, they
Figure 7.5: Development of anisotropy of the stresses as a function of the local contraction ratio in comparison with RDT prediction. Nozzles are located at $x_{cont} = 0.225$. 

Contraction location = 0.225 m

- $c=1.27$, M=5,7,10 mm, $U_{grid}=2.1$ m/s
- $c=3.69$, M=5,7,10 mm, $U_{grid}=2.2$ m/s
- $c=14.75$, M=5,7,10 mm, $U_{grid}=2.5$ m/s

RDT prediction with $R_0=1.7$
Figure 7.6: Comparison of the anisotropy development of the stresses through the nozzle with \( c = 14.75 \) for different locations of the nozzles away from the grid.
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Figure 7.7: Comparison of anisotropy development of the stresses through the nozzle with $c = 3.69$ for different initial grid Reynolds numbers and comparison with RDT predictions. $x_{cont} = 0.225$
Figure 7.8: Comparison of anisotropy development of the stresses through the nozzle with \( c = 1.27 \) for different initial grid Reynolds numbers and corresponding initial anisotropies. \( x_{cont} = 0.225 \).
7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

Figure 7.9: Development of off-diagonal stress component $\overline{uv}$ relative to the normal stresses.

can be considered to be axisymmetric.

Although the accuracy of $\overline{uv}$ fluctuations suffers due to the wire length effect, it is still interesting to observe the change in the anisotropy values when $\overline{uv}$ data are accounted for. The anisotropy values on the AI-map in Figure 7.10 depict that the flow represents kinematic axisymmetry. If this plot is compared with Figure 7.4, one can realize that there is no remarkable deviation from the left axisymmetry edge of the AI-map. Comparison of the anisotropy amplitudes, which is not presented here, also did not show any significant change.

Some representative figures of skewness and flatness factors for the streamwise velocity fluctuations for different measurements are provided in Figure 7.11. In this figure, two sets of data are presented: they differ only in the distance between the grid and the contraction. The skewness factors in Figure 7.11a show that turbulence after the grid enters the contraction before reaching a perfect symmetric distribution and becomes symmetric in the nozzles. However, when nozzles are situated further away from the grid, as can be seen in Figure 7.11c, the turbulence becomes symmetric before and along the nozzles. The flatness factors in Figures 7.11b and d display a similar behavior: the flatness factors for the nozzle located further downstream are more stable and have less tendency to increase compared with those in Figure 7.11b.

Mills & Corrsin (1959) observed an increase of skewness factor along the contraction and concluded that their contraction cannot be regarded as homogeneous. In the
7. Effect of Axisymmetric Strain on Grid-generated Turbulence

Figure 7.10: Courses of the second and third invariants of anisotropy stress tensor on the AI-map in converging nozzles with different contraction ratios when $\overline{uv}$ is included in the calculation: (a) $c=1.27$; (b) $c=3.65$; (c) $c=14.75$. $x_{cont} = 0.225$. Arrows indicate the flow direction.

In the present study, such a trend in skewness factor cannot be observed, so it is concluded that both distances between the nozzles and the grid are sufficiently large to have a homogeneous turbulent flow. Moreover, turbulent flows in the nozzles maintained their homogeneous state.

7.2.4 Development of Correlation Functions and Length Scales

As already mentioned in Chapter 2, in axisymmetric turbulence there are four correlation coefficients, $R_{uu_x}$, $R_{uu_y}$, $R_{vv_x}$, and $R_{vv_y}$. From these, four integral and four dissipation scales can be derived. In the present work, autocorrelation coefficients were measured and by using the frozen turbulence assumption of Taylor (1938) these were converted to the streamwise correlation coefficients $R_{uu_x}(x)$ and $R_{vv_x}(x)$. The integral length scales $L_{ux}$ and $L_{vx}$ and the dissipation length scales $\lambda_{ux}$ and $\lambda_{vx}$ were calculated by using these correlation coefficients. It should be noted that the correlation coefficients and the length scales were not corrected as was done for turbulent stresses. For the correction of correlation coefficients, different methods should be developed and applied. Hence the results at high contraction ratios should be interpreted with care. In order to reduce the uncertainty and ambiguity due to wire separation effects, only $v$ fluctuations were taken from X-wire measurements and the necessary $u$ fluctuations
7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

Figure 7.11: Examples of skewness and flatness factors of $u$ fluctuations along the nozzles: (a) and (b) for $x_{\text{cont}}=0.225$ m; (c) and (d) for $x_{\text{cont}}=0.425$ m and 0.525 m. Corresponding grid velocities are given in Figure 7.5.

were taken from SN-wire measurements.

It is well known from RDT and vortex stretching that the longitudinal length scales increase along the contractions. The amount of elongation of these length scales along the contractions is quantified by the following elongation parameters

$$
\frac{c L_{ux}}{L_{ux}(0)} = \frac{L_{ux}(x)}{L_{ux}(0)}, \quad \frac{c L_{vx}}{L_{vx}(0)} = \frac{L_{vx}(x)}{L_{vx}(0)}, \quad \frac{c \lambda_{ux}}{\lambda_{ux}(0)} = \frac{\lambda_{ux}(x)}{\lambda_{ux}(0)}, \quad \frac{c \lambda_{vx}}{\lambda_{vx}(0)} = \frac{\lambda_{vx}(x)}{\lambda_{vx}(0)}. \tag{7.14}
$$

The developments of the length scales in nozzles with $c = 3.69$ and 14.75 are presented in Figure 7.12. The increase in the integral length scale of $v$ fluctuations, $L_{vx}$, is the best indication of stretching in the flow direction and the elongation of this quantity is well predicted with the RDT. The integral length scale of $u$ fluctuations, $L_{ux}$, also increased.
and, as can be seen in Figure 7.12a, the amount of elongation can be approximated by $\sqrt{c}$. This increase with $\sqrt{c}$ was also predicted by the simple vortex stretching theory of Prandtl (1932, 1933), which was sketched earlier in Figure 2.5. The deviation of $c_{Lvx}$ from $\sqrt{c}$ at high contraction ratios is related to the increasing effect of flow rate pulsations in the measured correlation coefficients and the spatial resolution problems of the X-wire probe.

As can be seen in Figure 7.12b, the dissipation scales also develop similarly to the integral length scales. The elongation of $\lambda_{ux}(0)$ can be better approximated by local $c(x)$, whereas the elongation of $\lambda_{vx}(0)$ does not show a simple relation with the local contraction ratio $c(x)$. To a great extent, the developments of length scales along the tested nozzles confirm the vortex stretching concept.

The correlation coefficients at the inlet and outlet of the nozzles are compared in Figures 7.13a-c for the three applied contraction ratios. This comparison also clearly shows that the turbulence and especially transverse fluctuations maintain their correlation at longer distances as the contraction ratio increases. The correlation coefficients versus normalized distance with the corresponding integral length scales are shown on the right in Figure 7.13. From this figure, it can be concluded that the correlation coefficients can be scaled along the contraction by the corresponding integral scales, which again scales with the contraction ratio as shown before.
Figure 7.13: Development of measured correlation coefficients $R_{uu}(x)$ and $R_{vv}(x)$ along the contractions on the left as a function of distance on the symmetry axis and on the right as a function of normalized distance on the symmetry axis. (a) $c = 1.27$; (b) $c = 3.69$; (c) $c = 14.75$. 

7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

(a) $c=1.27, M=10 \text{ mm}, U_{grid}=2.1 \text{ m/s}, x_{cont}=0.225 \text{ m}$

(b) $c=3.69, M=10 \text{ mm}, U_{grid}=2.2 \text{ m/s}, x_{cont}=0.225 \text{ m}$

(c) $c=14.75, M=10 \text{ mm}, U_{grid}=2.5 \text{ m/s}, x_{cont}=0.225 \text{ m}$
7. Effect of Axisymmetric Strain on Grid-generated Turbulence

7.2.5 Development of One-dimensional Power Density Spectra

The one-dimensional power density spectra of $u$ and $v$ fluctuations $E_u(f)$ and $E_v(f)$ are among the measured quantities along the symmetry axis of nozzles. Formulation of the one-dimensional spectra has already been described in Section 5.3.

