Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture

<http://pib.sagepub.com/>

Tolerance analysis of systems in motion taking into account interactions between deviations

Michael Walter, Tobias Sprügel and Sandro Wartzack

Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 2013 227: 709

originally published online 9 April 2013

DOI: 10.1177/0954405412473719

The online version of this article can be found at:

<http://pib.sagepub.com/content/227/5/709>

Published by:

<http://www.sagepublications.com>

On behalf of:

[Institution of Mechanical Engineers](#)

Additional services and information for *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture* can be found at:

Email Alerts: <http://pib.sagepub.com/cgi/alerts>

Subscriptions: <http://pib.sagepub.com/subscriptions>

Reprints: <http://www.sagepub.com/journalsReprints.nav>

Permissions: <http://www.sagepub.com/journalsPermissions.nav>

Citations: <http://pib.sagepub.com/content/227/5/709.refs.html>

>> [Version of Record](#) - May 10, 2013

[OnlineFirst Version of Record](#) - Apr 9, 2013

[What is This?](#)

Tolerance analysis of systems in motion taking into account interactions between deviations

Proc IMechE Part B:
J Engineering Manufacture
227(5) 709–719
© IMechE 2013
Reprints and permissions:
sagepub.co.uk/journalsPermissions.nav
DOI: 10.1177/0954405412473719
pib.sagepub.com

Michael Walter, Tobias Sprügel and Sandro Wartzack

Abstract

A product's functionality depends largely on the interaction of its components and their geometries. Hence, tolerance analyses are used to determine the effects of deviations on functional key characteristics of mechanisms. However, possible interactions between the different deviations and the resulting effects on themselves as well as on the functional key characteristics have not yet been considered.

This article considers the extension of the existing “integrated tolerance analysis of systems in motion” approach. By means of the methodology, the interactions between appearing deviations can be identified and integrated into a tolerance analysis functional relation. Therefore, the appearing interactions are represented by meta-models that can be easily integrated into the functional relation. Consequently, the product developer is able to gain information about the effects of deviations on functional key characteristics, as well as the effects of the deviations among themselves. In order to show the methodology's practical use, the interactions between deviations of a nonideal crank mechanism inside a four-stroke combustion engine are considered. For this purpose, two different meta-modeling techniques are used: response surface methodology and artificial neural networks.

Keywords

Tolerance analysis, interactions, systems in motion, meta-modeling, artificial neural networks

Date received: 13 July 2012; accepted: 10 December 2012

Introduction

A successful and economical product development goes hand in hand with the ambition to ensure the product's functionality as early as possible during the product development process. Hence, geometric deviations of the considered system and its components need to be taken into account in order to ensure the system's functionality.¹ Deviations appear during every stage of the product's life cycle: manufacture, assembly and the system's use (operation time).²

Usually, a tolerance analysis is used to determine how the deviations affect a system's functional key characteristics (FKCs). However, the consideration of time-dependent systems in motion results in the following two major problems that the product developers have to face.

- The different kinds of deviations affect the FKCs in two different ways: random deviations (e.g. manufacturing-caused deviations of dimensions like length or height) result in a variation of the FKCs (Figure 1(a)) and systematic deviations (e.g. an

operation-dependent deformation due to inertial forces) cause a mean shift in the distribution of the FKCs (Figure 1(b)).

- The time-dependent motion behavior results in significant variable effects of the appearing deviations on the FKCs.

Stuppy and Meerkamm³ and Wartzack et al.⁴ present the “integrated tolerance analysis of systems in motion” approach. This approach takes into account the effects of manufacturing-caused and operation-dependent deviations on a system's time-dependent FKCs. Statistical tolerance analyses are usually performed several times with at least 10,000 samples.

Chair of Engineering Design, Friedrich-Alexander-University Erlangen-Nuremberg, Erlangen, Germany

Corresponding author:

Michael Walter, Chair of Engineering Design, Friedrich-Alexander-University Erlangen-Nuremberg, Martensstrasse 9, 91058 Erlangen, Germany.

