On uncertainties in simulations in engineering design: A statistical tolerance analysis application

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Abstract

Simulations not only facilitate new and unprecedented insights in highly sophisticated science areas, but also support product design in engineering in terms of improved functionality, cost and time issues. However, as a matter of fact, simulations examine limited excerpts of real systems with accompanying simplifications, abstractions and idealizations. Hence, there is a distinct need to be aware of upcoming risks in simulation outcomes caused by uncertainties. These influence every step of forward-thinking simulation design which is not only restrained by modeling practice but begins with reality perception itself.

The intention of the paper is to embed an awareness of uncertainty in the context of simulation by linking major classes of uncertainty with uncertainties within simulations in engineering design. Besides the decisive inclusion of reality as the starting point, mathematic approaches are also used to understand how those uncertainty classes evolve through exponential knowledge creation of systems. The transfer to statistical tolerance analysis shall finally put emphasis on the practical classification of uncertainty, starting from data preparation via concept design and mathematical implementation to result depiction. In the end, the reader's conception of possible uncertainties in special simulation cases shall be sharpened which, vice versa, shall lead to even better and well-thought-out simulation outcomes.

Keywords

Uncertainties in simulation, tolerance analysis, tolerance management, statistical tolerance simulation

1. Introduction

The use of simulations has increased exceptionally during recent decades. Simulations and the corresponding modeling processes can be found in nearly all fields of research. These applications of simulation are based on the reproduction and modeling of objects and processes of the surrounding reality. Consequently, understanding and knowledge of these objects and processes can be strengthened, extended and renewed by means of appropriate experimental imitation. Moreover, rapidly increasing computational performance causes these simulations, to an exceptional extent, to be carried out virtually.

The main reason for the prevalence of simulations in engineering design is that the product developer is able to analyze relevant characteristics of the products and processes, like functional fulfillment, quality requirements, or resulting risks and reliability, during product development—usually far ahead of production of the first prototypes. Furthermore, simulations assist the product developer in understanding complex systems behavior

and, thus, shorten the half-life of knowledge creation by means of causality and the effects of natural circumstances. (A system is complex if the degree and nature of interaction of its objects is imperfectly known. Its overall behavior is difficult to predict, even when the object's behavior is foreseeable.¹) Consequently, simulations are more flexible, detailed and cost-efficient alternatives to real experiments for gaining knowledge about a large field of applications at an early stage of the product life cycle.

However, the methodology on simulation shows several limitations indicating that a virtual simulation is not the panacea to optimize products and processes. This can mainly be traced back to simplifications, abstractions and idealizations of reality, which are essential to establishing

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an appropriate model. The corresponding interpretation of reality in mathematical and logic dependencies inevitably leads to assumptions-based simulation methodologies. Then again, these methodologies are just the results of the imperfect human perception. Consequently, all these models and simulations contain uncertainties. This is all the more serious, since an increasing number of the responsible parties' decisions are based only on the results of virtual simulations.

This paper emphasizes the product developer's awareness of the results of modeling and simulation in today's product development. Therefore, critical scrutiny is an essential task to ensure successful and reliable modeling and simulation. Consequently, a universal point of view on simulations and the corresponding uncertainties is adopted in this work.

First, simulation will be introduced in the context of virtual product development (Section 2.1), while the four stages of the methodology on simulation are detailed in Section 2.2. Considerable emphasis is placed on a clear presentation of the transfer of a real system into the virtual simulation environment. The "paradox of reality" is claimed to be the cause of uncertainties which are inevitably associated with simulation (Section 3). These uncertainties are considered in detail in the following section (Section 4), starting with a general classification of uncertainties in the two most relevant classes: aleatory uncertainties (Section 4.1) and epistemic uncertainties (Section 4.2). Moreover, a mathematical approach is presented, which enables the product developer to both differentiate appearing uncertainties of a simulation and to evaluate their effects on existing knowledge of the considered product or process (Section 4.3). Section 4 closes with a consideration of the different forms of errors as uncertainties linking aleatory and epistemic uncertainties. In Section 5, four major kinds of uncertainty in simulations are detailed in accordance with the given classification of uncertainties in the previous section (Section 4). The paper closes with the demonstrative transfer of the found uncertainties on a statistical tolerance analysis application and details how these uncertainties qualitatively affect the analysis' results.

2. Simulation

2.1. Simulation in virtual product development

Basically, the domain of simulation is constantly growing in significance. It has developed to the extent, that simulations are designated the "third branch of scientific method," accompanying the theoretical and experimental branches.^{2,3} According to Banks and Banks et al., a simulation is the "imitation of the operation of a real-world process or system over time."^{4,5} Simulations are used, if

- A system exists, but experimental investigations may be elaborate or even impossible (e.g. to answer "What if" questions).

- A system does not exist in reality, but is hypothetically or conceptually describable.

Consequently, the use of simulations is an appropriate way to analyze a product in every stage of the product development process. The knowledge gained in these simulations can then be used, for example, to evaluate and improve different concepts of the product and its corresponding functional relevant characteristics as well as to design and optimize required manufacturing and assembly processes.

Simulations close the gap between theory and experiment.⁶ They enable the product developer to gain knowledge and to draw conclusions about a potential real experiment of the considered product, based on simulation results. This benefit is particularly important if the considered system and its operational behavior are based on theories, concepts and assumptions, which are widespread in engineering design.

Simulations could also be defined as virtual experiments. The more complex the system and/or the derived problem and, thus, the less a real experiment seems to be the adequate method, the more the use of simulations should be recommended.⁷ Nevertheless, it must be obviously stated that a simulation cannot replace an experiment.⁶ Moreover, increasing complexity of the problem usually goes hand in hand with an increasing number of uncertainties, which essentially affect the simulation results. Consequently, the suitability of simulation to validate theories and assumptions concerning products and processes in engineering design is not universal.