In Figure 7.14, an example measured normalized one-dimensional spectrum at the inlet of the nozzle is compared with the theoretical spectrum of isotropic turbulence in equation (5.20). As already discussed in Chapter 5 and demonstrated in Figure 5.18, the measured spectrum deviates from the analytical spectrum at high wavenumbers and this difference increases with decreasing $Re_\lambda$. In order to check the effect of wire length on the measured spectrum, the measurements were corrected by using the correction suggested by Wynagard (1968). For the correction, the theoretical spectrum given by equation (5.20) was used. As can be seen in Figure 7.14, the difference between measured and corrected spectra is not significant, at least before the contraction. Nevertheless, in the subsequent spectrum analysis the wavenumber and the frequency corresponding to the wire length are depicted on each spectrum with a cross sign, i.e. spectra should be interpreted with caution for values higher than this critical quantity.

The spectra $E_u(f)$ and $E_v(f)$ are compared with each other for each nozzle in Figure 7.15, where the spectra are normalized with the corresponding turbulent stress values, so that the integrals of both spectrum curves give unity. Thus changes in shape
rather than amplitude can be compared. In agreement with RDT and previous measurements, for instance Warhaft (1980), \( u \) spectra are more affected than the \( v \) spectra, which hardly change in form. Moreover, the energies in \( u \) spectra shift to higher frequencies.

In order to see the distribution of the energy among wavelengths, it is common practice to look at \( E_u(k_1)k_1 \) and \( E_v(k_1)k_1 \). Such a comparison is provided in Figure 7.16. At first glance, it can be seen that the peaks of the spectra move towards lower wavenumbers along the contractions. This indicates that the eddies carrying most of the energy elongate. When \( E_u(k_1)k_1 \) and \( E_v(k_1)k_1 \) are compared, it can be seen that the peaks of \( E_v(k_1)k_1 \) spectra shift faster towards a lower wavenumber than those of \( E_u(k_1)k_1 \).

Note that these observations are indirect confirmations of the length scale developments given in Figure 7.12. For nozzles with \( c = 3.69 \) and 14.75, it can be seen that \( E_u(k_1)k_1 \) drops for all the wavenumbers, whereas the spectrum of transverse fluctuations \( E_v(k_1)k_1 \) shows a continuous decrease in the high wavenumber range and a continuous increase in the low wavenumber range. These results indicate that in addition to the lengths of the vortex filaments, the vorticity of the vortex filaments increases, as was predicted by RDT.

The anisotropy of the one-dimensional spectra can be examined when the ratio of \( E_u(k_1) \) to \( E_v(k_1) \) is plotted versus normalized wavenumber as shown in Figure 7.17. In the same plots the theoretical ratio for isotropic spectra, which is 1.333, is also provided (see Pope 2000, p. 229, for details). This figure clearly shows that the anisotropy is more emphasized in the low wavenumber range and turbulence tends to be isotropic in the high wavenumber range. In particular, the order of magnitude difference between the anisotropy values for \( c = 3.69 \) and 14.75 is significantly large. The high ratios at the exit of the nozzle having \( c = 14.75 \) indicate the vortex filaments are well aligned parallel to the symmetry axis and almost all of the energy of turbulence is transferred to these filaments, which are responsible for \( v \) fluctuations.

Fully normalized spectra are shown in Figure 7.16. Similarly to the correlation coefficients, the normalized spectra show almost no change along the nozzles. Thus, broadly speaking, for cases having \( S^* > 1 \) and nearly isotropic at the inlet of the contractions, the knowledge of the length scales, turbulent stresses, correlation coefficients and spectra at the inlet of converging nozzles is sufficient to predict the in development along the nozzles with the help of RDT.

As shown in Figure 7.19, especially the amplitudes of the normalized \( v \) spectra at the outlet of the nozzles are higher for higher \( \text{Re}_\lambda \) at high wavenumbers. However, except for the nozzle with \( c = 1.27 \), the anisotropy values for \( c = 3.69 \) and 14.75 in Figures 7.5-7.7 do not reflect a considerable dependence on \( \text{Re}_\lambda \). This comparison shows that turbulent stresses are extensively composed of the energies of eddies in the low wavenumber range and, as shown in Figure 7.17, the anisotropy occurs mainly in this range. Since \( \text{Re}_\lambda \) is a dissipation-related quantity, it is clear that the \( \text{Re}_\lambda \) dependence occurs first at high wavenumbers. Note that present measurements are in the \( \text{Re}_\lambda \) range 15–50; however, this dependency is expected to be greater for higher \( \text{Re}_\lambda \).
Figure 7.15: Development of one-dimensional velocity fluctuation spectra: $E_u(f)/\bar{uu}$ on the right and $E_v(f)/\bar{vv}$ on the left for nozzles with (a) $c = 1.27$, (b) $c = 3.69$ and (c) $c = 14.75$. 
7.2 Effect of Axisymmetric Contraction on Grid-generated Turbulence

Figure 7.16: Development of one-dimensional velocity fluctuation spectra as a function of wavenumber: $E_u(k_1)k_1$ on the right and $E_v(k_1)k_1$ on the left for nozzles with (a) $c = 1.27$, (b) $c = 3.69$ and (c) $c = 14.75$. 
Figure 7.17: Development of the ratio between $E_v(k_1)$ and $E_u(k_1)$ along the nozzles with (a) $c = 1.27$, (b) $c = 3.69$ and (c) $c = 14.75$. 

**Legend:**
- $c$: Strain parameter
- $M$: Nozzle gap
- $U_{grid}$: Grid velocity
- $x_{cont}$: Contour spacing
Figure 7.18: Normalized one-dimensional velocity fluctuation spectra: \( E_u(k_1)/\overline{uu}L_{x_u} \) on the right and \( E_v(k_1)/\overline{vv}L_{x_v} \) on the left for nozzles with (a) \( c = 1.27 \), (b) \( c = 3.69 \) and (c) \( c = 14.75 \).
Figure 7.19: Effect of Re_λ at the inlet on the normalized one-dimensional velocity fluctuation spectra: \( E_u(k_1)/uuL_{x_u} \) on the right and \( E_v(k_1)/vvL_{x_v} \) on the left for a nozzle with \( c = 14.75 \) (a) at the inlet of the nozzle and (b) at the outlet of the nozzle.
7.3 Effect of Axisymmetric Expansion on Grid-generated Turbulence

Axisymmetric turbulence at the right edge of the AI-map was investigated by using the negative axisymmetric strain of grid-generated turbulence. A negative strain rate was produced by an expansion (diffuser) having a total expansion angle of 7°, which prevented separation in the diffuser. As far as the rapidity of the strain is concerned, the expansion data presented here contribute expansion knowledge at a low $S^*$ range of around 0.2. In other words, the turbulent field generated does not represent a case for which RDT is valid. To the author’s knowledge, due to separation problems in diffusers, rapid distortion of axisymmetric turbulence in an expansion can only be achieved by DNS simulations; for instance, Lee (1985) performed DNS of axisymmetric expansion for $S^*$ ranging from 0.96 to 96. He showed that rapid expansion of axisymmetric turbulence delivers the minimum achievable anisotropy of stresses, which is in contrast to the rapid contraction.

7.3.1 Development of Reynolds Stresses and Their Anisotropies

Examples of $\overline{uu}$ and $\overline{vv}$ measurements along the expansion are shown in Figure 7.20. In contrast to the contraction cases, the changes in the stresses are very slow and look like ordinary decay of grid-generated turbulence. The continuous decay is also in contrast to the RDT prediction of an increase in $\overline{uu}$ and a decrease in $\overline{vv}$ and $\overline{ww}$ because of the the production terms $P_{11}$ and $P_{22}$ in equations (2.80) and (2.81). RDT also predicts that the rapid pressure-strain term $\Pi_{ij}$ damps the generation of anisotropy. Nevertheless, as mentioned earlier, applied strains are not rapid enough to neglect the dissipation and the slow pressure-strain terms. The slow strain expansion simulations of Lee (1986) showed that the contribution of the slow pressure-strain term is negligible, whereas the dissipation term is at the same order as the rapid pressure-strain term and their sum compensates the production of $\overline{uu}$. Hence the continuous decrease of the stresses in Figure 7.20 is not surprising for low (slow) strains.

The change of anisotropy experiencing negative strain can be seen better either on the AI-map in Figure 7.21 or as a function of $c(x)$ in Figure 7.22. Note that in Figure 7.22, the anisotropies do not follow a unique line like those of the converging nozzles with $c = 3.69$ and 14.75. Moreover, in both of the figures it can be seen that the change in the anisotropy is slower for higher Re$\lambda$. In other words, the higher the Re$\lambda$ of the initial isotropic turbulence, the harder it is for the turbulence to become anisotropic. Since dissipation terms are as dominant as the production and the rapid pressure-strain terms, the dependence of anisotropies on Re$\lambda$, which is a representative number for the dissipation process, ensues. In spite of the fact that Re$\lambda$ and the anisotropy range are not wide, these measurements correspond to important test cases for testing a turbulence since they exactly lie on the right edge of the AI-map.
7. Effect of Axisymmetric Strain on Grid-generated Turbulence

Figure 7.20: Stress development in the expansion for different grid Reynolds numbers.