Email: walter@mfk.uni-erlangen.de

Figure 1. Effects of deviations and interactions between the deviations on a system's functional key characteristic: (a) variation of the FKCs (due to random deviations), (b) variation (random deviations) and mean shift (systematic deviations) of the FKCs and (c) additional variation of the FKCs (due to interactions between deviations).
FKC: functional key characteristic.

Consequently, the consideration of operation-depending deviations is highly time-consuming since these deviations must be determined for each generated virtual sample of the system using, for example, computer-aided simulations.

In addition to the deviations' effects on a FKC, effects of the deviations among themselves also appear. In compliance with Shah et al.,⁵ who deemed a "fundamental understanding of geometric variations and how they interact" to be "the key missing ingredient" in tolerance analysis, these dependencies are henceforth termed "interactions between deviations." These interactions between the appearing deviations also affect a system's FKCs. Figure 1(c) illustrates the additional variation of a FKC, which is caused by the interactions between deviations. Consequently, a definition of these interactions can be derived: interactions are the dependencies between dimensional and geometrical deviations, which result in different effects on a FKC (variation and/or mean shift) in the cases of the deviations' independent, sequential (Figure 1(a) and (b)) and simultaneous appearance (Figure 1(c)). However, these interactions are not considered in existing approaches to the statistical tolerance analysis of systems in motion.

This article focuses on the extension of the existing "integrated tolerance analysis of systems in motion" approach.^{3,4} By means of the presented modified methodology, the interactions between deviations can be determined and integrated into a statistical tolerance analysis of a system in motion. Therefore, the tolerance analysis' functional relation needs to be modified because appearing interactions are not considered yet. Consequently, appropriate mathematical models are needed, which represent the operation-depending deviations as well as the interactions toward these deviations. Moreover, these models should be easily integrated into the functional relation. Therefore, meta-models will be used. In the context of engineering

design, a meta-model is the result of the meta-modeling procedure.⁶ According to Simpson et al.,⁶ this procedure includes three steps: the experimental design for generating data, the choice of a model to represent the data (the meta-modeling technique) and the fitting of the data to the observed data. A meta-model can describe dependencies (real or virtual) using mathematical and statistical methods, such as approximation, estimation, machine learning and artificial intelligence. Well-known meta-modeling techniques are the response surface methodology (RSM)⁶ and the training of artificial neural networks (ANNs).

Consequently, the product developer will be able to gain information about the effects of different deviations on a FKC, as well as the effects of these deviations among themselves. Furthermore, the time as well as the computational expense of a statistical tolerance analysis can be reduced significantly since far less samples are needed to generate/train the meta-model.

State of the art

In product development today, tolerance analysis and tolerance synthesis—being the two major objectives of dimensional management—are well known and widely used. The large diversity of analyzed products and the resulting requirements toward tolerance analysis and tolerance synthesis reflect in previous and current research activities that deal with many different aspects concerning products and processes. However, existing approaches to tolerance analysis do not integrate the specific aspects of mechanisms during their use. In this context, especially, the time-depending effects of deviations on the FKCs, the different kinds of deviations that appear during the product's life cycle and the entailed interactions between these deviations have to be emphasized. Moreover, in 1957, Morrison⁷ identified interactions between varying parameters as an essential aspect in reducing the variation of a system's FKCs.

Despite this potential, interactions between deviations have not drawn much attention in tolerance management and robust design.⁸ This prompts Hasenkamp et al.⁸ to say, in considering all stages of the product's life cycle, that the development of integrated methods is both a promising and needed aim of tolerance management and robust design.

