Quantum and Thermal Phase Escape in Extended Josephson Systems

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Preface

Since the description of the phenomena on coupled superconductors by Josephson [1], the dynamics of the quantum mechanical phase across a Josephson junction (JJ) has been studied extensively. A well known application of these effects is the superconducting quantum interferometer, which is a sensitive measurement device. The JJ in a SQUID is completely described by this single macroscopic variable, which behaves as a single degree of freedom. In the resistively-capacitively shunted junction model, a resistance acts as a damping element on this degree of freedom, while the capacitive part represents an effective mass. Because of the nonlinearity of the Josephson element, metastable states can exist. Fluctuations by thermal noise current in the resistive state of the junction were described by Dahm et al. [2] Due to this noise current metastable states in a small junction have a finite lifetime, examined by Fulton et al. [3] and Kukijärvi et al. [4]. Recently, Josephson junctions received attention in the context of quantum computation. The basis for this was set long ago [5, 6, 7] by observing macroscopic quantum behaviour of the superconducting phase difference.

In long Josephson junctions the soliton solution of the governing sine-Gordon equation corresponds to a circulating vortex of supercurrent. In this work thermal, quantum limited and RF induced activation from metastable states in long, annular Josephson junctions are presented. It is a continuation of the research on fluctuation induced activation in annular Josephson junctions done by A. Wallraff [8] which resulted in the experimental observation of macroscopic quantum tunneling of a vortex in sub-μm junctions, described in Ref. [9]. At this stage, a classical two-state system, formed by heart-shaped Josephson junctions was demonstrated [10, 11, 12], the use of which as a quantum bit was proposed in Ref. [13]. Hence another direction of research was open at the beginning of the work on this thesis.

Quantum tunneling in reproducible μm scale annular junctions is observed, similar to the results given in Ref. [9]. The samples used were of low critical cur-
rent density, and therefore physically shorter, and may behave as short Josephson junctions. The measurements presented here answer the following question: May the phase variable in a long Josephson junction tunnel homogeneously, even if it is twisted by a trapped quantum of magnetic flux? Depending on temperature and magnetic field a range of behaviours, like thermal vortex activation [14], thermal small junction activation [3], and quantum tunneling [5, 6, 7, 9] is observed. I interpret the results using a newly developed approximation of the phase distribution in the junction, derived from the one given in Ref. [15], but adapted to a single flux quantum trapped in a short annular Josephson junction limit. From the activated escape of the phase I proceed to another type of activation from a metastable state. The vortex-antivortex nucleation process, discussed by Fistul et al. [16], is another approximation of what is described for a short junction in Ref. [15].

Of major importance for applications, the parasitic barrier created by the two dimensional design of the heart-shaped Josephson junctions was to be determined by the means of thermal activation between bistable states. Only the calculations given in Ref. [17] explain the failure to observe any thermal activation. A changed vortex rest mass, a parasitic effect at strongly curved regions in the samples used, prohibited any systematic observation of thermal activation in heart-shaped Josephson junctions.

Although this leaves some perspective for an enhanced design of heart-shaped Josephson junctions, another bistable system, namely a microshort junction, seems more attractive. This kind of junction was examined before theoretically in Ref. [18]. The realization was prevented by the trilayer technology generally used for the production of the junctions, which allows for only a single critical current density. As a conclusion and a way ahead, we propose a novel production technique for microshorts, namely a width modulated junction. The use of this mechanism for potential engineering was discussed by Goldobin et al. [19]. The concept of this approach is that a small longitudinal region of the junction repels the vortex because of its enhanced width. The advantages of this approach are, besides the simplicity of the concept, the steepness of the potentials generated. Furthermore, it does not require sub-µm sample production, but only sub-µm level size control, which is within the limits of visible light photolithography. Results, including state preparation and readout protocol, will be published elsewhere in Ref. [20] and form the last experimental chapter of this thesis. Measurements done in collaboration with A. Price lead to the observation of quantum tunneling of a vortex through a barrier in a photo-lithographically produced 1 kA/cm² sample, which will be published elsewhere [21].
Chapter 1

Introduction and Theory

1.1 Superconductivity

Superconductors were discovered in 1911 by Heike-Kamerlingh Onnes. Below a critical temperature the resistance of superconductors drops to zero. This effect could not be explained until Bardeen, Cooper and Schrieffer [22] interpreted this as electrons of opposite spin\(^1\) bound together by phonons, the composite particle being similar to a boson. These are formed if the average thermal energy of the electron bath is low enough and condense into a collective ground state, lowering the total energy of the system. Any Fermi sea is unstable against this pair formation, given an arbitrarily small attractive force between the particles. At finite temperature elementary excitations on the whole system, called quasiparticles coexists to the Cooper pairs.

Because this collective ground state behaves as a single quantum particle, it is described by a macroscopic wavefunction. In a bulk superconductor, there exists no excitation other than a modulation of the phase of this wavefunction\(^2\). Thus the system is governed by

\[ \Psi = \Psi_0 e^{i\phi(\vec{x}, t)}, \]  

(1.1)

where \(|\Psi_0|^2\) is the amplitude of the ground-state wavefunction. The field \(\phi(\vec{x}, t)\)

---

\(^1\)Superconductors with Cooper pairs of spin 1 exist, called heavy-fermion superconductors. Cooper pairs in high \(T_C\) superconductors are also suspected to have another pairing mechanism.

\(^2\)For one and two dimensional superconducting structures, oscillations of the order parameter are possible, resulting in the existence of phase slip centers in thin wires [23] or phase slip lines in thin films [24].
is the only degree of freedom, if the energy is lower than the energy gap of the superconductor. For niobium, the superconductor used in the experiments reported in this thesis, the energy a single quasiparticle needs to acquire is $1.4 \text{ meV}$. If, at a Josephson junction the available energy per electron exceeds this value, a Cooper pair is removed from the ground state wavefunction and broken into quasiparticles. The amplitude of this wavefunction has a certain value $|\Psi_0|$ in the bulk superconductor, which decays exponentially outside the superconductor with the coherence length $\xi$, as shown in Fig. 1.1. Besides the lack of resistance

![Figure 1.1: Inside a superconductor, the magnetic field decreases exponentially as a function of coordinate $x$, while outside of the superconductor the amplitude of the wavefunction decreases exponentially.](image)

there are two features of superconductors deserving a closer look, namely the Meissner effect and the fluxoid quantization.

Magnetic flux is expelled from a bulk piece of superconductor, illustrated in Fig. 1.1. This state is called the Meissner state. It turns out that only a thin surface layer of the London penetration depth $\lambda_L$ carries a current, which compensates the external magnetic field inside of the bulk superconductor. Combining the second London equation,

$$\vec{h} = -c\Lambda \nabla \times \vec{J}_s,$$

with Ampere’s law yields

$$\nabla^2 \vec{H}_s = \frac{\vec{H}_s}{\lambda_L^2},$$

where $\vec{J}_s$ is the density of the supercurrent and the phenomenological parameter $\Lambda$ given by

$$\Lambda = \frac{4\pi\lambda_L}{c^2}. $$
1.1. SUPERCONDUCTIVITY

Solving this leads to an exponential decay of the magnetic field $\vec{H}_s$ inside the superconductor.

The Ginzburg-Landau theory, essentially an expansion of the free energy in the superconductor, describes the essential thermodynamic properties of the superconducting-normal phase transition. It can be used to predict that the difference between the coherence length and the London penetration depth determines the surface energy between the superconducting and the normal phase inside a superconductor. While a positive surface energy makes the minimal surface the equilibrium state, a negative surface energy supports the existence of normal conducting regions, separated from each other. These regions appear as separate treads of magnetic flux through the superconductor, Abrikosov vortices[25]. Superconductors with negative surface energy are called type-II superconductors. Niobium, the superconductor, which was used in the experiments presented in this work, is a type-II superconductor. Without careful magnetic shielding the superconductor is not in the Meissner state, but in a mixed state, a state first observed by Shubnikov et al.[26].

A peculiar effect happens in multiply connected superconductors, namely fluxoid quantization. A single-valued complex order parameter $\Psi$, requires the phase $\phi(\vec{x}, t)$ to change by a multiple of $2\pi$ when choosing any closed path in a multiply connected superconductor, such as a ring, depicted in Fig. 1.2.

The canonical momentum of cooper pairs in a bulk superconductor is given by

$$\vec{p} = \hbar \vec{\nabla} \phi = 2m_e \vec{v}_s + \frac{2e}{c} \vec{A},$$  \hspace{1cm} (1.5)

Deep inside superconductor, the velocity of Cooper pairs is zero, because the momentum of the paired electrons is opposite. Thus the total momentum of the pairs is determined by the vector potential $\vec{A}$. Thus the equality between the enclosed flux $\Phi$ and the integral value of the phase gradient,

$$\frac{\Phi_0}{2\pi} \oint_C \nabla \phi \, d\vec{l} = \oint_C 2m_e \vec{v} + \oint_C \vec{A} \, d\vec{l} = n2\pi = 0 = \Phi$$  \hspace{1cm} (1.6)

leads to quantization of the flux enclosed in the loop. The values which the flux $\Phi$ may take are multiples $\Phi_0$.

$$\Phi_0 = \frac{\hbar}{2e} = 2.07 \cdot \text{mV} \cdot \text{ps}$$  \hspace{1cm} (1.7)
1.2 Small Josephson Junctions

The Josephson effect was first predicted in 1962 by Josephson [1], and observed by Anderson and Roswell [27]. It occurs in a tunnel junction, in which the current across the tunneling barrier is carried by a supercurrent of Cooper pairs. Ambegaokar and Baratoff [28] analysed the tunneling processes including the temperature dependence and a relation between the normal resistance and the critical current of a tunnel barrier. Different types of weak links, such as narrow superconducting bridges between superconductors, or thin layers of normal metal or an insulator can act as barrier to Cooper pairs. In the experiments reported in this thesis only the latter type of junction was used. The wavefunctions of the two electrodes extend beyond the length scale of the coherence length of the order parameter, illustrated in Fig. 1.3.

There are two important relations, which relate the difference $\varphi = \phi_1 - \phi_2$ of the phases of the superconducting wavefunctions to the current and to the voltage across the barrier. In calculating the overlap integral explicitly, the coupling
1.2. SMALL JOSEPHSON JUNCTIONS

Figure 1.3: Overlap of the wavefunctions $\Psi_1$ and $\Psi_2$ causes a finite coupling energy. A current $I$ would cause a phase difference between the two electrodes.

energy is written as

$$\mathcal{H}_J = - \int_{\Omega} [(\Psi_1^* \Psi_2 + \Psi_2^* \Psi_1)] d\Omega,$$

(1.8)

where $\Omega$ is the region of overlapping wavefunctions.

This yields the phase dependent coupling energy

$$\mathcal{H}_J = -E_J \cos \varphi$$

(1.9)

where $E_J$ is the coupling energy for zero phase difference. From this[29], the DC Josephson relation

$$I = I_C \sin \varphi$$

(1.10)

can be derived. The critical current $I_C$ of the weak link is proportional to the coupling energy:

$$I_C = E_J \frac{2\pi}{\Phi_0}.$$  

(1.11)

If transport current exceeds the critical current, a voltage drop appears across the weak link. The AC Josephson relation relates the time derivative $\varphi_t$ to the DC voltage $V$ between the two electrodes:

$$V = \varphi_t \frac{\Phi_0}{2\pi}.$$  

(1.12)

Combining this with Eq. (1.10), yields a time dependent supercurrent

$$I = I_C \sin \left( \varphi_0 + tV \frac{2\pi}{\Phi_0} \right).$$  

(1.13)
The frequency $\nu$ is given by

$$\nu = V \frac{1}{\Phi_0}. \quad (1.14)$$

A real Josephson junction is described by the RCSJ model, discussed below, in which the ac current causes an ac voltage across the junction.

### 1.3 Quasiparticles and the Gap

The current-voltage characteristics for both, zero and non-zero voltage is plotted in Fig. 1.4. In Ref. [30] the different processes leading to this picture are discussed on a microscopic model. In the zero-voltage state the current is carried by the non-resistive Cooper pair tunneling. At $2\Delta_G$ enough energy is available to break a Cooper pair and inject one of the generated quasiparticles into the other electrodes quasiparticle band. At voltages below $2\Delta_G$ only quasiparticles already excited by temperature are available to carry the current. Setting up the equations of motion for $\varphi$ is done using the resistively-capacitively shunted junction (RCSJ) model shown in Fig. 1.5, where a Josephson junction is represented by a Josephson element fulfilling the DC Josephson relation, a capacitor representing the junction capacitance and a resistor, which represents the dissipative quasiparticle current.

The quasiparticle resistance plays a fundamental role in the thermal activation processes discussed later. It is the schematic, lumped element representation of the coupling of the superconducting order parameter to the heat bath. Around zero voltage (same chemical potential for quasiparticles on both sides) quasiparticles are only excited by the thermal energy. This follows the simple exponential behavior:

$$R_j = R_0 \exp \frac{\Delta_G(T)}{T k_B}, \quad (1.15)$$

$R_0$ being the normal resistance, when all current is carried by quasiparticles. At low temperatures, $\Delta_G$ can be assumed constant, otherwise approximated[28, 31] by different phenomenological formulas resembling the value calculated from the BCS theory.
1.3. QUASIPARTICLES AND THE GAP

Figure 1.4: Current-voltage characteristics of a small junction. Different tunneling processes contribute to the total current. At zero voltage (S-S tunneling) Cooper pairs tunnel from one electrode to the other. Above the gap the density of states \( D(E) \) is populated by quasiparticles in a finite energy region (Q-Q), depending on temperature.
Figure 1.5: Kirchhoff’s law requires the sum of currents into nodes (1) and (2) to be zero. Thus the external bias current distributes among the Josephson junction, the capacitance $C$ formed by the electrodes and the resistor $R$, representing the quasiparticle current. Analogue is a mechanical pendulum under the influence of gravitation, corresponding to the Josephson relation and with a mass, corresponding to the capacitance of the junction.

### 1.4 The RCSJ Model

Kirchhoff’s law, applied to node (1) in Fig. 1.5 is written as

$$I = ć \dot{U} C_j + \frac{U}{R_j} + I_C \sin \varphi.$$  \hspace{1cm} (1.16)

Rewriting this expression using the AC Josephson relation yields

$$I = \frac{\Phi_0}{2\pi} \dot{\varphi}_t C_j + \frac{\Phi_0}{2\pi} \varphi_t R_j + I_C \sin \varphi.$$  \hspace{1cm} (1.17)

The timescale of dynamical evolution is given by the plasma frequency of the junction,

$$\omega_P = \sqrt{\frac{I_C 2\pi}{\Phi_0 C_j}} = \sqrt{\frac{j_c 2\pi}{\Phi_0 C_j^*}},$$  \hspace{1cm} (1.18)

where $j_c$ and $C_j^*$ are the critical current density and the specific capacitance of the barrier. For a given production process and a given critical current density
the plasma frequency is independent of the junction size. Using $\omega_P$, Eq. (1.17) is rewritten to

$$-\varphi_{tt} \frac{1}{\omega_P^2} = \sin \varphi - \gamma + \alpha \frac{1}{\omega_P} \varphi_t,$$

(1.19)
as equation of motion for $\varphi$, where $\gamma = I/I_C$ and $\alpha = 1/(R_jC_j\omega_P)$. The role of the current $I$ is a driving force on the variable $\varphi$; $\alpha$ is a damping parameter. *Johnson noise* is omnipresent in resistive electrical circuits [32, 33, 34] as a consequence of the dissipation-fluctuation theorem. Therefore the parallel resistance to the junction also acts as a Gaussian current noise source. It can be defined most intuitively by the relation between resistance, temperature, observation bandwidth and RMS value.

$$I_n = \sqrt{\frac{4k_B T \nu_{BW}}{R_j}}$$

(1.20)

The time scale can be normalized as

$$t = \tilde{t} \omega_P^{-1},$$

(1.21)

so that Eq. (1.19) takes the form

$$-\varphi_{tt} = \sin \varphi - \gamma + \alpha \varphi_t.$$

(1.22)

Neglecting the quasiparticle damping by setting $\alpha$ to zero, this system is a Hamiltonian system with the equivalent Hamiltonian

$$H = (1 - \cos \varphi) + \frac{1}{2} \varphi_t^2 - \gamma \varphi$$

(1.23)
The energy scale $E_J$ of this Hamiltonian is given by Eq. (1.11)

The dynamics of a small junction in presence of quantum or thermal fluctuations, is more conveniently described after defining $\tilde{\hbar}_j = \hbar \omega_P/E_J$ to be the normalized Planck constant and $\tilde{k}_{B,j} = k_B/E_J$ to be the normalized Boltzmann constant. Rewriting $\tilde{\hbar}_j$ in terms of $j_c$ and the junction area $A$,

$$\tilde{\hbar}_j = \frac{\hbar}{A} \sqrt{\left(\frac{2\pi}{\Phi_0}\right)^3 \frac{1}{j_c C_j^*}}$$

(1.24)
shows that for a given junction fabrication process, the only way to increase $\tilde{\hbar}_j$ is to decrease the junction area.
1.5 The Two-Dimensional Sine-Gordon Equation

In an extended junction, such as illustrated in Fig. 1.6, supercurrent flows parallel to the Josephson barrier in the presence of a magnetic field. Taking into account that the magnetic flux in an inductive region corresponds to the product of the inductance and the current flowing, we use Eq. (1.6) and follow the dashed path, in Fig. 1.6. The phase gradient from points (1) to (2) and points (3) to (4) does not play a role due to the small thickness of this layer. If the path is chosen deep inside the electrodes we find the identity

$$\left(\phi_1 - \phi_2\right) - \left(\phi_3 - \phi_4\right) = \frac{2\pi}{\Phi_0} \left(\Phi_{\text{ext}} - L^* \vec{j}_{\text{inp}}(x_2 - x_1)\right), \quad (1.25)$$

where $L^*$ denotes the sheet inductance

$$L^* = \mu_0 d',$$ \quad (1.26)

$\vec{j}_{\text{inp}}$ the in-plane supercurrent density (Unit is $A/m$) in the x-direction, and $\Phi_{\text{ext}}$ the externally applied magnetic flux. The magnetic thickness $d'$ is given by $d' = t_J + 2\lambda L$, where $t_J$ is the thickness of the insulating barrier. Letting the distance over which the integration is carried out go to zero yields the first derivative $\phi_x$

$$\phi_x = \frac{2\pi}{\Phi_0} \left(H_{\text{ext}}\mu_0 d' - L^* \vec{j}_{\text{inp}}\right) \quad (1.27)$$

Generalizing this to two dimensions, where $\vec{H}_{\text{ext},V}$ consists of two in-plane components of the external magnetic field yields,

$$\nabla \phi = \frac{2\pi}{\Phi_0} \left(\vec{z} \times \vec{H}_{\text{ext},V}\right)\mu_0 d' - L^* \vec{j}_{\text{inp}}. \quad (1.28)$$

The supercurrent in the plane is given by

$$L^* \vec{j}_{\text{inp}} = \left(\vec{z} \times \vec{H}_{\text{ext},V}\right)\mu_0 d' - \frac{\Phi_0}{2\pi} \nabla^2 \phi. \quad (1.29)$$

Without an additional source of current, charge conservation in the sheet requires that

$$L^* \vec{j}_{\text{inp}} = \nabla\left(\vec{z} \times \vec{H}_{\text{ext},V}\right)\mu_0 d' - \frac{\Phi_0}{2\pi} \nabla^2 \phi = 0. \quad (1.30)$$
1.5. THE TWO-DIMENSIONAL SINE-GORDON EQUATION

![Diagram of a two-dimensional Josephson junction](image)

Figure 1.6: In a two-dimensional Josephson junction, the Josephson current and the displacement current act as local sources in the superconducting plane. The thickness of the barrier (bright) is not to scale. The dotted lines indicate the magnetic thickness, which is the barrier thickness plus twice the London penetration depth.

Using Kirchhoff’s current law, the Josephson current, the displacement current, the quasiparticle current, and the external bias current $\gamma j_c$ are added to the current corresponding to Eq. (1.30). This yields:

$$\Phi_0 \frac{1}{2\pi L^*} \nabla^2 \varphi + \nabla (\vec{z} \times \vec{H}_{\text{ext}, V}) = j_c \sin \varphi + \frac{\Phi_0}{2\pi} C_j^* \varphi_{tt} + \gamma j_c + \frac{1}{R^*} \frac{\Phi_0}{2\pi} \varphi_t, \quad (1.31)$$

where $R^*$ is the specific quasiparticle resistance.

This equation has a characteristic length scale, the Josephson length $\lambda_J$, which is given by

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi L^* j_c}}. \quad (1.32)$$

Normalizing the spatial coordinates to $\lambda_J$ by defining the normalized coordinate $\tilde{x} = x/\lambda_J$ and the time as in Eq. (1.21) yields the two-dimensional sine-Gordon equation in normalized units

$$\tilde{\Delta} \varphi - \varphi_{tt} = \sin \varphi - \gamma + \tilde{\nabla} \tilde{H}_{\text{ext}, V} + \alpha \varphi_t, \quad (1.33)$$

$^3$The surface impedance term, related to the AC voltage induced along the Josephson junction, only plays a role for high fluxon velocities and is neglected here.
where \( \tilde{\Delta} \) and \( \tilde{\nabla} \) denote the corresponding operators acting on normalized units, and \( h_{\text{ext}} \) is the normalized magnetic field, defined by

\[
\tilde{h}_{\text{ext},V} = (\vec{z} \times \tilde{\vec{H}}_{\text{ext},V}) / H_0, \quad H_0 = \frac{\Phi_0}{2\pi d' \lambda J \mu_0}.
\]

(1.34)

Typical values for \( H_0 \) are on the order of one gauss for the samples used. (see Appendix appendix C). Eq. (1.33) can be solved numerically or in the special case of a long junction be reduced to an one-dimensional model, where all properties of the junction are described as a function of the position along the junction. The electromagnetic wave propagation velocity (Swihart velocity) is given by

\[
\bar{c} = \frac{\lambda J}{\omega P}
\]

(1.35)

### 1.6 Long Josephson Junctions

If the transverse dimension is smaller that the Josephson length \( \lambda_J \) and the longitudinal dimension is larger than the Josephson length, then we are dealing with the special case of a long junction. Long Josephson junctions (LJJ) have received a lot of attention as an ideal model system for the one-dimensional sine-Gordon equation. Comprehensive overviews are given in Refs. [35, 36, 37, 38].

The reduction of the dimensionality takes place by assuming that \( \tilde{\Delta} \varphi \) can be replaced by \( \varphi \tilde{x} \tilde{x} \), where \( x \) denotes the coordinate along the junction, along the vector \( \vec{x} \), furthermore that \( \tilde{\nabla} \vec{h}_{\text{ext},V} \) can be replaced by \( \partial h_y / \partial \tilde{x} \). This is justified, if the width in \( y \)-direction is sufficiently smaller than the Josephson length. In this limit, the currents perpendicular to the longitudinal axis are assumed to be zero. This results in a one-dimensional transmission line model, as shown in Fig. 1.7, where the junction electrode is represented by a inductance. Furthermore this approximation requires that the functional determinant of the transformation from the junction coordinate system to the laboratory system is close to one, which essentially limits this approximation to junctions where the curvature of longitudinal axis is sufficiently small[17]. This condition is violated in the heart-shaped Josephson junctions[13], as discussed in sec. 5.1. After the simplifications, the one-dimensional sine-Gordon equation takes the form:

\[
\varphi \tilde{x} \tilde{x} - \varphi \tilde{t} \tilde{t} = \sin \varphi - \gamma + \frac{\partial h_{\text{ext}}}{\partial \tilde{x}} - \alpha \varphi \tilde{t}.
\]

(1.36)

For the \( \alpha = 0 \) case the system is Hamiltonian. The case of zero bias current and
Figure 1.7: An equivalent lumped element electrical circuit is for a long Josephson junction. The central assumption is, that the phase difference in Fig. 1.6 between the upper and lower electrode is the equal at both transversal edges of the junction \( y_1 \) and \( y_2 \). Hence the system can then represented by a inductance-Josephson transmission line along the JJ, carrying a current proportional to the longitudinal phase gradient.

zero external magnetic field

\[
\varphi_{\ddot{x}} - \varphi_{\ddot{y}} = \sin \varphi
\]

is called the unperturbed sine-Gordon equation.

The Hamiltonian consists of the Josephson energy,

\[
\mathcal{H}_J = \int_0^w \int_0^l (1 - \cos \varphi) \, d\tilde{x} \, d\tilde{y},
\]

the inductive energy

\[
\mathcal{H}_L = \int_0^w \int_0^l \frac{1}{2} \varphi_{\ddot{x}}^2 \, d\tilde{x} \, d\tilde{y},
\]

the capacitive energy,

\[
\mathcal{H}_C = \int_0^w \int_0^l \frac{1}{2} \varphi_{\ddot{y}}^2 \, d\tilde{x} \, d\tilde{y},
\]

an energy term describing the driving force of the bias current

\[
\mathcal{H}_\gamma = - \int_0^w \int_0^l \varphi \gamma \, d\tilde{x} \, d\tilde{y}
\]
and the interaction with the magnetic field

\[ \mathcal{H}_H = \int_0^w \int_0^l \varphi \frac{\partial h_{\text{ext}}}{\partial \bar{x}} \, d\bar{x} \, d\bar{y}. \] (1.42)

The total Hamiltonian

\[ \mathcal{H}(\varphi, \varphi_\ell) = \mathcal{H}_J + \mathcal{H}_L + \mathcal{H}_C + \mathcal{H}_\gamma + \mathcal{H}_H \] (1.43)

can be used for deriving effective potentials using perturbation theory. Assuming constant width \( w \), Eq. (1.43) is written as

\[ \mathcal{H}(\varphi, \varphi_\ell) = w(h_{\text{ext}}(0)\varphi(0) - h_{\text{ext}}(l)\varphi(l)) + \]
\[ w \int_0^l \left( \frac{1}{2} \varphi_\ell^2 + \frac{1}{2} \varphi^2 + \varphi_\gamma + \varphi_\ell h_{\text{ext}} - \cos \varphi \right) \, d\bar{x} \] (1.44)

If the assumption of a homogeneous junction is weakly violated, but the width of the junction is smaller than \( \lambda_J \) it is possible to assume \( \varphi \) to be constant in the \( y \) direction. I use this later to derive the potentials for changing width and finite radii.