Figure 7.21: Courses of the second and third invariants of anisotropy stress tensor on the AI-map in expansions for different grid Reynolds numbers: (a) Re=1720; (b) Re=1650; (c) Re=2310; (d) Re=3300. $x_{exp} = 0.325$ m. The arrow indicates the flow direction.
7.3 Effect of Axisymmetric Expansion on Grid-generated Turbulence

Figure 7.22: Development anisotropy of the stresses in the expansion with $c = 0.72$ for different inlet $Re_\lambda$ as a function of the local contraction ratio. $x_{exp} = 0.325$ m.
7.3.2 Development of Correlation Functions and Length Scales

Figure 7.23 shows that the longitudinal integral length scales $L_{ux}$ are elongated along the contraction. The dissipation scales $\lambda_{ux}$ and $\lambda_{vx}$ are the second and third effected scales, whereas $L_{vx}$ values are almost not effected. According to the vortex stretching approach, all length scales in the longitudinal direction should become shorter. However, applied strains were not rapid enough for vortex stretching approach to be valid. Anyhow, under the dominating action of dissipation, the integral and dissipation length scales elongate similar to those of the decaying nearly isotropic turbulence presented in Chapter 5, but the elongation of length scales are relatively suppressed in the expansion when compared to the decay of grid-generated turbulence.

In Figure 7.23, after $c(x) \approx 0.72$, which corresponds to $x = 0.75$ m, the developments of length scales show anomalies because of end effects. Therefore, the correlation coefficients and spectra will be analyzed up to that point. The correlation functions at the inlet and outlet of the expansion are presented for one experimental case in Figure 7.24. The longitudinal correlation coefficient $R_{uu}(x)$ changed most in its shape. As can be seen in Figure 7.24b, when the distance $x$ is normalized with the corresponding integral scale, all correlation functions almost collapse on each other.

7.3.3 Development of One-dimensional Power Density Spectra

The spectra normalized with the turbulence stresses in Figure 7.25 depict the effect of dissipation as a decreasing contribution of the high-frequency fluctuations and slightly
7.4 Effect of Successive Axisymmetric Straining on Grid-generated Turbulence

In this section, investigations on successive straining are reported. Successive straining was generated by attaching a symmetric diffuser (expansion-2) with \( c = 0.7 \) to the exit of the contraction with \( c = 3.69 \). The non-dimensional strain \( S^* \) in the contraction...
7. Effect of Axisymmetric Strain on Grid-generated Turbulence

Figure 7.25: Development of one-dimensional velocity fluctuation spectra along the expansion with $c = 0.72$: (a) $E_u(f)$ and (b) $E_v(f)$.

Figure 7.26: Development of one-dimensional velocity fluctuation spectra as a function of wavenumber along the expansion with $c = 0.72$: (a) $E_u(k_1)k_1$ and (b) $E_v(k_1)k_1$. 
7.4 Effect of Successive Axisymmetric Straining on Grid-generated Turbulence

Figure 7.27: Development of the ratio between $E_v(k_1)$ and $E_u(k_1)$ along the expansion with $c = 0.72$.

part and in the expansion are on average 4.5 and 3.5, respectively. Compared with $S^* \approx 0.2$ of the pure expansion case, $S^* \approx 3.5$ generated in the expansion part is relatively rapid. According to equation (7.11), such an order of magnitude difference in $S^*$ cannot be obtained for nearly isotropic grid-generated turbulence; nevertheless, the turbulence at the exit of the contraction is anisotropic and has low dissipation values, which increases the value of $S^*$ according to its definition (7.6). The Re$_\lambda$ range of the experiments was 10–25. The main objectives of successive symmetric strain investigations are to check the path of the anisotropy on the AI-map and determine out the peculiarities of the expansion of anisotropic axisymmetric turbulence which would lie on the left edge of the AI-map prior to expansion.

7.4.1 Development of Reynolds Stresses and Their Anisotropies

Overall development of the mean velocity and stresses is shown in Figure 7.30. Immediately after contraction, the $\bar{v}\bar{w}$ component starts to decrease rapidly and the $\bar{u}\bar{w}$ component starts to increase slowly. The production terms $P_{11}$ and $P_{22}$, already given in equations (2.80) and (2.81)

$$P_{11} = -2\bar{u}_1\frac{\partial \bar{U}_1}{\partial x_1}, \quad P_{22} = \bar{u}_2\frac{\partial \bar{U}_1}{\partial x_1},$$
7. Effect of Axisymmetric Strain on Grid-generated Turbulence

Figure 7.28: Development normalized one-dimensional velocity fluctuation spectra along the expansion with $c = 0.72$: (a) $E_u(k_1)/\overline{uu}L_{x_u}$ and (b) $E_v(k_1)/\overline{vv}L_{x_v}$.

explain the stress development in the expansion part, when $\frac{\partial U_1}{\partial x_1} < 0$; $P_{11}$ becomes a source term whereas $P_{22}$ behaves like a sink term. However, the faster decrease of $\overline{vv}$ (i.e. $u_2 u_2$) shows that the pressure-strain term and the dissipation term damp the increase in $\overline{uu}$ and augment the decrease in $\overline{vv}$. It is known from RDT that, in addition to the dissipation term, the rapid part of the pressure-strain term in the transport equation for $\overline{vv}$ becomes a sink term when turbulence is expanded (see Pope (2000), p. 418). Nevertheless, more insight into the effect of these terms can be gained only from DNS data. The corresponding developments of $a_{11}$ along the straining sections are presented in Figure 7.31.

Since homogeneous turbulence constitutes an initial value problem, the effect of expansion on anisotropic turbulence can be treated separately. Thus, the path of anisotropy invariants on the AI-map are separately plotted for the contraction and the expansion in Figure 7.31. Broadly speaking, expansion pulls the anisotropic turbulence on the left edge of the AI-map back to the isotropic state along the left edge. For better comparison with the pure-expansion case $a_{11}$ is also plotted as a function of $c(x)$ in Figure 7.33 along each straining section. When compared with Figure 7.22, it can be seen that the absolute change in the anisotropy is much greater than in the pure-expansion case. This difference between the two expansions reflects either the effect of the higher $S^*$ values and/or the higher initial anisotropy values. However, since both the production and the rapid pressure-strain terms are proportional to $\frac{\partial U_1}{\partial x_1}$, most probably the increased rate of change is principally due to the higher $S^*$ values.

Again, when Figure 7.33 is compared with Figure 7.22, in the pure-expansion case the dependence of $a_{11}$ values on $Re_\lambda$ is visible whereas in the present case they follow more or less the same path. Whereas turbulence becomes less anisotropic downstream of expansion, the anisotropy values do not show any dependence on $Re_\lambda$, as can be
7.4 Effect of Successive Axisymmetric Straining on Grid-generated Turbulence

7.4.2 Development of Correlation Functions and Length Scales

The length scale developments plotted in Figure 7.34 clearly show the effects of contraction and the expansion which can be deduced from the vortex stretching concept sketched in Figure 2.5: first the integral scales in \( L_{ux} \) and \( L_{vx} \) are stretched in the contraction and then shortened in the expansion. The amplification of the \( L_{ux} \) and \( L_{vx} \) downstream of expansion are signs of low-frequency pulsation due to the exit flow effects. Note that the influence of such global pulsations on evaluated data is corrected only for the stresses. Figure 7.34a and b show that in the contraction \( L_{vx} \) and \( \lambda_{vx} \)
Figure 7.30: Measured mean quantities along the axisymmetric successive straining duct for different grid Reynolds numbers: (a) mean velocity and (b) Reynolds stresses.

Figure 7.31: Development of stress anisotropy $a_{11}$ along the axisymmetric successive straining duct.
Figure 7.32: Development of invariants of stress anisotropy tensor on the AI-map along the axisymmetric successive straining duct: (a) along the contraction part and (b) along the expansion part. Arrows indicate the flow direction.

Figure 7.33: Development of stress anisotropy $a_{11}$ along the axisymmetric successive straining duct as a function of contraction ratio calculated separately for the contraction part and the expansion part. Arrows indicate the flow direction.
become longer in the contraction than those of $u$ fluctuations and they start to cease as soon as turbulence is expanded. The ratio of the longitudinal to transverse length scales in Figure 7.34c and d depicts the clear difference between the strained turbulence and the isotropic turbulence in terms of the length scale development.