2.2. Practice of simulation

Simulations are dynamic procedures that consider parts of reality—henceforth named systems—in simplified models. These animated or dynamic model states, behaviors and corresponding characteristics are subject to change during simulation time.

A state consists of at least one physical value, which describes the system at any point in time of simulation. These values depend on the given objective of simulation. The four stages to set up a simulation are shown in Figure 1. Each stage is detailed in the following remarks.

Figure 1. Procedure to set up a simulation according to Roza.⁸

2.2.1. Reality. Reality provides the foundation for each simulation. The systems, whose operational behaviors should be modeled and simulated, are extracted from reality. Reinhardt defines reality as “the substantiality concerning all present, actually existing and representational in contrast to only thought and imagined.”⁹ Thereby, solely material systems can be investigated using simulations. However, this definition is limited and thus, needs to be extended since also simulations of non- or not-yet-material systems can be performed. In particular, such systems can be found during product design. These systems are usually already well defined in the virtual computer-aided-design (CAD) environment far ahead of their material realization in manufacture and assembly. So, simulations of non-material systems are an essential part of today’s predictive engineering.

Consequently, reality is divided into material reality and imaginary reality (Figure 2). This definition is compliant with the conventions of the Simulation Interoperability Standards Organization.¹⁰ Material reality includes the tangible, cognizable surroundings with its objects. These objects can be analysed by means of experimental investigations. Imaginary reality is based on theories, creativity and the laws of nature. Objects of the imaginary reality are not tangible and only partially analyzable by means of simulations (as for climate change).

There is no clear demarcation of material reality and imaginary reality. For instance, a current imaginary system

can become a material system due to manufacturing of its parts and the following assembly of the entire system.

2.2.2. Reference of reality. According to the schematic illustration in Figure 2, reality can be understood as a space with objects. These objects can be both material and imaginary. Related objects are summarized to systems (framed with red dashed lines in Figure 3). However, a system can also consist of only one object. Usually, systems contain several elements, mostly having connections among them.^{4,10}

The first step of the procedure (see Figure 1) includes the definition of the so-called reference of reality. Therefore, the reality’s objects, that are (expected to be) irrelevant for the considered simulation problem, are ignored. Consequently, only a limited part of reality will be taken into account and thus, has to be further specified by the product developer. This part of reality is called the “reference of reality” since it is the essential basis for the following modeling step, as depicted in Figure 3 (according to Roza⁸).

2.2.3. Model. The complexity of the reference of reality is further reduced during definition of a model since additional simplifications, idealizations and abstractions are needed to derive the required model. According to Ryschkewitsch, abstraction is “the process of selecting essential aspects of a reference system to be represented in a model or simulation while ignoring those aspects that are not relevant to the purpose of the model or simulation.”¹² Hence, the higher the level of abstraction, the more the fidelity of the model declines. Figure 3 details the loss of fidelity during the procedure for an arbitrarily system.

An initial loss of fidelity is caused by ignoring object *B* for modeling purposes. Additional characteristics of the objects, that are defined as irrelevant by the product developer, are simplified (e.g. ignoring the Young’s modulus’

Figure 2. Material and imaginary reality, according to Gross.¹⁰

Figure 3. Decline of fidelity of a system to be simulated.

dependence on the temperature) or even omitted. Accordingly, the real object *A* will be simplified to an idealized object *a*.

The overall objective of modeling is to describe a former, current or anticipated condition of a system or a set of systems in reality. Usually, this condition can be mathematically formulated by differential equations, which depend on the objects' characteristics.^{5,11} The product developer has to define appropriate values for each characteristic of the objects during product development. Hence, the definition of a model varies significantly during the entire product development process, since the system's design is constantly extended and additional information (e.g. on appropriate manufacturing processes) is available.

In addition to connections among the objects within a system, also interactions of the model with its real surroundings (e.g. a human user, other models or even another model's single objects) can appear. These interactions are caused by events. Depending on the direction of caused interactions, events can be classified into two classes: inputs and outputs.

The principle is established that a model should be as simple as possible, but as complex as required.¹³ The model and, thus, the corresponding simulation have to satisfy the given requirements against the backdrop of time, required resources and resulting costs. Furthermore, it is essential to respect that a model is the functional description of a system, that is, that no results can be determined solely by means of the model. Nevertheless, a model is the essential basis for any following simulation.⁴

2.2.4. Simulation. Roza defines simulation as the analysis of a physical, symbolic or computer model that imitates the time-dependent variation of a system's reference of reality.⁸ Consequently, simulations are always at least one-step processes with time-dependent (see Figure 3) conditions, as well as varying characteristics and events of the considered model. In today's product development, the product developer can choose from a variety of different computer-aided simulation tools, which provide specific mathematical methods to determine the considered model's conditions. For instance, models of structural mechanical problems can be simulated by means of finite-element (FE) simulation. Hence, simulation is a "highly effective tool to help engineers make decisions" at an early stage of the product development process.¹⁴

3. Paradox of reality

As stated, both material and imaginary reality are the essential basis for each modeling and simulation activity during the entire product development process. These simulations serve as sources of information, which are required to observe systems and/or their objects,

investigate appearing interactions and to draw conclusions on the resulting time-dependent conditions of a system.

The perception of reality and, thus, each drawn conclusion are highly subjective and always non-static. Furthermore, the perception depends on the sociocultural as well as intellectual attitude of each individual and his/her state of knowledge. For instance, an engineer has a significantly different point of view on a given product design than a humanities scholar who has, upon reversion, a different understanding of social and political phenomena. The philosopher Locke called this the "way of ideas," which is unique for every human being.¹⁵

Concerning the use of simulation in engineering design, this causes the definition of the reference of reality to be largely dependent on the simulation expert as well as the entire interdisciplinary team. Obviously, each individual will consider different objects, characteristics and interactions relevant for a given simulation task. Nevertheless, the correlation of the point of view of a system increases significantly with similar professional attitudes of the team members. Furthermore, this encourages comprehensible decisions and definitions since a common denominator appears. However, the higher the complexity of the problem and, thus, the more heterogeneity of reality that appears, the more the importance of a common denominator diminishes.