Tolerance analysis of a mechanism

The kinematic behavior of a mechanism is essentially affected by dimensional and geometric deviations of its components. These deviations can be traced back to, for example, manufacturing discrepancies. Therefore, a time-depending tolerance analysis of the effects of geometric deviations is necessary. Several publications have considered manufacturing deviations for mechanisms with both lower^{9–11} and higher kinematic pairs.¹² In addition to manufacturing-caused deviations, operation-depending deviations also appear. The operation-depending displacement of components due to joint clearance is considered in the studies by Sacks and Joskowicz¹³ and Muvengi et al.,¹⁴ while Dupac and Beale,¹⁵ Imani and Pour¹⁶ and Guowei et al.¹⁷ take into account the appearing deformation of components due to the forces resulting from the system's motion. Further research considers manufacturing-caused as well as operation-depending deviations.^{18,19} However, the time-dependency of the mechanism, and thus of the deviations, remains unconsidered. A robust design approach is presented in the study by Huang and Zhang,²⁰ which enables the product developer to analyze a system in motion with manufacturing deviations and imperfect joints. Furthermore, Stuppy and Meerkamm³ and Wartzack et al.⁴ present the “integrated tolerance analysis of systems in motion” allowing the tolerance analysis of a mechanism with both manufacturing and operation-depending deviations (deformation and displacement due to joint clearance). In contrast to Huang and Zhang,²⁰ the “integrated tolerance analysis” is not limited to only normal distributions concerning the deviation's statistical distributions. An appropriate visualization of the results of the “integrated tolerance analysis” is shown in the study by Wartzack et al.⁴

However, all the listed research focuses on the effects of manufacturing and/or operation-depending deviations on the FKC's of a technical system. The aspect of possible interactions between the different deviations and the resulting effects among themselves, as well as the effects on the FKC's, has not yet been considered.

Use of meta-modeling in tolerance management and robust design

In 2002, Hong and Chang²¹ stated with confidence that meta-modeling (especially ANNs) could pave the way to “a systematic method which automates this procedure, incorporating the domain specific knowledge as

well as the geometry and process knowledge.” Despite this promising potential, the use of meta-modeling techniques in tolerance management is currently still limited. However, Dantan et al.²² commented on an extended use of meta-modeling in tolerance management in recent years. Moreover, Wynn and Ograjensek²³ detail that meta-modeling techniques are the needed methods to “model very large data sets with almost unlimited commercial applications.” They also quote that the upcoming years will be “the golden age for statistics.”

Meta-models are mainly used in tolerance management in two ways: on the one hand, the dependencies of tolerances and the resulting manufacturing costs are described using meta-models. Especially in the case of tolerance synthesis, the tolerance–cost relations are usually unknown, and therefore, approximated using, for example, response surfaces^{24,25} or ANNs.^{26,27} On the other hand, the functional relations between varying parameters (e.g. deviations) and the system's response (e.g. FKC's) are formulated. Due to the large diversity of meta-modeling techniques, a variety of research considers the tolerance-related application of different meta-modeling techniques. For instance, Wang et al.²⁸ use support vector regression (SVR) to predict the failure probability of a nonideal car door assembly. The widely used RSM is used to approximate the deformation of a beam in bending,²⁹ as well as the springback of sheet metal resulting from a deep-draw process.³⁰ Watrin et al.³¹ use the RSM to approximate the noise level of a car's rear axle bevel gear, which depends on the manufacturing-caused deviations of the gear components. The objective function of a tolerance synthesis is replaced by approximated response surfaces in the works of Huele and Engel³² and Kim et al.³³ The effects of manufacturing-caused deviations on the assembly behavior of a technical system can also be represented using ANNs, as shown in the study by Andolfatto et al.³⁴ Furthermore, Kopardekar and Anand³⁵ compared the results of a tolerance synthesis based on an ANN with the results of a vector-chain-based tolerance synthesis. They conclude that the meta-models can be used for tolerance allocation problems. In addition to the investigation of tolerances (analysis and synthesis), meta-modeling techniques can be found in terms of the representation of tolerances and the measurement of deviations. Barbato et al.³⁶ use Kriging models to predict the optimal measuring points of parts that underlie flatness deviations.