The correlation coefficients are shown in Figure 7.35 only for the expansion part to avoid repetition of the results in Section 7.2. The shortening of the length scales from the inlet to the outlet of the expansion can clearly be seen in Figure 7.35a. Similar to the previously investigated cases, the fully normalized correlation coefficients in Figure 7.35b imply that the development of the correlation coefficients can be well approximated when the corresponding integral length scales are known or modeled.

### 7.4.3 Development of One-dimensional Power Density Spectra

The peculiarities of the one-dimensional spectra are analyzed only within the expansion part. In contrast to the contraction case, normalized $v$ spectra slightly change their shape along the straining section, as shown in Figure 7.36. The spectra plotted as a function of wavenumber in Figure 7.39 imply that while $u$ fluctuations gain energy at all wavenumbers (at all scales), $v$ fluctuations lose energy of the large scales (in low wavenumber range) and the small scales (at high wavenumbers) gain energy along the expansion. Remember from Figure 7.16 that an opposite development occurs along the contraction, i.e. $u$ fluctuations lose energy at all wavenumbers (at all scales), whereas $v$ fluctuations lose energy of the small scales (at high wavenumbers) and the large scales (at low wavenumbers) gain energy along the contraction.

The development of the ratio of the spectra $E_v(k_1)$ to $E_u(k_1)$ shown in Figure 7.38 implies that the turbulence returns back to an isotropic state mainly at the low normalized wavenumbers, to be specific at the wavenumber range around the integral length scale. Similarly to the contraction and pure expansion cases, fully normalized spectra along the expansion part collapse on each other as shown in Figure 7.39. Hence the development of the spectrum along the expansion can be approximated when the initial spectrum is known and the change in integral length scales is known or modeled.

In Figure 7.40, the effect of $Re_\lambda$ on the fully normalized spectra is examined. The $Re_\lambda$ dependence of the spectra at the outlet of the expansion becomes weaker compared with the dependence at the inlet of the expansion (outlet of contraction). As expected, the dependence is at the high-wavenumber range.
Figure 7.34: Development of measured (a) integral length scales $L_{u_x}$ and $L_{v_x}$, (b) dissipation scales $\lambda_{u_x}$ and $\lambda_{v_x}$, (c) the ratio of integral length scales and (d) the ratio of dissipation scales, along the axisymmetric successive straining duct.
Figure 7.35: Development of measured correlation coefficients $R_{uu}(x)$ and $R_{vv}(x)$ along the expansion ($c = 0.70$) located after the contraction ($c = 3.69$) as a function of (a) distance on the symmetry axis and (b) normalized distance with corresponding integral length scale.

Figure 7.36: Development of one-dimensional velocity fluctuation spectra along the expansion ($c = 0.70$) located after the contraction ($c = 3.69$): (a) $E_u(f)/\overline{uu}$; (b) $E_v(f)/\overline{vv}$. 
Figure 7.37: Development of one-dimensional velocity fluctuation spectra as a function of wavenumber along the expansion ($c = 0.70$) located after the contraction ($c = 3.69$): (a) $E_u(k_1)k_1$; (b) $E_v(k_1)k_1$.

Figure 7.38: Development of the ratio between $E_v(k_1)$ and $E_u(k_1)$ along the expansion ($c = 0.70$) located after the contraction ($c = 3.69$).
Figure 7.39: Development of normalized one-dimensional velocity fluctuation spectra along the expansion \((c = 0.70)\) located after the contraction \((c = 3.69)\): (a) \(E_u(k_1)\bar{u}u/L_{x_u}\); (b) \(E_v(k_1)\bar{v}u/L_{x_v}\).
7.4 Effect of Successive Axisymmetric Straining on Grid-generated Turbulence

Figure 7.40: Effect of Reλ on the normalized one-dimensional velocity fluctuation spectra, \( E_u(k_1)/\nu L x_u \) (right) and \( E_v(k_1)/\nu v L x_v \) (left): (a) at the inlet of the expansion (\( c = 0.70 \)) located after the contraction (\( c = 3.69 \)) and (b) at the outlet of the same expansion. Given Reλ values are calculated at the inlet of the contraction.
Chapter 8

Predictions with Various Turbulence Models and Comparison with Experiments

8.1 Selected Test Cases and Details of Predictions

It was shown in Chapter 6 that hot-wire measurements need corrections, especially when measured along contractions having high contraction ratios. Correction methods are also described in this chapter. In Chapters 5 and 7 the corrected measurements are analyzed. Thus, one of the major goals of the present study, name by the generation of axisymmetric turbulence data for the validation of various turbulence models, has been achieved. Next, the measured data are used to check the prediction performance of the anisotropy-invariant model (AI-model) of Jovanović et al. (2003) for axisymmetric turbulence. The AI-model was constructed in order to account for the anisotropy of turbulence. In order to emphasize this feature of the AI-model, predictions are also performed with the RDT and $k-\epsilon$ model, and all predictions are compared with measured data. The RDT of Sreenivasan & Narasimha (1978) was selected, since it accounts for the initial anisotropy turbulence. The $k-\epsilon$ model of Spalding & Launder (1971) and Jones & Launder (1972) was selected to demonstrate the performance of this most commonly used turbulence model for the axisymmetric turbulent flows investigated in this study.

Unstrained and strained axisymmetric turbulent flow cases were selected for predictions. The selected data sets are listed in Table 8.1. Since $Re_\lambda$ is not constant in the streamwise direction, only the initial $Re_\lambda$ of the selected data sets are provided. The geometric details of the straining ducts can be found in Table 3.1.

Necessary details of the AI-model, $k-\epsilon$ model and the RDT are provided in Chapter 2. For the predictions, the mean velocity field, the initial values of Reynolds stresses
Table 8.1: The selected test cases for the comparison with turbulence models.

and total dissipation were taken from the measurements. The initial total dissipation was calculated by using the turbulent kinetic energy equation (2.12) and the measured Reynolds stresses. It is shown in Chapter 4 that in grid-generated turbulence ±5% inhomogeneity of Reynolds stresses exists. Therefore, throughout the predictions, the initial values of Reynolds stresses and total dissipation were set free to change by ±5% and ±10%, respectively, in order to obtain the best possible prediction downstream of the initial location.

8.2 Decay of Grid-generated Turbulence

The measurements are those performed in the wind tunnel and already analyzed in Chapters 4 and 5. A detailed comparison for $Re_M = 2640$, $\bar{U}_{grid} = 5 \text{ m/s}$ is shown in Figure 8.1. Not surprisingly, both models predict $q^2$ almost perfectly. The $q^2$ predictions in Figure 8.2 show that the AI-model and $k - \epsilon$ model reproduce the decaying grid-generated turbulence measurements with the same degree of agreement in a wide range of $Re_M$. In contrast to the $k - \epsilon$ model, the AI-model is capable of resolving the development of $\overline{u^2}$ and $\overline{v^2}$ separately and of capturing the development of anisotropy $a_{11}$ (Figures 8.1b and c). The decrease in initial anisotropy in $k - \epsilon$ predictions is due to the isotropization effect of the Boussinesq approximation (2.44). The small anisotropy values predicted by the $k - \epsilon$ model in Figure 8.1c are due to the small mean velocity gradient.

8.3 Decay of Anisotropic Axisymmetric Turbulence

This turbulent flow case was generated by the straining of grid-generated turbulence through a 3.69:1 contraction and relaxing it downstream of the contraction in a constant-diameter duct. Measurements for this turbulent case are comprehensively analyzed in Section 5.4. A small velocity gradient remains downstream of the contraction as shown
Figure 8.1: Predictions of the decaying grid-generated turbulence at $U_{grid} = 5$ m/s: (a) $q^2$; (b) $uu$ (dashed lines) and $vv$ (solid lines); (c) $a_{11}$.

Figure 8.2: $q^2$ predictions of the decaying grid-generated turbulence at different $U_{grid}$ with (a) the AI-model and (b) the $k-\epsilon$ model.
in Figure 8.3a. The \( q^2 \) prediction of the AI-model shown in Figure 8.3b is slightly better than that of the \( k - \epsilon \) model. The comparison of Reynolds stresses and resulting anisotropy shows the superiority of the AI-model (Figure 8.3c and d). In spite of the problems related to the modeling of the \( \psi \) term in the AI-model (see Figure 5.21), it can fairly be argued that the AI-model performs well for the unstrained axisymmetric turbulent flow cases.