According to Roza, all existing systems (including all their objects, characteristics and interactions) have to be investigated based on the perception of all individuals taking into account all possible surroundings to establish a unique and universal understanding of reality.⁸

This impossible task clarifies that a unique definition of surrounding reality with its systems and objects cannot exist. Figure 4 illustrates this. Based on the individual point of view of the considered system of reality (bars) either three or four bars can be seen. Basically, both answers ("three bars" and "four bars") are correct. Consequently, a unique answer (corresponding to a universal solution for a given task) cannot be defined. Moreover,

Figure 4. Two different (but still correct) points of view on several bars.

Figure 5. Types of uncertainty (according to Engelhardt et al.¹⁸) and definition of aleatory and epistemic uncertainties.

this indicates that an all-embracing understanding of reality never exists.⁶

Furthermore, the psychologist Wilson's quote sums it up quite well: "The opinion that the human philosophy of life apprehends reality is ... as naive as the assumption that a pocket rule details more reality than a voltmeter."¹⁶ Hence, the fundamentals of modeling and simulation are just approximated by the human intellect. For instance, validation methods of simulations are highly critical since these are based on the assumption that the perceived reality—as the last resort—is absolutely trustworthy. However, since this is not ensured, the approximation itself must be validated, too. In the context of product development these conflicts are hereinafter referred to as the "paradox of reality."

The implication of the paradox is that all methods of modeling and simulation underlie uncertainties a priori. Further, additional limitations, abstractions and idealizations increase the level of uncertainty in modeling and simulation. Zickgraf even speaks about the "sword of

Damocles of uncertainty" hovering over all models and simulations.¹⁷

4. Uncertainty

Uncertainty appears when information about systems and their surroundings is unknown or at least vague. Uncertainty originates from the limitation of the human perception of reality and thus, increases due to simplifications, idealizations and abstractions during modeling and simulation. According to Engelhardt et al., uncertainty can be divided into three types (see Figure 5):¹⁸

1. The effects and the corresponding probability as well as the resulting divergences of **stochastic uncertainties** can be sufficiently quantified (using, e.g., the theory of probability).
2. **Estimated uncertainties:** Their effects are known but cannot be quantified entirely. Usually, quantification is mostly limited to the definition of lower

and upper specification limits, since corresponding probability distributions cannot be completely derived from available information.

3. Both effects and resulting divergences of **unknown uncertainties** are assumed or unknown.

Furthermore, the detailed three types of uncertainty can be further classified into aleatory and epistemic uncertainties.^{19–21} The significant dependence of uncertainty on the known information is also detailed in Figure 5. The decreasing level of known (or at least vague) information results in an increasing level of uncertainty.⁷ Consequently, aleatory uncertainties contain more information than epistemic uncertainties.

4.1. Aleatory uncertainties

Aleatory uncertainties are characterized by a comparably high level of information. These uncertainties are usually described statistically and, thus, the uncertainty is known and can be sufficiently quantified. In literature, the term “variability” is often used when considering aleatory uncertainties.^{7,12,22} According to Ryschkewitsch, variability is the “estimated amount or percentage by which an observed or calculated value may differ from the true value.”¹² Hence, products and processes which are affected by varying characteristics, such as manufacturing and assembly processes, underlie aleatory uncertainties. Furthermore, mathematical formulations of variability can be used to quantify aleatory uncertainties.

Aleatory uncertainty is caused by random effects. These effects are inevitable and, thus, can never be reduced to zero. However, on the one hand, with exponentially increasing effort, aleatory uncertainties can be minimized significantly.²³ On the other hand, aleatory uncertainty increases during the product development process since the level of knowledge about the considered product or process increases constantly. At some stage of product development, the aleatory uncertainty is reduced no further because of the reasonable balancing of benefits and efforts.^{21,22}

4.2. Epistemic uncertainties

Epistemic uncertainties result from incomplete, insufficient and non-existing information, which restrict the obtainable knowledge about reality. These uncertainties are dynamic, that is, current and future research activities result in new knowledge and better methods and tools. Hence, epistemic uncertainty is converted into knowledge and—if possible—aleatory uncertainty. Consequently, epistemic uncertainty can be avoided. However, the required effort should be carefully evaluated.^{7,23}

4.3. Aleatory vs. epistemic uncertainties and their effects on knowledge—A mathematical approach

A mathematical approach is presented which summarizes presented facts on aleatory and epistemic uncertainties in correlation with knowledge creation for improved understanding. Furthermore, it supports the product developer in analyzing and evaluating the current level of uncertainty of a product or a process during product development.

Therefore, the following starting point is assumed: at the beginning of its product development process neither information nor knowledge about the considered system is available. According to Nielsen the knowledge $K(t)$ exponentially increases during the progressing product development process (time t).²⁴ Based on this dependence and taking into account the system’s complexity c , knowledge is:

$$K(t) = 1 - e^{-\frac{t}{c}} \quad (1)$$

The complexity c varies in the interval $]0; 1]$. A complexity of $c = 1$ corresponds with an exceptionally high complexity of the considered system. In consequence, the knowledge $K(t)$ approaches asymptotically $K_{\text{asymptote}} = 1$. This illustrates, that universal knowledge about a system can never be achieved. On the contrary, the remaining lack of knowledge

$$U(t) = \overline{k(t)} = U_{\text{epis}}(t) = 1 - K(t) = e^{-\frac{t}{c}} \quad (2)$$

quantifies the missing knowledge and, thus, epistemic uncertainty $U_{\text{epis}}(t)$. Equation (2) demonstrates that epistemic uncertainty $U_{\text{epis}}(t)$ decreases during product development and finally approaches asymptotically $U_{\text{asymptote}} = 0$. Consequently, epistemic uncertainty can never be avoided entirely. This does not correspond to its definition (as detailed in Section 4.2), but is reasonable due to the limitations of human perception and the “paradox of reality” defined in this work.