The energy scale \( E_0 \) of this Hamiltonian is given by

\[ E_0 = \frac{\Phi_0}{2\pi} w \lambda_J = E_J \frac{1}{l}, \] (1.45)

the Josephson energy of a small junction of width \( w \) and length \( \lambda_J \). Together with the time normalization, this results in a normalized Planck’s constant for a LJJ Hamiltonian

\[ \tilde{\hbar} = \frac{\hbar}{E_0} \omega_P = \tilde{\hbar}_j l \] (1.46)

and a normalized Boltzmann constant

\[ \tilde{k}_B = \frac{k_B}{E_0} = \tilde{k}_{B,j} l. \] (1.47)

The difference from Eq. (1.24) is that reducing the minimum feature size in a lithographic process only reduces the width, while the length scale is set by the Josephson length. Therefore the minimum feature size enters in the power \(-1\) instead of \(-2\).

\[ ^4 \text{Integration by parts} \]
1.6. LONG JOSEPHSON JUNCTIONS

1.6.1 Soliton Solutions

For the time-independent (static) case without magnetic field and bias current Eq. (1.36) is reduced to the Ferrell-Prange equation:

$$\varphi \dddot{x} = \sin \varphi.$$  \hspace{1cm} (1.48)

For infinitely long systems there exist soliton solutions

$$\varphi_f(x, \tilde{x}_0) = \pm 4 \arctan \left( e^{\tilde{x} - \tilde{x}_0} \right),$$  \hspace{1cm} (1.49)

where $\tilde{x}_0$ is the center of mass position of the excitation. The corresponding phase profile is plotted in Fig. 1.8a. A two-dimensional plot of the energy density of the position and phase dependent terms of Eq. (1.43), as depicted in Fig. 1.9 visualizes that the solution $\varphi_f$ corresponds to a chain connecting one Peierls valley [39] to another.

1.6.2 Small Wave Excitations

Small amplitude perturbations around a static state $\varphi_0(\tilde{x})$ are possible. The ansatz

$$\varphi(\tilde{x}, \tilde{t}) = \varphi_0(\tilde{x}) + \varphi_\delta(\tilde{x}, \tilde{t}),$$  \hspace{1cm} (1.50)

placed in Eq. (1.36) yields, at zero dissipation:

$$\varphi_0 \dddot{x} + \varphi_\delta \dddot{x} - \varphi_\delta \dddot{t} = \sin(\varphi_0 + \varphi_\delta).$$  \hspace{1cm} (1.51)

For small amplitude fluctuations $\varphi_\delta$ this takes the form

$$\varphi_0 \dddot{x} + \varphi_\delta \dddot{x} - \varphi_\delta \dddot{t} = \sin \varphi_0 + \varphi_\delta \cos \varphi_0,$$  \hspace{1cm} (1.52)

which after subtracting the Ferrel-Prange equation reads

$$\varphi_\delta \dddot{x} - \varphi_\delta \dddot{t} = \varphi_\delta \cos \varphi_0.$$  \hspace{1cm} (1.53)

Rewriting leads to

$$-\varphi_\delta \dddot{t} = \varphi_\delta \cos(\varphi_0) - \varphi_\delta \dddot{x}.$$  \hspace{1cm} (1.54)

Since this is a linear differential equation, the general solution takes the form

$$\varphi_\delta = \sum_{i=1}^{N} A_i \exp(\omega_i \tilde{t}) \varphi_{\delta_i}(\tilde{x}),$$  \hspace{1cm} (1.55)

$\varphi_{\delta_i}$ being the eigenmodes of Eq. (1.54) and $A_i$ the amplitude of the small waves.
CHAPTER 1. INTRODUCTION AND THEORY

Figure 1.8: (a) The phase profile of a vortex located at $x_N = 0$, (b) the magnetic field in the barrier (normalized).

Plasmons

Small waves in a homogeneous long Josephson junctions are called plasmons. The constant choice $\varphi_0 = 0$ yields

$$\varphi_0 = 0$$

an inhomogeneous wave equation. The solutions

$$\varphi_\delta(k, \omega_0(k)) = \exp(i(k\tilde{x} - \omega_0(k)\tilde{t}))$$

of this are small amplitude sinusoidal waves. The dispersion relation $\omega_0^2(k) = 1 + k^2$ is graphed in Fig. 1.10 for an infinitely long Josephson junction. Modes of frequencies $\omega < 1$ correspond to evanescent modes in a waveguide or low frequency waves reflected upon incidence with a plasma. The transmission of energy via such modes drops exponentially with the length of the system, as known from textbooks[40], with a damping constant of $\alpha_{\text{plasma}} = \frac{2\omega_P}{\tilde{c}}$. In normalized units, $\tilde{\alpha}_{\text{plasma}} = 2$, corresponding to an attenuation of 8.7dB per
1.6. LONG JOSEPHSON JUNCTIONS

Figure 1.9: The vortex phase profile connects Peierls valley 0 with Peierls valley 1. For infinite or annular systems, this defines the solution sector topologically.

Josephson length. For a homogeneously biased Josephson junction $\varphi_0$ takes the value $\arcsin \gamma$, and the cutoff frequency as well as the damping constant are changed. Nonlinear PDE simulations of LJJs embedded in superconducting circuits[41], to evaluate the use as tunable filters for qubit control, verify the scaling of the filter properties with the bias current.

1.6.3 Idle Region Effects

The long junctions used in this work were produced using trilayer technology. Due to limited resolution of photo lithography the junctions have a passive region, as depicted in Fig. 1.11. The junction itself has an active region of width $w$, and electrodes outside the junction of width $w_2$. Such junctions have been discussed in Refs. [42, 43]. The effective inductance per unit length is in this
case not given by the sheet inductance, divided by the width but rather by

\[ L_{\text{eff}}^* = L^* \frac{1}{\frac{2w_2d}{wd'} + 1}. \]  

(1.58)

A detailed experimental investigation of these effects was presented in Ref. [44]. The idle region leads to an increase of the Josephson length \( \lambda_J \) to an effective value of

\[ \lambda_{\text{eff}} = \lambda_J \sqrt{1 + \frac{2w_2d}{wd'}}. \]  

(1.59)

1.7 Annular Junctions

One particular geometry of Josephson junctions is the annular geometry, illustrated in Fig. 1.12 and schematized in Fig. 1.13. The flux quantization allows the change of the superconducting phase \( \varphi \) around an annular junction circumference to be a multiple of \( 2\pi \) only. This imposes a special kind of boundary condition on annular junctions. The difference in the number of trapped flux quanta in the upper and lower superconducting ring is called the vorticity \( n_v \). In terms of \( n_v \) the boundary conditions between the phases at position 0 and \( l \) take the form

\[ 2\pi n_v = \varphi(l) - \varphi(0). \]  

(1.60)
1.7. ANNULAR JUNCTIONS

\[ d = t + 2 \lambda_L \]

Figure 1.11: A cross-section through a lithographically produced sample, with the Josephson active region of width \( w_1 \) and a passive region of width \( w_2 \).

Figure 1.12: An annular Josephson junction consists of two superconducting ring electrodes separated by a thin isolating barrier. Around each electrode a persistent current, corresponding to a multiple of \( \Phi_0 \), can flow. The difference in the magnetic flux trapped in the superconducting rings threads the barrier.
and, because the magnetic field in the junction is continuous,

\[ 0 = \varphi_x(l) - \varphi_x(0). \]  

(1.61)

The boundary condition Eq. (1.60) is represented in Fig. 1.13 as an ideal phase source\(^5\). Such a source corresponds to an infinitely small inductance, across which an external current generates a phase drop of \(2\pi\). A non-ideal implementation of such an element are current injectors [45].

For example, annular boundary conditions for \(n_v = 0\) and \(n_v = 1\) correspond to a mechanical model as rendered in Fig. 1.14 (vorticity \(n_v = 0\)) and in Fig. 1.15 (vorticity \(n_v = 1\)). A chain of pendula coupled via torsion springs is hanging on a ring. To introduce a change in the vorticity, the chain is twisted by the appropriate number of turns. For long systems the vortex approximation, valid for infinitely long Josephson junctions, is valid. For shorter systems the solution is given by cnoidal waves, which represent a dense vortex chain, or alternatively a single vortex in a finite annular Josephson junction, as described in the next chapter. The topological charge of the excitations is fixed, thus the driving force of a bias current. Within a homogeneous system, the solutions are invariant under translation along the junction length.

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\(^5\)Standard 2D PDE packages, such as the PDE toolbox in MATLAB do not support such boundary conditions out of the box, but could be modified with a moderate effort.
Figure 1.14: The mechanical analog of an annular Josephson junction [46] with vorticity $n_v = 0$. The pendula (black) correspond to a section of the Josephson junction. The springs between the pendula are represent a torsion coupling between the pendula. The situation depicted corresponds to a junction bias close to $\gamma = 1$ under a small applied magnetic field.

Figure 1.15: The mechanical analog of an annular Josephson junction, vorticity $n_v = 1$. The junction length is $20\lambda_J$ (three pendula per Josephson length). The junction is unbiased and at zero magnetic field.
1.7.1 Perturbation Theory: Vortex Effective Potentials

The application of perturbation theory[47] treats statics and dynamics of LJJ vortex states. In this section, effective potentials for a conservative vortex equation of motion are derived. This is based on the field Hamiltonian of the system, including the bias current and the external magnetic field. The effect of dissipation is discussed in the next section.

For an annular Josephson junction, the external magnetic field term takes the form

$$h_{\text{ext}}(\tilde{x}) = \tilde{h}_{\text{ext}} \cos(\tilde{x}/\tilde{r}),$$

with \(\tilde{r}\) being the normalized radius of the junction, and \(\tilde{h}_{\text{ext}}\) the externally applied magnetic field.

For annular junctions Eq. (1.44) can be rewritten as

$$H(\varphi, \varphi_t) = \int_0^l \left( \frac{1}{2} \varphi_{\tilde{x}}^2 + \frac{1}{2} \varphi_{\tilde{t}}^2 - \gamma \varphi + \varphi_{\tilde{x}} \cos(\tilde{x}/\tilde{r}) \tilde{h}_{\text{ext}} - \cos \varphi \right) d\tilde{x}$$

The first term of Eq. (1.44) is independent on \(\varphi\) and therefore removed.

Solutions have to fulfill the boundary conditions (1.60) and (1.61). After setting \(h_{\text{ext}}\) and \(\gamma\) to 0, the system is totally homogeneous. In the derivation of the effective potentials it is assumed that neither \(h_{\text{ext}}\) nor \(\gamma\) can change the solutions of the unperturbed sine-Gordon equation, Eq. (1.37). A necessary, but not sufficient condition for this, is that for a given unperturbed solution the energy change of the total system due to these terms is small in comparison to the rest energy of the solution itself.

A static solution \(\varphi_f\) of Eq. (1.37) is generalized by Lorentz transformation to a set of solutions

$$\varphi(\tilde{x}, \tilde{t}) = \varphi_f \left( \frac{\tilde{x} - \tilde{x}_0 - \tilde{v} \tilde{t}}{\sqrt{1 - \tilde{v}^2}} \right).$$

In the non relativistic limit \((\tilde{v}^2 \ll 1)\), the only effect of the transformation is a spatial translation of the solution.

$$\varphi(\tilde{x}, \tilde{v}) = \varphi_f(\tilde{x} - \tilde{x}_0 - \tilde{v} \tilde{t}).$$

The time derivative \(\varphi_t\) is given by

$$\varphi_t(\tilde{x}, \tilde{t}) = \tilde{v} \varphi_{f,\tilde{x}}(\tilde{x} - \tilde{x}_0 - \tilde{v} \tilde{t}).$$
The varying terms of the Hamiltonian are given by

\[
H(\varphi, \varphi_t) = \ln \gamma + \int_0^l \frac{1}{2} \varphi_{t, \bar{x}}^2 d\bar{x} + \int_0^l -\gamma \varphi_{t, \bar{x}} \bar{x} d\bar{x} + \int_0^l \varphi_{t, \bar{x}} \cos(\bar{x}/\bar{r}) \hat{h}_{\text{ext}} d\bar{x}
\]

(1.67)

with \( \kappa_f \) being the first Fourier component of \( \varphi_f \). In general, the last term in Eq. (1.67), is a convolution of the local magnetic field. For the solution defined by Eq. (1.49), corresponding to the magnetic profile

\[
\varphi_{f, \bar{x}}(\bar{x}, \bar{x}_0) = 2 \text{sech} (\bar{x} - \bar{x}_0),
\]

(1.68)

\( \kappa_f = 2 \pi \text{sech} \left( \pi^2/l \right) \) is found [48], resulting in the potential \( U_{h}(\bar{x}_0) \)

\[
U_h = -\hat{h}_{\text{ext}} 2 \pi \text{sech}(\pi^2/l) \cos(\bar{x}_0/\bar{r}) = -2 \pi \hat{h} \cos(\bar{x}_0/\bar{r}),
\]

(1.69)

where \( \hat{h} = \kappa \hat{h}_{\text{ext}} \).

### 1.7.2 Vortex Dynamics

The equation of motion for \( \bar{x}_0 \) is given by

\[
\tilde{m} \ddot{\bar{x}}_0 = 2 \pi \gamma - \frac{\partial U_h}{\partial \bar{x}_0},
\]

(1.70)

where the mass \( \tilde{m} \), associated with a translation of the excitation is given by

\[
\tilde{m} = \int_0^l \varphi_{\bar{x}}^2 d\bar{x}.
\]

(1.71)

For the solution given by Eq. (1.49) in an infinite junction,

\[
\tilde{m} = \int_{-\infty}^{\infty} \varphi_{f, \bar{x}}^2(\bar{x}) d\bar{x} = 8.
\]

(1.72)

To take into account the effects of the dissipative quasiparticle term in Eq. (1.36), the corresponding Rayleigh dissipation function[49] can be constructed. which describes the power dissipated in the system and, therefore,
the change in the total energy of the modified Hamiltonian system. For the gen-
eralized force, given by
\[ \gamma_\alpha = -\alpha \varphi_{\tilde{t}}, \]  
(1.73)
the Rayleigh dissipation function \( F \) is then given by
\[ F = \int_0^l \frac{1}{2} \alpha \varphi_{\tilde{t}}^2 d\tilde{x}. \]  
(1.74)
After inserting \( \varphi_{\tilde{t}}, \tilde{x} \), the dissipated energy is given by
\[ F = \frac{1}{2} \tilde{v}^2 \int_0^l \varphi_\tilde{x}^2 d\tilde{x}. \]  
(1.75)
The vortex equation of motion Eq. (1.70) is modified to
\[ \tilde{m} \tilde{v}_{\tilde{t}} = -\frac{dU_1(\tilde{x}_0)}{d\tilde{x}_0} - 2\pi \gamma - \tilde{m}\alpha \tilde{v}. \]  
(1.76)
Any excitation residing in a small or large Josephson junction, for which the local expression is given by Eq. (1.19), in especial any excitation discussed in this thesis obtains dynamical mass by the capacitive energy, which is governed by the same term as the Rayleigh dissipation function. It is therefore not a co-
incidence that the mass term appears on the RHS of Eq. (1.76). The normalized damping \( \eta \) is determined by \( \eta = \alpha \), as long as the resistance \( R_j \) is the only source of dissipation.

The average voltage at a chosen point of the junction is proportional to the change of the flux in the barrier. A vortex with the limiting Swihart velocity \( \bar{c} \) of electromagnetic waves in the junction generates voltage \( V_0 \), given by
\[ V_0 = \frac{\bar{c}\Phi_0}{2\pi r}. \]  
(1.77)
The specific shape of the current-voltage characteristics is determined by per-
turbation theory developed by McLaughlin and Scott [47]. Perturbation theory results and experiments are compared in detail in Ref. [50]. In the relativistic limit, the friction force is related to the equilibrium velocity \( \tilde{v}_\infty \) by
\[ F = 8 \frac{\tilde{v}_\infty}{\sqrt{1 - \tilde{v}_\infty^2}} \left( \alpha + \frac{\beta}{2 - (1 - \tilde{v}_\infty^2)} \right), \]  
(1.78)
where $\beta$ is the surface impedance damping parameter. Since bias current $\gamma$ exerts a force of $2\pi$ on the vortex, Eq. (1.78) defines the current-voltage characteristics of a long junction. An example current-voltage characteristic for vorticity $n_v = 1$ is plotted in Fig. 1.16. Residual pinning due to nearby trapped Abrikosov vortices causes a finite zero-voltage current. For a perfectly homogeneous annular Josephson junction a zero depinning current is expected. Voltage corresponding to the Swihart velocity $\bar{c}$ is only a fraction of the gap voltage of the superconductor. Inhomogeneities in the system cause the vortex to emit electromagnetic waves, which cause the system to generate additional vortex-antivortex pairs. In this limit the single vortex approximation is not suitable any more.
Figure 1.16: A current-voltage characteristic for a vortex trapped in an annular Josephson junction. At point (1) the vortex is depinned, at point (2) the vortex solution becomes unstable and the whole junction switches to the resistive state. The dashed line shows a fit of Eq. (1.78) to the experimental dataset. The vertical dotted lines show the voltage corresponding to the Swihart velocity.
Chapter 2
Experimental Technique and data evaluation

2.1 Measurement Scheme

The most typical electric measurement scheme schematized in Fig. 2.1. The circuit consists of a voltage-controlled current source, and a low-noise preamplifier (high dc impedance), which together form a four wire measurement. At the maximum sensitivity of the setup, differential resistances on the order of mΩ can be determined without lock-in techniques. Details on the filtering circuits and electronic error estimates are described in appendix A.

For measurements of the current voltage characteristics, the voltage output and current monitor signals are measured by an AD converter and the control voltage for the current source is generated by a DA converter. Both are located on a buffered synchronous data acquisition card[51, 52, 53] in the controlling computer. Time-series of both current and voltage are measured and evaluated by software. This allows the definition of complex waveforms, which can be used to test state preparation and readout schemes.

An alternative mode of operation of the setup allows for the control voltage to be generated by an analog function generator. The voltage signal is compared to a threshold value, chosen to detect the transition of the system from the superconducting (zero-voltage) state to the running (finite voltage) state by triggers. The time delay between the trigger edges of the current and the voltage trigger is acquired by a high-precision time interval counter [54]. A detailed description of
the predecessor setup (Ref. [55]) and appendix B describe the noise properties of the room-temperature data acquisition equipment, while appendix A describes the Johnson noise in the biasing resistors.

### 2.2 Vortex Injection

Vortices in annular Josephson junctions may be trapped during the transition of the superconducting electrodes through the critical temperature. Another, more controllable and reversible way of introducing the vortices into an annular junction is to use a pair of current injectors which form a local current dipole [45, 56].

The geometry of the annular Josephson with injectors is illustrated in Fig. 2.2. The corresponding lumped element model\(^1\) is schematized in Fig. 2.3. In addition to the current source homogeneously biasing the Josephson junction, two local current sources inject positive and negative current of the same amplitude to two neighboring nodes. According to the lumped element model, this cell can be biased to contain an additional quantum of magnetic flux. This cor-

---

\(^1\)This scheme represents the situation used in our experiments presented in this chapter. Instead of an isolated current source [45], two current sources of opposite polarity are utilized. From the viewpoint of electronics, this offers significant advantages, see appendix B for a detailed explanation.
2.2. VORTEX INJECTION

Figure 2.2: Sample geometry of the samples used. The top metalization layer is green/hatched, the bottom metalization layer is yellow/solid. In the top layer, current injectors are made to create a local twist in the phase.

Figure 2.3: Lumped element model of an injector, using two separate current sources which locally inject the additional currents $I_{\text{inj}}$ and $-I_{\text{inj}}$. This produces an artificial jump in the phase across the inductance $L_1$. 
responds to a phase jump of $2\pi$ over the element $L_1$ in Fig. 2.3. Since the total vorticity of the junction is conserved, the phase difference of $-2\pi$ distributes along the rest of the junction. This corresponds to the phase drop induced by the annular boundary conditions. A formed vortex can move freely along the Josephson junction. Thus, by injecting an appropriate current, the vorticity of the annular Josephson junction state can be changed without leaving the superconducting state. If the distance between the injectors is on the order of a $\lambda_J$ or less, the net current flowing perpendicular through the junction is negligible.

The statics and dynamics of injected vortices are very similar to that of vortices trapped during cooldown. A comparison of the current-voltage characteristics of both is displayed in Fig. 2.4. In the zero voltage state additional pinning at the locations of the injectors is observed. Figure 2.5 compares the two-dimensional $I_C(H)$ patterns[10] for both types of vortices, acquired on sample AJJ-INJ (see appendix C). Since the injected current generates magnetic flux in the junction barrier, an additional pinning occurs at the injectors.
2.2. VORTEX INJECTION

Figure 2.5: Comparison of the depinning current of a vortex trapped during cooldown (dashed white lines) and an artificially inserted one (density plot, black contour). Current difference between isolines is 0.5 mA. At location (1) the patterns deviate due to parasitic pinning by the magnetic flux by the barrier at the injectors, while at (2) and (3) the critical currents coincide.
CHAPTER 2. EXPERIMENTAL TECHNIQUE AND DATA EVALUATION

2.3 Escape Field Distributions

Under applied bias current the zero voltage state of a Josephson junction is a metastable state. To examine fluctuations of the escape current due to thermal excitations or quantum mechanical effects, we use the well known\cite{3, 4, 5} technique of repeatedly ramping the bias field with an analog function generator. A common model for the activation is the depinning of a point-like particle of mass $m_{\text{eff}}$ from a potential well, depicted in Fig. 2.6. It is convenient to define the reduced bias field $\epsilon = (1 - \gamma/\gamma_{\text{cr}})$. Thermal or quantum fluctuations cause the lifetime, defined as the inverse of the escape rate $\Gamma_{\text{esc}}$, of the trapped state to be finite. For a wide class of potentials, both the thermal and quantum escape rates are modelled as a function of $\epsilon$ by the general form

$$\Gamma_{\text{esc}}(\epsilon) = A\epsilon^{a+b-1} \exp(-B\epsilon^b).$$

The type of activation and the damping limit of the system determines $a$ and $b$, while and $A$ and $B$ depend on sample parameters, temperature and external field or current biases. In the case of thermal activation $B$ is the ratio of the unbiased barrier height, and the thermal energy

$$B = \frac{\overline{U}}{k_B T},$$

and $b = 3/2$. The meaning of $A$ and $a$ is specific to the different damping regimes, as discussed in the next section. In the experiments the bias current $\gamma$ is increased at the constant normalized ramp rate $\gamma_{\tilde{t}}$

$$\gamma = \gamma_{\tilde{t}} \tilde{t}.$$

The normalized ramp rate is related to the derivative of the bias current $\dot{I}_b$ as

$$\gamma_{\tilde{t}} = \frac{\dot{I}_b}{I_C \omega_P},$$

where $I_C$ and $\omega_P$ are not necessarily known to a high precision. Ideally both are determined using spectroscopy. Otherwise both are estimated by a measurement of the critical current (density) and an estimation of the junction capacitance. The values for $\gamma_{\tilde{t}}$ are in our setup typically on the order of $10^{-10} \cdot \cdot \cdot 10^{-7}$. Due to this slow ramp the system remains in equilibrium, as opposed to the intentional excitation technique used by Silvestrini et al.\cite{57}. Here $\Gamma_{\text{esc}}(t)$ is defined by Eq. (2.1) or more generally as a function of $\gamma(\tilde{t})$ only.
2.3. ESCAPE FIELD DISTRIBUTIONS

Figure 2.6: Potentials depending on reaction coordinate $q$ (curves are offset for visibility), for different reduced bias fields $\epsilon$. The dot denotes the stable minimum position. If $\epsilon$ reaches zero, the potential has no stable position any more. In a stable minimum, the particle oscillates with the small oscillation frequency $\omega_0$. For the unbiased potential, the potential barrier height $U_0$ is indicated.
When the system is activated, the junction switches to the voltage state. The time interval between the moment of the bias current \( I_b \) to be zero and of this transition to the voltage state is recorded and the bias current is set to zero. The ensemble of these time intervals forms the observations of a decay from an occupation probability \( P = 1 \) for zero time (all members of the ensemble in zero voltage state), which decays to zero close to the critical current\(^2\).

Solving the differential equation

\[
\dot{P} = -P \Gamma_{esc}(\gamma(t)),
\]

with the initial value \( P(0) = 1 \), yields the cumulative distribution function (CDF). The probability distribution function (PDF) is then given by \(-\dot{P}\). In this thesis the specific solution of Eq. (2.5) for a thermal escape is referred to as the Kurkijärvi-Fulton-Dunkelberger (KFD) \([3, 4]\) distribution. Dividing Eq. (2.5) by \(-P\) and substituting \( P \) by \( 1 - \int_{\tau=0}^{\tilde{t}} \dot{P}(\tau) \)

\[
\Gamma_{esc}(\gamma(t)) = \frac{\dot{P}(\tilde{t})}{(1 - \int_{\tau=0}^{\tilde{t}} \dot{P}(\tau)d\tau)}
\]

is obtained. Experimental observations can be binned into finite intervals to approximate the PDF by a histogram like displayed in Fig. 2.7 (Sample AJJ-SHORT (see appendix C)), from which a rate estimate is calculated. The corresponding rate estimate is depicted in Fig. 2.8. The advantage of this method that it is easy to understand. A standard technique is to fit the rate model \( \Gamma_{esc}(a, b, A, B) \) to the estimate \([3, 5, 6, 8]\) using least-squares fitting. This method leaves the freedom of choosing an arbitrary binning to the experimenter as does \( \chi^2 \) fitting of the probability distribution function \([58]\) to the histogram. Another method is the maximum likelihood estimation (MLE) \([59]\), which finds the most likely parameter set for the model defining the KFD PDF and an observation ensemble. This method relies on iterative optimization algorithms and a probability model of the observations distribution itself. In the case of large datasets, the statistical errors in the \( \chi^2 \) are negligible and both methods (MLE and \( \chi^2 \)) are equivalent. If the histogram exhibits long tails, the MLE method is expected to have a better performance.