Work methodology

This article considers a main modification of the existing “integrated tolerance analysis of systems in motion” methodology.^{3,4} Therefore, in order to take into account interactions between deviations, the necessary modifications to the methodology will be detailed in the upcoming section.

Figure 2. Modifications of the “integrated tolerance analysis of systems in motion” methodology. EFAST: Extended Fourier Amplitude Sensitivity Test.

In order to show the modified approach’s practical use, interactions between deviations of a crank mechanism will be considered. The crank mechanism’s components are subject to both random manufacturing-caused and systematic operation-dependent deviations, during a motion sequence consisting of two crank rotations. The deviations affect the piston’s position (FKC) as well as the combustion ratio during the system’s motion, in addition to causing possible collisions of the piston with additional components (e.g. piston/valve).

According to the modified methodology, the mathematical relation between the FKC and the system’s characteristics and the appearing deviations has to be defined. This usually requires that a time-dependent vector-chain be established. In order to take into account the appearing interactions, appropriate meta-models will be determined that represent the affected deviations. These meta-models can be approximations, which can be integrated into the already formulated vector-chain-based functional relation. Finally, the prediction qualities of the meta-models will be determined to ensure an effective and reliable tolerance analysis.

Tolerance analysis of systems in motion considering interactions between deviations

The existing “integrated tolerance analysis” methodology does not consider possible interactions between the different deviations and the resulting effects among themselves, as well as the effects on the FKCs. The tolerance analysis of a system in motion can be divided into three main steps. First, the time-dependent mathematical relations between the system’s varying characteristics and the FKCs are needed. Therefore, time-dependent vector-chains are established, which include the system’s characteristics as well as the appearing dimensional and geometric deviations. The second step includes the tolerance analysis method.

The existing methodology uses a Monte Carlo simulation to determine the FKCs for a defined number of samples of the system. The representation and interpretation of the results is the final step.^{3,4}

The extension of the existing methodology toward considering interactions between appearing deviations requires two main modifications. First, the interactions need to be taken into account when formulating the functional relation. Therefore, appropriate approximations (meta-models) of the affected deviations replace the vectors (representing the terms of these deviations) in the relation’s vector-chain. Consequently, the numerical expense of the tolerance analysis can be reduced significantly. Second, the interactions and their effects on the FKCs need to be taken into account during the representation and interpretation of the tolerance analysis results. Therefore, the common contributor analysis (e.g. high–low–median) is replaced by a global sensitivity analysis in order to identify the deviation’s main and total effects. Figure 2 details the modifications of the methodology.

Because the second modification can be realized quite easily, this article will focus on the first modification: the determination and integration of appropriate meta-models into the functional relation.

Based on the definition of the considered system, the appearing deviations must be defined and the corresponding interactions identified. The systematic and random deviations, which affect a system’s FKCs, can be classified according to the stage of the product’s life cycle in which they appear (manufacture, assembly and use). According to this classification, the interactions can be identified considering the deviation’s appearance in time. Interactions always appear between deviations if there is a difference in time between these deviations’ appearances. For example, because manufacturing-caused deviations appear earlier in the product’s life cycle, they also affect the assembly-caused and the operation-dependent deviations. The effects of different kinds of deviations on the FKCs, as well as the corresponding interactions among themselves (dotted and dashed arrows), are shown in Figure 3.

Figure 3. Interactions (dotted and dashed arrows) between the appearing deviations of a mechanism in use.

Figure 4. Methodology for the integration of interactions between deviations in the tolerance analysis' functional relation. CAX: computer-aided x; MBD: multibody dynamics; FEM: finite element method.

Furthermore, two different forms of interactions appear due to the characteristics of the deviations: systematic and random deviations. The interactions toward random deviations (dashed arrows) can be considered in a Monte Carlo-based tolerance analysis by correcting the covariance matrix (entries aside the principal diagonal). However, the integration of the remaining form of interactions (the effects of systematic and random deviations on systematic deviations of later life cycle stages, dotted arrows) can be much more complex and, therefore, requires a separate methodology, as detailed in Figure 4.