According to the given definition, aleatory uncertainty always exists, but is known and can be quantified. Hence, aleatory uncertainty $U_{\text{alea}}(t)$ is part of knowledge $K(t)$, expressed in equation (3). The remaining part of knowledge $K_{\text{secured}}(t)$ corresponds to unambiguous and secured knowledge, which has been gained during the product development process. For instance, secured knowledge about manufacture includes a definition of the required manufacturing processes, while aleatory uncertainty can be traced back to imperfections of the manufactured parts which are affected by dimensional and geometrical deviations.

$$K(t) = U_{\text{alea}}(t) + K_{\text{secured}}(t) \quad (3)$$

Figure 6. $K(t)$, $U_{alea}(t)$, $U_{epis}(t)$ and $K_{secured}(t)$ during the product development of a system of high complexity ($c = 0.9$).

Assuming, that a system's complexity is directly proportional to the corresponding variation of the system, the aleatory uncertainty and the secured knowledge are:

$$U_{alea}(t) = (1 - e^{-\frac{t}{c}}) \cdot c \quad (4)$$

$$K_{secured}(t) = (1 - e^{-\frac{t}{c}}) \cdot (1 - c) \quad (5)$$

The insertion of equation (3) into the lack of knowledge $U(t)$ (equation (2)) yields the epistemic uncertainty $U_{epis}(t)$:

$$U_{epis}(t) = 1 - [U_{alea}(t) + K_{secured}(t)] \quad (6)$$

Equation (6) enables the product developer to differentiate appearing uncertainties and to evaluate their effects on existing knowledge about the considered products or processes.

Upon revision, the insertion of equations (4) and (5) into equation (6) results in equation (2). Hence, the presented mathematical formulation of epistemic and aleatory uncertainties in simulations is consistent. Figure 6 details time-dependent aleatory and epistemic uncertainties as well as the knowledge and the secured knowledge of a system with high complexity ($c = 0.9$) during the system's product development process.

The assumption of direct proportionality between the system's complexity c and aleatory uncertainty $U_{alea}(t)$ is not a mandatory claim. However, the consistency of the presented equations is still given for systems with different dependences and, thus, a modified equation (4).

4.4 The error considered an uncertainty

Uncertainties based on human failure, such as oversights or abstractions respectively idealization, need further distinction and classification. Those uncertainties can be completely avoided and, thereby, are not allowed to be

Figure 7. Overview: Four types of uncertainties in simulation.

distinctly classified as aleatory. Due to the fact that they can also evolve on a very high knowledge level on which state-of-the-art methods can hardly bring new insights, it is conflicting to range them as absolutely epistemic. Thereby, a third intermediate class is necessary and, according to Oberkampff et al., introduced as the “error.”²¹

Errors can be further sub-divided into “acknowledged errors” and “unacknowledged errors”. While acknowledged (and quantifiable) errors can rather be associated to aleatory uncertainties, unacknowledged errors tend towards epistemic uncertainties. The latter kind of error always exists and goes hand in hand with human nature (like the mentioned oversights).

The line between a quantifiable acknowledged error and an aleatory uncertainty is blurred and often hard to grasp. For instance, an error becomes aleatory if no further actions can be taken to reduce the corresponding variability (e.g. due to financial, numerical and computational expense or feasibility). By means of the successive acquisition of information in validation and verification, unacknowledged errors (e.g. due to human failure) can be identified. Hence, such errors are avoidable, even though these errors are not caused by a lack of knowledge necessarily. The epistemic character of unacknowledged errors is clearly apparent. However, only acknowledged errors identify the potential for improvements.

5. Uncertainties in simulations

After presenting the different uncertainty classes and their characteristics, the identification of special uncertainties aroused in simulations occurs afterwards. The linkage to defined classes gives a framework which can lead to better control methods because of knowing how to handle their characteristics. Figure 7 provides an appropriate overview for orientation.

5.1. Uncertainty in data

Based on the given problem, the intended implication and systems' definition for modeling and simulating the

needed kinds of data have to be acquired. These are required to mainly determine the parameters of the objects (e.g. Young's modulus), external conditions of use (e.g. surrounding temperature), inputs (e.g. heat flux), and initial conditions to start a simulation. The gathering and the data itself are subject to:

Variation: From the definition it is classified as aleatory. Several alternatives for a parameter or input are feasible for which unavoidable reasons, like abrasive wear in machining for example, are seen.⁷ In the case of one or more parameter(s)/input(s) following a defined probability distribution, a set of methods supports the analysis over all the statistical variations on the system. As a powerful tool the numeric Monte Carlo simulation can be named.²⁵ If there is no probability distribution available, for example, in the case of a lack of data in the early design stages, intervals can be used for uncertainty descriptions and linked by arithmetical mathematics. Finally, if data are gathered whose variation is not quantifiable but described in linguistic ways, fuzzy logic is an appropriate tool to transfer them for mathematical purposes.¹⁹ Variation can only be accompanied by unacknowledged errors if all acknowledged errors are transferred to aleatory uncertainty and less variation is not intended.

Vagueness: Vagueness originates from lack of knowledge and is classified as epistemic. This concerns data whose determination underlies unclear conditions like undefined metering noise. Moreover, a lack of confidence in the data can also be identified, for example, if data is based on less representative samples. Additionally, Kruse and Meyer name knowledge-based systems which combine knowledge of a group of experts as vague.²⁶ Vagueness is to be avoided by repeating and improving data collection and thereby generating information and consequently knowledge. In the event of there being no possibility of knowledge improvement, its epistemic character increases the aleatory uncertainty of variable data. Besides unacknowledged errors, acknowledged errors can also be found due to vague data collection because of an intentionally relieved workload.