![Figure 8.3: Predictions of decaying anisotropic axisymmetric turbulence: (a) measured \( \bar{U} \); (b) \( q^2 \); (c) \( \bar{u}\bar{u} \) (dashed lines) and \( \bar{v}\bar{v} \) (solid lines); (d) \( a_{11} \).](image)

### 8.4 Axisymmetric Contraction

The three selected data sets with contractions of 1.27:1, 3.69:1 and 14.75:1 correspond to slow, moderately rapid and rapid axisymmetric contraction (strain) cases, respectively (see Figure 7.1). This selection is expected to show the reaction of different models to different levels of mean velocity gradients.

For the slow axisymmetric contraction case (Figure 8.4a-d), the best prediction of \( q^2 \) is
obtained with the AI-model, whereas the $k - \epsilon$ model also delivers good predictions and the RDT fails as the strain is not rapid enough. The superiority of the AI-model can be seen in the comparison of the Reynolds stress. It is interesting that although the RDT cannot deliver the development of the Reynolds stresses, it predicts the development of $a_{11}$ comparatively well. The direct dependence of $u_i u_j$ on the mean velocity gradients (2.44) in the $k-\epsilon$ model causes initially a drastic change in the anisotropy value, which results in a total disagreement in $a_{11}$ downstream of the contraction.

For the moderately rapid, axisymmetric, 3.69:1 contraction case (Figure 8.4a-d), the differences between the RDT predictions and the experiments are reduced. The AI-model underpredicts the $v v$ and $a_{11}$ absolute values downstream of contraction, whereas anisotropy is captured fairly well by the RDT. The $k-\epsilon$ model overshoots the turbulent energy. In addition, $u u$ takes negative values and, consequently, the anisotropy development drops below its minimum allowed value, namely the two-component isotropic limit (= -1/3). The Reynolds stresses predicted by the $k-\epsilon$ model are not plotted because of their non-physical extreme values.
8. Predictions with Various Turbulence Models and Comparison with Experiments

The best agreement between the experiments and the RDT is obtained for the rapid contraction case with a 14.75:1 contraction (Figure 8.6). The good agreement of $q^2$, Reynolds stresses and anisotropy implies that in the experiments turbulence was generated as described and solved by the RDT. Similar to the moderately rapid contraction case, the AI-model undepredicts $\bar{v}v$ and overpredicts $\bar{u}u$, which consequently delivers a lower $a_{11}$ (absolute value). The $k-\epsilon$ model delivers an extremely high negative $\bar{u}u$ and positive $\bar{v}v$, which yield $a_{11}$ values far below the two-component isotropic limit.

In order to clarify the ambiguities in $k-\epsilon$ model predictions, they are plotted on the AI-map in Figure 8.7. Except for the predictions for the slow contraction case, predictions for the moderately rapid and rapid contraction cases leave the AI-map. In other words, turbulences predicted by the $k-\epsilon$ model are not realizable for these cases. It should be noted that the conditions generated in the experiment, i.e. contraction shape and the turbulence state at the inlet of the contractions, are not that marginal when compared with many technological applications. The cliché “blind application of any turbulence

Figure 8.5: Predictions of axisymmetric strained turbulence through a 3.69:1 contraction: (a) measured $\bar{U}$; (b) $q^2$; (c) $\bar{u}u$ (dashed lines) and $\bar{v}v$ (solid lines); (d) $a_{11}$. 
8.4 Axisymmetric Contraction

Figure 8.6: Predictions of axisymmetric strained turbulence through a 14.75:1 contraction: (a) measured $\overline{U}$; (b) $q^2$; (c) $\overline{uu}$ (dashed lines) and $\overline{vv}$ (solid lines); (d) $a_{11}$.

model should be avoided” fits well the presented state of the $k-\epsilon$ model predictions.

8.4.1 Modeling Aspects for Rapid Pressure-Strain Term

As the RDT yields the analytical solution to the rapid straining of homogeneous turbulence, the turbulence models are expected to reproduce the results of the RDT under the conditions where the RDT is valid. Therefore, the rapid contraction experiments along a 14.75:1 contraction will be exploited here in order to determine the problematic term in the AI-model.

In the RDT, only the production and the rapid pressure-strain are the active processes. By switching off the slow pressure-strain term $\Pi_{ij}$ and the dissipation term $\epsilon_{ij}$ in equation (2.61), the AI-model is forced to function similarly to the RDT. For this purpose, predictions with the RDT and the modified AI-model were performed starting
from the same location, which was close to the inlet of the 14.75:1 contraction. As can be seen in Figure 8.8, AI-model predictions in the absence of slow pressure-strain and the dissipation term still cannot reach to those of the RDT.

This observation readily implies that the rapid pressure-strain is one important failing component of AI-model. The rapid pressure-strain term predicted by the AI-model and the RDT were extracted by using the corresponding transport equations of Reynolds stresses. Extracted $\Pi_{ij}$ data are normalized with the local $q^2$ and the corresponding strain rate $S_{ij}$, as was done by Jovanović et al. (2003), and compared in Figure 8.8 as a function of the second invariant $II_a$ of the $a_{ij}$ tensor. The positive normalized value suggests that $\Pi_{ij}$ takes the sign of the corresponding strain rate, i.e. it acts like a source for $\overline{uu}$ and sink for $\overline{vv}$ along a contraction. Hence, the comparison immediately shows that the modeled $\Pi_{ij}$ keeps its value at a much higher level than those predicted by the RDT and consequently prevents $\overline{vv}$ from increasing more and $\overline{uu}$ from decreasing further. Further consequences of this failure in $\Pi_{ij}$ are the damping of the anisotropy and the generated turbulent energy, even though $\Pi_{ij}$ and the trace of it do not directly appear in the transport equation of the turbulent energy.

This kind of artifact in the models of $\Pi_{ij}$ has already been extensively discussed by Johansson & Hallbäck (1994). According to their work, $\Pi_{ij}$ of the AI-model is close to a second-order model in terms of $a_{ij}$ and they showed that $\Pi_{ij}$ should be a fourth-order function of $a_{ij}$. This issue will not be elaborated further here; however, the author is convinced that $\Pi_{ij}$ can be easily corrected without changing the frame of the AI-model.
Figure 8.8: Comparison of AI-model and the RDT predictions when slow pressure-strain and dissipation terms in AI-model are set to zero.

Figure 8.9: Predicted development of the normalized fast pressure-strain term with the RDT and AI-model.
8.5 Axisymmetric Expansion

All the axisymmetric expansion cases analyzed in Section 7.3 are examples of slow straining. Hence, as shown in Figure 8.10, both the AI-model and the $k - \epsilon$ model predicted the energy development well except in the downstream region where the AI-model starts to deviate from the measurements.

![Graphs showing predictions with various turbulence models and comparison with experiments](image)

Figure 8.10: Predictions of axisymmetric strained turbulence through a 1:0.72 expansion: (a) measured $\overline{U}$; (b) $q^2$; (c) $\overline{uu}$ (dashed lines) and $\overline{vv}$ (solid lines); (d) $a_{11}$.

8.6 Axisymmetric Successive Strain

This test case is selected to test the models under moderate strain which changes its sign. The $q^2$ and the $\overline{vv}$ values are well predicted by the AI-model, as shown in Figure 8.11b and c. These quantities are overpredicted by the RDT, whereas the RDT
predicts the $a_{11}$ development very well. The $k - \epsilon$ model does not yield reasonable predictions.

Figure 8.11: Predictions of successively strained axisymmetric turbulence first through a 3.69:1 contraction then a 1:0.72 expansion: (a) measured $\bar{U}$; (b) $q^2$; (c) $\bar{u}\bar{v}$ (dashed lines) and $\bar{v}\bar{v}$ (solid lines); (d) $a_{11}$.

In general it can be said that the AI-model in its present form delivers good predictions for unstrained and low and moderate by strained axisymmetric flows. However, correction of the modeling of the fast pressure-strain term would increase its accuracy for larger strain rates. Moreover, only after this correction can dissipation and slow pressure-strain terms be attacked for a more refined model, since there is no strained turbulent flow in which they can be treated isolated from the rapid pressure-strain term.
Chapter 9

Conclusions and Outlook

9.1 Conclusions

Experimental and numerical investigations were performed in the present study in order to study the dynamics of unstrained and strained axisymmetric turbulence, which is considered to represent the backbone of the anisotropy-invariant models of turbulence. The decay of nearly isotropic grid-generated turbulence, decay of anisotropic axisymmetric turbulence, axisymmetric contraction, axisymmetric expansion and successive axisymmetric strain were among the experimentally generated axisymmetric turbulent flow cases. As these turbulent flow cases cover the left limiting edge and partially the right limiting edge of the anisotropy-invariant map, their investigations gave invaluable insights into the modeling of axisymmetric turbulence.