As mentioned earlier, the terms of the systematic deviations in the functional relation have to be replaced by appropriate models (meta-models) that represent the appearing deviations as well as the interactions toward these deviations. At first, a defined number of samples of the nonideal mechanism's deviations and varying parameters, which cause the considered interactions, must be generated. Usually, this requires far less samples than the generation of virtual samples of the system during the second step of the tolerance analysis. Therefore, methods of the design of experiments (DoE) can be used. In the case of technical systems in motion,

the focus should be on Monte Carlo-based sampling methods (e.g. Latin Hypercube Sampling) because the interactions toward random deviations (first form of interactions) can also be considered. Subsequently, the time-depending systematic deviations can be determined for each of the generated nonideal systems using CAX tools. For instance, the deformation of a part can be determined using a flexible multibody dynamics (MBD) simulation (coupling of MBD and finite element analysis (FEA)). Because these deviations essentially affect the system's FKC (by causing a mean shift), an expert is needed to set up the necessary simulation models. This ensures a realistic and reliable data set. Based on the determined data set, the meta-models can be generated/trained. In addition to the well-known RSM,³⁷ a variety of additional meta-modeling techniques can be used. As detailed in section "Use of meta-modeling in tolerance management and robust design," tolerance-related publications deal inter alia with ANNs, Kriging and SVR. The data set is split into two different sets. One is used to train the meta-model, and the remaining (test) samples are used to evaluate the prediction quality of the meta-model according to the so-called goodness-of-fit parameters,³⁸ such as the

mean squared error, R^2 —the coefficient of determination³⁸ or the coefficient of prognosis (COP).³⁹ To obtain the training and the test samples, different validation strategies can be applied, which differ in the way that the data set is split. The split validation splits the data set in the commonly used ratio of 70 training samples to 30 test samples. The 10-fold cross validation uses a ratio of 90:10. However, the separation is done 10 times, ensuring that each sample belongs to the test data set once. Additional validation strategies are based on a random constellation of the training and test data sets. If a satisfying prediction quality of the meta-models can be achieved, these meta-models can finally replace the terms in the functional relation, which represent the systematic deviations. Subsequently, with the modified functional relation, the product developer can proceed to the second step of the tolerance analysis, the application of the tolerance analysis method.

Demonstrator: crank mechanism

The precise motion of the crank mechanism is affected by appearing deviations. These deviations can cause collisions of the components among themselves (e.g. piston/crankshaft) or collisions with additional engine parts (e.g. piston/valves). Moreover, the engine's combustion ratio is affected due to the mechanism's varying position of the piston (FKC). The crank rotates at 3000 r/min resulting from a time-dependent piston pressure with a maximum of 180 bar.³ A motion sequence of the crank mechanism consists of two crank rotations with a total crank angle of $\varphi = 720^\circ$. In addition to varying operation parameters, different deviations also appear:

- manufacturing deviations (crank radius r , con-rod link length l , position deviation of the piston);
- operation-dependent deviations (deformation of the crankshaft, displacement due to joint clearance $s = 0.06$ mm in the lower con-rod bearing).

Table 1 details the varying parameters with the corresponding specification limits and distributions. The crank mechanism and the appearing manufacturing deviations are shown in Figure 5.

The defined specification limits of the engine oil viscosities result from a selection of 12 different oils (viscosity class: 5W30). The limits of the crankshaft's Young's modulus and density are based on the specifications of the material 42CrMo4 (US system: AISI 4140) taken from material databases as well as from the manufacturer's material specifications.

Integration of interactions between the crank mechanism's deviations into the functional relation

In order to show the modified methodology's practical use, the functional relation of the nonideal crank mechanism will be determined. As detailed, the mechanism is subject to both manufacturing-caused and operation-dependent deviations, during a motion sequence consisting of two crank rotations.