5.2. Uncertainty in model and simulation

Basically, this uncertainty leads to deviations from conditions in reality and its framed reference due to model creation and its application within a simulation. In this context, it is separated into the concept, the mathematical model and its numeric, the programming and the presentation of simulation results.²⁷

Concept: The starting point is the mentioned specification of the reference of reality. The system to be examined by simulation has to be extracted with its relevant objects from reality. The more extensive and/or complex the system is, the more permanent the lack of knowledge remains

(compare Figure 6). Hence, epistemic uncertainty prevails right from scratch in engineering design.

With the design of the concept, the model abstraction and idealization leads to both epistemic and acknowledged errors. Epistemic is caused by abstraction, respectively idealization, as a result of ignorance or non-availability of data about objects, interactions or conditions. Acknowledged errors arise from abstraction and idealization for intended simplicity purposes.

Mathematical model: The formulation of differential equations provides epistemic uncertainty. The reason for this mainly lies in restrained or lacking physical expertise about the on-going physical processes and interactions. Also, errors occur, for example, via simplification of mathematical principles (e.g. rounding of figures), named structural errors.²¹ Furthermore, approximation through transfer of non-linear differential problems in discrete difference equations causes acknowledged quantifiable errors if convergence discrepancies are mathematically expressed. Especially, coupling of non-linear interactions in systems, like turbulent streams, caused by explosions can exhibit significant discrepancies in the continuous and discrete problem description. Yee and Sweby provide anomalies of implicit approximation procedures.²⁸

Programming: Mainly unknown mistakes occur which are traced back to human failure. Principally, Hatton's study on syntax errors in commercial software based on language C (basic population: 3.3 million program lines) claims that 8 errors per 1000 lines occur on average.²⁹ Those influence the simulation outcome without having indicated a compilation error before. Finally, epistemic uncertainty appears because different programmers develop different source codes with discrepancies in outputs. Thereby, confidence in programming can be critical and accreditation is highly recommended.

Visualization of results: The continuous representation of simulation results on discrete calculus requires approximations and thus causes an acknowledged error.²¹ This can be described by means of statistics and is able to be transferred into aleatory uncertainty.

5.3. Phenomenological uncertainty

Ignorance about events and future influences cause this type of uncertainty. It is not foreseeable and thereby cannot be considered. In the literature it is classified as "unknown Unknowns."⁷ For instance, this uncertainty concerns data which is far in the future. This intensifies vagueness of this data, but cannot be recognized in data preparation since it is unknown in the present. Moreover, this uncertainty can also affect administrative affairs, for example, unexpected changes in problem formulations or resources. It is epistemic with the specialty of not being reduced by intentional research and development (R&D) but by incidence.

5.4. Uncertainty in human behavior

This kind of uncertainty accompanies all other kinds and every step of the entire simulation process and can be reduced by verification. This uncertainty goes further than unacknowledged errors, as a result of oversights for example, since it can be aligned towards concrete actions:

Interpretation: The analysis of a simulation's input data as well as corresponding simulation results underlie the subjective knowledge and mind of the examiner. This also influences the input determination and thereby affects data uncertainty.⁷ Epistemic uncertainty predominates here.

Decision making: Being closely related to interpretation, decision-making can be named as an uncertainty source. However, this largely depends on the corresponding decision-maker's personality. Decisions based on interpretations do not claim to be reasonable and, thus, include reservation, risk aversion or political/administrative influences in model creation and simulation implementation.³⁰ In this way, deliberately ignoring important objects within the specification of the reality reference could take place to reduce complexity and work load. The intermediate character leads to classification as acknowledged or unacknowledged error.

Ambiguity: Imprecise declarations and information provoke uncoordinated activities and, as follows, inappropriate results. A reason can be lack of knowledge, what is epistemic, or generalization due to intended simplicity, what entails acknowledged errors. Clear and definite problem and target formulation has to be undertaken especially in the early stages of simulation projects to avoid uncertainty of ambiguity and thereby high costs for belated compensation.

In conclusion, especially epistemic uncertainty, that is, lack of knowledge, has a dominating influence on modeling and simulation. According to Figure 6, skepticism is advisable towards complex physical systems because the amount of epistemic uncertainty is more difficult to transfer into knowledge. Validation is an appropriate tool to deal with this uncertainty. This needs some expertise which relies on measured data from reality. On the contrary, validation of simulated imaginary future systems proves to be very difficult because of the insecure data base, which could be estimated at best.

Aleatory uncertainty prevails during data collection and preparation and is quantifiable. The effects of the propagation of different aleatory sources have to be evaluated throughout the system.

Errors can be acknowledged or unacknowledged, whereas acknowledged errors are also quantified especially when it comes to propagation. This is not possible, if the reference for error determination is missing. Besides epistemic uncertainty, unacknowledged errors are the main source for incorrect simulation results but can be reduced by systematic verification.

6. Statistical tolerance analysis

Deviations in the parts are inevitable due to the axiom of manufacturing imprecision.³¹ Consequently, dimensional and geometric deviations which appear must be limited by appropriate tolerances to ensure the functional capabilities of a system. Therefore, statistical tolerance simulations are widely used in engineering design to support the product developer during tolerance design. These simulation procedures correspond to the detailed procedure according to Roza (see Figure 1).⁸

According to Salomons et al., three essential activities are required for the design of tolerances during product development. First, tolerances for each part are defined to limit the system's deviations (tolerance specification). Then the effects of these tolerances on the system's functional key characteristic (FKC) and, thus, the fulfillment of given functional requirements are analysed with a numerical (usually statistical) tolerance analysis. Finally, based on the results of a tolerance analysis, the tolerances are modified to optimize the system concerning aspects such as functional fulfillment, assemblability and cost-efficiency (tolerance synthesis).³²