During each MLE iteration step, Eq. (2.5) has to be integrated. A numerical integration of Eq. (2.5) is time consuming\(^3\). It is quite easy to see that Eq. (2.5)

\(^2\)For very high ramp rates the decay takes place (in a classical picture) after crossing the critical force, equivalent to having a finite population at \( \epsilon = 0 \).

\(^3\)Typical iteration counts are on the order of 100. Integration using an adaptive Runge-Kutta ODE
is a homogeneous differential equation, where the solution takes the simple form

\[ P(t) = 1 - \exp(s). \] (2.7)

The persistence probability exponent \( s \) is given by

\[ s(t) = - \int_0^t \Gamma_{\text{esc}} dt. \] (2.8)

For rates of the form of Eq. (2.1) the persistence probability \( s \) can be developed into a series\(^4\) if the PDF is negligible small everywhere besides \( \epsilon \ll 1 \). Using \( s \), the PDF and CDF (Cumulative distribution function) can be derived as closed form expressions, which are suitable for MLE fitting. The series developed for by Garg [60] is the following:

\[ s(\epsilon) = \frac{A}{|\gamma_i|} \gamma_{\text{cr}} e^a \chi^B Bb \left( 1 + \frac{a}{b} \chi + \frac{a(a - b)}{b^2 B^2} \chi^2 + O(\chi^3) \right), \] (2.9)

\(^4\)By partial integration.

---

Figure 2.7: Histograms (1200 samples) for two different temperatures. At the higher temperature the histograms is wider due to the lower Arrhenius exponent.
where \( \chi = \exp(\epsilon^{-b}) \). In our experiments \( \mathcal{B} \gg 1 \) always holds. The physical meaning of this is that an unbiased or moderately biased system is not activated with any measurable rate. Using terms up to the first order in \( \chi \) in Eq. (2.9), the mean value of the reduced bias field at which the system switches is given by

\[
\langle \epsilon \rangle^b = \frac{\bar{B}}{\mathcal{B}},
\]

with \( \bar{B} \) given by

\[
\bar{B} = \log \left( \frac{\gamma_{cr}}{\gamma_i} A \frac{1}{b \mathcal{B}^{(1+a/b)}} \right). \tag{2.11}
\]

At the mean escape current, \( \bar{B}^{1/b} \) is the exponent in \( \Gamma_{esc} \). The standard deviation is given by

\[
\sigma^2 = \frac{1}{6} \left( \frac{\bar{B}^{(-2+\frac{2}{b})}}{\mathcal{B}^{2/b^2}} \right). \tag{2.12}
\]
2.4 Thermal Escape in a Washboard Potential

In the following all escapes are treated for a one dimensional washboard potential. A washboard potential is an effective potential expressed as a function of a generalized coordinate \( q \) in the form

\[
U = -U \cos \left( \frac{q}{r} \right) - \gamma q,
\]

(2.13)

where \( U \) is the potential barrier height at zero bias current, and \( r \) sets the length scale of the potential well. In our systems \( r \) is either the normalized radius of an annular junction, or \( r \) equal to one in the case of a small junction. In this potential, a metastable state exists as long as the condition \( \gamma < \gamma_{cr} \) remains fulfilled, with \( \gamma_{cr} \) given by

\[
\gamma_{cr} = \frac{U}{r}.
\]

(2.14)

After determining the equilibrium position, the potential barrier at a given bias current is calculated to be

\[
U_0 = U \sqrt{1 - \left( \frac{\gamma}{\gamma_{cr}} \right)^2}.
\]

(2.15)

A cubic approximation (\( O(3) \) in \( q \)) of \( U_0 \) around \( \gamma = \gamma_{cr} \), i.e. \( \epsilon = 0 \), yields

\[
U_0 = \bar{U} \epsilon^{3/2}
\]

(2.16)

where \( \bar{U} = U \sqrt{2^2/3} \). The coefficient is due to the cubic approximation of the cosinusoidal potential. The equation of motion with friction is given by

\[
\ddot{q} + \gamma m_{eff} \dot{q} - \eta \dot{q} = 0,
\]

(2.17)

where \( m_{eff} \) is the effective mass associated with \( q \). This equation of motion leads to a small oscillation frequency \( \omega_0 \) in the metastable state

\[
\omega_0 = \Omega_0 \left( 1 - \left( \frac{\gamma}{\gamma_{cr}} \right)^2 \right)^{1/4},
\]

(2.18)

where \( \Omega_0 \) is given by

\[
\Omega_0 = \frac{1}{r} \sqrt{\frac{U}{m_{eff}}}.
\]

(2.19)
Eq. (2.18) can be developed to the first order around \( \epsilon = 0 \):

\[
\omega_0 = \Omega_0 \epsilon^{1/4}.
\] (2.20)

Classical and quantum escape dynamics are, for a single event, modelled [61] using the variable parameters \( \omega_0 \) and \( U_0 \), and the constant temperature \( T \), damping \( \eta \) and mass \( m_{\text{eff}} \). Thermal fluctuations cause an escape rate from a metastable well which depends exponentially on the negative inverse of the temperature \( T \). This is due to the probability of finding an instance of the thermal ensemble at the barrier energy. Thus,

\[
\Gamma_{\text{esc}} = A \exp \left( -\frac{U_0}{k_B T} \epsilon^{3/2} \right) = A \exp \left( -B \epsilon^{3/2} \right),
\] (2.21)

where \( A \) is the Arrhenius factor of the system, which depends on damping, as discussed below. Physically \( B = \bar{U}/k_B T \) corresponds to the Arrhenius exponent of the unbiased system, and the exponent for use in Eq. (2.1) \( b = 3/2 \).

### 2.5 Damping Regimes

There are different damping limits, as realized already by Kramers.

1. The simple transition state theory (TST) neglects damping and assumes a thermalized occupation of phase space. In this case a particle on a given phase space trajectory arrives one time per small oscillation at its highest potential energy. A phase space diagram without damping is plotted in Fig. 2.10a. If the thermal energy exceeds the barrier height, the particle escapes from the well. In this case the prefactor \( A \) in Eq. (2.21) is given by the attempt frequency (the small oscillation frequency)

\[
A = \frac{\omega_0}{2\pi},
\] (2.22)

 equivalent to (see Eq. (2.20)):

\[
A = \frac{\Omega_0}{2\pi} \epsilon^{1/4} = A \epsilon^{1/4},
\] (2.23)

with \( A = \Omega_0/2\pi \) and \( a = -1/4 \).

2. The low damping limit is very similar to the first case discussed above, but it is necessary to reconsider the condition of thermal equilibrium of
the ensemble. For low damping, the system is nearly Hamiltonian. Thermalization happens by a fluctuating force, which in electrical circuits occurs due to the Johnson noise (see Eq. (1.20)), existent according to the dissipation-fluctuation theorem. In the limit of high resistance (low damping) the escape is limited by the transport in the energy direction, because during a single small oscillation period the noise current does not redistribute the particle state in the well, but only causes a slow diffusion in the energy direction. This was already recognized by Kramers [61]. For the very low damping limit \( \eta U_0/k_B T \ll 1 \)

\[
A = \frac{18}{\pi} \eta B e^{3/2},
\]

resulting in \( A = \frac{18}{\pi} \eta B \) and \( a = 1 \).

3. In the strong damping limit, the momentum of the particle, needed to overcome the barrier, is larger because of the friction. For the same starting points as in Fig. 2.10a, Fig. 2.10b shows the strong damping case. The expected flux over the barrier (integrating over all trajectories from the saddle point to infinite momentum, weighted by the thermal ensemble) is reduced by the damping. This case is not relevant for the experiments described in this thesis.

These three cases correspond to a transport in the phase space limited in different ways. In the moderate damping regime, the limit the transport of an ensemble of along the bound phase space trajectories - the noise is strong enough to redistribute the particles after some oscillations. In the low damping limit, the noise can not redistribute an ensemble across all bound states after a small number of oscillations. The coefficients for the different cases are listed in Table 2.1.

Calculating the PDF for the corresponding cases shows that for typical parameters in our experiments, only the weak damping limit imposes a severe change on the distribution, compared to the TST case. For the same value of \( U \), with a ramp rate of \( \gamma t = 10^{-11} \), Fig. 2.9 shows the calculated distributions. Relevant to the experiments presented here are the weak and the moderate (TST) damping regimes. These cases are bridged in Ref. [62]. The TST expression holds for a moderate damping, but a constant correction to the escape rate has to be carried out. According to Ref. [62], the correction to the TST rate \( \Gamma \) is given by

\[
\frac{\Gamma_{\text{esc}}}{\Gamma_{\text{esc}}(\text{TST})} = \frac{\sqrt{1 + 4\alpha YY^{-1} - 1}}{\sqrt{1 + 4\alpha YY^{-1} + 1}},
\]

(2.25)
Mechanism | Damping | $A$ | $B$ | $a$ | $b$
---|---|---|---|---|---
Thermal | Low | $\frac{18\pi}{5\pi} B_{th}$ | $B_{th}$ | 1 | $3/2$
Thermal | Moderate | $\frac{\Omega_0}{2\pi}$ | $B_{th}$ | $-1/4$ | $3/2$
Thermal | High | $\frac{\Omega_0^2}{2\pi\eta}$ | $B_{th}$ | 0 | $3/2$
Quantum | None | $\left(\frac{30B_{qu}}{\pi}\right)^{1/2}$ | $\Omega_0$ | $5/8$ | $5/4$

Table 2.1: Coefficients (table taken from Ref. [60]) for the three damping limits for thermal activation and the quantum limit. In the thermal limit, the three different regimes correspond to the following situations: Escape rate over the barrier depletes the phase space, because the coupling to the bath does now allow for a thermalization (low damping). System thermalizes on the order of the expected lifetime of the metastable state (moderate damping). The damping hinders the system to escape over the barrier (high damping). In the vortex case, the $B_{th}$ and $B_{qu}$ are given by $B_{th} = \frac{U}{k_B T} = \frac{4\pi \hbar}{k_B T} \frac{2^{3/2}}{2\sqrt{2}}$, $B_{qu} = \frac{36U}{5\Omega_0 \hbar} = \frac{36}{5} \frac{4\pi}{h} \sqrt{8\hbar}$.

Figure 2.9: Comparison of three different switching current distributions for the same potential, the same ramp rate of $\gamma_t = 10^{-11}$, but two different values of the damping $\eta$ and for the TST result, respectively.
where $\alpha$ is a constant of order 1 and $\Upsilon$ is given by

$$\Upsilon = \frac{\eta S_b}{k_B T} = \frac{3 \cdot 2^{5/4}}{10} \frac{\eta S_0}{k_B T} \epsilon^{5/4},$$

(2.26)

where $S_b$ is the action of the path from the top of the barrier to the bottom and back for a given value of $\epsilon$. $S_0$ is the action of this path at $\epsilon = 1$. For the washboard potential, $S_0 = 16\sqrt{m_{\text{eff}} U}$ [62, Eq. 2.9].

### 2.6 Parameter Estimation

It is not possible to determine the a priori unknowns ($\eta, I_C, \Omega_0, \mathcal{U}$) as independent parameters. Ideally, $\gamma_{\text{cr}}$ is determined independently from a spectroscopic measurement. If this is unavailable\(^5\), it is suitable (in the small JJ case) to proceed as follows:

1. Estimate $\Omega_0$ from the nominal sample parameters.

2. Perform MLE fitting of datasets from different temperatures, using $A$ as fixed and $B$ and $\gamma_{\text{cr}} I_C$ as free parameters. Take the ratio of the barrier height $B$ in units of the thermal energy to the Josephson energy given by Eq. (1.11) for the corresponding $I_C$. For moderate damping the $B T / I_C$ ratio matches the TST value.

3. Fit the datasets with $A$ as a free parameter using the assumption of a fixed $B T / I_C$ ratio. Compare to the theory by Büttiker et al.\[62\]. The result of this procedure can be seen in Fig. 2.11, where the analytical line was drawn using Eq. (2.25), for the 1 kA sample SJJ-DAMP (see appendix C). Neglecting the strong and weak damping regimes leads to an underestimation of the barrier height\(^6\). It must be stressed that leaving $I_C$ and $B$ as free independent parameters in any TST-like fit can compensate for not taking the change of $A$ into account and still lead to acceptable fits, since the freedom to shift and scale the distribution reproduced the effect of a changed damping.

---

\(^5\)On some of the experiments presented, the coupling between the antenna and the sample was not sufficiently flat.

\(^6\)This leads either to an overestimation of the escape temperature, which may be an alternative explanation for [8, Fig. 6.28(b), Fig. 6.22], or an underestimated standard deviation, which may influence the interpretation of [63, Fig. 3]
2.7 Conclusion

Two dimensional $I_C(H_x, H_y)$ patterns[10] were used to examine injected vortices. A comparison of the injected vortex $I_C(H_x, H_y)$ pattern to the pattern of a vortex trapped during cool down shows the locality of the residual potential created by current injector [45, 56]. This demonstrates that in extended circuits, a vortex injector does not influence the vortex behavior at distances substantially larger than the Josephson penetration depth $\lambda_J$.

To prove the setups performance we use the well known experimental technique of ramping the bias current and determine the critical current and the damping parameter of a small Josephson junction. A turnover from the moderate to the weak friction regime is observed, and the dependence of the damping on temperature agrees qualitatively with earlier observations [58], despite of a different data evaluation scheme, based on a direct maximum likelihood esti-
2.7. CONCLUSION

Figure 2.11: Bottom: MLE fitting result using TST scaling for critical current for experimental data from $1 \text{kA/cm}^2$ SJJ of area 100 $\mu\text{m}^2$. The only free parameters where $A$ and $I_C$. Using Eq. (1.11), $B = \frac{\Phi_0}{2\pi k_B T} I_C$. The critical current density using the temperature dependent superconducting gap for the calculation of the critical current density using the Ambegaokar-Baratoff theory and approximated temperature-dependence of the gap [28, 31], with a critical temperature of $T_c = 6.6$ K, which fitted for unknown reasons the experimental data.

Top: The prefactor $A$ represents the normalized plasma frequency $\omega_P$, modified by weak-to-moderate damping. The solid line represents the scaling of $A$, according to Eq. (2.25) (evaluated at the average value of $\epsilon$) and Eq. (1.15), where the resistance was put to be approximately $10^2$ times smaller than the value expected from the normal resistance (compares well to Ref. [58]). The data points at the highest temperatures were subjected to a fluctuation of the temperature, and not taken into account.
mation of escape events. We observe a shunting resistance to be significantly smaller than the predicted quasiparticle resistance. This is most likely due to the influence of the finite impedance of the electromagnetic environment [64].
Chapter 3

Metastable Vortex States

States of vorticity $n_v = 1$ with a single flux quantum trapped between the superconducting rings of an annular Josephson junctions were first investigated experimentally in Ref. [65]. The experimental results presented in this chapter where achieved during the investigation of the escape of vortices from a metastable state formed by magnetic field and bias current. For this, ramp-type ensemble measurements are used, in the same way as in [9, 8] and described in the previous chapter. The predicted scaling of the small oscillation frequency with magnetic field and bias current is verified by resonant spectroscopy. In this chapter, spectroscopic measurements on intermediate length ($l \approx 10\lambda_J$) and thermal activation measurements on short ($l \approx 4\lambda_J$) junctions are presented. Both junctions are $\mu$m-scaled and were produced using photo-lithography at a commercial foundry[66]. The question of how far the elementary excitations in this finite systems coincide with soliton excitations in infinite junctions is treated in terms of a harmonic model, which coincides for short junction lengths with the cnoidal wave solution, (see e.g. [67, 68, 69]).

3.1 Metastable Vortex States

A single, resting Josephson vortex can undergo a transition from a static, pinned state (zero-voltage) to a dynamic, moving state (finite voltage). This process is equivalent to the activation of a small junction, but the activated variable is not the phase difference, but the center of mass position of a vortex.

The fluctuation-free depinning current $\gamma_{cr}$, is found by equating the magnetic
CHAPTER 3. METASTABLE VORTEX STATES

field force $F_h$ maximum

$$F_h = -2\pi \frac{\hbar}{\tilde{r}} \sin(\tilde{x}_0/\tilde{r})$$  \hspace{1cm} (3.1)

to the bias current driving force $2\pi \gamma_{cr}$. In the fluctuation free case the metastable state vanishes at

$$\gamma_{cr} = \frac{\hbar}{\tilde{r}}$$  \hspace{1cm} (3.2)

and the system switches to the voltage state as discussed in sec. 1.7.2. In the presence of finite temperature or quantum fluctuations, the escape happens from a metastable state, as depicted in Fig. 2.6, with the vortex coordinate $\tilde{x}$ as the dynamic variable. In addition to the minimum $\tilde{x}_0$ a potential maximum at position $\tilde{x}_1$ exists as a metastable equilibrium point. The positions of these are given by

$$\tilde{x}_{0,1} = \tilde{r} \left( \frac{\pi \pm \pi}{2} \pm \arcsin \left( \frac{\gamma \tilde{r}}{\hbar} \right) \right).$$  \hspace{1cm} (3.3)

The minima for different bias currents are marked in Fig. 2.6 by a dot.

In the region of validity of the vortex perturbation theory, the critical current is linear in the magnetic field. In order to describe the dynamics of the system in the presence of fluctuations, the barrier height $U_0$ and the small oscillation frequency in the well $\omega_0$, indicated in Fig. 2.6, suffice to describe the dynamics of the system. Evaluating the potential at the positions calculated in Eq. (3.3) results in the barrier height

$$U_0 = 4\pi \frac{\hbar}{\tilde{r}} \sqrt{1 - \left( \frac{\gamma_{cr}}{\gamma} \right)^2}.$$  \hspace{1cm} (3.4)

The small oscillation frequency (in normalized units), is given by

$$\omega_0 = 2\pi \frac{1}{\tilde{r}} \sqrt{\frac{\hbar}{m} \frac{4}{\Omega_0(\tilde{h})} \sqrt{1 - \left( \frac{\gamma_{cr}}{\gamma} \right)^2}}.$$  \hspace{1cm} (3.5)

This relation is verified by resonant spectroscopy in the next section, where for each given magnetic field, $N$ measurements $(\omega_0, \omega_P, \gamma_i I_C), i = 1..N$ form the set of observed values \(^1\) with a model parameter vector $(\Omega_0, \omega_P, I_C \gamma_{cr})$. The

\(^1\)Independent variable: $\omega_0, \omega_P$, dependent variable: $\gamma_i I_C$. In real experiments in the next section there exists the power of the resonant probing as a hidden variable, which is not modelled.
3.2. SPECTROSCOPY ON AN ANNULAR JUNCTION

The dependence of $\omega_0$ on the bias current can be verified by adding an rf bias to the dc bias and observing the rate of decay as a function of the bias current, a technique applied earlier to small junctions [70], and to vortices in narrow Josephson junctions [9]. In the classical interpretation, small oscillations of the vortex position are excited [71, 72]. The amplitude of these depends on the detuning of the rf from the small oscillation frequency of the system, and the applied power. At a single value of $\epsilon$, the small oscillation frequency corresponds to the externally applied frequency $\nu$

$$\nu = \omega_0 / 2\pi \epsilon^{1/4}$$  \hspace{1cm} (3.10)
and the oscillation amplitude reaches its maximum.

For frequencies small in comparison to $\Omega_0$, a rate enhancement is observed, given by the ratio of the decay rate with rf bias to the decay rate of the dc only biased system. A Lorentzian peak in the rate enhancement $(\Gamma_{\text{esc,rf}} - \Gamma_{\text{esc}})/\Gamma_{\text{esc}}$, where $\Gamma_{\text{esc}}$ is the decay rate without rf bias and $\Gamma_{\text{esc,rf}}$ is the decay rate with rf bias, is expected to be centered around the resonant frequency. In the measurements presented next, high frequencies were used. In this case the resonant peak is clearly separated from the ground state peak, as shown in Fig. 3.1, acquired on sample AJJ-MS2 (see appendix C). In this limit a double peak in the switching current distribution is observed. Although the theoretical criterion for selecting the rf amplitude is arbitrary, the automated data acquisition software used for the measurements presented in this chapter chooses the power such that both peaks have approximately the same probability.

A typical resonance curve of the depinning process in the annular junction AJJ-MS2 (see appendix C) can be seen in Fig. 3.2. When changing the frequency the position of the resonant peak shifts. The maximum of the resonant peak is determined; so that a relation between the bias current an the frequency can be determined. At this point, $\gamma_{cr}$ is still unknown, as well as the unbiased small
oscillation frequency $\Omega_0$. So it is a priori impossible to determine the value of $\epsilon$. Since the relation $\omega_0 = \Omega_0(1 - (\gamma/\gamma_{cr}))^{1/4}$ is a nonlinear one, $\gamma_{cr}I_C$, as well as $\Omega_0\omega_P$ can be interpreted as free fitting parameters. This fitting was carried out in Fig. 3.2 to find the most likely parameters. The resulting resonance curve is indicated by the dashed line. After the fluctuation free depinning current $\gamma_{cr}I_C$ is known, it is possible to determine the corresponding values of the reduced bias field, plotted on the y-axis in Fig. 3.2. In Fig. 3.2, there are some systematic deviations, and some random outliers. In order to understand these, it must be kept in mind that the rf amplitude varies strongly with frequency. A clear distinction must be made: the rf amplitude varies due to the electric environment of the system, and the rf amplitude required for the vortex escape from the metastable state also varies. The latter depends on the height of the barrier to be surpassed, and thus on the frequency. In Fig. 3.2, the effects of both power variations are evident. Around 13 GHz, the coupling to the junction of the generator varied strongly. Another pronounced structure is the transition at 10 GHz. This may be attributed to a effect of the finite temperature at 150 mK. The crossover temperature $T^* = \hbar\nu/k_B T$ is calculated to be 72 mK. Whether the step in the mean switching current is step is bound up with $T^*$ could be verified by measurements at different temperatures. The mechanism by which such a step could be explained is that the escape from the resonant peak is supported by thermal fluctuations down to the crossover temperature, lowering the required power, while below the crossover temperature, thermal activation is suppressed, so the rf amplitude must be increased in order to drive the vortex resonantly over the barrier. But at small values of the reduced bias field, the frequency, and therefore the crossover temperature drops to zero. Hence non-resonant activation depends on the rf signal and the thermal noise.

When repeating the procedure several times at different magnetic fields, the relation Eq. (3.6) between the depinning current and the small oscillation frequency at zero bias is found experimentally. In this experiment three different magnetic field amplitudes and three angles of magnetic field are chosen. For each a series of histograms exhibiting a double peak was acquired for different frequencies. By fitting to this data the fluctuation free depinning current $\gamma_{cr}I_C$ and the unbiased small oscillation frequency are determined. In Fig. 3.3 the result of this procedure is compared to the theoretical prediction. The slope of the indicated line was chosen to fit the data.
3.3 On Classical Resonances and Quantum Transitions

The number of levels in a well is determined by $\epsilon$, which in turn is determined by the resonant frequency $\nu_{\text{trans}}$. In a well of potential depth $U$ and small oscillation frequency $\omega_0$ we find approximately $n$ quantum levels

$$n = \frac{U_0}{\hbar \omega_0},$$

assuming the dynamics of a harmonic oscillator. Rewriting in terms of the reduced bias field, and the potential height $U$ and small oscillation frequency $\Omega_0$ at zero bias yields

$$n = \frac{U \epsilon^{3/2}}{\hbar \Omega_0^{1/4}} = \frac{U}{\hbar \Omega_0} \epsilon^{5/4}.$$
Figure 3.3: Several measurements on sample AJJ-MS2 (see appendix C), at different angles of the applied magnetic field for three different magnetic field amplitudes. The two independent fitting parameters, obtained from measurements as presented in Fig. 3.2, are plotted. The expected dependence, estimated from specific capacitance (chosen to match the threshold voltage of a vortex step), and critical current density (approximated by the maximum switching current in the absence of field at $n_v = 0$), is indicated by the dashed line.
Utilizing the relationship between the small oscillation frequency and the reduced bias field, we arrive at the simple formula

\[ n = \frac{1}{\tilde{h}} \frac{U}{\Omega_0} \left( \frac{2\pi \tilde{\nu}_{\text{trans}}}{\Omega_0} \right)^5, \]  

(3.13)

where \( \tilde{\nu}_{\text{trans}} \) is the normalized resonant frequency. This can be written for a vortex in an annular junction as:

\[ n = \frac{1}{\tilde{h}} \frac{U}{\Omega_0^6} (2\pi \tilde{\nu}_{\text{trans}})^5 = \frac{1}{\tilde{h}} \frac{U}{\sqrt{\tilde{\nu}_{\text{trans}}}} (2\pi \tilde{\nu}_{\text{trans}})^5 = \frac{(m_{\text{eff}} r)^3}{\tilde{h} U^2} (2\pi \tilde{\nu}_{\text{trans}})^5 \]

(3.14)

The resonant frequency, which corresponds to \( n \) levels in the well, is now given by

\[ 2\pi \tilde{\nu}_{\text{trans}} = \left( \frac{n \tilde{h} U^2}{(m_{\text{eff}} r)^3} \right)^{1/5} \]

(3.15)

Again, \( U \) can be replaced by \( \gamma_{\text{cr}} r 2\pi \) in order to relate this equation directly to the experiment.

\[ 2\pi \tilde{\nu}_{\text{trans}} = \left( \frac{n \tilde{h} (2\pi \gamma_{\text{cr}})^2}{m_{\text{eff}}^3 r} \right)^{1/5} = \left( \frac{\tilde{h} (2\pi)^2}{m_{\text{eff}}^3 r} \right)^{1/5} \gamma_{\text{cr}}^{2/5} n^{1/5} \]

(3.16)

For junctions of 3 \( \mu \text{m} \) width, \( \tilde{h} \approx 7 \cdot 10^{-4} \) at a specific capacitance of 40 fF/\( \mu \text{m}^2 \). At \( j_c = 100 \text{ A/cm}^2 \) and a junction radius of 50 \( \mu \text{m} \) the normalized radius is \( r = 1.3 \). In this case \( 2\pi \tilde{\nu}_{\text{trans}} = 0.13 \gamma_{\text{cr}}^{2/5} n^{1/5} \). The resonant frequency vs number of levels at resonance is plotted for \( \gamma_{\text{cr}} = 2/3 \) in Fig. 3.4 In Fig. 3.2 the number of levels in the well at the time of the activation is large for most of the frequencies, so that the dynamics of the system is not described by a single level transition, but either by multi-level quantum dynamics or by classical dynamics. Typically, at least 4 – 5 levels in the well prevent the system from being correctly described by a single transition. This depends on the activation rate from the first excited state. The highest states have decay rates so fast that they are not observable in practice.
3.3. ON CLASSICAL RESONANCES AND QUANTUM TRANSITIONS

Figure 3.4: The resonant frequency of a vortex pinned with a critical current of $\gamma_{cr} = 2/3$ in a junction of width $w = 3 \mu m$, radius $50 \mu m$ and critical current density $j_c = 100 A/cm^2$. 
3.4 Thermal Activation of a Vortex

When varying the external magnetic field strength, the effective potential barrier height for a vortex is changed. This means that it is possible to change the Arrhenius factor $B$ without changing the temperature. So it is possible to independently examine the influence of both quantities on the standard deviation.