Hot-wire anemometry was employed for the measurements, which were conducted in two different flow facilities: the wind tunnel at LSTM-Erlangen and the axisymmetric strain tunnel (AST). The AST was specially constructed in this work for the investigation of strained axisymmetric turbulent flow. Measures were taken during the design of the AST to keep the background turbulence intensity in the AST below 0.05%.

In the initial phase of this study, special attention was given to the axisymmetric contraction, because of the lack of qualitative and quantitative agreement between the theoretical, numerical and experimental investigations for contractions with high contraction ratios. Increasing longitudinal velocity fluctuations downstream of contractions having high contraction ratios were always reported in the literature. In contrast to these “experimental observations”, an increase in longitudinal fluctuations cannot be predicted by rapid distortion theory (RDT) and was also not obtained by direct numerical simulations. Therefore, the measured increase in longitudinal fluctuations in contractions was accepted as being anomalous and referred to as the high contraction ratio anomaly in the present study. Extensive experimental investigations of axisymmetric contraction were carried out in the AST and three important sources of contaminations, which occur simultaneously and cause the anomaly, were identified. The first one is the
mass flow rate fluctuations in the flow, which are caused by various parts of the flow facility and amplified as the flow is strained by a contracting nozzle. The second source is the electronic noise of the measurement system. The third and most important is the finite wire length and the distance between the two wires of the X-wire probe, which may not be sufficiently small to resolve turbulent structures undergoing a thinning in the lateral and transverse directions as they flow through the contractions.

A correction method, applying two-point correlation measurements with two single normal wires (SN wires), was proposed to separate longitudinal turbulent velocity fluctuations from the raw measurements, contaminated with non-turbulent velocity fluctuations. The proposed method was successfully tested for strained turbulent flow with and without artificial flow rate fluctuations. With the proposed method, the random unsteadiness of the flow facility can be determined exactly, hence the turbulence fluctuations can properly be extracted from the raw measurements. It was shown that this method delivered the mean square of the longitudinal turbulent velocity fluctuations without any increase downstream of the nozzle having a high contraction ratio when it was measured with SN wires. Although the effect of acoustic disturbances was not separately discussed, it is obvious that the proposed correction method cleans the contribution of acoustic disturbances. The contaminations due to the imperfect spatial resolution of the X-wire and SN-wire probes were corrected with a second method which already incorporated the former method. The effect of wire separation of X-wire probes was eliminated by using single normal and inclined wire measurements at the same location. The effect of the wire length was corrected by adopting the correction methods proposed for isotropic turbulence, which are available in the literature, to the anisotropic turbulent flows investigated in this work.

In order to utilize the two developed correction methods simultaneously, a probe head consisting of two SN wires and one X wire was constructed and final measurements of all the turbulent flow cases were conducted with this probe head. The corrections to mass flow rate fluctuations and electronic noise were successfully applied for all measurements, whereas corrections for imperfect spatial resolution of hot-wire probes were carried out only for the axisymmetric contraction case. The corrections applied in the axisymmetric contraction case removed the anomalous increase of the longitudinal velocity fluctuations and, at the same time, enhanced the transverse and the lateral velocity fluctuation components. This finding was the major experimental achievement in the present study.

Investigations on the contaminations in raw experimental data suggested that the qualitative discrepancies in the published experimental data were induced by the difference between the flow qualities of wind tunnels, signal-to-noise ratios of measurement instrumentation, size of the hot-wire probes relative to the turbulence length scales and the level of turbulence at the inlet of the contractions. The proposed measurement methods should be employed in flows where low turbulence levels are present and/or the length scales become highly anisotropic.

Homogeneity is one of the basic properties of axisymmetric turbulence. Therefore, prior to the detailed measurements of axisymmetric turbulence, the assumption of the
homogeneity of grid-generated turbulence was questioned and experimentally investigated. Contrary to the common belief that turbulence generated by low-solidity grids becomes homogeneous after some distance from the grid, it was shown that inhomogeneous turbulence generated in the vicinity of the grid is conserved at long distances away from the grid and showing ±5% inhomogeneity in Reynolds stresses and ±20% inhomogeneity in anisotropy of Reynolds stresses. In addition, the effect of strain on inhomogeneous grid-generated turbulence was investigated and it was observed that as turbulence becomes isotropic in a contraction it becomes more homogeneous. This observation indirectly suggests that mixing occurs best in the isotropic turbulent state. No measures were taken to make the grid-generated turbulence more homogeneous and the measurements of axisymmetric turbulence were conducted with those grids. Nevertheless, this inevitable inhomogeneity induced by the grid was taken into consideration during the analysis of nearly isotropic turbulence and numerical predictions for experiments with various turbulence models.

After resolving experimental ambiguities, specific to axisymmetric turbulence, measurements of the aforementioned five different kinds of axisymmetric turbulence were performed and analyzed in terms of the development of the Reynolds stresses, length scales, correlation functions and spectra. The important conclusions made for each case are recapitulated below.

**Decay of Nearly Isotropic and Anisotropic Turbulence**

1. The isotropy of the grid-generated turbulence, measured for the range $15 < \text{Re}_\lambda < 60$, was questioned. No tendency towards a long-lasting isotropic state was observed on looking at the anisotropy of the Reynolds stresses, the turbulent length scales and the spectra. The anisotropy of grid-generated turbulence was reduced by straining it along the 1.27:1 contraction. Even though the measured turbulence downstream of the contraction had less anisotropy, the dissipation term and the ratios between the longitudinal and transverse length scales did not yield a picture of completely isotropic turbulence.

2. Two special forms of the von Kármán & Howarth equation were derived with which the isotropy of measurements can be easily checked in terms of their decay characteristics and length scales at the same time. Employing these equations, it was shown that both the grid-generated turbulence and slightly contracted axisymmetric turbulence were far from being isotropic.

3. As regards the inhomogeneity of grid-generated turbulence, it was demonstrated that depending on the location of the measurement axis behind the turbulence-generating grid, constants of the power-law decay of grid-generated turbulence vary strongly.

4. At a fixed position downstream of the grid, the increase in $\text{Re}_\lambda$ resulted in an increase in energy in the high wavenumber range in the normalized one-dimensional spectra. This increase was accompanied by the development of an inertial subrange in the spectra.

5. The values of the $\psi$ term, which appears in the modeling of the dissipation equation, were evaluated from the measurements of decaying nearly isotropic and an-
isotropic turbulence. The comparison of experimental data with different models of $\psi$ revealed that almost all models are equally good in the vicinity of the isotropic limit, whereas for the decay of anisotropic axisymmetric turbulence no agreement on $\psi$ could be established out between the different $\psi$ models as well as between the present experiments and the experiments reported in the literature.

**Axisymmetric Contraction**

1. Owing to the removal of the high contraction ratio anomaly, for the first time, the rapid distortion theory could be validated for symmetric nozzles having high contraction ratios ($c > 14$). It was shown that even for moderately rapid contractions, rapid distortion theory predicts the development of Reynolds stress anisotropy and the length scales in a satisfactory manner. This finding legitimates the application of the RDT in the calibration of the rapid pressure-strain term that appears in the transport equation of Reynolds stresses.

2. Again for the first time, the two-component isotropic limit of turbulence on the AI map could be achieved for axisymmetric contractions acting on grid-generated turbulence.

3. In agreement with RDT, the greatest change in the shape of one-dimensional spectra occurred for the longitudinal fluctuations: a shift of energy in spectra was observed towards higher frequencies along the contraction.

4. The one-dimensional spectra of transverse velocity fluctuations depicted that, whereas the higher wavenumbers lose energy along the contraction, the lower wavenumbers gain energy. In other words, the increase in transverse Reynolds stress is mainly due to the increase in energy in the energy-containing wavenumber range.

5. The increasing anisotropy of Reynolds stresses along contractions is linked to the development of the anisotropy of one-dimensional spectra in the energy-containing wavenumber range.

6. It was shown that the two-point correlation curves and the spectra along the symmetry axis of each contraction reduce to one curve when normalized with the corresponding integral length scales and Reynolds stresses. This property can be used together with RDT for a full description of turbulence in contractions when the spectra of turbulence at the inlet of the contractions are known.