Tolerance analysis—being the major task of dimensional and variation management—is well known and widely used.³³ A statistical tolerance analysis can be divided into three steps (Figure 8). First, a mathematical relation between appearing deviations or tolerances and the system's FKC must be established.³⁴ Therefore, usually vector-chains are used. Afterwards, the FKC is determined for a destined number of virtual systems (usually $n > 10,000$ according to a Monte Carlo simulation). These n virtual systems differ in their characteristics which underlie deviations. The final step includes determination and representation of the results, such as the resulting probability distribution of the FKC as well as a quantification of the contribution of each tolerance to the resulting FKC's variation.³⁵

According to the classification of uncertainties (Section 4) manufacturing-caused deviations of the components are aleatory uncertainties.³⁷ These deviations can be quantified by corresponding probability distributions but cannot be avoided entirely. By means of statistical tolerance simulations, the product developer gains knowledge and, thus, reduces epistemic uncertainty of the product development process. However, as detailed in Section 5, each tolerance simulation underlies several uncertainties (see Figure 9). A selection of these uncertainties is further detailed in the following remarks to emphasize the product developer's awareness of the results of statistical tolerance analyses in engineering design.

Appearing uncertainties depend largely on the degree of maturity of the considered system. Tolerance simulations are usually carried out in the early and late embodiment design stage.³⁸ Hence, uncertainties which affect a

Figure 8. Procedure of a statistical tolerance analysis (according to Walter et al.³⁶).

Figure 9. Uncertainties which affect the results of a statistical tolerance analysis.

statistical tolerance analysis differ significantly during the product development process. During embodiment design, the percentage of epistemic uncertainties rapidly decreases. This can be traced back to a significant lack of knowledge as well as quantified aleatory uncertainties in the beginning of embodiment design. This causes, inter alia, uncertainties in data such as variation and vagueness.

6.1. Variation and vagueness

Vague assumptions on probability distributions of manufacturing-caused deviations, limited information about the system's geometry and operation conditions as well as about manufacturing processes require radical simplifications of the tolerance simulation model. In

particular, these simplifications can be found in early embodiment design.

Due to incomplete information about the corresponding statistical probabilities of the appearing deviations (such as the four Pearson parameters: mean, standard deviation, skewness and kurtosis), Monte Carlo-based tolerance simulations are either questionable to a large extent or even impossible to perform. The resulting probability distributions show inherent variability since they just approximate a non-deterministic experiment. This approximation causes aleatory uncertainty and acknowledged errors (if improvements are still possible), respectively. Furthermore, the computational expense of a Monte Carlo simulation is significantly higher compared with classic tolerance simulation techniques such as

worst-case-analysis and root-sum-square.^{39,40} In addition to missing statistical information, usually only substantially geometrical dimensions of the system's parts are well defined in early embodiment design. The geometrical definition (including minor geometrical design parameters) is completed at the end of the embodiment design stage. Hence, according to the current state of the product development, only dimensional and geometrical deviations which are already adequately defined can be taken into account in tolerance simulations. For instance, the consideration of the operation-dependent deformation of a part depends on this part's stiffness and on its actual geometry, and, thus, on the current stage during product development.⁴¹

During later embodiment design, additional information is available and specified by the product developer. Hence, epistemic uncertainty decreases while knowledge and aleatory uncertainty increase. Nevertheless, in particular the specification of the required manufacturing processes still causes epistemic uncertainty. These specifications are often merely based on experience and lessons learned and, thus, highly subjective and hard to comprehend, and underlie unacknowledged errors.

The following remarks in Sections 6.2–6.4 focus on tolerance simulations in the later embodiment design.

6.2. Uncertainties of concept

The formulation of the mathematical relation between FKC and appearing deviations is based on the derived reference of reality of the system. Therefore, the real assembly-chain is separated and only relevant parts (chain-links) are considered when establishing the functional relation for tolerance simulations. This is mainly undertaken to reduce complexity. However, the effects of these excluded parts and their corresponding manufacturing-caused deviations and tolerances on the system's FKC are ignored. This results in an acknowledged and quantifiable error.

Furthermore, additional assembly-caused and operation-dependent deviations appear and affect the system's FKCs. Due to simplifications and idealizations of the reference of reality and the corresponding simulation model, several deviations are not taken into account. In particular, time-variant operation-dependent deviations (e.g. wear, thermal expansion and vibrations) are ignored and, thus, cause an epistemic uncertainty, since their integration into a statistical tolerance analysis is still the subject of current research.^{36,42}

6.3. Uncertainties of the mathematical model

With regard to Monte Carlo simulations, in particular the number of generated samples must be highlighted. Stuppy illustrates the convergence of the results of a statistical tolerance analysis with increasing numbers of samples.⁴³

However, this goes hand in hand with exceptionally increasing computational expense, in particular, for complex systems. Hence, the definition of a number of samples leads to an acknowledged error.

For the sake of completeness, attention is drawn on the determination of operation-dependent deviations by means of numerical computer-aided-engineering tools (CAE-Tools), such as FE-Analysis, to determine the deformation of non-ideal parts. Moreover, the implementation of these tools underlies additional uncertainties (as already detailed in Section 5.2), such as an acknowledged error caused by the discretization of the mathematical problem in time, that is, the definition of the time increment. The more the time increment increases, the more the temporal resolution of the FKCs decreases. By means of interpolation this error can be reduced. However, it cannot be avoided entirely.

Furthermore, the gained results of numerical computer simulations differ with the varying initial settings. For instance, the optimum determined by means of a local optimization algorithm is not necessarily the global optimum. Moreover, with varying initial settings of the optimization algorithm the optimum will differ from the optimum determined in a previous run. Consequently, multiple runs with varying initial settings of a numerical computer simulation are highly recommended to quantify the appearing error.