The standard deviation of the critical force of a depinning process from a biased washboard potential, as discussed in sec. 2.3, is given by

$$\sigma^2 = \frac{1}{6} \left( \frac{B^{-2+\frac{2}{3}}}{B^2/b^2} \right).$$

While the specific role of $\bar{B}$ depends on the damping, to be discussed below, $b$ and $B$ have the same meaning for all thermal activation process, namely $B = 2\bar{U}/\tilde{k}_B T$, and $b = 2/3$ being the exponent of the potential $\epsilon$ for $\epsilon \ll 1$. The cubic approximation sets $\bar{U} = 2/\sqrt{2}\bar{U}$, where $\bar{U}$ is the unbiased potential barrier height. Explicitly, this means that unless $\bar{B}$ depends significantly on $B$, a simple power law scaling holds between the standard deviation of the reduced bias field at the time of activation and the potential height. The magnetic field generated potential $U_h$ in a long Josephson junction is given by

$$U_h = 2\pi\bar{h},$$

which can be written as a function of the junction radius

$$U_h = 2\pi\gamma_{cr}\tilde{r}.$$ (3.19)

The Arrhenius factor $B$ is then given by

$$B = \frac{\gamma_{cr}^{4/3}\pi\sqrt{2}\tilde{r}}{\tilde{k}_B T}.$$ (3.20)

If the reduced bias field $\epsilon$ is small, the mean value of the switching currents in a single histogram is a good approximation for $\gamma_{cr}I_C$. If the normalized Boltzmann constant $\tilde{k}_B$, the normalized junction radius $\tilde{r}$, and $\bar{B}$ are known, it is possible, without any further information, to compare the expected standard switching current distribution width to the experiment. The quantities $\tilde{r}$ and $\tilde{k}_B$ can be estimated from the critical current density $j_C$ of the junction. Figure 3.5 contains such a comparison. For arbitrary values of $\bar{B}$ the scaling of the switching current distribution width is indicated by a dashed lines. The experimental data, consisting of datasets containing 1200 switching events, was
acquired on an annular junction of width $w = 2 \mu m$, radius $50 \mu m$, and critical current density $25 A/cm^2$ at temperatures from $300 \text{mK}$ to $3.5 K$. The values of $\bar{B}$ are indicated on the dashed line. Before the different regimes are discussed, it is necessary to consider the dependence of $\sigma$ on $\bar{B}$. The factor of 5 between $\bar{B} = 2$ and $\bar{B} = 10$ corresponds to approximately the same factor in $\sigma$ as to the 1000-fold increase between $\bar{B} = 10$ and $\bar{B} = 10000$. For thermal activation, $\sigma$ scales as $\sigma \propto \bar{B}^{-1/2}$. This means that for large arguments of the logarithm in Eq. (2.11), the specific value of $\bar{B}$ cannot be determined by such a technique, since it becomes exponentially imprecise. The numeric value of $\bar{B}$ is found by

$$\bar{B} = \log \left( \frac{\gamma_{cr}}{\gamma_{\tilde{t}}} \frac{1}{bB^{(1+a/b)}} \right),$$

(3.21)

where $b = 3/2$ for thermal activation, $\gamma_{\tilde{t}}$ is the normalized ramp rate, $A$ has the meaning from table 2.1. Solving Eq. (3.20) for $\gamma_{cr}$ yields

$$\gamma_{cr} = B\frac{\tilde{k}_B T}{4/3\sqrt{2\pi \gamma_{\tilde{t}}}}.$$

(3.22)

Hence, Eq. (3.21) can also be written as

$$\bar{B} = \log \left( \frac{\tilde{k}_B T}{4/3 \pi \sqrt{2\tilde{r} \gamma_{\tilde{t}}} \frac{1}{bB^{(a/b)}}} \right).$$

(3.23)

It is now obvious that $\bar{B}$ changes with $B$, but this change is normally small in high temperature limit.

The prefactor $A$ is listed from the different cases in Table 2.1, which is taken from [60]. Using the prefactor for the moderate damping, $\bar{B}$ is determined to be $\bar{B} = \log(2.8 \cdot 10^{-6})$, for $\gamma_{\tilde{t}} = 10^{-9}, B = 10^3, \tilde{r} = .55$ and $\tilde{k}_B = 1/800 K^{-1}$ and $T = 3.5 K$. This fits to Fig. 3.5. To match the low-temperature datasets, the low-damping regime is appropriate. At a damping of $\eta = 10^{-4}, T = 300 \text{mK}$ and $B = 2 \cdot 10^3, \bar{B} = \log(17)$. The agreement in Fig. 3.5 is acceptable, taking into account that the standard deviation is systematically increased by the experimental noise.
CHAPTER 3. METASTABLE VORTEX STATES

Figure 3.5: Logarithmic plot of the standard deviation of the reduced bias field $\epsilon$ vs. $B$, which was estimated from the mean switching current and the estimated junction length ($3.5 \lambda_J$). The datasets were obtained at different temperatures ($3.5, 3, 2.5, 2, 1.5, .3$) K. Within each dataset the potential barrier height was varied by varying the external magnetic field. The dashed lines indicates the scaling of the standard deviation for five values of $\bar{B} = (\log(1.2), \log(2), \log(10), \log(1000), \log(10000))$, as indicated on the line.
3.5 From the Low Damping Regime to the Quantum Regime

The damping in the low temperature limit, is assumed not to be dominated by the quasiparticle tunneling given by Eq. (1.15). In this limit, the dominating sources of damping are the real part of the environments admittance at the small oscillation frequency of the system [64] or the dielectric loss in the insulating layer of the window region. Decoherence considerations [74] and later spectroscopic measurements were brought into agreement by Martinis et al. [75] with dielectric loss measurements by microwave absorption [76], where bath is formed of two-level systems (TLS) formed by defects in the dielectric. Since TLSs can be saturated, the temperature and power dependence of this dissipation source is nontrivial (see Figure. 1 of [76]), exhibiting a saturation. Spectroscopic measurements on small Josephson junctions [75], where only a few polarizable resonators reside in the junction barrier and window region can provide explicit statistics of two level systems in the barrier. The specific method of deposition of the dielectric has significant influence of the behavior of the TLS [75, 77]. In the experiments presented in this work, no system was designed for a high isolation [64] from the environment at high frequencies, needed to observe resonances with a low frequency splitting, so that no direct estimation of $\eta$ at in this limit is possible.

In Fig. 3.6, measurements carried out at temperatures on AJJ-SHORT (see appendix C) between 28 mK and 302 mK are presented and compared to the low-damping thermal escape calculations. The datasets are restricted to the limits of validity of the vortex approximation, see sec. 3.6. At a damping value of $\eta = 4 \cdot 10^{-4}$ the predicted standard deviation at a temperature of 200 mK and 302 mK matches to the experimental results. The Josephson energy is extrapolated from the high-temperature regime. The barrier height is estimated from the mean switching current and the junction radius. Both are estimated from the critical current density. The scaling for the thermal activation measurement, is indicated as solid lines for the two temperatures. What is most interesting in Fig. 3.6 is that for the two datasets at a temperature of 133 mK and 28 mK the scaling (slope in the logarithmic plot) is changed in comparison to the other two datasets. To verify that we are dealing with a crossover to the quantum regime, we examine this regime closer. The usual criterion[78, 79], in the TST regime for the crossover is

$$T^* = \frac{\hbar \omega_B}{2\pi k_B},$$

(3.24)
Figure 3.6: Standard deviation of switching current vs. $B$ (thermal) plotted on logarithmic axes. The datasets were limited to a single magnetic field polarity and switching current values $I_C$ from 30 $\mu$A (standard deviation limited by electronic noise) to 80 $\mu$A (validity of vortex approximation). Lines and symbols correspond to set temperatures of 28, 133, 200, 302 mK, as indicated. Solid lines indicate the low-damping limit thermal activation prediction. Dashed lines indicate the predicted ground state quantum tunneling distribution widths; to match the data, the latter ones are scaled by a constant.
where $\omega_B$ is the oscillation frequency in the flipped over barrier potential. In the cubic potential $\omega_B = \omega_0$ this condition is equivalent to the thermal energy being small in comparison to the energy level spacing.

Experimentally quantum tunneling causes a saturation of the standard deviation of the switching current distribution for low temperatures. This is equivalent to a saturation of the mean of the switching current, thus the reduced bias field $\epsilon$. Due to $1/f$ noise in the electronics, the saturation of the distribution width is easier to observe (see appendix B). As a matter of fact, both conditions are quite easily related by comparing Eq. (2.10) and Eq. (2.12). This yields a relation between $\langle \epsilon \rangle$ and $\sigma(\epsilon)$:

$$\sigma(\epsilon) = \langle \epsilon \rangle \tilde{B}^{-1} \frac{\pi}{b \sqrt{6}}$$

(3.25)

Thus, if $b \approx 1$ for all processes involved and the value of $\tilde{B}$ is similar for the quantum and the thermal process activation, a larger value of $\sigma(\epsilon)$ is associated with a larger value of $\langle \epsilon \rangle$. It is necessary to keep in mind that $\tilde{B}$ is given by Eq. (2.11), using the low-damping value for $A$ in the thermal case, while the WKB approximation is used in the quantum case,

$$\tilde{B} = \log \left( \frac{\gamma_{cr}}{\gamma_t} \sqrt{\frac{30}{\pi} \frac{\Omega_0}{B_{qu}}} \right),$$

(3.26)

where $B_{qu}$ is given by

$$B_{qu} = \frac{36\mathcal{U}}{5\Omega_0\hbar} = \frac{36}{5} \frac{4\pi \sqrt{8\hbar}}{\hbar}.$$  

(3.27)

In the WKB approximation $B_{qu}$ is proportional to the action of the trajectory from the bottom of the well to the top of the well. In order to determine the temperature and magnetic field of the crossover for a given ramp rate and damping, $\langle \epsilon \rangle$ is compared for the quantum and the thermal case:

$$\frac{\langle \epsilon \rangle_{qu}}{\langle \epsilon \rangle_{th}} = \left( \frac{B_{qu}^{4/5}}{B_{th}^{2/3}} \right) \cdot \left( \frac{B_{th}^{2/3}}{B_{qu}^{4/5}} \right) = \log \frac{\gamma_{cr}}{\gamma_t} \sqrt{\frac{30}{\pi} \frac{\hbar}{m_{eff}}} \cdot \frac{188}{2} \left( \frac{k_B T}{\mathcal{U}} \right)^{3/2} \cdot \frac{180}{2} \left( \frac{5 \hbar \Omega_0}{k_B T} \right)^{4/5}$$

(3.28)

The first factor diverges at very low temperatures, indicating that the thermal escape breaks down, and approximation by Garg Eq. (2.12) does not hold any
more. In the limit of low ramp rates and high damping the change of the logarithmic factor may be negligible. In our the term $\eta k_B U / T$ dominates, and the logarithm in the denominator changes rapidly. For a damping of $\eta = 10^{-5}$ the saturation temperature is approximately 150 mK. In Fig. 3.6 the change of the scaling takes place between 133 mK and 200 mK. The temperature calculated above for the crossover lies between these temperatures. The scaling of the expected standard deviation, calculated using $\tilde{B}_{qu}$, $B_{qu}$, and $\tilde{\hbar}$ using Eq. (2.12) is indicated using dashed lines in Fig. 3.6. Yet the absolute standard deviation in the quantum case is roughly 7 times lower than the observed one. There are several different possible explanations for that; the most simple one, which is commonly used as an alibi for unaccounted results is to claim that the resolution of the experimental setup limits the minimum distribution width acquired. Obviously, Fig. 3.6 shows the scaling for data points of all plotted standard deviations; the measurement noise for this ramp rate is on the order of 40 nA. There is the possibility of a saturation of the bath temperature (due to the changing critical current and dissipated power), which would have a strong dependence on the bias current, but not on temperatures below the crossover. Yet this hypothesis has to be excluded fully in future experiments. Another, more sophisticated explanation is based the constant temperature of parts of the filtering system (see appendix A for details). If a part of the Johnson (white) noise spectrum is not filtered on the way to the junction, it biases the junction with colored, broadband noise. This effect can not be measured in terms of standard deviation, but would appear as an additional heat source to the junction. One alternative explanation could be given by the fact that the vortex is not a localized particle, but, in our case distributed along the whole junction. This may be taken into account using nonlocal models of the system [80], or by adopting to the short junction limit the path integral approach used previously for the long junction limit [81, 82]. A simpler possible explanation is, that the experiments show quantum tunneling, where not only the ground state is populated. External sources can be held responsible for this, e.g. the finite ramp rate, or colored noise from the biasing circuitry. A likely reason for the discrepancy is neglecting the influence of temperature and damping at the time of escape. This can be explained in a hand waving approach by the fact that the standard deviation caused by the thermal fluctuations is changed. The physical reason for this is a non-thermal distribution of energies in the well in the low-damping limit. This non-thermal distribution is equivalent in the quantum limit to a non-thermal occupation probability of the states - in a way that the upper states are populated stronger than they should be. For an explanation of a possible way to go I refer the reader to section IX.D, IX.E, and Fig. 43 of reference [78] and [83].
3.6 Crossover to the Small Junction Case

Solutions of Eq. (1.36) for systems of finite length are found as cnoidal waves[67, 69],

\[ \varphi_c(\tilde{x}, \tilde{x}_0) = \pi - 2 \arccos \left( \frac{\text{cn} \left( \frac{\tilde{x} - \tilde{x}_0}{k}, k \right)}{k} \right). \]  (3.29)

For a given separation \( l \) between the vortices an elliptic modulus \( k \) is determined by the implicit equation \( 2K(k)k = l \). The corresponding phase profile and the magnetic flux density is plotted in Fig. 3.7. For a single vortex in an annular junction, the distance corresponds to the junction length. By inserting \( \varphi_c \) into Eq. (1.67) perturbation theory yields, the magnetic field coupling coefficient \( \kappa \) and the effective mass \( m_{\text{eff}} \) in Eq. (1.67).

Numerical integration yields these coefficients displayed in Fig. 3.8 as a function of the junction length. Junctions shorter than \( 2\pi \) exhibit, in contrast to the fluxon model Eqs. (1.71,1.69), a strong change of both \( m_{\text{eff}} \) and \( \kappa \). Whereas effective mass \( m_{\text{eff}} \) decreases below the fluxon value of \( m_{\text{eff}} = 8 \) with decreasing junction length, the coupling coefficient \( \kappa \) increases.

It can be understood intuitively that the vortex solution does not fulfill the annular boundary conditions for junctions of finite size, and does not vary the phase as strongly as would be required to make up for the total phase jump of \( 2\pi \) along the junction length. At the junction length of the experiments presented in this chapter \( l = 3.5 \), the mass \( m_{\text{eff}} \) is reduced to approximately 7.5, and the coupling coefficient \( \kappa \) is increased by 25%.

At high magnetic fields, the perturbation approach based on the collective coordinate \( x_0 \) as the single degree of freedom breaks down, and a crossover to phase-activated “small junction” like behavior is visible. This behavior is explained in terms of Fig. 3.9. A bias current drives the string towards a higher phase. For a long junction, the potential landscape of which is depicted in Fig. 3.9(b) it is favorable for the string to overcome the barrier formed by the magnetic field in the \( \tilde{x} \) direction, since otherwise it would have to be lifted over a large length from the Peierls valley. For the short junction potential shown in Fig. 3.9(a) the energy cost of this transition is much lower. The introduction of a second degree of freedom in the perturbation approach, namely a “phase offset” \( b \), can be used to describe this effect quantitatively. The phase is given by

\[ \varphi(\tilde{x}, \tilde{x}_0, b, a) = \frac{2\pi}{l} n_v(\tilde{x} - \tilde{x}_0) + \varphi_{\text{harm}}(\tilde{x} - \tilde{x}_0, a) + b, \]  (3.30)

with \( a \) as an inner degree of freedom, which is chosen in such a way that the total energy for a \( v = 0 \) is minimized. Eq. (3.30) is the general form of the harmonic
approach[15]. A harmonic function $\varphi_{\text{harm}}$ fulfills the boundary conditions of the system. The term $2\pi n_v / l$ cancels the necessity to describe the twist $\varphi$. The non-constant terms of the Hamiltonian in this case take the form

$$H = \int_0^l \frac{2\pi n_v}{l} \varphi_{\text{harm},\tilde{x}}(\tilde{x}, a) + \frac{1}{2} \varphi_{\text{harm},\tilde{x}^2}(\tilde{x}, a)$$

$$+ \varphi_{\text{harm}}(\tilde{x} - \tilde{x}_0, a) h_{\text{ext}}(\tilde{x})$$

$$- \cos \left( \frac{2\pi n_v}{l} \tilde{x} + \varphi_{\text{harm}}(\tilde{x}, a) + b \right) d\tilde{x}$$

$$- \gamma b - 2\pi \gamma n_v \tilde{x}_0. \quad (3.31)$$

A few harmonic components suffice to approximate $\varphi_{\text{harm}}$ in the short junction ($l < 2\pi$) limit. For the junction studied (length $l = 3.5$), the approximation by a
3.6. CROSSOVER TO THE SMALL JUNCTION CASE

Figure 3.8: Dependence on the normalized length of: (a) the maximum pinning force per magnetic field unit of the cnoidal wave solution ($\kappa_{cn}/\tilde{r}$). (b) the ratio of the magnetic field coupling coefficient $\kappa$ calculated numerically for the cnoidal wave solution to the coupling of a vortex solution (solid), the ratio of $\kappa$ calculated numerically using the harmonic approximation (Eq. (3.30)) to coupling of a the vortex solution (dashed), and (c) the effective mass of the cnoidal wave approximation (solid) and the harmonic approximation (dashed).

A single harmonic component yields satisfactory results.

$$\varphi_{\text{harm}}(\tilde{x}, a) = a \sin(\tilde{x}/\tilde{r}).$$

While the magnetic field energy takes a trivial form, the Josephson energy term
Figure 3.9: The potential landscape in a junction of $l = 4\lambda_J$ (subplot (a)) and $l = 10\lambda_J$ (b), with magnetic field applied. The string (indicated as a line with circles) resembles a vortex in an infinite junction. The contour represents the potential (white: maximum), and the circles represent the phase in the annular JJ.
3.6. CROSSOVER TO THE SMALL JUNCTION CASE

is expressed as

\[
\int_{0}^{l} - \cos \left( \frac{2\pi}{l} \tilde{x} + a \sin(\tilde{x}/\tilde{r}) + b \right) \, d\tilde{x} = \\
\cos(b) \int_{0}^{l} \cos \left( a \sin \left( \frac{\tilde{x}}{\tilde{r}} \right) + \frac{\tilde{x}}{\tilde{r}} \right) \, d\tilde{x} = \\
\cos(b) \tilde{r} \int_{0}^{2\pi} \cos(a \sin(\tilde{x}) + \tilde{x}) \, d\tilde{x}. \tag{3.33}
\]

The integral, defined in terms of the Bessel function \( J_1(a) \), depends on the single parameter \( a \) only. In total, the potential energy as a function of \( \tilde{x}, b \) and \( a \) is given by

\[
\mathcal{H} = \frac{1}{2} \pi \frac{a^2}{\tilde{r}} + h_{\text{ext}} a \sin \frac{\tilde{x}_0}{\tilde{r}} - (b \tilde{r} + \tilde{x}_0) \gamma 2\pi - \tilde{r} \cos b J_1(a). \tag{3.34}
\]

Fixing \( b \) to 0 results in a potential resembling Eq. (1.69) as a function of \( \tilde{x}_0 \)

\[
U(\tilde{x}_0) = 2\pi \left( \frac{1}{2} h_{\text{ext}} a \sin \frac{\tilde{x}_0}{\tilde{r}} - \tilde{x}_0 \gamma \right). \tag{3.35}
\]

In the small field limit, \( a \) is determined by

\[
\frac{\pi}{\tilde{r}^2} a = J_1(a)'. \tag{3.36}
\]

From this the field coupling \( \kappa \) is derived

\[
\kappa = a \pi \tag{3.37}
\]

In Fig. 3.8 ((b), dashed line), the correction to the fluxon model to field coupling \( \kappa \) is plotted versus the junction length.

The cnoidal wave solution is valid for any junction length. For a length below \( \approx 2\pi \) the single harmonic approximation is valid, above \( \approx 2\pi \) the soliton solution is valid. The calculation of the mass in Fig. 3.8 ((c), dashed line) shows a larger deviation from the cnoidal wave approximation (solid line). While the higher harmonics do not play a role in the coupling of the vortex to the magnetic field, each harmonic neglected in the estimation of the rest mass \( m_{\text{eff}} \) contributes to the mass of the particle. This is easily seen by the fact that the rest mass term is
quadratic, so that the integral over the products of different harmonics are zero. All products of a specific harmonic with itself are positive. That means that no matter how high the harmonic is, it will still contribute and, since all terms are zero or positive there is no possibility for a negative term compensating the increase in this term. The used junction AJJ-SHORT (see appendix C) is in the short junction limit.

Stable phase string configurations pinned in the potential described by Eq. (3.35), vanish at the depinning current $\gamma_{cr}$. The simplest conditions for an activation is that, for one of the two variables $\tilde{x}_0$ or $b$, the maximum pinning force exerted in this direction is overcome. This simple possibilities exist for the activation: The phase string is driven either in the spatial direction along the junction or the total junction is driven in the phase direction. Fig. 3.10 compares the potential Eq. (3.35) for two different magnetic fields as a function of $\tilde{x}_0$ and $b$. The potential was minimized in respect to $a$, which does not change significantly. The limits of a string being pinned in the phase direction, or by magnetic field are found by deriving Eq. (3.35). The condition for the activation in the spatial direction at a fixed phase offset $b$ is

$$\gamma < \gamma_{cr} = \frac{1}{2} a(h_{ext} = 0) h_{ext},$$

(3.38)

which is just the depinning condition of a vortex. Activation in the phase direction is given by

$$\gamma < \gamma_{cr} = \frac{J_1(a(h_{ext}))}{2\pi},$$

(3.39)

equivalent to the case of a small junction of with Josephson energy $lE_{J, reduced}$, where $E_{J, reduced} = J_1(a(h_{ext}))$ is the reduced Josephson energy. If either of the conditions Eq. (3.38), Eq. (3.39) is violated, the system enters into the running state. For lower magnetic fields it is favorable to move over the saddle point in the spatial ($\tilde{x}_0$) direction, but for higher fields the pinning potential in this direction is so strong that state moves in the phase direction. In Fig. 3.10(a) this corresponds to the phase offset $b$ slipping from the potential minimum at $(b \approx 1, \tilde{x}_0 \approx .5)$ to larger, where $b$ passes the saddle point to the right. In Fig. 3.10(b) the vortex position $\tilde{x}_0$ slipping from the potential minimum at $(b \approx 0.7, \tilde{x}_0 \approx .5)$ to a smaller value of $\tilde{x}_0$ and passes the saddle point toward the bottom of the plot.

These conditions are compared in Fig. 3.11(a) with experimentally acquired mean switching currents on sample AJJ-SHORT (see appendix C) at a single temperature. The linear lobe corresponds to the condition given in Eq. (3.38), the curved lobe corresponds to Eq. (3.39). The lesser of the two is indicated
3.6. CROSSOVER TO THE SMALL JUNCTION CASE

Figure 3.10: (a). High-field case: Potential for $h_{\text{ext}} = 2\pi \cdot 0.6$. The saddle point is lying in the positive $b$-direction from the minimum.

Subfigure (b). Low-field case: Potential for $h_{\text{ext}} = 2\pi \cdot 0.4$. The saddle point is lying in the negative $\tilde{x}_0$-direction from the minimum. Minima are indicated by (min).
by the solid line. In the region where Eq. (3.38) is violated the particle starts
to move in the spatial direction. In the region shaded in gray, beyond the field
$h_{crossover}$, where Eq. (3.39) is violated before Eq. (3.38), the activation happens
in the direction of the phase.

The measured critical current varies smoothly, in contrast to the simple prediction made here. In Fig. 3.11(b) an experimentally determined standard deviation is plotted. The scales in Fig. 3.10 are the canonical ones, namely the spatial offset and the phase offset.

The potential barrier height in the phase direction is given by

$$U_0 = 2lE_{J,\text{reduced}}\sqrt{1 - \left(\frac{\gamma}{\gamma_{cr}}\right)^2}, \quad (3.40)$$

When writing an equation of motion in this variable

$$\varphi_{\parallel\parallel}m_{\text{ph}} = -U_{\parallel}, \quad (3.41)$$

comparing the coefficients in the Hamiltonian Eq. (3.31) yields the effective
mass $m_{\text{ph}}$ in the phase direction to be $m_{\text{ph}} = l$ This results in a reduced plasma frequency

$$\omega_0 = \sqrt{\frac{E_{J,\text{reduced}}}{l}}\sqrt{4 \sqrt{1 - \left(\frac{\gamma}{\gamma_{cr}}\right)^2}}, \quad (3.42)$$

of the junction, because the effective Josephson energy in Eq. (3.33) is reduced, while the mass is not.