**Axisymmetric Expansion**

1. The rate of anisotropy increase becomes slower with increasing $\text{Re}_\lambda$.

2. Since the applied strain was slow, the length scales, correlation functions and spectra developed similar by to those of decaying grid-generated turbulence. However, in contrast to grid-generated turbulence, the elongation of the length scales was suppressed in the expansion.
Successive Axisymmetric Strain

1. The anisotropy increased along the contraction on the left edge of the AI-map towards the two-component isotropic limit and decreased during expansion, again on the left edge, towards the isotropic limit. Hence, the axisymmetry of turbulence was preserved.

2. Due to the reduced dissipation at the exit of the contraction, which was the first straining element, the mean strain rate parameter in the following expansion had a relatively high value and expansion acted like a moderately rapid expansion. Consequently, the anisotropy of Reynolds stresses followed more or less the same path for different $Re_{\lambda}$. In addition, in contrast to the single expansion case, the length scales shortened, as RDT predicted.

3. In the expansion part, the increase in longitudinal Reynolds stress occurred at all wavenumbers, whereas the measured decrease in the transverse Reynolds stress occurred only in the energy-containing range of the corresponding one-dimensional spectra.

From all the measurements, one data set was selected from each of the investigated axisymmetric turbulent flow cases, and Reynolds stress predictions for these data sets were performed with the anisotropy-invariant model (AI-model) of Jovanović et al. (2003), RDT of Sreenivasan & Narasimha (1978) and $k-\varepsilon$ model of Spalding & Launder (1971) and Jones & Launder (1972). The comparison of the experiments with these predictions revealed that:

1. The AI-model and $k-\varepsilon$ model can predict the turbulent kinetic energy relatively well for unstrained and slowly strained axisymmetric turbulent flows. In addition to the prediction of turbulent kinetic energy, the AI-model resolves Reynolds stresses better than the $k-\varepsilon$ model, as one would expect.

2. For turbulence exposed to a moderate contraction ($c = 3.69$), the turbulent kinetic energy and the transverse Reynolds stress were underpredicted by the AI-model and overpredicted in the RDT computations. Despite the mismatch of its Reynolds stress prediction, RDT delivered the best anisotropy prediction. The $k-\varepsilon$ model overpredicted turbulent kinetic energy and yielded negative values of Reynolds stresses and corresponding unphysical anisotropy values during moderate-rapid strain.

3. The best agreement was achieved when RDT was applied for the prediction of the rapid contraction case ($c = 14.75$). The AI-model again delivered underestimated turbulent kinetic energy, transverse Reynolds stress and corresponding Reynolds stress anisotropy.

4. The good agreement between the RDT and the experiments suggested that the turbulence models should reproduce the predictions of RDT at rapid distortion limits. Accordingly, by utilizing the RDT and rapid contraction measurements, it was possible to show that the rapid pressure-strain term in the AI-model damps the increase in anisotropy by pumping more energy than necessary from the more energetic Reynolds stresses to the lower energetic stresses.
9.2 Outlook

The anisotropy-invariant turbulence model of Jovanović et al. (2003) offers a simple structured approach to turbulence modeling. In this approach, the knowledge of anisotropy-invariant functions at limiting states of turbulence can be elegantly used to construct a turbulence model. The experimental and numerical investigations in the present study showed that this model can be improved for anisotropic decay and rapid strain of turbulence. The effect of $Re_\lambda$ on strained turbulence could not be extensively investigated in the literature, or in the present study, because of the experimental limitations on the generation of high-$Re_\lambda$ turbulence with static grids.

In order to obtain a better anisotropy-invariant model which may be used at very low and very high $Re_\lambda$ ($\approx 500$) and, at the same time, reacts to slow and rapid strains accurately, a research approach is proposed below.

Three basic axisymmetric flows, given in Table 9.1, are the main turbulent flow cases to be investigated in the suggested approach. Experimental and/or numerical investigations should be performed in such a way that the left and right edges of the AI-map are covered up to the two-component isotropic and one-component limiting states of turbulence. According to Table 9.1, rapid axisymmetric strain is the simplest case with one unknown term, namely the rapid pressure-strain ($\Pi_{ij}^f$). In the anisotropic decay of axisymmetric turbulence, only the dissipation correlation ($\epsilon_{ij}$) and the $P^1_\epsilon - \gamma = \psi$ term are significant. For axisymmetric strain cases, which are not rapid, all terms in the transport equations become significant.

First, the rapid axisymmetric strain can be studied and the rapid pressure-strain term can be modeled analytically by using RDT, as Johansson & Hallbäck (1994), Kassinos & Reynolds (1995) and Sjögren & Johansson (2000) have already done. In the context of the proposed approach, only the anisotropy-invariant functions are suggested to be modeled.

In order to measure quantitative by the decay of anisotropic axisymmetric and strain of axisymmetric turbulence, the pioneering work of Lindborg (1995), which allows separate measurements of the slow and fast pressure-strain terms indirectly from the

<table>
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Table 9.1: The significant and insignificant terms in the transport equations of homogeneous turbulence for different axisymmetric turbulent flows.
correlation coefficients, is suggested to be employed. This technique was successfully utilized by Sjögren & Johansson (1998) for nozzles with moderately high contraction ratios. Thus, as second step, decay of anisotropic axisymmetric turbulence can be investigated. Since $\Pi_{ijs}$ is measured separately, components of the dissipation tensor ($\epsilon_{ij}$) can be determined from the experiments and the partition of the total dissipation into the components of the dissipation tensor can be studied. As $\psi$ is the only significant term in the transport equation of total dissipation, its model can be refined or reconstructed. After the steps mentioned so far, except the $P_1 + P_2$ term, other terms in the transport equations should have been properly modeled. In the third step, first the measurements of fast and slow pressure-strain terms can be used to check the constructed models for them. Later, using the model of $\psi$ and the dissipation term extracted from the measurements, the model for $P_1 + P_2$ can be constructed.

In order to achieve $\text{Re}_\lambda$ around 500 in a flow facility of medium size, an active grid can be utilized to generate nearly isotropic turbulence. This technique was successfully used for the investigation of high-$\text{Re}_\lambda$ isotropic turbulence by Mydlarski & Warhaft (1996). As demonstrated in the present study, all these measurements should be performed in a sufficiently large wind tunnel with a sufficiently large grid size in order to avoid contaminations due to imperfect spatial resolution of hot-wire probes. Experimental realization of an axisymmetric expansion case, in which turbulence approaches the one-component limit, is very difficult to achieve because of separation problems in symmetric diffusers. Therefore, in addition to experiments, direct numerical simulations can also be performed, especially for the axisymmetric expansion case. Direct numerical simulations, may also be employed in the investigation of other cases; however, one cannot reach very high-$\text{Re}_\lambda$ in simulations. Moreover, due to stretching and thinning of turbulent eddies in strained highly anisotropic flows, additional grid resolution requirements are imposed on the simulations. Hence the most fruitful approach would be a combination of experiments and numerical simulations, which would also allow one to cross-check the results obtained by these two methods. As a result of such a research approach, highly anisotropic axisymmetric flows with a wide range of Reynolds number can be modeled. Using the developed experimental and numerical techniques, more complicated problems such as plane distortion can be studied and, in this manner, the model can be further refined towards a generally applicable turbulence model. One should note that this approach is similar to that followed by the Stanford and KTH groups, although they are not explicitly mentioned anywhere, but may be concluded by studying the sequence of their research publications. In contrast to their studies, the straining of high-$\text{Re}_\lambda$ turbulence is suggested here.
References


[111] Tan-atichat, J., Nagib, H. M. and Drubka, R. E.: Effects of Axisymmetric Con-

[112] Tavoularis, S., Jimenez, J. and Leuchter, O. “A selection of test cases for the val-
idation of large-eddy simulations of turbulent flows”, AGARD Advisory Report

196-212, 1921.

421, 1935.

91-96, 1935.

1937.


[120] Tsugé, S.: Effects of flow contraction on evolution of turbulence, Phys. Fluids,
vol. 27(8), 1948-1956, 1984.


[123] Uberoi, M. S.: Effect of wind-tunnel contraction on free-stream turbulence, J.

[124] Uberoi, M. S.: Equipartition of energy and local isotropy in turbulent flows, J.

[125] Uberoi, M. S. and Kovasznay, L. S. G.: On mapping and measurement of random

[126] Uberoi, M. S. and Wallis, S.: Small axisymmetric contraction of grid turbulence,

[127] von Kármán, Th.: The fundamentals of the statistical theory of turbulence, J.

[128] von Kármán, Th. and Howarth, L.: On the statistical theory of isotropic turbulence,


Experimentelle und numerische Untersuchungen der axialsymmetrischen Turbulenz

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Zusammenfassung


achtet, sondern auch die Zunahme von Reynoldsspannung in Querrichtung korrekt erfasst. Diese Messmethoden wurden anschließend angewandt zur Vermessung der ausgewählten axialsymmetrischen turbulenten Strömungsfälle.