6.4. Visualization of results

Finally, the result visualization and, thus, the following interpretation by the product developers are subject to uncertainties. For instance, the resulting probability distributions of FKCs are usually approximated and visualized by Gaussian distributions. However, this assumption is only justified for a large number of samples (according to the central limit theorem—law of large numbers).⁴⁴ Against the backdrop of two diverging requirements (computational expense vs. accuracy), tolerance analyses are usually performed with more than 10,000 samples. Consequently, the visualized Gaussian distribution goes hand in hand with an acknowledged and quantifiable error.

7. Closing remarks

According to ISO 31000:2009 uncertainties always cause risks.⁴⁵ These risks affect the results of simulations and, in particular, the derived decisions. Hence, the following rules can be formulated:

- An increasing number of uncertainties goes hand in hand with steadily growing risks.
- The more precisely uncertainties can be identified as well as quantified, the more easily the risks can be identified and reduced by appropriate actions.

Basically, uncertainties are an essential motivation for innovations in engineering design. This is not merely restricted to the domain of modeling and simulation, but can be noticed in all fields of research and development. For instance, the innovation of the electronic stability program (ESP) of cars is the result of the product designer's aim to control uncertainties in the car's driving behavior. As already detailed in Section 4.2, originally existing epistemic uncertainties were transferred into knowledge and measurable aleatory uncertainties during the product development process of ESP.

In today's research simulations, essentially methods to investigate, predict and ensure the behavior of products and processes are needed. Quite often these simulation results serve as the basis of decision making, disregarding the fact that those results were not gained in real experimental investigations. However, the consequences can be catastrophic. Crichton proposes attaching simulation results with a warning label, saying "Warning: Computer simulations may be erroneous and unverifiable."⁴⁶

This work details that each step of the procedure to set up a simulation is essentially affected by uncertainties. Aleatory uncertainties can be ascertained well, and, thus, their effects on a simulation's results can be quantified (e.g. by means of statistical methods, such as statistical process control and Monte Carlo simulation). However, the effects of epistemic uncertainties on the procedure are still of high interest in several research activities such as those by Roza, Choi and Oberkampff et al.^{8,19,21} A promising trend in engineering design is the so-called knowledge-based engineering, which assists the interdisciplinary team with appropriate methods and tools such as data mining and engineering assistance systems.⁴⁷

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References

1. Sussman JM. *Ideas on Complexity in Systems—Twenty Views*. Cambridge, MA: MIT Press, 2000, <http://web.mit.edu/esd.83/www/notebook/20ViewsComplexity.pdf>
2. Taylor IJ, Deelman E, Gannon DB, et al. *Workflow for e-Science*. Berlin: Springer, 2007.
3. Stodden V. *Computational research and scientific method: A third branch?* Yale Statistics Department Seminar, 2010, <http://www.stanford.edu/~vcs/talks/YaleStats2010-VictoriaStodden-2.pdf>
4. Banks J. *Handbook of Simulation: Principles, Methodology, Advances, Applications, and Practice*. 1st edition. New York: John Wiley & Sons, 1998.
5. Banks J, Carson JS, Nelson BL, et al. *Discrete-Event System Simulation*. 5th edition. Upper Saddle River, NJ: Prentice Hall, 2009.
6. Mouroulis P, Ferrero M, Roman P, et al. Readers elaborate on fashion and truth, fact and theory. *Phys Today* 2003; 12: 13–14.
7. Thunnissen DP. *Propagating and mitigating uncertainty in the design of complex multidisciplinary systems*. PhD Thesis, California Institute of Technology, Pasadena, CA, 2005.
8. Roza ZC. *Simulation fidelity theory and practice. A unified approach to defining, specifying and measuring the realism of simulations*. PhD Thesis, Delft University of Technology, Delft, The Netherlands, 2005.
9. Reinhardt M. *Philosophie und Technik. Realität und Wirklichkeit*. Project thesis, University of Applied Science Düsseldorf, Düsseldorf, Germany, 2008. Available at: et.fh-duesseldorf.de/home/philotec/data/reinhardt.pdf
10. Gross DC. Report from the Fidelity Implementation Study Group. In: *Proceedings of the Spring simulation interoperability workshop*, Orlando, FL, 14–19 March 1999, paper no. 167.
11. Imboden DM and Koch S 2003. *Systemanalyse: Einführung in die mathematische Modellierung natürlicher Systeme*, 1st edition. Berlin: Springer, 2003.
12. Ryschkewitsch MG. *Standard for Models and Simulation—NASA Technical Standard NASA-STD-7009*, National Aeronautics and Space Administration, Washington, USA, 2008, <https://standards.nasa.gov/documents/detail/3315599>.
13. Zellner A. Keep it sophisticatedly simple. In: Heuzenkamp M, McAleer M and Zellner A (eds) *Simplicity, Inference and Modelling: Keeping it Sophisticatedly Simple*. Cambridge: Cambridge University Press, 2001, pp.242–262.
14. Williams EJ. Supply chain simulation and analysis with SimFlex. In: *Proceedings of the 2003 Winter simulation conference*, New Orleans, LA, 7–10 December 2003, pp.231–237.
15. Yolton JW. *Locke and the Way of Ideas*. South Bend, IN: St. Augustine's Press, 1956.
16. Wilson RA. *Quantum Psychology: How Brain Software Programs You and Your World*. 2nd edition. Las Vegas, NV: New Falcon, 1992.
17. Zickgraf A. Unter dem Damoklesschwert der Unsicherheit (Under Damocles' sword of uncertainty), *Telepolis*, 21 March 2011, <http://www.heise.de/tp/artikel/34/34358/1.html>.
18. Engelhardt RA, Eifler T, Mathias J, et al. Linkage of methods within the UMEA methodology: An approach to analyze uncertainties in the product development process. In: *Proceedings of the 18th international conference on engineering design—Vol. 10*, Copenhagen, Denmark, 15–18 August 2011, pp.433–442.
19. Choi H-J. *A robust design method for model and propagated uncertainty*. PhD Thesis, Georgia Institute of Technology, Atlanta, GA, 2005.
20. Hofer E. When to separate uncertainties and when not to separate. *Reliab Eng Syst Safe* 1996; 2–3: 113–118. doi: 10.1016/S0951-8320.
21. Oberkampff WL, DeLand SM, Rutherford BM, et al. *Estimation of Total Uncertainty in Modeling and Simulation*. Sandia Report SAND2000-0824, Sandia National Laboratories, Albuquerque, NM, 2000, infoserve.sandia.gov/sand_doc/2000/000824.pdf.
22. Pulkkinen U and Simola K. *Uncertainties of Phenomenological Models in PSA*. NKS Report NKS-45,