### 3.7 Conclusion

Experiments using the methods demonstrated in the previous chapter and microwave spectroscopy on vortex-like metastable states in macroscopic ($314 \mu m \times 2/3 \mu m$) annular Josephson junctions of low critical current density ($30 \text{ A/cm}^2$, $100 \text{ A/cm}^2$), show underdamped thermal activation and quantum tunneling. The spectroscopic results agree quantitatively with the predicted scaling of the small oscillation frequency with the determined depinning current. We observe thermal activation in a temperature range from $200 \text{ mK}$ to $3.5 \text{ K}$, where at each temperature the scaling is as predicted by the TST theory, which assumes a thermalized population of the phase space. However, the prefactor between the datasets clearly changes. In the low temperature limit, this change is attributed to a low damping value. At and below temperature of $133 \text{ mK}$ a transition from
underdamped thermal activation to quantum tunneling is observed, as was before on sub-$\mu m$ wide Josephson junctions[9]. The experiments presented in this chapter show, for the first time, the crossover of a experimentally determined fluctuation scaling exponent for a junction width larger than 1 $\mu m$. The prefactor of the tunneling rate leaves open questions for a nonlocal model of the string escape or an extended model of the non equilibrium distribution of states in the metastable well.
Figure 3.11: (a) Standard deviation of experimentally obtained ensemble of critical currents. (b) Comparison of the experimental $I_C(h)$ pattern (dots) to the fluxon approximation (linear lobe) and the small junction limit of a twisted phase (curved) lobe. The solid line is the analytical prediction for the depinning current, normalized units on the left axis/bottom, physical units on the right axis. Gray region indicates phase escape. The dotted lines indicate the escape criterion with the higher depinning current.
Chapter 4

Phase Escape In Extended Systems

The escape of a one dimensional string pinned in a single valley of a sine-Gordon potential into either a neighboring Peierls valley, or the escape from a metastable state has received considerable theoretical attention. It can be used to describe charge-density waves, and it is important to understand the escape of the phase in a LJJ under the influence of an external bias. Even though nearly all regimes important for the treatment of long Josephson junctions have been discussed, experiments on this field are rare. Most of the treatments deal with localized escape via the nucleation of vortex-antivortex pairs in an infinite system. Among the theoretically treated regimes are the nucleation by thermal fluctuations in the overdamped regime [84, 85], dissipative tunneling under the influence of large damping [86], dissipative tunneling in general [73], and tunneling between stable states at negligible damping [87]. These references assume a homogeneous system, which is clearly not the case in an annular Josephson junction under the influence of a magnetic field. Non homogeneous systems were considered in the case of negligible damping for the thermal and quantum limits [88, 89, 16]. Refs. [88, 89] discuss a point-like inhomogeneity, which could be examined using a system similar to the microshort junctions presented in chapter 5. The specific case of an inhomogeneous bias created by a homogeneous magnetic field acting on an AJJ was discussed in [16].

While fluctuation free statics and dynamics of states with vorticity $n_v = 0$ were examined theoretically and experimentally in Refs. [90, 91, 92, 93], the
first fluctuation measurements of ensembles of critical currents on such states were presented in [16]. The experimental results therein were interpreted as fluctuation induced nucleation of vortex-antivortex (VAV) pairs.

In this chapter the method of RF induced phase activation is applied to a long junction in order to probe for the small oscillation frequency of the phase field in a junction biased close the critical current. The results of this were also presented in [94]. This enables examination of the scaling of the critical nucleus size with magnetic field and bias current. The scaling is predicted to be significantly different from the small junction case and the vortex case.

Another issue treated in this chapter is the application of a harmonic approximation as discussed in [15] to calculate the small oscillation frequency, as well as the characteristic barrier height at depinning for short to intermediate length junctions. Similar schemes have been used earlier [90, 91, 92], but without extracting the parameters of interest for the fluctuation induced escape.

4.1 Harmonic Approximation

The short junction limit can be treated using the harmonic approximation [91]. Ref. [15] extends the approximation to the problems discussed here.

Inserting the harmonic ansatz (see Eq. (3.30)), into the long Josephson junction Hamiltonian (Eq. (1.44)), and setting $n_v = 0$ yields the potential energy as a function of two variables, the first harmonic $a$ and the phase offset $b$ of the phase profile at a constant $\tilde{x}_0 = \tilde{r}2\pi/4$.

\[
U = \frac{a^2}{2r^2} + ah - 2b\gamma - 2J_0(a) \cos(b) \tag{4.1}
\]

We are interested in configurations $(a, b)$ with a vanishing potential gradient in respect to both variables, since these are the equilibrium positions. This translates to the following two equations,

\[
h_{ext} + a\frac{1}{r^2} + 2J_1(a) \cos(b) = 0 \tag{4.2}
\]

and

\[
\gamma = J_0(a) \sin(b). \tag{4.3}
\]

Each of the $(a, b)$ tuples fulfilling Eq. (4.3) and Eq. (4.2) corresponds to either a local minimum in the potential energy, a local maximum in the potential energy, or a saddle point. Each minimum represents a stable state, around which the
system performs small oscillations. These small oscillations approximate a certain eigenmode of the phase profile. To find the small oscillation frequency and the eigenmodes of the small oscillations, the Hessian matrix of second partial derivatives \((U_{aa}, U_{ba}, U_{ab}, U_{bb})\), is used. Writing the eigenvalue equation

\[
0 = \left| \begin{pmatrix} U_{aa} & U_{ab} \\ U_{ba} & U_{bb} \end{pmatrix} - \lambda_j \hat{E} \right|,
\]

where \(\hat{E}\) denotes the identity matrix, we find the set of eigenvalues \(\lambda_j\). At a minimum, all \(\lambda_j\) are positive, while at a saddle point, one of the eigenvalues is negative, denoting an unstable dimension. The dynamics relevant to the phase escape of the system are now mapped to a one-dimensional model. In this model it is assumed that a local minimum together with a nearby saddle form a one-dimensional line, along which the system escapes. Without any fluctuations, the system escapes from the metastable state when the local minimum it is trapped in, meets with the corresponding saddle point. This happens when the smallest eigenvalue \(\lambda_1\) at the minimum and the single negative eigenvalue at the saddle become zero. This yields an additional equation by setting setting \(\lambda = 0\) in Eq. (4.4), namely the determinant of the Hessian being zero:

\[
0 = \left| \begin{pmatrix} U_{aa} & U_{ab} \\ U_{ba} & U_{bb} \end{pmatrix} \right|.
\]

After inserting the second derivatives, the following equation is found

\[
0 = -4J_1(a)^2 + 2J_0(a) \cos(b) k^2 + (2J_0(a)^2 - 2J_2(a)J_0(a) + 4J_1(a)^2) \cos^2(b).
\]

The critical current \(\gamma_{cr}\) is found by numerically solving the system of equations defined by Eq. (4.3), Eq. (4.2) and Eq. (4.6). A non-vortex trapped pattern on sample AJJ-VAV (see appendix C) was plotted in Fig. 4.1. The system of equations formed Eq. (4.3), Eq. (4.2) and Eq. (4.6) was solved numerically for a normalized radius of \(\tilde{r} = 10.5/2\pi\), and the resulting critical bias \(\gamma_{cr}\) was multiplied by the critical current of the junction. The shape of the innermost lobe of the \(I_C(H_{ext})\) pattern is reproduced by the numerical calculation to high precision. Fig. 4.1 indicates that the harmonic model works quite well for moderate junction lengths. There are more findings which can be extracted using this method. In order to estimate the rate of escape in the case of quantum or thermal fluctuations, the same properties of the potential have to be examined in the limit of small \(\epsilon\). The small oscillation frequency can again be written as a power law

\[
\omega_0 = \Omega_0 \epsilon^{1/4}.
\]
This implies that in the vicinity of the critical point, at the critical bias $\gamma_{cr}$ the cubic approximation holds, i.e. that the fourth derivative is sufficiently small. In the case of Eq. (4.1) the exponent of $\frac{1}{4}$ can be either derived by symbolic calculations, or checked numerically. This is done by selecting a set of small reduced bias fields, using

$$\gamma = (1 - \epsilon)\gamma_{cr},$$

(4.8)

and solving Eq. (4.3) and Eq. (4.2) to find the static positions.

At the chosen $\epsilon$, the small oscillation frequency is calculated from the smallest eigenvalue $\lambda_1$ found using Eq. (4.4) at a minimum. The small oscillation frequency is then given by the relation

$$\omega_0 = \sqrt{\frac{\lambda_1}{m_{eff}}}$$

(4.9)

For the same $\epsilon$, the one-dimensional line of activation along the minimum and the saddle point is found, and the potential barrier height is determined and found to have the form

$$U_0 = U\epsilon^{3/2}.$$  

(4.10)
Due to the nonlinear nature of the potential there is no a priori equivalence between the $\Omega_0$ and the real small oscillation frequency at zero current, and $\mathcal{U}$ and the physical potential height at zero current. The most definite experimental result presented in this chapter is the non-trivial scaling of $\Omega_0$. For a small Josephson junction, the term $a/\tilde{r}^2$ in Eq. (4.3) dominates the LHS. Specifically, the last term on the LHS of Eq. (4.3) has a maximum slope of 2, at $a = 0, b = 0$. If $1/\tilde{r}^2$ is much larger than 2, then $a$ does not change significantly with small variations of $b$ around zero. Physically this means that the phase is twisted much more strongly than could be achieved by the force exerted by the Josephson coupling; in the language of the mechanical model this corresponds to an external torque which far exceeds the gravitational torque of the pendula. Thus, $a$ is, for all $b$, close to its value for $b = 0$, which is found to be

$$|h_{ext}| = a \frac{1}{\tilde{r}^2} + 2J_1(a).$$

(4.11)

Equation 4.3 determines the critical current by setting $b = \pi/2$, the location of the maximum of $\sin(b)$:

$$\gamma_{cr} = J_0(a).$$

(4.12)

On the other hand, reordering Eq. (4.3), and adding the inertial force, the equation of motion is easily found to be

$$b_{\ddot{t}} = -J_0(a) \sin(b) - \gamma.$$  

(4.13)

Comparing Eq. (4.12) and Eq. (4.13) that Eq. (4.13) describes a washboard potential of potential height $-J_0(a)$. Because the critical current is linear in $-J_0(a)$, as is the curvature of the unbiased potential, the small oscillation frequency at zero bias is proportional to the square root of the critical normalized current; Since $a$ is assumed to be constant under variation of the bias current, the small oscillation frequency behaves just as in the case of the one dimensional washboard potential, and the small oscillation frequency derived at the critical current matches the scales as it scales at zero bias current. Thus a small junction behavior is expected if $1/\tilde{r} > 2$.

The expected scaling of $\Omega_0$ versus magnetic field, calculated by the harmonic approach for a short ($l = 4\lambda_f$) AJJ is graphed in Fig. 4.2. In Fig. 4.2(a) the critical bias, determined numerically in the harmonic approximation is compared to that of a small junction. The dependence of the critical current on magnetic field is nearly the same for both cases. The small oscillation frequency, determined by the harmonic approach is plotted in Fig. 4.2(b), and compared to a $\Omega_0 \propto \gamma_{cr}^{1/2}$ and a $\Omega_0 \propto \gamma_{cr}^{1/4}$ scaling. The first one is expected for an SJJ. For
small magnetic fields, the AJJ curve resembles the SJJ scaling, while for large magnetic fields it shows a considerable deviation. The same type of plot for an intermediate length \((l = 10\lambda_J)\) junction is shown in Fig. 4.3. The scaling of the small oscillation frequency in Fig. 4.3(b) clearly agrees with the \(\Omega_0 \propto \gamma_{cr}^{1/4}\) scaling, where the prefactor was chosen arbitrarily. The \(\Omega_0 \propto \gamma_{cr}^{1/4}\) was derived in Ref. [63] to hold in the limit of a long junction. While it is valid to assume the SJJ scaling of the small oscillation frequency for junctions of a length smaller than \(2\pi\lambda_J\), for longer junctions the scaling of \(\Omega_0 \propto \gamma_{cr}^{1/4}\) is verified in sec. 4.3.

4.2 VAV Dissociation

If \(\bar{r}\) is larger than unity, system does not escape homogeneously, but rather begins escaping in a localized region: for the case of an infinitely extended sine-Gordon system, discussed in [84, 85], there exists a small amplitude nucleus, which provides the energetically most favorable path through the barrier. The size of this nucleus scales with the unbiased potential barrier height separating the Peierls Valleys. A similar form of nucleus for an inhomogeneously biased sine-Gordon field in the case of an AJJ was presented in [16]. The essence of this reference is that for the specific case (moderate magnetic field) under consideration a bias current applied to an annular Josephson junction drives the phase homogeneously close to the critical point \(\varphi_Z = \pi/2\). Therefore the potential can be developed into a Taylor series. At \(\varphi_0 = \pi/2\), the first-order term in Eq. (1.52) vanishes. The nonlinearity caused by the second order term has to be taken into account. Including the magnetic field perturbation and the bias current, the sine-Gordon equation can be reformulated for a small variable part \(\varphi_\delta\):

\[
\varphi_{\delta,\bar{t}} - \varphi_{\delta,\bar{x}\bar{x}} - \frac{\varphi_\delta^2}{2} = -(1 - \gamma) - \hbar_{\text{ext}} \cos \left( \frac{x}{r} \right),
\]

(4.14)
corresponding to a cubic potential with a spatially varying bias force.

The particular solution ansatz chosen in Ref. [16] supposes a pinned VAV pair in this cubic potential, approximated by

\[
\varphi_{\delta,\text{vav}}(|\bar{x} - \bar{x}_0|, a) = \sqrt{2} \sqrt{1 - \gamma} \left( \frac{3}{\cosh^2 \left( \frac{|\bar{x} - \bar{x}_0| + a}{2} \right) (\gamma_{cr} \epsilon)^{1/4} - 1} \right).
\]

(4.15)

The effective coordinate \(a\) represents the separation between the vortex and the antivortex.
4.2. VAV DISSOCIATION

Spatially the center of this VAV-pair nucleus is located at $\bar{x}_0$, where the potential barrier height is lowest. For a constant magnetic field orientation, $\bar{x}_0$ is constant, since the driving force of the bias current on the vortex and the antivortex are of equal size, but opposite polarity. Thus the net force is zero, and the VAV pair center is fixed by the magnetic field. By inserting $\varphi_{\delta,\text{vav}}$ into Eq. (1.44), the total energy is obtained as[16]

$$H = (a, \dot{a}) = \frac{m_{\text{eff}}(a)}{2} \dot{a}^2 + U(a),$$

(4.16)

where the effective mass $m_{\text{eff}}(a)$ is given by

$$m_{\text{eff}}(a) = 18(1 - \gamma)^{3/2} \int_{-\infty}^{a} \frac{\sinh^2(x^{\frac{1}{8}}\sqrt{1 - \gamma})}{\cosh^6(x^{\frac{1}{8}}\sqrt{1 - \gamma})}.$$  

(4.17)

The potential $U(a)$ is found to be

$$U(A) = m_{\text{eff}}(a) - 12\sqrt{2}\sqrt{1 - \gamma} h \tanh(a^{\frac{1}{2}}\sqrt{1 - \gamma}).$$

(4.18)

Without fluctuations, the critical value of $a$ at a given bias $\gamma$ is defined by

$$\sinh((2(1 - \gamma))^{1/4}a/2) = 1.$$  

(4.19)
If the separation of the vortex pair is larger, it is not a bound state. In the next section, we examine the transition from such a bound state by means of microwave spectroscopy. For that, we are interested in the specific dependence of the small oscillation frequency. Of greatest experimental interest is the behavior close to the switching bias $\gamma_{cr}$. The critical current can be found by minimizing $E$ and is approximated by $\gamma_{cr} = 1 - 2h/3$. The potential can be expanded around $a_0$ and $\gamma_{cr}$ to yield an equation of motion for the small variable part $a_\delta$:

$$E(a) = \frac{\chi h^{5/4} a_\delta^2}{2} + \frac{3^{3/2}\sqrt{h}}{2} \gamma_{cr} \epsilon a_\delta - \frac{h^2}{6} a_\delta^3,$$

with $\chi$ being a constant coefficient of order one.

Fluctuations can now be treated by considering the dynamics of this small variable part only, yielding a potential barrier $U_0$, whose form is equivalent to Eq. (2.16)

$$U_0 = 2 \cdot 3^{5/4} h^{-1/4} \gamma_{cr}^{2/3} \epsilon^{2/3},$$

Figure 4.3: (a) For junction length $10 \lambda_J$, the switching current $\gamma_{cr}$ predicted by the harmonic approach[15] is compared to that for a SJJ. (b) The unbiased small oscillation frequency $\Omega_0$ as a function of magnetic field. Dashed lines indicate a $\gamma_{cr}^{1/4}$ (black line) and a $\gamma_{cr}^{1/2}$ (red line) scaling. The $\gamma_{cr}^{1/4}$ was scaled to fit for high fields.
and a small oscillation frequency in the form of Eq. (2.18)

$$\omega_0(\gamma, h) = \Omega_0(h)\epsilon^{1/4}. \tag{4.22}$$

The small oscillation frequency $\Omega_0$ is approximated by

$$\Omega_0 = \omega_P \frac{3^{3/8}}{\sqrt{\chi}} \gamma_{cr}^{1/4}. \tag{4.23}$$

### 4.3 RF Induced Decay

In Ref. [94] we presented measurements of the small oscillation frequency versus magnetic field for a Josephson junction of length $l = 10.5 \lambda_J$ (sample AJJ-VAV (see appendix C)). We apply rf to the sample to induce small oscillations of the phase. As for the vortex case, (see Fig. 3.1) when the rf frequency matches with the small oscillation frequency, the amplitude of the small oscillation increased due to the resonance. The system is activated at this current with a higher probability than elsewhere. For different values of magnetic field, we measured the dependence of the current at which the resonance occurs on the applied rf frequency. Experimental data for zero magnetic field and $H = 0.2$ Oe, are displayed in Fig. 4.4. They compare well to Eq. (4.22). Least-squares fits of Eq. (4.22) as the model equation for each magnetic field to the data to yields both $I_c(H)$ and $\Omega_0(H)$.

The experimentally determined magnetic field dependence of the zero bias oscillation frequency $\Omega_0(H)$ is fitted to Eq. (4.23) in Fig. 4.5. We determine the characteristic frequency $3^{3/8}\omega_0/(2\pi\sqrt{\chi})$ from the least-squares fits to be 74.7 GHz. The field dependence of the zero bias oscillation frequency extracted from experimental data is in good agreement with the predictions based on the model for VAV dissociation. This model also agrees with the harmonic model concerning the scaling $\Omega_0 \propto \gamma_{cr}^{1/4}$ for this junction length. The expected oscillation frequency for small junction like escape of the phase (dotted line in Fig. 4.5) clearly disagrees with the experimental data, which emphasizes the relevance of our long-junction approach.
Figure 4.4: The current position of the resonant peak versus microwave frequency. Experimental data for zero magnetic field (solid circles) and 0.2 Oe (open circles) are compared to the model (solid line and dashed line). The respective critical currents are indicated by the dotted vertical lines. In the inset two switching-current probability distribution $P(I_{sw})$ are indicated with the same current scale. The right peak is the escape without microwave applied. The left peak corresponds to the resonant escape at a microwave frequency of 36 GHz.
Figure 4.5: Field dependence of zero bias oscillation frequency $\Omega_0$ extracted from measured data (error bars), $\Omega_0(H)$ calculated from $I_c(h)$ using the VAV model (dashed line) and $\Omega_0(H)$ of the small-junction model (dotted line).
4.4 Thermal Activation Measurements on Short Rings and the Crossover to the Quantum Regime

We performed phase escape distribution measurements on sample AJJ-SHORT (see appendix C). The switching current, calculated numerically using the harmonic approximation method compares well to the experimentally determined mean switching current in Fig. 4.6. The corresponding standard deviation, for fields up to $200 \mu T$ is plotted in Fig. 4.7, where the scaling versus the magnetic field is compared to the values of $B$ found by the harmonic approximation. For low temperatures the scaling predicted for the thermal activation deviates strongly from the experimental results, indicating the crossover to the quantum regime. The absolute experimental values however required a correction to $B$ by a factor of approximately three. It may be supposed that the deviations are due to either neglecting the two-dimensional nature of the potential, neglecting an additional degree of freedom by fixing the position of the localized phase escape, or quantum corrections to the escape rate.

The procedure described in sec. 4.1 can be used to calculate the prefactor and the exponent of the potential scaling in Eq. (4.10) numerically. In the limit of a small Josephson junction, a scaling like that of the vortex case is expected. Whereas in the vortex case, the junction radius enters into the ratio of the potential barrier height and the depinning current

$$\frac{U}{\gamma_{cr}} = \frac{2}{3} \sqrt{2E_0} 2\tilde{r} 2\pi$$

(4.24)

in the small junction case, we find using Eq. (1.11)

$$U = 2E_J \frac{2}{3} \sqrt{2} = 2I_C \frac{2}{3} \sqrt{2} \frac{\Phi_0}{2\pi}.$$  

(4.25)

Yet, the relation

$$2\pi \tilde{r} E_0 = E_J$$

(4.26) holds, too. When using this on Eq. (4.24) to rewrite the potential barrier in terms of $E_J$, we find

$$U = \gamma_{cr} 2E_J \frac{2}{3} \sqrt{2} = 2\gamma_{cr} I_C \frac{2}{3} \sqrt{2} \frac{\Phi_0}{2\pi}.$$  

(4.27)

Since $\gamma_{cr} I_C$ is nothing else but the switching current, a junction with a vortex trapped resembles Eq. (4.25) from the viewpoint of the relation between the potential barrier height and the switching current. The difference between the cases
Figure 4.6: The measured mean switching current on sample (AJJ-SHORT (see appendix C)) as used in the previous chapter for the vortex activation measurements. A length of $3.5\lambda_J$ was assumed to calculate the line using the harmonic approximation; an offset in the field and a self-field correction factor were chosen to match the data.

is the small oscillation frequency, which is influenced by the junction length. The other parameters (normalized ramp rate, damping) are the same as for the vortex case, and $\bar{B}$ which depends logarithmically on these parameters, is the same. Hence it is possible to compare the ratio of the saturation temperature for the vortex case, discussed in sec. 3.5 directly to the saturation temperature of the small junction case. The ratio of the unbiased potential barrier $U$ and the small unbiased small oscillation frequency suffices for comparing the second factor in Eq. (3.28). The small oscillation frequency of a vortex is given by

$$\Omega_0 = \frac{1}{r} \sqrt{\frac{2\pi}{8} \frac{\gamma}{\gamma_{cr}}},$$  \hspace{1cm} (4.28)$$

of a SJJ by

$$\Omega_0 = \sqrt{\frac{\gamma}{\gamma_{cr}}},$$  \hspace{1cm} (4.29)$$
Considering the relevant part of Eq. (3.28), the ratio of crossover temperatures in the same annular junction, for the same switching current is

\[
\frac{T^*_{\text{vortex}}}{T^*_{\text{SJJ}}} = \left(\frac{1}{r} \sqrt{\frac{2\pi}{m_{\text{eff}}}}\right)^{12/10}.
\] (4.30)

For short junctions \( l \approx 4\lambda_J \) the effective mass is reduced to \( m_{\text{eff}} \approx 7 \) (Compare Fig. 3.8), so that the ratio of saturation temperatures is approximately given by the inverse of the junction radius. The ratio of the switching current to the critical current enters only as a square root. This means, for AJJ-SHORT (see appendix C) we expect a lower crossover temperature for the homogeneous escape than in the vortex case. The width of the switching current distribution in sample AJJ-SHORT (see appendix C) without a vortex trapped was determined at the same temperatures as used for the measurements displayed in Fig. 3.6. It is displayed in Fig. 4.8 for selected fields. At the lowest magnetic field, the saturation temperature is 100 mK. The crossover for the vortex case in the same junction happened at approximately 160 mK, which is compatible with the considerations above. The ratio calculated using Eq. (4.30) is \( \approx 1.5 \).
4.5 Conclusion

Short and intermediate length junctions were examined by switching current distribution measurements. The small oscillation frequency scaling for VAV dissociation process[63] was verified experimentally by stimulating the system with a microwave bias of frequencies. For these junctions the VAV model agrees with a harmonic model[15] extended to predict the small oscillation frequency. The same model predicts the critical current of a short AJJ with a high accuracy, while the calculated effective barrier height did not accurately describe the magnitude of fluctuations induced during thermal escape, but only reproduced the scaling with magnetic field. The crossover temperatures, for vortex activation measurement and the vortex free phase activation have a ratio compatible to a simple estimation. Further experiments on intermediate length LJJJs are needed to verify the crossover between short and long LJJJs and to probe the scaling of the small oscillation frequency and the potential barrier as a function of the junction length.

Figure 4.8: For several selected fields from Fig. 4.7, a total \(\sigma(T)\) dependency is shown, including temperatures, which were skipped in Fig. 4.7 for visibility. A clear saturation is observed for some magnetic fields, at temperatures of approximately 100 mK.
Chapter 5

Double-Well Potentials in Long Josephson Junctions

One of the objectives of this work is to examine the possibility of creating a double-well potential for vortices. Such a potential should be suitable for creating a qubit based on spatially distinct vortex states. The realization of a double-well potential in a long Josephson junction requires a method to create a localized potential barrier. During the qubit operation both, barrier height and symmetry of the potential must be tunable independently.

A crucial point is that the quantum state of a system decoheres due to dissipation, which manifests as a rapidly increasing degree of entanglement with the oscillators forming the heat bath. The criteria for coherent evolution depend on damping and temperature. In a quartic double well potential with ohmic dissipation, the minimum requirement for observing any coherent effect is given [95], [73, P. 322] by

$$\alpha_c = \frac{m_{\text{eff}} \eta \Delta \tilde{x}_0}{2\pi \hbar} < \frac{1}{2}$$

(5.1)

where $\Delta \tilde{x}_0$ is the separation between the classical equilibrium positions of the particle in the double well. The value $\alpha_c$ is a dimensionless coupling strength of the system to the bath. In the case of long Josephson junctions, this condition translates to a simple criterion for a working qubit

$$\Delta \tilde{x}_0 \ll \frac{\hbar}{\eta}.$$  

(5.2)
CHAPTER 5. DOUBLE-WELL POTENTIALS

The simple relation in Eq. (5.1) is replaced by the stronger criterion in Eq. (5.2), since Eq. (5.1) corresponds to the condition of observing any coherent effect, (including a single coherent oscillation), while for a working quantum computation system, many coherent oscillations are required.