Da in der vorliegenden Studie Turbulenz mit Gittern erzeugt wurde, und die Homogenität eine Grundeigenschaft axialsymmetrischer Turbulenz ist, wurde die räumliche Inhomogenität der Turbulenz im Nachlauf eines Gitters detailliert untersucht. Es konnte gezeigt werden, dass das inhomogene Turbulenzfeld, das in der Nähe des Gitters generiert wurde, auch in großem Abstand vom Gitter erhalten blieb, selbst für Gitter mit niedriger Versperrung. Der Grad an Inhomogenität an Positionen mehr als 40 Maschenweiten stromab des Gitters wurde mit ±5% bezogen auf die Reynoldsspannungen und ±20% bezogen auf die Anisotropie der Reynoldsspannungen gemessen. Ähnliche Homogenitätsmessungen wurden in axialsymmetrischen Kontraktionen und Expansionen durchgeführt. Es stellte sich heraus, dass die Inhomogenität der gittergenerierten Turbulenz erst verschwindet nachdem sie sich in einer Kontraktion zur Isotropie hin entwickelte.

Nach der Beseitigung der oben genannten experimentellen Artefakte wurden Messungen der Reynoldsspannungen, der Längenskalen, der Geschwindigkeitskorrelationsfunktionen und der Geschwindigkeitspektren für ungedehnte und gedehnte axialsymmetrische turbulente Strömungsfälle durchgeführt. Es zeigte sich, dass gittergenerierte Turbulenz annähernd isotrop in Bezug auf die Reynoldsspannungen ist, jedoch hat sie keine Tendenz einen lang anhaltenden isotropen Zustand zu erreichen während sie zerrüttelt. Ähnliche Beobachtungen wurden für die gittergenerierte Turbulenz gemacht, deren Anisotropie sich reduziert, wenn sie leicht kontrahiert wird. Was die Inhomogenität der gittergenerierten Turbulenz betrifft, so stellte sich heraus, dass die Konstanten des Zerfalls im Exponentialgesetz von der Position der Messung relativ zum Gitter abhängig sind. Der ψ-Term in der Dissipationgleichung des AI-Modells wurde in ungedehnten turbulenten Strömungen gemessen und sowohl mit jenen aus der Literatur als auch mit anderen Modellierungen verglichen. Es konnte keinerlei Übereinstimmung gefunden werden.

Alle gemessenen turbulenten Strömungsfälle wurden in Bezug mit verschiedenen statistischen Größen analysiert. Besonders detailliert betrachtet wurde die Anisotropie des Reynoldsspannungstensors für Turbulenz, die in Strömungsrichtung gedehnt wird. Wegen der in den Messungen angewandten Korrekturen konnte die RDT für axialsymmetrische Kontraktionen mit hohen Kontraktionsverhältnissen erstmals experimentell bestätigt werden. Es wird gezeigt, dass die RDT die Anisotropie entlang starker Kontraktionen sehr gut, entlang moderater Kontraktionen relativ gut, vorhersagt.

Vorhersagen der Reynoldsspannungen für die gemessenen turbulenten Strömungsfälle wurden mit dem AI-Model, der RDT und dem \( k – \epsilon \) Modell durchgeführt. Das AI-Model bringt gute Voraussagen für ungedehnte und langsam gedehnte Fälle, versagt aber für starke axialsymmetrische Dehnungen. Es konnte gezeigt werden, dass die schnelle Druck-Verformungsgeschwindigkeitskorrelation im AI-Model verbessert werden kann, indem sie mit der RDT kalibriert wird.
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Kapitel 1
Einführung

1.1 Einleitung


Nach der isotropen Turbulenz ist die axialsymmetrische Turbulenz, eine Unterklassedierhomogenen turbulenten Strömungen, die zweiteinfachste Art der Turbulenz. In diese Klasse gehören sowohl die axialsymmetrische gedeihnte homogene Turbulenz, als auch der Abklingvorgang isotroper und anisotroper axialsymmetrischer Turbulenz. Eine turbulente Strömung wird axialsymmetrisch genannt, wenn die Korrelationen der fluktuierenden Turbulenzgrößen, gemessen an beliebig vielen Punkten, bei Verschiebung invariant sind, und bei Rotation und Spiegelung um eine Symmetrieachse. Trotz ihrer im Vergleich mit der isotropen Turbulenz gesteigerten Komplexität, machen die analytischen Eigenschaften der axialsymmetrischen Turbulenz und die leichte experimentelle Generierung sie für die Turbulenzmodellierung attraktiv.


Die AI-Karte in Abbildung 1.1 zeigt, dass es fünf Unterklassen axialsymmetrischer Turbulenz gibt: (1) axialsymmetrische Dehnung (Kontraktion und Expansion) (2) anisotrope axialsymmetrische Turbulenz ohne Dehnung (Rückkehr zur Isotropie), (3) isotrope Turbulenz, (4) isotrope Zweikomponenten-Turbulenz, (5) Einkomponenten-Turbulenz. Die Untersuchungen dieser axialsymmetrischen turbulenten Strömungen bilden wertvolle Quellen für der Verständnis der Turbulenz und die Validierung von Turbulenzmodellen.


Die axialsymmetrischen turbulenten Strömungen werden in dieser Arbeit experimentell und numerisch untersucht. Die Hauptziele der Untersuchungen sind:

i. Die Bereitstellung zuverlässiger Messungen axialsymmetrischen Verzerrungen isotroper Turbulenz und axialsymmetrischer Turbulenz mit hoher Anisotropie

ii. Die Bereitstellung zuverlässiger Messungen abklingender isotroper und anisotroper axialsymmetrischer Turbulenz

iii. Die Beurteilung verfügbare Modelle axialsymmetrischer Turbulenz
Diese Ziele wurden hauptsächlich durch die experimentellen Untersuchungen der folgenden Arten von axialsymmetrischen Strömungen erreicht:

- Abklingvorgang von beinahe isotroper gittergenerierter Turbulenz.
- Abklingvorgang anisotroper axialsymmetrischer Turbulenz.
- Axialsymmetrische Kontraktion beinahe isotroper gittergenerierter Turbulenz.
- Axialsymmetrische Expansion beinahe isotroper gittergenerierter Turbulenz.
- Axialsymmetrische Kontraktion gefolgt von einer axialsymmetrischen Expansion (sukzessive axialsymmetrische Dehnung).

Um quantitativ zuverlässige Daten für solche Strömungen zu erreichen, wurden zuerst beträchtliche theoretische und experimentelle Arbeiten unternommen, um mögliche Artefakte in den Messungen herauszufinden und zu beseitigen. Deshalb wurden, zusätzlich zu den der oben genannten Untersuchungen, zwei weitere Themen zwangsläufig zu wichtigen Teile der vorliegenden Studie:

- Die Ursachen für widersprüchliche Ergebnisse in den Messungen der axialsymmetrischen Kontraktion beinahe isotroper gittergenerierter Turbulenz mit Düsen des hohen Kontraktionsverhältnisses herauszufinden und zu beseitigen.
- Die Homogenität der gittergenerierten Turbulenz in ungedehnten und gedehnten Turbulenzfeldern zu analysieren.


In Kapitel 6 wird eine detaillierte Literaturstudie über experimentelle und numerische Arbeiten zur Wirkung axialsymmetrischer Kontraktion auf Gitterturbulenz bereitgestellt. Die Anomalie bei hohen Kontraktionsverhältnissen, die in der Literatur und in den vorliegenden Experimenten erscheint, wird herausgearbeitet. Theoretische und
experimentelle Ursachen für diese Anomalie und experimentelle Methoden, mit denen diese beseitigt werden kann, werden aufgezeigt.

In Kapitel 7 werden eigene experimentelle Untersuchungen der axialsymmetrischen Kontraktion, Expansion und sukzessiven axialsymmetrischen Dehnung anhand von Reynoldsspannungen, deren Anisotropien, Korrelationsfunktionen, Turbulenzlängen- skalen, und Spektren analysiert. In Kapitel 8 sind Datensätze aus den gemessenen turbulenten Strömungsfällen ausgewählt, und werden verglichen mit Vorhersagen, die mit dem AI-Modell, der RDT und \( k - \epsilon \) Modell berechnet wurden. In letztem Kapitel sind die Ergebnisse und Schlussfolgerungen der vorliegenden Arbeit zusammengefasst, und ein Forschungsansatz für die Verbesserung des AI-Modells ist aufgezeigt.
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