- VTT Automation, Finland, 2001, http://www.nks.org/en/nks_reports/.
23. Daneshkhah AR. *Uncertainty in probabilistic risk assessment. A review*. Working Paper, University of Sheffield, UK, 2004.
 24. Nielsen J. *Usability Engineering*. San Diego, CA: Academic Press, 1994.
 25. Binder K and Heermann DW. *Monte Carlo Simulation in Statistical Physics*. 4th edition. Berlin: Springer, 2002.
 26. Kruse R and Meyer KD. *Statistics with Vague Data*. 1st edition. Dordrecht, The Netherlands: Reidel, 1987.
 27. Kreye ME, Goh YM and Newnes LB. Manifestation of uncertainty—A classification. In: *Proceedings of the 18th international conference on engineering design—Vol. 6*, Copenhagen, Denmark, 15–18 August 2011, pp.96–107.
 28. Yee HC and Sweby PK. Global asymptotic behavior of iterative implicit schemes. *Int J Bifurcat Chaos* 1994; 6: 1579–1611. doi: 10.1142/S0218127494001210.
 29. Hatton L. The T-Experiments: Errors in scientific software. *IEEE Comput Sci Eng* 1997; 2: 27–38. doi: 10.1109/99.609829.
 30. Eriksson J and Brannemo A. Coping with deviation and decision-making. In: *Proceedings of the 18th international conference on engineering design—Vol. 1*, Copenhagen, Denmark, 15–18 August 2011, pp.429–440.
 31. Zhang M, Anwer N, Mathieu L, et al. 2011. A discrete geometry framework for geometrical product specifications. In: Bernard A (ed) *Global Product Development—Proceedings of the 20th CIRP Design Conference*. Berlin: Springer, 2011, pp.142–148.
 32. Salomons O, van Houten F and Kals H. Current status of CAT systems. In: El Maraghy HA (ed) *Geometric Design Tolerancing: Theories, Standards and Applications*. Dordrecht, The Netherlands: Springer, 1998, pp.438–452.
 33. Wartzack S, Meerkamm H, Stockinger A, et al. Lifecycle-oriented tolerance simulation. *Konstruktion* 2011; 6: 63–67.
 34. Walter M, Sprügel T and Wartzack S. 2012. Tolerance analysis of mechanism taking into account the interactions between deviations using meta-models. In: *Proceedings of the 9th Norddesign conference*, Aalborg, Denmark, 22–24 August 2012, pp.587–594.
 35. Ziegler P, Breitsprecher T, Walter M, et al. Application of sensitivities on simulation-data-based metamodels during the product synthesis. In: *Proceedings of the 12th international design conference 2012*, Dubrovnik, Croatia, 21–24 May 2012, pp.589–598.
 36. Walter M, Sprügel T and Wartzack S. Tolerance analysis of systems in motion taking into account interactions between deviations. *J Eng Manuf* 2013; 5: 709–719. doi: 10.1177/0954405412473719.
 37. Dantan JY, Gayton N, Qureshi AJ, et al. Tolerance analysis approach based on the classification of uncertainty (aleatory/epistemic). In: *Proceedings of the 12th CIRP conference on computer aided tolerancing*, Huddersfield, UK, 18–19 April 2012, paper no. 30.
 38. Pahl G and Beitz W. *Engineering Design—A Systematic Approach*. 3rd edition. London: Springer, 2007.
 39. Chase K and Parkinson A. 1991. A survey of research in the application of tolerance analysis to the design of mechanical assemblies. *Res Eng Des* 1991; 1: 23–37. doi: 10.1007/BF01580066.
 40. Epstein B. Tolerances on assemblies. *American Machinist* 1946; (1): 119–121.
 41. Walter M, Breitsprecher T, Gruber G, et al. Simulation based generation of an initial design taking into account geometric deviations and deformations. In: *Proceedings of the 18th international conference on engineering design—Vol. 10*, Copenhagen, Denmark, 15–18 August 2011, pp.78–90.
 42. Armillotta A and Semeraro Q. Critical operating conditions for assemblies with parameter-dependent dimensions. *J Eng Manuf* 2013; 5: 735–744. doi: 10.1177/0954405412472640.
 43. Stuppy J. *Methodische und rechnerunterstützte Toleranzanalyse für bewegte technische Systeme (A methodology on computer-aided tolerance analysis of systems in motion)*. PhD Thesis, Friedrich-Alexander-University Erlangen-Nuremberg, VDI Fortschrittsberichte Reihe 20 Nr. 433, VDI Verlag, Düsseldorf, Germany, 2011.
 44. Wilks SS. Determination of sample sizes for setting tolerance limits. *Ann Math Stat* 1941; 1: 91–96. doi: 10.1214/aoms/1177731788.
 45. ISO (International Organization for Standardization). Risk management—Principles and guidelines DIN ISO 31000, 2009.
 46. Crichton M. *State of Fear*. New York: Harper Collins, 2004.
 47. Wartzack S. *Predictive Engineering—Assistenzsystem zur multikriteriellen Analyse alternative Produktkonzepte*. PhD Thesis, Friedrich-Alexander-University Erlangen-Nuremberg, VDI Fortschrittsberichte Reihe 1 Nr. 336, VDI Verlag, Düsseldorf, Germany, 2000.

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