Since the bath is not only entangling with the qubit, but subjected to a thermal distribution of its own state, there also exists a condition on temperature. This condition is related to the level splitting \( \Delta_0 \) of the lowest energy eigenstates in the double well and the average thermal energy \( k_B T \) of a fluctuator in the bath

\[
\frac{\alpha_c k_B T}{\Delta_0} \ll 1. \tag{5.3}
\]

In a hand-waving interpretation, Eq. (5.1) determines how quickly quantum information leaks to the heat bath, while Eq. (5.3) determines how much quantum information per unit time is transferred from the heat bath to the qubit. It is striking that the different factors in Eq. (5.1) and Eq. (5.3) depend on different steps of the production of the qubit. The damping \( \eta \) is set by the quality of the dielectric in the junction and its surroundings, \( \hbar \) is set by the lithographic resolution. The energy scale \( k_B T \) by the cryogenic equipment. So it is up to the designer of the potential, and the experimenter, to set \( \delta \tilde{x}_0 \) and \( \Delta_0 \) appropriately. In order to fulfill Eq. (5.1) it is essential that both \( \delta \tilde{x}_0 \) and \( \Delta_0 \) are controlled at the time of the experiment. To operate a qubit, it must be prepared in the classical regime by intentionally violating Eq. (5.1) while reducing of the coupling, thereby projecting the state. The operation frequency of the control circuit is set by the coherent oscillation period \( \tau_{coh} = \hbar / \Delta_0 \). During a time considerably smaller than \( \tau_{coh} \), the control circuit changes the qubit’s potential from a regime fulfilling Eq. (5.1) and Eq. (5.3) to a regime where Eq. (5.1) is violated and \( \Delta_0 \) vanishes, projecting the qubit state to a classical state. When reading out the classical state to which the original qubit’s state has been projected it is essential that no thermal activation takes place in the meantime\(^1\).

In long Josephson junctions there exist several independent ways of generating potentials for vortices. Among the terms in Eq. (1.43) we group \( H_J \) and \( H_L \), which both contribute to the vortex rest mass and \( H_H \), the external magnetic field interaction energy. The first energy term \( H_J + H_L \) is parametrized as by the junction width, the critical current density \( j_c \) and the junction curvature. Variation of the rest mass along the junction can be used to form a potential for the vortex. Previously microshorts, small areas of increased critical current and microresistors, small areas of decreased critical current, density were the-

\(^1\)Due to possibly different time scales, this condition is independent from Eq. (5.3).
5.1. PARASITIC POTENTIALS

oretically discussed in relation to this [96, 82]. Although it is difficult to vary
the critical current density $j_c$ as a function of the position using a standard tri-
layer process, microshort qubits based on this microshorts have been proposed
[18]. Relaxation and decoherence times for vortex qubits based on microresis-
tors were estimated [97], considering the $\frac{1}{j}$ fluctuations of the critical current
and effect of temperature.

An alternative approach is based on varying the junction width [19, 98]. This
thesis presents the first results of experiments studying the effect of localized
width changes designed to produce a double-well potential.

The $H_H$ term is parametrized by the local external magnetic field. A lo-
calization can be created either by generating a local magnetic field, or an ex-
ternal homogeneous field, where the vortex-field interaction is defined by the
angle between the junction’s centerline and the external magnetic field. To cre-
ate a double-well potential using this method, heart-shaped Josephson junctions
[13, 10, 11, 12] have been proposed. It is worth mentioning that a curvature
change causes parasitic potential barriers in heart-shaped junction. This is dis-
cussed briefly using a model for a curved Josephson junction in Ref. [17] in the
limit of junction widths smaller than a Josephson length.

The bias current interaction term $H_\gamma$ in Eq. (1.43) can be used to tilt the
potential. To observe coherent quantum effects, a particularly well controlled
potential tilt is needed, since its fluctuations determine the speed of the quantum
mechanical phase evolution.

5.1 Parasitic Potentials in Heart-Shaped Joseph-
son Junctions

One possibility to shape a controllable double well potential is to engineer the
direction of the centerline of the junction such that the vortex interaction with
the external magnetic field depends on vortex position. Bending the junction di-
rection changes the mutual inductances of the junction segment with each of the
orthogonal (x,y) external in-plane magnetic field components. Bistable states
in heart-shaped Josephson junctions have been demonstrated in Ref. [11, 12].
Regrettably, the spatial separation between the classical states in the two poten-
tial minima, is set at production time by the geometry and critical current of the
junction [10], which is imprecisely produced. For a normalized Plancks constant
$\hbar$ on the order of $10^{-4}$ to $10^{-3}$, corresponding to junctions of $\mu$m to moderate
sub-$\mu$m width, it seemed reasonable to create a systematic spread of the junction
parameters at design time to manufacture an acceptable yield of junctions fulfill-
ing Eq. (5.1) and simultaneously capable of considerable tunneling rates [10]. A electron beam micrograph of a junction designed for a test in the thermal regime is shown in Fig. 5.1.

The potential created by a curved segment in a long Josephson junction without magnetic field is derived in Ref. [17]. If the junction width is smaller than a single Josephson length, the effect of the curvature is described by first order perturbation theory, equivalent to the interaction of a vortex with a magnetic field. The parasitic potential is generated by the variation of the energies associated with $\mathcal{H}_J$ and $\mathcal{H}_L$. The static vortex solution does not only fulfill the sine-Gordon equation, but also stores equal energy densities in the Josephson coupling and the magnetic field. Since both energy density distributions contribute equally to the rest mass, and are proportional to the junction width, it is possible to define the “rest mass profile” $\check{\mu}$ of a static vortex at position $x_0$ as:

$$\check{\mu}(\check{x}_0) = 4\text{sech}^2(\check{x}_0). \quad (5.4)$$

Using this, the vortex rest mass $\check{m}$ takes the form of a simple convolution

$$\check{m}(\check{x}_0) = \mu(\check{x}) \ast \check{\mu}(\check{x} - \check{x}_0), \quad (5.5)$$

where $\mu(\check{x})$ is the mass density per unit length and per squared magnetic field unit, normalized to $E_0$

$$\mu(\check{x}) = \frac{\int_0^w \frac{1}{2} \varphi \check{x}^2 + (1 - \cos \varphi) \, dy}{\int_0^{w_0} \frac{1}{2} \varphi \check{x}^2 + (1 - \cos \varphi) \, dy}. \quad (5.6)$$

Compared to the case of convolution of the magnetic field with the magnetic field sech-kernel [48, 99, 10], the rest mass convolution sech$^2$-kernel is narrower, corresponding to a higher curvature for the same potential barrier height. A comparison between both kernels is shown in Fig. 5.2.

The rest mass density $\mu$ is determined by the junction shape. This is due to the deformation of the fluxon in comparison to the one dimensional sine Gordon equation model. For the heart-shaped Josephson junctions discussed in this thesis, only a small part of the junction has a radius small enough to change the potential energy considerably. In the micrograph in Fig. 5.1, all parts of the junction except for the bend in the enlarged region are assumed to have infinite radius. In this case [17], the local rest mass density of the field in normalized units takes the form

$$\mu(\check{x}) = \left( \frac{R}{w} \log \frac{2R + w}{2R - w} - 1 \right) \frac{\Theta(a + \check{x}) + \Theta(a - \check{x})}{2} + 1, \quad (5.7)$$
Figure 5.1: Micrograph of a sample AJJ-HEART2 (see appendix C). The external magnetic field orientation and the designed vortex states (1) and (2) are indicated. The enlarged region is intended to provide a magnetic field dependent barrier. The inset shows an enlarged region, in which the junction curvature is high.

where \( \tilde{x} = -a \) is the beginning, and \( \tilde{x} = a \) the end of the curved region. In Fig. 5.1, \( a = r\beta \), the. The convolution of the sech\(^2\) kernel with the step-function can be carried out analytically (since the integral of the kernel is known), yielding the first-order perturbation potential

\[
U(\tilde{x}_0) = \mu_c \frac{\tanh(\tilde{x}_0 - a) - \tanh(\tilde{x}_0 + a)}{2}.
\]

(5.8)

In the junctions AJJ-HEART1, AJJ-HEART2, AJJ-HEART3 (see appendix C), \( R = 2 \mu m, w = 3.5 \mu m, \) and \( \lambda_J = 14 \mu m, \) for angles \( \beta \) of 26\(^\circ\), 37\(^\circ\), and 43\(^\circ\). The influence of lateral vortex deformation\([17]\) is neglected by setting the variation parameter \( B \) from\([17]\) to zero. To determine whether a lateral variation of the phase should be considered, the deformation energy for a vortex\([17, \text{ see Eq.27}]\) is calculated to first order by

\[
U = \frac{4\pi^4}{24w^2} B^2,
\]

(5.9)
where $B$ is the first harmonic component of the lateral deformation[17]. This means that the vortex shape remains unperturbed, and $B = 0$ is an adequate approximation. If $w < 1$, the energy attributed to this deformation is high.

Fig. 5.3 contains measured depinning currents, minimized by varying external magnetic fields as well as the depinning current obtained from the convoluted potential Eq. (5.8) as a function of the angle $\beta$.

Some measurements not included in Fig. 5.3 contain other pinning sources like Abrikosov vortices. These are identified by too large zero field pinning current, or a changing zero-field depinning current during repeated measurements. These measurements were carried out without injecting vortices. Theory and experiment agree acceptably, considering that neither the effect of the irregular finite idle region at this location on the magnetic field distribution is known nor the mask alignment error, which causes an asymmetry in the idle region width(see Fig. 5.1, enlarged region) is taken into account. Both effects could give rise to a moderate change in the total inductance along the junction. Using the same parameters as in Fig. 5.3, the potential barrier height at zero field can be derived. The result is shown in Fig. 5.4. For a junction of width $3.5 \mu m$, $j_c = 1 kA/cm^2$ and radius of curvature $2 \mu m$, this barrier dominates the mag-
5.1. PARASITIC POTENTIALS

Figure 5.3: The minimal pinning currents, determined without active state preparation, for junctions of different bending angle \(\beta\). The line is the prediction for the pinning current using the estimation given Ref. [17], a radius of 2 \(\mu\text{m}\) and a junction width of 3.5 \(\mu\text{m}\) at a critical current density of \(j_c = 1 \text{kA/cm}^2\). Dots/open circles correspond to different current polarities.

netic field induced barrier even at moderate fields. This parasitic barrier makes it impossible to observe thermal activation in any of the produced junctions. To achieve a working qubit, barrier heights on the order of one Kelvin were intended. Thus the the possibility of characterizing the magnetic field induced barrier in the thermal regime on junctions with a spatial scale achievable using photo-lithography is excluded. While this hinders systematic progress on developing a working vortex qubit based on a barrier formed by magnetic field, the concept may still work in the limit of very narrow junctions. Evaluating Eq. (5.7) for different junction widths shows that, for a fixed radius of 2 \(\mu\text{m}\) at a critical current density of \(j_c = 1 \text{kA/cm}^2\), the relative size of the effect is reduced by two orders of magnitude for a junction of a width of \(\approx 0.2 \mu\text{m}\).
Figure 5.4: The potential barrier height in Kelvin, caused by the curvature only, versus angle $\beta$. 
5.2 Lithographic Microshorts

A microshort is a small area in a long junction, which has an increased critical current density. In a one-dimensional model, this means that the critical current density along the length of the junction is modulated. Usually this is thought to be caused by a variation of the barrier thickness, as sketched in Fig. 5.5. In the simple case that the modulation has a spatial extent which is small in comparison to $\lambda_J$, it can be described by a $\delta$-like inhomogeneity of the critical current:

$$j_c(x) = (1 + \mu \delta(x - x_\mu))j_{c0}$$ (5.10)

Here $x_\mu$ is the position of the microshort and $j_{c0}$ is the unperturbed critical current density. $\mu$ describes the relative strength of the microshort, which is the product of the change in the critical current density with the spatial extent of this change. Such inhomogeneities contribute a perturbation to the sine-Gordon equation

$$\varphi_{xx} - \varphi_{tt} = (1 + \mu \delta(x - x_\mu)) \sin(\varphi),$$ (5.11)

which was discussed in the perturbation theory of Ref. [47]. For the motion of a vortex the effective potential was derived in to be

$$U = 2\mu \text{sech}^2(x - x_\mu).$$ (5.12)

If $\mu$ is negative it corresponds to an increased thickness of the barrier, and a local decrease of the critical current density. The whole perturbation is called a microresistor in this case. Few instances of long Josephson junctions with a spatially artificially modulated critical current density have been previously realized experimentally [100, 101]. All previous experimental realizations of modulated critical current density involved regions of zero critical current density created by replacing the trilayer with an insulating barrier (insulator-type microresistors). Fluctuation induced activation on an annular junction containing a single insulator-type microresistor to create a vortex pinning potential were reported in [8]. A very relevant result of Ref. [8] is the fact that the capacitance change of an insulator-type microresistor is considerable and plays a role in the experiment. At the structuring resolutions available (on the order of $1 \mu m$ for an reliable insulation), one finds the minimal potential depth of the well to be on the order of $500 K$ for $j_c = 1 \text{kA/cm}^2$. Still, since the potential is convoluted with the fluxon shape, the use of spatially close insulator-type microresistors to create a small barrier in between could be feasible. This would result in a double well potential, however up to now the preparation and readout of bistable states in such a structure has not been demonstrated. Without considering the
experimental realization, the observation of quantum effects on microshorts as barriers was theoretically described earlier [81]. Specifically the tunneling of a vortex through a single barrier and the resonant tunneling through a double barrier formed by two microshorts was considered there. Using a long Josephson junction containing a microshort in an external magnetic field as a qubit was proposed in Ref. [18]. No trilayer process allowing the reproducible production of microshort junctions is available. The difficulty is that the trilayer is formed by oxidation, a process very sensitive to external parameters, especially the partial pressure of oxygen. Hence it is difficult to perform the lithographic step, required for spatially defining the areas of increased or reduced critical current density.

The rest of this chapter examines an alternative approach to the problem of forming a well defined microshort-like structure with which to build a vortex qubit. We use local variation of the junction width to create a localized barrier. Such width modulation was considered before [19], with the intention of creating a microscopic ratchet potential in one-dimensional Josephson junctions. To the authors knowledge no working realization of width modulated junctions was presented. Our junction design is laid out in Fig. 5.6. In the subsequent sections such width modulated junctions are discussed and the first experimental results are presented.

The width variation is defined by $\Delta w(x) = w(x) - w_0$, where $w_0$ denotes the unperturbed width. In the junction designs tested experimentally $\Delta w(x)$ was
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defined by a step function

\[ w(x) = w_0 + w_{ms} \Theta(|x| - l_{ms}/2), \]

(5.13)

where \( w_{ms} \) is the extra width at the step, and \( l_{ms} \) the length of the microshort. The complete layout is shown in detail in Fig. 5.7. In our case, \( w_{ms} = 0.5 \mu m \), and \( l_{ms} = 2 \mu m \), and \( w_0 = 3 \mu m \). If the maximum width of the junction is smaller than \( \lambda_J \), the phase is laterally constant \( \varphi(\tilde{x}, y) = \varphi(\tilde{x}) \). Hence the complete model is still defined by a one-dimensional field equation of motion.

Figure 5.6: An annular Josephson junctions active area, with laterally applied magnetic field. The finite region of larger width, used to form the double well potential, is enlarged. The stable positions of the fluxons with applied external magnetic field are indicated by (1) and (2).

The most important difference in the derivation of the field equation of motion for a long junction including a microshort is that the whole Lagrangian is multiplied by a single factor

\[ L = \left( 1 + \frac{\Delta w(\tilde{x})}{w_0} \right) \left( \frac{1}{2} \varphi_{\tilde{x}}^2 + (1 - \cos \varphi) - \varphi_{\tilde{t}} \right). \]

(5.14)

A perturbed equation of motion is found by applying the Lagrangian formalism

\[ \frac{w_{\tilde{x}}(\tilde{x})}{w(\tilde{x})} \varphi_{\tilde{x}} = \sin \varphi - \varphi_{\tilde{x}} \varphi_{\tilde{x}} + \varphi_{\tilde{t}} \]

(5.15)
It is striking that the LHS of this equation of motion has the same function as a bias current perturbation to a SGE. The physical meaning is obvious: The current flowing through the junction electrode is proportional to its width and the phase gradient. It is easy to perform the thought experiment of connecting several identical junctions in parallel, each threaded by the same magnetic field. If the magnetic field at a given point changes smoothly, this means that at an inductance change the lateral current along the electrode will change; since the Kirchhoff law holds, this means that current flows through the junction. Such parametric inductance changes in sine-Gordon systems have been considered before [69, 19]. Usually it is assumed that the change is slow in comparison to the characteristic length. In this case the first-order perturbation theory equivalent to [47] can be applied. The derivative of the junction width \( \tilde{w}_x(\tilde{x}) \) is in our design is as sharp as the production process allows. It was not possible to examine the final junction using electron beam microscopy, since the dimensions of the lithographic microshort were on the order of the thickness of the metalization and insulation layers above, so that the real \( \tilde{w}_x(\tilde{x}) \) is unknown. Hence the data needed to determine if \( \tilde{w}_x(\tilde{x}) \) is small enough to ignore the influence of the perturbation on the fluxon shape is lacking. Assuming that the phase profile of a vortex located at \( x_0 \) is unchanged by the perturbation (equivalent to assuming a small \( \tilde{w}_x(\tilde{x}) \)) it can be substituted into the changed energy density \( H = H_0 w(x)/w_0 \). Again, from the relevant terms in Eq. (1.43) we group \( H_J \) and \( H_L \), which are the only two terms which play a role for an unbiased, magnetic field-free Josephson junction.

Since \( l_{ms} \) is shorter than the Josephson length the effect of the modulation is
approximated in first order perturbation theory by

$$\frac{w(\tilde{x})}{w_0} = 1 + \delta(\tilde{x})\mu_{ms},$$  \hspace{1cm} (5.16)

where $\mu_{ms}$ is the total junction area in the extra region. For this case the potential predicted by the perturbation theory is given by

$$U(\tilde{x}_0) = 4\mu_{ms}\text{sech}^2(\tilde{x}_0)$$  \hspace{1cm} (5.17)

### 5.2.1 Bistable States in Microshort Junctions

This potential can be added to the effective potential described by Eq. (1.69), created by a magnetic field of strength $\vec{h}_{ext,V}$ in the vertical direction and $\vec{h}_{ext,H}$ in the horizontal direction, and a driving bias current term $\gamma$, resulting in the potential

$$U(\tilde{x}_0) = 4\mu_{ms}\text{sech}^2\tilde{x}_0 - 2\pi \left( \gamma\tilde{x}_0 - \cos\left(\frac{\tilde{x}_0}{r}\right) \vec{h}_{ext,V} + \sin\left(\frac{\tilde{x}_0}{r}\right) \vec{h}_{ext,H} \right).$$  \hspace{1cm} (5.18)

The position of the microshort is set to be $\tilde{x}_0 = 0$. If a vertical magnetic field (oriented as in Fig. 5.6) is applied, Eq. (5.18) a double-well potential is created. For various vertical magnetic fields the resulting potential is plotted in Fig. 5.8. The barrier height of this potential is predetermined during design time by the microshort strength $\mu_{ms}$. When a vertical magnetic field magnetic field is applied, the barrier is reduced. In order to determine the criterion for zero barrier height, it must be determined whether the potential given by Eq. (5.18) has a minimum or a maximum at $\tilde{x} = 0$. Obviously the slope at this point is 0, if $\vec{h}_{ext,H} = 0$. The criterion for a minimum or a maximum is given by the total curvature at this point; so the curvature of the magnetic field created potential and the microshort created potential are added. We define the orientation of the vertical field so that $\vec{h}_{ext,V} > 0$ for the orientation of the magnetic field in Fig. 5.6. The criterion for the two curvatures adding to zero is thus given by

$$\frac{2\pi\vec{h}_{ext,V}}{r^2} = 8\mu_{ms}.$$  \hspace{1cm} (5.19)

The critical magnetic field strength $h_{ext,cr}$ at which the potential vanishes is given by

$$h_{ext,cr} = \frac{r^28\mu_{ms}}{2\pi}.$$  \hspace{1cm} (5.20)
Thus, there are three possibilities depending on $\vec{h}_{\text{ext},V}$: For $\vec{h}_{\text{ext},V} < 0$, only a single maximum is found at $\tilde{x} = 0$, and a single minimum is found at $\tilde{x} = l/2$. For $0 < \vec{h}_{\text{ext},V} < h_{\text{ext,cr}}$, a maximum exists at $\tilde{x} = 0$, and two minima, which form a double well potential. At $\vec{h}_{\text{ext},V} > h_{\text{ext,cr}}$, only a single minimum exists, and the double well potential ceases to exist.

Operating the system as a qubit happens close to this value of magnetic field. At this point, the barrier height and distance of the minima are reduced to small values, and the potential can be approximated by a quartic approximation. In this framework, thermal activation is considered in sec. 5.2.6 and the coherence conditions Eq. (5.1) and Eq. (5.3) can be checked.

### 5.2.2 Readout

Under the influence of a field in the horizontal and vertical field, qualitatively different situations arise, depicted in Fig. 5.9. There are, four principal directions of applied magnetic field. One of them (Fig. 5.9(a)), leads to the double-well potential, as discussed in the previous section. Inverting the field polarity
(Fig. 5.9(b)) leads to the situation where the magnetic field is repulsive at the position of the microshort. So only a single stable position exists. Applying a field in the perpendicular direction (Fig. 5.9(c,d)) leads to the situation, where one stable position is (for a long Josephson junction) the magnetic field induced one. The microshort is far away, and its influence is practically zero. A second stable position does exist, which is more obvious when comparing the corresponding potentials depicted in Fig. 5.10. If the microshort pinning is strong enough, it can locally compensate for the slope of the magnetic field induced potential and create a second minimum. Under horizontal field the potential shows neither an point-like symmetry nor an mirror symmetry. Still under the inversion of the magnetic field polarity the potential is mirrored, so that the situations (c) and (d) in Fig. 5.10 and Fig. 5.9 are equivalent, if the junction coordinate is reversed. This is more closely discussed in sec. 5.2.5.

Now we examine the depinning process further. The simplest situation is the symmetric double well potential, plotted in Fig. 5.11. The figure also indicates the points of the maximum slope of the potential. If a vortex is trapped in either minimum, a bias current with a magnitude smaller than \( \max(U_x/(2\pi)) \), where
\[ U = 1 \quad c \quad d \quad b \]

\[ \lambda_J \]

\[ \mu_{ms} = 0.033 \]

\[ \max \] denotes the closest maximum in the positive direction is needed to free the vortex. If the bias current exceeds this value, the vortex is driven beyond this point. From there the vortex is accelerated by the bias current. This acceleration gives a rise to the kinetic energy of the vortex. The increase in velocity is proportional to the driving force and the distance the vortex travelled since depinning. So the total energy of the vortex is given by a straight line, which starts at the potential energy at the position of depinning and rises with a slope equal to the depinning force. Thus the tangent to the potential at the location of maximum slope represents the total energy of the depinned vortex as a function of position. If this tangent does not intersect the potential a second time, the vortex overcomes all maxima of the potential, since its energy is the sum of its starting potential energy and the energy gained from the driving force. Otherwise, the vortex is pinned again at the next maximum. Fig. 5.11 shows a situation, where neither of the tangents cuts the potential again. A a vortex in either well is liberated at the corresponding current. Since the two critical currents are different, this can be used for readout of the state.

The two critical currents, corresponding to the microshort pinning potential and the magnetic field induced pinning potential can be found numerically. For the case of zero magnetic field, the microshort depinning current can be calcu-
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Figure 5.11: Depinning positions in a symmetric potential (dots). The lines are the tangents to the potential in these points. None of the tangents cut the potential a second time in the direction of depinning.

\[ \gamma_{cr} = \frac{4}{2\pi \mu_{ms}} \frac{8 \sqrt{\left(1 - \frac{2}{\sqrt{5}}\right) \left(1 + \frac{2}{\sqrt{5}}\right)}}{5} \]  

(5.21)

If the particle is trapped at the magnetic field barrier, it is depinned at

\[ \gamma_{cr} = h_{ext}/r \]  

(5.22)

At small magnetic field, the two depinning currents differ strongly. Since the microshort barrier disappears at the magnetic field determined by Eq. (5.19), of the corresponding well is reduced to zero at this field. There is no closed-form expression for the depinning from this well as a function of magnetic field, but it is a monotonous function, so at some magnetic field the field induced depinning current overcomes the microshort induced depinning current.

\(^{2}\)Remove the root sech² from the second derivative to find the equation for the positions of the absolute maxima of the force sech\((x_0) = 2 \tanh(x_0)\).


5.2.3 State Preparation

State preparation can be performed by the “shaker” method. This method makes use of the fact that in general the retrapping of the system, after turning off the bias current, happens stochastically. A current is chosen to have a magnitude between the microshort induced depinning current and the magnetic field induced depinning current. A biascurrent pulse train, switching between zero current and this current is applied to the junction (phase (1) in Fig. 5.12). If the system resides in the state of the lower pinning current, the vortex is depinned, otherwise no switching happens. If retrapping happens stochastically, the vortex settles after each pulse to the state of the higher depinning current with some probability. Thus, the probability of being in the state associated with the lower depinning current decreases exponentially with the number of pulses.

5.2.4 Experimental Test of State Preparation and Readout

The overall experimental biasing sequence is depicted in Fig. 5.12. After the preparation of the system in phase (1), phase (2) changes the magnetic field to suppress the potential. This is the phase where experiments are carried out. Before proceeding to these experiments, a test of the preparation and readout scheme is done. The readout is done at the end of the sequence in phase (3) in Fig. 5.12 by a current ramp. If the magnetic field is chosen correctly, the situation is as in Fig. 5.11, where the two pinned states of the vortex differ in the depinning current, and can be read out. In order test the state preparation and readout, the pulse height in phase (2) is set to zero, and the magnetic field in vertical direction is swept.

The result of this preparation and readout, without the suppression pulse, is displayed in Fig. 5.13, where the different symbols denote the polarity of the shaker pulses. The potential without bias current is symmetric around the microshort position; inverting the polarity of the shaker pulses just tilts the potential in the other direction. The same applies to the readout. A state pinned by the microshort under positive bias current polarity is the same state, as appears to be pinned by magnetic field under negative bias polarity. Using shaker pulses and reading out in the same bias current polarity we will find the higher of both depinning currents, while, when reading out in the opposite polarity we expect to find the lower depinning current. Qualitatively the predicted existence of the bistable states for the positive field polarity is verified in Fig. 5.13, including the relations between the polarity of readout and shaker pulses. This identifies the microshort-like depinning on one polarity to be associated with the same state...
as the magnetic field like depinning current of the other polarity. For the negative field polarity, the microshort branch is the dominant pinning (both forces are adding up). To understand the situation better fully, consider Fig. 5.14. For vertical magnetic field, the potential landscape is symmetric. A change of the read-out polarity flips the role of the states between (a) and (b). For negative field polarities there exists only a single stable state at zero bias current, the preparation is without effect, even if there are two possible depinning position in Fig. 5.14(c) and (d). The second depinning position appears at a higher bias current, if the magnetic field strength is high enough. The effect of this can be seen at high negative magnetic field for negative bias currents in Fig. 5.13, where the depinning changes abrupt from a microshort-like branch to a magnetic field-like branch; yet the preparation has, as predicted, no effect on the measured current.

In the same figure, lines indicate the pinning calculated for a $\mu \approx 0.06$ and an magnetic field asymmetry correction. For the junction reported here the expected depinning current from the microshort at zero field is $\approx 0.2 \text{ mA}$, according to Eq. (5.21). In Fig. 5.13 the experimentally measured zero field depinning current of was $2.2 \text{ mA}$. The origin of this factor of approximately 10 remains
Figure 5.13: Depinning current versus vertical magnetic field. Solid lines: calculated depinning current (magnetic field pinning), see sec. 5.2.7, with a parasitic x-component of magnetic field added. Dashed lines: calculated depinning current (microshort pinning). Solid triangles: Positive shaker pulse preparation (+1.5 mA) before measurements. Open squares: Negative shaker pulse (−1.5 mA) preparation before measurement.

unclear; there are two possibilities which are currently under investigation

1. The microshort perturbs the sine-Gordon equation so strongly that the phase profile itself is changed; this is briefly touched upon in sec. 5.2.7.

2. A magnetic field is trapped through both rings. This corresponds to current circulating along the ring. At the microshort, this current redistributes strongly due to the inductance change.

Moreover, also the repinning features are qualitatively reproduced. Fig. 5.15 illustrates for two different magnetic field strength, that the probability of retrapping in a given well depends on the horizontal magnetic field. The polarity
Figure 5.14: Depinning positions in a microshort Josephson junction. The yellow arrows mark the driving bias force. Note that for magnetic field pinning the fluxon is depinned at a point a quarter of the junction length away from the location of the unbiased minimum. The depinning from the microshort happens at a comparatively constant location, nearly independent of the magnetic field strength. The signature of a localized pinning potential is the symmetry of the observed depinning process. While for vertical field direction (parallel to the symmetry axis) the same absolute values of depinning currents are expected for the different bias current polarities (a) and (b), in the case of the horizontal magnetic field, an opposite bias current requires the magnetic field polarity also to be reversed to achieve the same absolute value of depinning current, like for the pair (e)-(h). The situations (a),(b),(e),(h) correspond to observable bistable states. In (g) and (f) only a single stable state exists. For (c) and (d) the distance between the location of depinning from the magnetic field induced potential is too close to the next maximum, which is not overcome.
Figure 5.15: For two different horizontal magnetic field strengths, the potentials are plotted (offset for greater visibility), including the vortex energy trajectories. The depinning depends on the strength of the magnetic field, since at the lower magnetic field the microshort created maximum is not overcome. Hence only the depinning from the microshort can be observed below a certain field threshold. In order to visualize the periodicity, the potential is drawn beyond the junction boundary.

chosen is equivalent to situations (e) and (h) in Fig. 5.14. The opposite polarities (f) and (g) in the same figure are shown in Fig. 5.17. In cases (f) and (g) the well from which the vortex escapes is more shallow, so that a vortex depinned from the microshort will be repinned in the magnetic field potential well. These considerations agree with the observations of bistable states in Fig. 5.16.

5.2.5 Symmetry of the Patterns

As done in the previous section, the depinning processes can be also understood qualitatively for the horizontal magnetic field direction. The expected approximate depinning positions for all four field directions are illustrated in Fig. 5.14. Under reversal of both the bias current polarity and the horizontal magnetic field polarity, the depinning current stay the same. When viewing at the horizontal magnetic field pattern in Fig. 5.16 to the schematic situations, one finds that the symmetry is matched.
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5.2.6 Thermal Activation over a Suppressed Barrier

To examine the thermal and quantum activation over the bistable potential well at zero bias current, a quartic approximation to the potential is used. This is derived from the Taylor series of the magnetic field potential, and the microshort potential, respectively. At $h_{\text{ext}} \approx h_{\text{ext,cr}}$ (see Eq. (5.20)) the potential is well approximated by a quartic polynomial.

$$\hat{U} = a\hat{x}_0^2 + b\hat{x}_0^4 - 2\pi\gamma\hat{x}_0$$

where for the case of the (zeroth order) microshort potential, $a = 2\pi\frac{1}{2\gamma}\left(-h_{\text{ext}} + h_{\text{ext,cr}}\right)$ and $b = 2\frac{3}{\gamma}\mu_{\text{ms}}$. The unbiased barrier height evaluates to

$$\mathcal{U} = \frac{a^2}{4b} = \frac{\left(-h_{\text{ext}} + h_{\text{ext,cr}}\right)^2\pi^2}{4r^4\mu_{\text{ms}}}$$

and the small oscillation frequency inside a single well to

$$\Omega_0 = \sqrt{\frac{2a}{m_{\text{eff}}}} = 2\sqrt{\pi}\sqrt{\frac{h_{\text{ext,cr}}}{m_{\text{eff}}r^2}} \sqrt{1 - \frac{h_{\text{ext}}}{h_{\text{ext,cr}}}}.$$
Figure 5.17: In an potential, equivalent to Fig. 5.15, but with opposite magnetic field polarity the pinning well is slightly more shallow, allowing only the observation of the microshort-pinned state. In order to visualize the periodicity, the potential is draw beyond the junction boundary.

For a fixed value of $b = 0.03$, both expressions are plotted in Fig. 5.18 for small $a$.

As a function of the control parameter $a$, the thermal switching rate of the system across the well in one direction is estimated in the underdamped limit as

$$
\Gamma_{\text{th}} = B \eta \exp(-B),
$$

where $B = U/\tilde{k}_B T$. In Fig. 5.19 the result for parameters, similar to those for the experiments is shown for an assumed damping of $\eta = 10^{-5}$ and a temperature of 100 mK. To perform thermal activation between the wells a Gaussian magnetic field pulse (see phase (2) in Fig. 5.12) suppresses the barrier. The specific parameters cannot be determined, as no matching analytic formula for the potential is known. Substituting the rate (Fig. 5.19) into Eq. (2.5) and integrating numerically yields the probability of the particle to pass over the barrier. Depending on temperature, for a damping of $\eta = 10^{-5}$, Fig. 5.20 displays the calculated and the experimentally determined probability of the system having switched on the Gaussian height for temperatures between 100 mK and 900 mK. Depending on the temperature, the particle is activate with a certain probability
at a given magnetic field pulse height. The higher the temperature, the lower is the magnetic field strength required to suppress the barrier to an energy on the order of the thermal energy. In a symmetric potential, particles undergo multiple transitions over the barrier, so it is expected that in experiment the probability to find a particle in a certain well equals to $\frac{1}{2}$ after a long time. Obviously an asymmetry in the experiment described here prevented this, since the switching probability trends to 1 for high temperatures and low barriers; this suggests the presence of a parasitic field component like that used to match the data plotted in Fig. 5.13. At a temperature of $100 \text{ mK}$, the curvature of the S-curve changes no longer. This change could be due to the crossover to the quantum regime (see Fig. 5.19); yet a verification requires demonstration of a saturation of the width of the S-curve.

Figure 5.18: Unbiased potential height $\mathcal{U}$ (solid) as a function of $a$. Small oscillation frequency $\Omega_0$ (dashed) inside a single well as a function of control parameter $a$, for $b = 0.03$. 
Figure 5.19: The thermal activation rate $\Gamma$ for $T = 100 \text{ mK}$ (dashed line) and the quantum tunneling rate estimated from the WKB approximation vs. the quadratic component $a$ of the quartic potential. The normalized Planck’s constant $\tilde{\hbar}$ and Boltzmann constant $\tilde{k}_B$ were estimated from the nominal junction parameters of the tested junction, and a quartic component expected from the first order perturbation theory.
After preparing the system in state (1) the magnetic field was decreased to suppress the barrier using a Gaussian magnetic field pulse. Plotted is the probability of finding the system in the state in which it was prepared versus the height of the magnetic field pulse, for different temperatures. It obtained by averaging 10 discrete events per point. A higher temperature enables the system to overcome a higher barrier, so the suppression pulse is smaller. The symbols and error bars on the solid lines denote the experimental data, while the dashed lines are calculated theoretically. The theoretical curves were shifted to match the critical field.
5.2.7 Second Order Perturbation

Even if there is some uncertainty in the production process, it is much smaller than the strong discrepancy between the experimental results and the first order perturbation theory. In particular the discrepancy between the expected depinning current and the experimentally determined depinning current, which is a factor of approximately ten can not be explained by lack of precision during the definition of the area of the lithographic microshort. One explanation might be the influence of flux trapped through the holes in both electrodes, which corresponds to a screening current. Experiments to verify or exclude this argument are ongoing. Meanwhile I propose an second order perturbation scheme, which may be helpful in explaining the data. I remind the reader what approximations were made in order to calculate the depinning current, and what they physically mean.

- The width of the junction is much smaller than the characteristic length: so the system is described by a one dimensional model.
- The extension of the microshort is small, so that it is described by a Dirac delta function.
- The change of the width happens slowly: this means that at all local point the LHS of Eq. (5.15) is much smaller than one.

The last condition may have been violated during the experiments. The maximum normalized phase gradient induced by the vortex is 2, at the center of the vortex. this means that per Josephson length $\lambda_J$ a relative junction width change of 0.5 is the maximum rate of change, at which the first-order perturbation scheme holds. At a Josephson length of $\lambda_J = 14 \mu m$, this corresponds to

$$\frac{w(x)}{w(x)} = 0.036^{1}/\mu m.$$  (5.27)

Assuming that the junction patterning process has a resolution of $0.125\mu m$, the relative junction width step of $0.5\mu m/3.0\mu m$ results in a local current density through the junction of $\gamma(\bar{x} = l_{ms}/2) = 1.33$. Since the current distribution in the microshort is likely to be influenced by the junction width, this is the maximum estimate for the local current and most likely a gross overestimation. The details can only be determined by a 2D-PDE solution of the problem at the lithographic microshort. Still a qualitative model can be deduced, even without knowing the exact current distribution. The microshort consists of two width changes of opposite polarity. So a constant phase gradient induces a current
dipole. Since the dipole has a finite spatial extent, there is, if the phase gradient is not constant, a difference in the magnitude of the locally induced currents of opposite polarity. This difference is proportional to the change in width at this point and, for small distances, proportional to the product of the spatial derivative of the phase gradient. The force exerted on the vortex is proportional to the local magnetic field at the position of this monopole. So that the net force is proportional to \[ F \propto \mu \varphi_x \varphi_x, \] (5.28)
a result consistent with the potential Eq. (5.17) derived above. In this approach the strength of the induced current dipole is forgotten and only the monopole, existing due to a non-vanishing second derivative of the phase profile, is taken into account. But if the dipole is strong enough it can induce a local phase twist at the microshort, which causes a linear distribution of the phase over a small distance. This corresponds to a local magnetic field.

A straightforward extension is a perturbation approach where not only the center of mass position is allowed to vary, but also an additional phase field in close vicinity of the microshort. The ansatz
\[
\varphi(\tilde{x}, \tilde{t}) = \varphi_0(\tilde{x} - \tilde{x}_0) + \varphi_1(\tilde{x}),
\]
(5.29)
is substituted into Eq. (5.15), yielding a boundary value problem for the perturbation \( \varphi_1(\tilde{x}) \) similar to Eq. (1.51). For large distances from the microshort, the boundary condition \( \varphi_1 = 0 \) is set. This corresponds to the perturbative effect of the microshort being localized. In Fig. 5.21 the numerically calculated second order magnetic field profile perturbation \( \varphi_1(\tilde{x}) \) is plotted for a vortex at a distance of \( 1.5\lambda_J \) from the microshort center. At the microshort a strong magnetic field of positive polarity appears, while in the rest of the junction, a opposite magnetic field compensates this one. The parameters are chosen in such a way that the experimental results are reproduced at least qualitatively. Realistic changes of the microshort length and width are assumed, which result in a relative change of the microshort strength of 2.5. Using this microshort strength, the zero field depinning current is reproduced. The zero field depinning current is found by substituting the resulting total phase profile back into Eq. (1.43) and calculating the effective potential. This potential is compared to the first order potential in Fig. 5.22 for the same microshort strength. The second order perturbation phase profile is even narrower than the \( \text{sech}^2 \) profile. It is striking that the potential barrier height itself is not modified strongly. Yet the force, and thus the depinning

\[ ^3 \text{The build-in boundary value problem solver in Mathematica[102] was used.} \]
current is increased considerably. Imagining this on the pendula chain-model is done by making a few pendula heavier and the connections between them more stiff. Before these turn over when one end of the chain is twisted the twist outside this region becomes stronger. So energy is stored in the twist, which is later released. Using this effective potential, a numerical calculation of the depinning current was carried out and compared to the experimental data in Fig. 5.13 Fig. 5.16. While the horizontal field polarity $I_C(H_{\text{ext,x}})$ pattern matches the expected symmetry of exchanging bias current polarity and field polarity\cite{10}, the $I_C(H_{\text{ext,y}})$ pattern violates the expected symmetry. In the case of the y-field a parasitic x-field component must be assumed. Later measurements \cite{103} performed after cycling the system through the critical temperature, show more symmetric results. This indicates that an Abrikosov vortex was trapped nearby.
5.3 Discussion

In this chapter two systems were discussed, which can be used to generate a localized barrier.

Heart-shaped junctions showed zero-field depinning currents, corresponding to parasitic potentials. The order of magnitude of these potentials is comparable to the one predicted by an analytic model [17], when applied to the bend of the smallest radius in the junction. The potential heights expected suggest that this approach cannot be easily studies systematically, since even the characterization of junctions in the thermal regime is difficult.

A new design for a vortex qubit is proposed, and first results, including state preparation, readout, and thermal activation in the low temperature regime are presented. A considerable difference between the expected depinning current and the measured depinning current was found. This disagreement cannot be explained by the production size margins of the junction. While one possibility is the presence of a magnetic field trapped through both electrodes, another is the neglect of the fact that the width change in this junction does not happen.
slowly. A numerical first order perturbation theory scheme was used to enhance the agreement of the predicted depinning current with the experiment. This first order scheme brings a correction to the pinning force much stronger than unity. Which of these possibilities causes the discrepancy is to be investigated by further experiments. At temperatures down to 100 mK thermal activation from one bistable state to the other is observed, indicating good controllability of the potential barrier height and the possibility to observe MQT.
Chapter 6

Discussion, conclusions and outlook

In this thesis different mechanisms of generating potentials suitable for vortex qubits in long Josephson junctions have been examined in detail. Experimentally escape current distributions from metastable states were acquired using a time-of-flight measurement technique.

States in annular Josephson junctions of different topological charge (vorticity), critical current density and geometry for different temperatures and applied magnetic fields were used. These distributions adhere to an extreme variate statistic, with a certain critical current and a scaling parameter (distribution width). The scaling of both parameters with magnetic field and temperature is compared with theories for point-like particles in a one-dimensional washboard potential, the effective potential of which is derived in a perturbation theory approach.

For vorticity one possible metastable escape process corresponds to the transition (depinning) of a vortex pinned by any inhomogeneity in magnetic field or junction geometry. This results in a tuneable potential for a vortex; a clear transition from the thermally dominated escape to the quantum escape is observed. The identification of the latter escape mechanism happened by an changing of the scaling with magnetic field. While the exact prefactor needs attention, the observed damping regime turnover in the thermal regime of the very system suggests that a refined treatment of the WKB rate is necessary; most likely an escape from the first excited state has to be considered. Further understanding
of this system is wished, while the understanding of the low-temperature scaling of the escape is a considerable progress. Quantum escape of a vortex has of for the first time in a photolithographically manufactured \((R = 50 \, \mu \text{m}, \, w = 2 \, \mu \text{m})\), commercially available \(^1\) junction.

A simple theory, describing a crossover from the escape of a magnetically pinned vortex in a short system of vorticity 1 to a small junction like phase escape was developed.

For systems of vorticity 0, two different decay mechanisms have been studied, and a crossover\(^{[88, 15]}\) is observed. For short junctions and small magnetic fields, the decay mechanism is identified to be a nearly homogeneous string escape of the phase. Quantum tunneling is clearly observed. For high magnetic field, a breakdown of the string escape model is found, and a SJJ-like temperature scaling while the scaling with the mean switching current is unalike the theory. For longer Josephson junctions the escape is described as a vortex-antivortex nucleation process. The small oscillation frequency is measured by a resonant decay. Extensive spectroscopy verifies quantitatively the predictions of the VAV-model, which turns out to be equivalent in respect to the small oscillation frequency scaling to a model more suitable for short rings\(^{[15]}\). A saturation is observed here too, and the predictions of the thermal decay and the quantum regime match the observations, when parameter values estimated from the vortex activation measurements are used. In particular, the correction of the prefactor for the quantum escape is the same as for the vortex case.

The last part of the thesis describes measurements on potentials created by junction inhomogeneities. For the heart-shaped junction a model of curved non-linear resonators \(^{[17]}\) predicts the pinning current for heart-shaped Josephson junctions of different geometry correctly. The estimated potential heights were uncontrolled by the junction design and higher than the allowed scale, wherefore negative results obtained on this junction design are explained. Preliminary tests on one possible replacement system are shown. The first one uses an injector for generating a locally variable pinning potential shows satisfactory agreement for the measured depinning current. The latter one, based on a width modulated Josephson junction, acts quzalitatively as predicted, and is a well controllable system. For a quantitative description it can only be stated, that first order perturbation theory breaks down. A second order scheme, like \(^{[17]}\), together with a more detailed analysis of the microshort geometry, probably would resolve the difference in the pinning strength between experiment and theory.

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\(^1\)The junction clearly violated the design rules of \(^{[104]}\); the yield on the ordered samples was not 100%. Hypres Inc. does not guarantee for junctions designed in this way.
Summary

In this work I examine phase escape in long annular Josephson tunnel junctions. The sine-Gordon equation governs the dynamics of the phase variable along the junction. This equation supports topological soliton solutions, which correspond to quanta of magnetic flux trapped in the junction barrier. For such Josephson vortices an effective potential is formed by an external magnetic field, while a bias current acts as a driving force. Both together form a metastable potential well, which the vortex is trapped in. When the driving force exceeds the pinning force of the potential, the vortex escapes and the junction switches to the voltage state. At a finite temperature the driving force fluctuates. If the junction’s energy scale is small, the phase variable can undergo a macroscopic quantum tunneling (MQT) process at temperatures below the crossover temperature. Without a vortex trapped, the metastable state is not a potential minimum in space, but a potential minimum at zero phase difference.

I present spectroscopic measurements for both cases, verifying the predicted small oscillation frequencies. In the vortex case a localized escape of a particle in a one dimensional potential is found. The escape without a trapped vortex exhibits a small oscillation frequency scaling with the magnetic field as predicted for a nucleus of a variable size. I present for both cases, with and without a vortex, measurements on 2 \( \mu m \) wide junctions showing macroscopic quantum tunneling. For the vortex case I demonstrate a change in the scaling of the measured fluctuations as a function of magnetic field from a MQT like scaling to a Brownian motion like scaling. This change of the scaling takes place at the temperature predicted by theory. For high magnetic fields, I observe a crossover in the measured fluctuations of the vortex depinning current. I attribute this, using a simple model, to homogeneous escape of the system, as for a small Josephson junction.

In the last chapter I explain the negative results for the heart shaped Josephson junction. This geometry was a vortex qubit candidate by means of a parasitic
potential created by strong curvature changes of the junction centerline. I propose and test a new type of vortex qubit, based on the lithographic fabrication of microshort-like structures. Qualitatively the experiment verifies the existence of bistable states and demonstrates their preparation and readout. Quantitatively, the depinning current from the lithographic microshort is not well understood. It differs by a factor of approximately 10 from the expected value. I suggest a perturbation theory approach to explain this. Additionally, a parasitic magnetic field component was observed. These disagreements between theory and experiments require further investigation. I verify thermal activation down to 300 mK, and thereby demonstrate control of the potential barrier height. I estimate the effect of the bias control on the thermal decay of a prepared state and on the level splitting in this double well potential. It appears feasible to observe incoherent quantum effects in this bistable system using the existing setup. The demonstration of coherent effects in this system seems to be a realistic goal for future experiments.
Zusammenfassung


Im letzten Kapitel erläutere ich die negativen experimentellen Ergebnisse.
Appendix A

Biasing

The presented experiments use customized analog electronics, which are battery powered. Galvanic isolation, achieved by optical fibers in this setup, prevents currents through ground loops in the signal processing electronics, which can cause low frequency, correlated offsets between the amplifier and the trigger and between the signal generators and the current source, respectively. In general, there are four kinds of noise sources in the setup, which have to be addressed separately.

1. Electromagnetic interference (EMI)
2. Thermal noise in the biasing circuits
3. $1/f$ noise in the biasing circuits (amplifier noise or noise from low quality resistors).
4. Limited resolution in amplification and signal processing of high-level signals

In practice 4 is generated by 3 and 2, but since the dominant part comes from amplifier noise, which is specified in the datasheets of the operational amplifiers, these circuits are considered as black boxes. For a description of the measurement limitations associated with the signal processing related electronics, see appendix B, and Ref. [55]. EMI is, due to presence of a shielding room, not an issue the experiments.

The problem discussed in this appendix is the contribution intrinsic to the biasing and measurement scheme on a physical level, when direct biasing via a
current source and a resistive network. For given sample resistance (zero in this case) and an allowed power dissipation at a given temperature, a certain noise current is expected.

**A.1 Biasing Scheme**

In Fig. A.1 the biasing circuit is schematized. The general scheme in the measurements consists of three levels of filtering. At room temperature, a \( \pi \) filter (feedthrough, 3 nF) with a cutoff frequency on the order of 10 MHz is used is used to block possible EMI from radio stations. Experience shows that this filter has an considerable influence if the system is operated outside the shielded room or if there are high-frequency noise sources present inside the shielded room, but nearly no effect otherwise. We always operate the system always with these filters, since they do not limit the total bandwidth. The fact that these filters have a strong effect, even if the cutoff frequency of the following stages is lower, can be attributed to crosstalk between lines in the cryostat, some which were not filtered using RC-filters. These lines share lengths on the order of 1 m inside the cryostat, corresponding to the frequency suppressed by the \( \pi \)-Filters. Higher frequencies do not couple into these lines efficiently, and are filtered out using filters mounted at the mixing chamber. Lower frequencies, on the other hand, are not susceptible to capacitive cross-talk. At the 1 K-pot a second stage of filters is mounted. These are standard RC-filters, referenced to the cryostat body at that point. These filters limit the signal bandwidth to a maximum \( \approx 10 - 20 \text{ kHz} \). It should be mentioned that the \( RC \) filters on the voltage and current lines differ; while the voltage line filters have a relatively high resistance of 1 k\( \Omega \), the current lines use a resistance of 220 \( \Omega \) with a shunt resistor of 47 \( \Omega \). The effect of the current divider is to reduce EMI from the outside by the corresponding resistance ratio. From here on the biasing scheme is single ended, using the cryostat body as the current return path, whereas the voltage measurement scheme is differential. The local current for the current dipole in the LJJ is filtered using the same scheme as the bias current. At the mixing chamber, resistive thermocoax coaxial line filters and copper-powder filters supress the high frequency range beyond 1 GHz. The advantage of the resistive coaxial line filters is their flat transmission characteristic. The disadvantage and the reason why these filters are only used on the voltage lines, is that their resistivity of \( \approx 50 \Omega \) is too high. The critical currents of the junctions used range to several mA at which the heat load of these filters is too high for stable operation at the mixing chamber at low temperatures. In this case we use copper powder filters, which consist of a
considerable length of wire embedded in a suspension of copper powder in a varnish suitable for low temperatures. Current passing along the wire induces eddy currents in the copper grains, which decay. The advantage of copper powder filters is that if correctly wound they are of low inductance and can be made out of superconducting wire which has zero resistance for DC signals. Their main disadvantage is, that it is difficult to prevent resonances. A typical filter characteristic, acquired on one of the filters in cryostat, is depicted in Fig. A.2. Strong suppression of the filtered signal starts at 1 GHz. As far as of noise calculations are concerned it can be assumed that this filter does not attenuate signal below 1 GHz, and blocks signals above this frequency.

A.2 Power Dissipation of the Biasing System

The maximum power loads at different stages of the cryostat are $10 - 200 \mu W$ on the mixing chamber depending on temperature and $\approx 250 \text{ mW}$ at the 1 K pot stage, as verified by a test. The current dividers at the 1 K pot used to divide the biasing signals limit the current which can be sent to the junction. While the bias current to the junction itself is usually low that the related dissipation does not play any significant role, the current injectors used to inject the vortex into the junction are driven with currents of up to 6 mA.

The current divider is schematized in Fig. A.3. A simple calculation of the dissipated power starts from the power dissipated in each resistor:

$$P = I_B^2 R_1 + I_{Sh}^2 R_2.$$  \hspace{1cm} (A.1)
Figure A.2: The transmission on the filter used. For estimating the noise bandwidth it is, despite the resonances save to assume 1 GHz.

Figure A.3: The current divider as used in the experiments. R2 is the shunting resistor, R1 is the resistor connected in series to the junction. R2 acts effectively as a voltage source.
The ratio of the resistors is the inverse ratio of the currents, so that the dissipated power is given by

\[ P = I_B^2 R_1 + I_B^2 \left(\frac{R_1}{R_2}\right)^2 R_2 = I_B^2 R_1 \left(1 + \frac{R_1}{R_2}\right). \] (A.2)

This formula has a simple interpretation. At a fixed ratio of the current divider, the dissipated power is proportional to the biasing resistor. This means, that if the bias current is decreased by a factor of 2, \( R_1 \) and \( R_2 \) may be increased by a factor of 4 for the same power dissipation. The RMS Johnson current noise in a resistor is given by

\[ V_n = \sqrt{\frac{4k_B T}{R} \sqrt{f_{BW}}}, \] (A.3)

where \( \sqrt{f_{BW}} \) denotes the bandwidth. If a decrease in the maximum bias current by a factor of 2 allows us to scale the resistor values by 4, this means that the observed noise current decreases by a factor of 2. So the signal to noise ratio is constant for a constant power dissipation and current divider ratio. The divider ratio itself should be selected in such a way that the current noise on the input cause electronics noise or Johnson noise at the room temperature resistors reduced to less than the level of Johnson noise induced by biasing resistor \( R_1 \).

The bandwidth is limited by an capacitor at the input or, if \( R_1 \) is sufficiently large, the capacitance of the wiring itself. In this setup, \( R_1 = 220 \, \Omega \) and the capacitor at the input is 47 nF. This capacitor, forms together with \( R_1 \), the filter at the mixing chamber and the sample the loop, in which the Johnson noise current of \( R_1 \) circulates. For calculating the Johnson noise of this loop, the bandwidth of the copper-powder filter is relevant. We find that this loop contributes 19 nA of noise current to the sample, consistent with the minimum noise current measured of 25 nA. The filters were realized using SMD technology on a mylar foil, which was glued onto a copper support. Improved filters with enhanced high-frequency properties are described in[105].
Appendix B

Electronics

B.1 Computer Control

All experiments are computer-controlled. Three different ways of interfacing the measurement electronics with the computer are used in our setup. For characterization measurements, and for testing new pulse sequences like the shaker preparation technique, National Instruments data acquisition cards[53] are used. These are capable of generating a timed triggered waveform output, and at the same time perform buffered synchronized multi-channel data acquisition. The RF-generator and the temperature controller are connected by an IEEE488-bus. In low-noise measurements serially interfaced 16-bit digital to analog DA converters [106] with galvanic isolation achieved by optical fibers [107]. Pulse-width modulation was used for the transfer of the digital data. Decoding to standard serial programming interface (SPI) synchronous serial signals is done using clockless logic [108]. Standard SPI daisy chaining is used share a single digital transmission channel for an arbitrary number of analog channels\(^1\). The temperature controller and the resistance measurement bridge are galvanically isolated by an integrated optical fiber picobus interface[109]. The RF-generator output is grounded to the shielding room feedthrough connector, but the rf ground and signal are galvanically isolated by a DC break. The temperature controller is battery powered like the analog electronics. A later test showed equivalent performance when using a properly filtered power supply unit.

\(^1\)This is the functional description of the latest version of the DA, namely the triple DA module[108]. Earlier versions did not have the possibility of daisy chaining.
B.2 Noise Estimation of the Ramp-Type Experiments

The resolution of the critical current measurements in ramp-type experiments is limited by several factors. The basic measurements involves current and voltage. Both are scaled, by either the monitor resistor in the current source or the voltage preamplifier to a full-scale (−5 V . . . 5 V) signal for further signal processing. The noise of the voltage preamplifier and the current monitor amplifier are the relevant sources of noise at this point. In our standard measurement further signal processing consists of a trigger for each channel. This trigger is the only nonlinear and hysteretic signal processing element. All white noise sources on each channel can be summed up geometrically with the input noise of the trigger. At the trigger the analog input noise is translated into a timing error, known as jitter, which is proportional to the RMS input noise. The proportionality between the jitter and the RMS noise value is given by the inverse slope of the input signal at the triggering point. Fig. B.1 illustrates the situation. This is equivalent to the usage of the error propagation formula for a single measured variable. We find the relation of voltage noise to the jitter

\[ \sigma(t) = \sigma(U)/\dot{U}. \]  

(B.1)

A calculation of \( \dot{U} \) is given below. The timing error itself translates to an error in the detected switching current. For a constant slope of the current ramp, the timing error is multiplied by the ramp rate:

\[ \sigma(I) = \sigma(t)\dot{I}. \]  

(B.2)

Thus we get a voltage signal processing path related current noise of

\[ \sigma(I) = \frac{\dot{I}}{\dot{U}}\sigma(U). \]  

(B.3)

It is obvious that if the voltage noise is the limitation on the current measurement precision, a change of the ramprate can remedy the problem at the cost of longer measurement times. While a longer measurement time may be undesirable, decreasing ramprate influences observable rates of escape. Since the rate of escape is directly related to the small oscillation frequency, a change in the ramprate has the effect of shifting the measurement to another, non equivalent, bias point. On the signal path from the sample to the digital trigger output, displayed in Fig. B.2, there are three elements which set the slew rate of the signal,
Figure B.1: A sketch of the error propagation of the voltage noise to the detected time of the switch. The red dash-dotted line represents the signal at the junction, while the black solid line represents the voltage at the trigger input. Projecting the error in the voltage measurement onto the tangential of the voltage signal yields jitter.
or contribute to the noise. The resistors $R_+$ and $R_-$ have a resistance of 1 kΩ. The Johnson noise voltage contribution from these resistors is negligible. The RC time constant is measured to be $1/12$ kHz, as measured. A voltage step of $V_0$ causes a maximum slope $\dot{U}$ at the time of the step, when the capacitor has zero voltage, so that $V_0$ drives a current of $I = V_0/R$ through the resistor. This results in a rate of voltage change across the capacitor of

$$\dot{U} = \frac{1}{RC} V_0.$$  \hfill (B.4)

The preamplifier (INA111) contributes to the noise considerably. Its low-frequency noise contributes $1 \mu V$ peak to peak noise, and the white noise density of $\approx 10 - 13$ nV per $\sqrt{\text{Hz}}$ contributes up to $\approx 10 \mu V$ RMS noise at a gain of 1000 over the amplified band. The trigger has a input noise of $20 \mu V$, which corresponds to $20 \text{nV}$ at the input of the amplifier. Putting in typical values for our setup $\frac{1}{RC} = 12$ kHz, $\dot{I} = .02$ A/s, and $V_0 = 2.5$ mV, we find a voltage processing path related current noise of $8$ nA. When detecting the jump to the vortex step $V_0 = 80 \mu V$, which would result in $250$ nA of RMS noise, the output bandwidth of this amplifier was limited to $20$ kHz. For this value we find $35$ nA of noise.

Estimating the noise of the current source, the relevant parts of which are schematized in Fig. B.3, is straightforward. Again the Johnson noise of the monitor resistor is negligible, due to the band selected by the RC-filters at the 1 K-pot. Only the noise of the instrumentation amplifier monitoring the current
is relevant and the same as in the voltage path. Hence the noise is given by

\[ \sigma(I) = \frac{\sigma(U)}{R_{\text{mon}}}, \]  

(B.5)

which yields approximately 10 nA for \( R_{\text{mon}} = 1 \, \text{kΩ} \). This noise is divided by the cold current divider.
Figure B.3: The parts of the voltage controlled current source relevant for noise calculations, including the monitor resistor $R_1$, the feed-back loop on IC$_2$, and the instrumentation amplifier IC$_1$. 
Appendix C

Table of Samples and Measurements

C.1 Samples Used

All samples investigated are manufactured using a tri-layer technology. In this technology, the junction barrier is not produced during the structuring unlike in the shadow evaporation technique. Instead a homogeneous bottom electrode film is grown on which a homogeneous barrier is formed, and a top electrode is deposited. The advantages of this method are a high reproducibility and independence of the junction parameters and the structuring steps. Most of the investigated samples, which are listed in table C.2, come from an external foundry, Hypres, which structures using photolithography. The design rules and restrictions can be found in [66]. Only junction widths down to 3 µm are guarantied by Hypres. However, a violation of the design rules yields working 2 µm wide junctions. For the sample SJJ-DAMP, the trilayer was produced by IPHT Jena, and the structuring was performed using e-beam lithography by Y. Koval. The nominal trilayer critical current densities are listed in table C.1.

C.2 Table of Measurements

Table C.4 contains the measurements presented in this thesis. The measurement type is either “Ensemble” (repeated measurement of $I_C$ as a function of parame-
### Table C.1: Trilayer identification, including the manufacturer. All trilayers produced at Hypres are also structured there, while the trilayer provided at IPHT was structured using e-beam lithography by Y. Koval.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Wafer</th>
<th>Width</th>
<th>Radius (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJJ-SHORT</td>
<td>Hypres KL700</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-VAV</td>
<td>IPHT Jena, 200 A/cm²</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-MS1</td>
<td>Hypres KL698</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-MS2</td>
<td>Hypres KL699</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-HEART1</td>
<td>Hypres KL519</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-HEART2</td>
<td>Hypres KL519</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-HEART3</td>
<td>Hypres KL519</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>AJJ-INJ</td>
<td>Hypres KL366</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>SJJ-DAMP</td>
<td>Hypres KL699</td>
<td>10x10</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table C.2: Dimensions of the junctions investigated. All samples besides SJJ-DAMP are topologically annular Josephson junctions, some which have a heart-shaped geometry or contain additional features, as listed in table C.3.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Comments</th>
<th>Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJJ-SHORT</td>
<td>Annular, Injectors</td>
<td>3,4</td>
</tr>
<tr>
<td>AJJ-VAV</td>
<td>E-beam lithography</td>
<td>4</td>
</tr>
<tr>
<td>AJJ-MS1</td>
<td>Microshort 2 µm × 0.5 µm</td>
<td>5</td>
</tr>
<tr>
<td>AJJ-MS2</td>
<td>Microshort 8 µm × 0.5 µm</td>
<td>3</td>
</tr>
<tr>
<td>AJJ-HEART1</td>
<td>Heart $\beta = 26^\circ$</td>
<td>5</td>
</tr>
<tr>
<td>AJJ-HEART2</td>
<td>Heart $\beta = 37^\circ$</td>
<td>5</td>
</tr>
<tr>
<td>AJJ-HEART3</td>
<td>Heart $\beta = 43^\circ$</td>
<td>5</td>
</tr>
<tr>
<td>AJJ-INJ</td>
<td>Injectors</td>
<td>2</td>
</tr>
<tr>
<td>SJJ-DAMP</td>
<td>Small junction</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table C.3: Details of the junctions investigated.
ters), “Spectroscopy” (Ensemble using RF-irradiation). The electronics are either “Tunnelelektronik-I” (TE-I) or “Tunnelelektronik-II” (TE-II), and can have the following attributes: SR - shielded room; GND - ground-referenced signals in the cryostat, CD - current dividers at 1K-pot, NIDAQ - waveform generation directly on the NIDAQ card; “NIDAQ - filtered” - waveform filtered by standard electromagnetic interference suppression filters on the control signal. Common to all measurements at temperatures below 4.2 K are copper-powder and high resistance coaxial line filters on the mixing chamber. All measurements not designated by NIDAQ feature galvanic insulation of the analog electronics from the computer.
### Table C.4: Measurements presented in this thesis.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Vorticity</th>
<th>Measurement type</th>
<th>Temp. [mK]</th>
<th>Electronics</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJJ-SHORT</td>
<td>0</td>
<td>Ensemble($T, H_x$)</td>
<td>28-400</td>
<td>TE-II (SR, GND, CD)</td>
</tr>
<tr>
<td>AJJ-SHORT</td>
<td>11</td>
<td>Ensemble($T, H_x$)</td>
<td>28-400</td>
<td>TE-II (SR, GND, CD)</td>
</tr>
<tr>
<td>AJJ-INJ</td>
<td>11, 1</td>
<td>IV, $I_C(H)$, $I_C(H_x, H_y)$</td>
<td>4200</td>
<td>TE-I (NIDAQ)</td>
</tr>
<tr>
<td>AJJ-VAV</td>
<td>1</td>
<td>Ensemble($H_x, H_y$), Spectr.</td>
<td>Spectr.</td>
<td>500</td>
</tr>
<tr>
<td>AJJ-VAV</td>
<td>0</td>
<td>Spectr.</td>
<td>28</td>
<td>TE-I (SR)</td>
</tr>
<tr>
<td>AJJ-HEART1</td>
<td>1</td>
<td>$I_C(H_x, H_y)$, $I_C^P(H_x, H_y)$</td>
<td>4200</td>
<td>TE-I (NIDAQ)</td>
</tr>
<tr>
<td>AJJ-HEART2</td>
<td>1</td>
<td>$I_C(H_x, H_y)$, $I_C^P(H_x, H_y)$</td>
<td>4200</td>
<td>TE-I (NIDAQ)</td>
</tr>
<tr>
<td>AJJ-HEART3</td>
<td>1</td>
<td>$I_C(H_x, H_y)$, $I_C^P(H_x, H_y)$</td>
<td>4200</td>
<td>TE-I (NIDAQ)</td>
</tr>
<tr>
<td>AJJ-MS2</td>
<td>1</td>
<td>Spectr.</td>
<td>150</td>
<td>TE-II (SR)</td>
</tr>
<tr>
<td>AJJ-MS1</td>
<td>1</td>
<td>$I_C(H_y)$, barrier suppression</td>
<td>100-800,</td>
<td>TE-II (SR, GND, CD, NIDAQ-Filtered)</td>
</tr>
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</table>

Table C.4: Measurements presented in this thesis.
## Appendix D

### Table of variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ₀</td>
<td>Magnetic flux quantum</td>
<td>V·s</td>
</tr>
<tr>
<td>e</td>
<td>Elementary charge</td>
<td>C</td>
</tr>
<tr>
<td>μ₀</td>
<td>Magnetic vacuum permeability</td>
<td>N/A²</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light</td>
<td>m/s</td>
</tr>
<tr>
<td>kₜ</td>
<td>Boltzmann constant</td>
<td>J/K</td>
</tr>
<tr>
<td>mₑ</td>
<td>Electron mass</td>
<td>kg</td>
</tr>
</tbody>
</table>

### Physical constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cₖ</td>
<td>Specific (junction) capacitance</td>
<td>F/m²</td>
</tr>
<tr>
<td>R*</td>
<td>Specific resistance</td>
<td>Ωm²</td>
</tr>
<tr>
<td>Δ₉</td>
<td>Superconducting gap</td>
<td>eV</td>
</tr>
<tr>
<td>L*</td>
<td>Inductance per square</td>
<td>H</td>
</tr>
<tr>
<td>jₖ</td>
<td>Critical current density</td>
<td>A/m²</td>
</tr>
<tr>
<td>λₗ</td>
<td>Josephson length</td>
<td>m</td>
</tr>
<tr>
<td>λₗ</td>
<td>London penetration depth</td>
<td>m</td>
</tr>
<tr>
<td>d'</td>
<td>Magnetic thickness</td>
<td>m</td>
</tr>
<tr>
<td>tₗ</td>
<td>Barrier thickness</td>
<td>m</td>
</tr>
<tr>
<td>ωₗ</td>
<td>Plasma frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>c</td>
<td>Swihart velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>$H_0$</td>
<td>Field normalization constant</td>
<td>G</td>
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### Junction properties

<table>
<thead>
<tr>
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<tr>
<td>$E_J$</td>
<td>Josephson energy</td>
<td>J</td>
</tr>
<tr>
<td>$I_C$</td>
<td>Critical current</td>
<td>A</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Junction capacitance</td>
<td>F</td>
</tr>
<tr>
<td>$R_j$</td>
<td>Quasiparticle resistance</td>
<td>Ω</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Ohmic resistance</td>
<td>Ω</td>
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### Experimental parameters and observables

<table>
<thead>
<tr>
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<th>Meaning</th>
<th>Unit</th>
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<tr>
<td>$\nu$</td>
<td>AC Josephson frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$\Gamma_{\text{esc}}$</td>
<td>Decay rate</td>
<td>Hz</td>
</tr>
<tr>
<td>$\Gamma_{\text{esc,rf}}$</td>
<td>Decay rate in presence of rf</td>
<td>Hz</td>
</tr>
<tr>
<td>$\Gamma_{\text{qu}}$</td>
<td>Quantum decay rate</td>
<td>Hz</td>
</tr>
<tr>
<td>$\Gamma_{\text{th}}$</td>
<td>Thermal decay rate</td>
<td>Hz</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Crossover temperature</td>
<td>K</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$\vec{H}_{\text{ext}}$</td>
<td>External magnetic field</td>
<td>G</td>
</tr>
<tr>
<td>$H_{\text{ext}}$</td>
<td>External magnetic field amplitude</td>
<td>G</td>
</tr>
<tr>
<td>$h_{\text{ext,cr}}$</td>
<td>External critical magnetic field amplitude</td>
<td>norm.</td>
</tr>
<tr>
<td>$I_b$</td>
<td>Bias current</td>
<td>A</td>
</tr>
<tr>
<td>$I_n$</td>
<td>RMS of Johnson/Nyquist noise cur-</td>
<td>A</td>
</tr>
<tr>
<td>$I_{C_P}$</td>
<td>Critical current, determined after state preparation</td>
<td>A</td>
</tr>
<tr>
<td>$I_{\text{inj}}$</td>
<td>Injected current</td>
<td>A</td>
</tr>
<tr>
<td>$\Phi_{\text{ext}}$</td>
<td>External magnetic flux</td>
<td>Vs</td>
</tr>
<tr>
<td>$V$</td>
<td>Voltage</td>
<td>V</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Vortex step voltage</td>
<td>V</td>
</tr>
<tr>
<td>$\vec{j}_{\text{inp}}$</td>
<td>In plane supercurrent density</td>
<td>A/m</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Small oscillation frequency</td>
<td>norm.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
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<tr>
<td>$\Omega_0$</td>
<td>$\omega_0(\gamma = 0)$</td>
<td>norm.</td>
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<tr>
<td>$\gamma$</td>
<td>Bias current density</td>
<td>norm.</td>
</tr>
<tr>
<td>$h_{\text{ext}}$</td>
<td>Normalized external magnetic field, projection on</td>
<td>norm.</td>
</tr>
<tr>
<td>$h_{\text{ext}}$</td>
<td>Normalized external magnetic field, projection on normal</td>
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</tr>
<tr>
<td>$\tilde{h}_{\text{ext}}$</td>
<td>Normalized external magnetic field</td>
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<td>$\tilde{h}$</td>
<td>Renormalized magnetic field</td>
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<td>$w_{\text{ms}}$</td>
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<td>$\tilde{x}_1$</td>
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<td>$\tilde{x}$</td>
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<td>$d\tilde{x}$</td>
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<td>$\tilde{x}$</td>
</tr>
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<td>$l$</td>
</tr>
<tr>
<td>$l_{\text{ms}}$</td>
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<tr>
<td>$r$</td>
</tr>
<tr>
<td>$\theta'$</td>
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<td>$\tilde{r}$</td>
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<td>$\varphi_c$</td>
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<tr>
<td>$\varphi_{\text{harm}}$</td>
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<td>$\varphi_0$</td>
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<tr>
<td>$\varphi_f$</td>
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<tr>
<td>$\varphi_{\text{sol}}$</td>
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<tr>
<td>$\varphi_{\delta}$</td>
</tr>
<tr>
<td>$\varphi_{\delta,vav}$</td>
</tr>
<tr>
<td>$\varphi$</td>
</tr>
<tr>
<td>Symbol</td>
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<tr>
<td>(\phi)</td>
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**Operators and functions**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
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<tr>
<td>(\Delta)</td>
<td>Laplacian in normalized coordinates</td>
<td>norm.</td>
</tr>
<tr>
<td>(\nabla)</td>
<td>Nabla operator, normalized units</td>
<td>norm.</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Dirac function</td>
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<tr>
<td>(K)</td>
<td>Elliptic integral of first kind</td>
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</tr>
<tr>
<td>(J_1)</td>
<td>First order Bessel function, real part</td>
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**Energies**

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<tr>
<th>Energy</th>
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<th>Unit</th>
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<tr>
<td>(E_{J,\text{reduced}})</td>
<td>Josephson energy reduction factor</td>
<td>norm.</td>
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<tr>
<td>(E_0)</td>
<td>Potential barrier height</td>
<td>J</td>
</tr>
<tr>
<td>(U_0)</td>
<td>Barrier height</td>
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<tr>
<td>(U)</td>
<td>Barrier height at zero bias</td>
<td>norm.</td>
</tr>
<tr>
<td>(\bar{U})</td>
<td>Barrier height, approximation</td>
<td>norm.</td>
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<tr>
<td>(U_{h})</td>
<td>Field generated potential</td>
<td>norm.</td>
</tr>
<tr>
<td>(U)</td>
<td>Barrier height (unbiased)</td>
<td>norm.</td>
</tr>
<tr>
<td>(U)</td>
<td>Potential energy</td>
<td>norm.</td>
</tr>
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<td>(F_h)</td>
<td>Magnetic field pinning force</td>
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**Hamilton functionals**

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<tr>
<th>Hamiltonian</th>
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<tr>
<td>(H)</td>
<td>Small Josephson junction Hamiltonian</td>
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<tr>
<td>(\mathcal{H})</td>
<td>Josephson energy (functional)</td>
</tr>
<tr>
<td>(\mathcal{H}_C)</td>
<td>Charging Hamiltonian</td>
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<tr>
<td>(\mathcal{H}_H)</td>
<td>Field interaction Hamiltonian</td>
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<tr>
<td>(\mathcal{H}_\gamma)</td>
<td>Bias current Hamiltonian</td>
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<tr>
<td>(\mathcal{H}_J)</td>
<td>Josephson Hamiltonian</td>
</tr>
<tr>
<td>(\mathcal{H}_L)</td>
<td>Self inductance Hamiltonian</td>
</tr>
<tr>
<td>(\mathcal{H}_0)</td>
<td></td>
</tr>
<tr>
<td>(\mu)</td>
<td>Hamiltonian functional</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Spatial rest mass distribution</td>
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**Other variables**

<p>| | |</p>
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<tbody>
<tr>
<td></td>
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<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
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<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>Normalized reduced Planck's constant</td>
</tr>
<tr>
<td>$\tilde{k}_B$</td>
<td>Normalized Boltzmann constant</td>
</tr>
<tr>
<td>$\tilde{\hbar}_j$</td>
<td>Normalized reduced Planck's constant, small JJ</td>
</tr>
<tr>
<td>$\tilde{k}_{B,j}$</td>
<td>Normalized Boltzmann constant, small JJ</td>
</tr>
<tr>
<td>$\alpha_{\text{plasma}}$</td>
<td>Damping constant</td>
</tr>
<tr>
<td>$\tilde{\alpha}_{\text{plasma}}$</td>
<td>Damping constant (normalize)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Normalized damping coefficient</td>
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<tr>
<td>$\gamma_{\text{cr}}$</td>
<td>Activation current, normalized</td>
</tr>
<tr>
<td>$\gamma_{\text{bi}}$</td>
<td>Barrier suppression bias</td>
</tr>
<tr>
<td>$\gamma_{\text{cr}}$</td>
<td>Thermal noise current</td>
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<tr>
<td>$\tilde{\hbar}_{\text{crossover}}$</td>
<td>Crossover field</td>
</tr>
<tr>
<td>$a$</td>
<td>Inner freedom</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Coupling to magnetic field</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>Coupling to magnetic field (fluxon approximation)</td>
</tr>
<tr>
<td>$m_{\text{eff}}$</td>
<td>Effective mass</td>
</tr>
<tr>
<td>$m_{\text{ph}}$</td>
<td>Effective mass, phase-direction</td>
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<tr>
<td>$\mu$</td>
<td>Microshort strength</td>
</tr>
<tr>
<td>$b$</td>
<td>Phase offset</td>
</tr>
<tr>
<td>$\tilde{\mu}$</td>
<td>Vortex rest mass profile</td>
</tr>
<tr>
<td>$\rho_{\alpha}$</td>
<td>Barrier transparency</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Reduced bias field</td>
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<tr>
<td>$\tau_{\text{rabi}}$</td>
<td>Rabi oscillation period</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$\tilde{t}$</td>
<td>Time, normalized</td>
</tr>
<tr>
<td>$\tilde{v}$</td>
<td>Velocity, normalized</td>
</tr>
<tr>
<td>$\tilde{v}_{\infty}$</td>
<td>Equilibrium velocity, normalized</td>
</tr>
<tr>
<td>$\tilde{v}_{\infty}$</td>
<td>Acceleration, normalized</td>
</tr>
<tr>
<td>$\tilde{m}$</td>
<td>Vortex mass, normalized</td>
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<tr>
<td>$n_v$</td>
<td>Vorticity</td>
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## APPENDIX D. TABLE OF VARIABLES

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<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
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<tr>
<td>$A_{qu}$</td>
<td>KFD quantum prefactor</td>
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</tr>
<tr>
<td>$A_{th}$</td>
<td>KFD thermal prefactor</td>
<td></td>
</tr>
<tr>
<td>$B_{qu}$</td>
<td>KFD quantum exponent</td>
<td></td>
</tr>
<tr>
<td>$B_{th}$</td>
<td>KFD thermal exponent</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{A}$</td>
<td>KFD prefactor</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>Arrhenius factor</td>
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</tr>
<tr>
<td>$B$</td>
<td>KFD exponent</td>
<td></td>
</tr>
<tr>
<td>$S_b$</td>
<td>Barrier action, see [62]</td>
<td></td>
</tr>
<tr>
<td>$S_0$</td>
<td>Barrier action at zero bias, see [62]</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>Reaction coordinate</td>
<td></td>
</tr>
<tr>
<td>$H_s$</td>
<td>Magnetic field in superconductor</td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>Small amplitude nucleus size</td>
<td></td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>Unstable barrier frequency</td>
<td></td>
</tr>
<tr>
<td>$\dot{U}$</td>
<td>Signal slew rate, input referred</td>
<td></td>
</tr>
<tr>
<td>$\sigma(U)$</td>
<td>Signal rms noise, input referred</td>
<td></td>
</tr>
<tr>
<td>$\sigma(t)$</td>
<td>Jitter</td>
<td></td>
</tr>
<tr>
<td>$\dot{I}$</td>
<td>Current ramp rate</td>
<td></td>
</tr>
<tr>
<td>$\sigma(t)$</td>
<td>Current noise</td>
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[79] U. Weiss, Quantum dissipative systems (World Scientific, 1999), 2nd ed.


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