Therefore, a time-dependent vector-chain is set up, including the characteristics of the crank mechanism's components. The appearing deviations are included in the vector-chain by additional vectors. Hence, the relation between the position of the piston (FKC) and the appearing deviations can be formulated. For a detailed derivation of the functional relation, see the study by Stuppy and Meerkamm.³ However, because this relation does not consider the interactions toward the operation-dependent deviations (deformation and displacement), it has to be modified according to the presented methodology (Figure 4).

DoE

In order to generate/train appropriate meta-models, the effects on the operation-dependent deviations of the crank mechanism need to be investigated for several nonideal crank mechanisms. Therefore, analyses based on the Monte Carlo Sampling are usually used. However, a large number of samples are necessary, which can be computationally expensive and thus time-consuming.⁴⁰ The samples for the simulations will be generated using Latin Hypercube Sampling, which requires far fewer samples to combine plausibly parameter values according to their corresponding distributions.⁴¹

Table 1. Varying parameters of the crank mechanism.

Parameter	Mean	Lower specification limit	Upper specification limit	Distribution
Crank radius r	45 mm	44.98 mm	45.02 mm	Triangle
Con-rod link length l	138 mm	137.95 mm	138.05 mm	Trapeze
Position deviation e (along Y_{glob})	0 mm	-0.02 mm	0.02 mm	Normal ($\pm 3\sigma$)
Crankshaft density (at 100 °C)	7.74 kg/dm ³	7.73 kg/dm ³	7.75 kg/dm ³	Uniform
Crankshaft Young's modulus	206 GPa	205 GPa	207 GPa	Uniform
Oil viscosity (kinematic, 40 °C)	68.4 mm ² /s	64.0 mm ² /s	72.8 mm ² /s	Uniform
Oil viscosity (kinematic, 100 °C)	11.1 mm ² /s	10.0 mm ² /s	12.2 mm ² /s	Uniform
Oil temperature	100 °C	98 °C	102 °C	Uniform

Figure 5. Crank mechanism with appearing deviations, coordinate systems (X_{glob} – Y_{glob} ; X_{rot} – Y_{rot}) and displacement parameters ε and δ .

Figure 6. Difference of the force $F_{X_{rot}}$ between S_1 – S_5 and the ideal system.

The number of samples for the simulations depends on two diverging requirements: In order to achieve good and reliable results, many samples should be generated. However, a higher number of samples lead to an increase in the simulation time. In this case, ANNs and response surfaces (second order) are used to generate/train the meta-models. ANNs do not have any restrictions concerning a minimum sampling size. However, according to equation (1) with eight varying input parameters, 45 samples are necessary in order to define a second-order response surface. Additional samples will be used to determine the response surface's approximation error (usually, a 50% oversampling is recommended) as well as to evaluate the prognosis quality of these meta-models. Consequently, 100 samples of the nonideal crank mechanism are generated.

Determination of the operation-depending deviations

Both the deformation of the crankshaft and the displacement of the con-rod are caused by the forces on the crankshaft appearing during the crank mechanism's use. In order to investigate the effects on the operation-depending deviations, first a multibody simulation of the nonideal crank mechanism has to be performed in order to determine the corresponding forces on the crankshaft for each of the 100 samples.⁴² The differences between the force components of five randomly chosen samples (samples S_1 – S_5) and the ideal mechanism are shown in Figure 6.

Deformation of the crankshaft. In order to determine the elastic deformation as a function of time during a motion sequence of the crank mechanism, additional 144 FEAs (5°-steps of the crank's rotation) were performed for each sample. Figure 7 displays the components of the resulting deformation of the ideal mechanism's crankshaft according to the rotating coordinate system X_{rot} – Y_{rot} , which is fixed to the crankshaft. The varying deformations ($Def_{X_{rot}}$) of the nonideal crank mechanism S_1 – S_5 are shown in Figure 8.

Displacement of the con-rod due to joint clearance. Due to the clearance of the revolute joint and the appearing forces, a displacement of the con-rod relative to the crankshaft appears. The determination of each sample's time-depending displacement is based on a hydrodynamic consideration of the revolute joint's behavior.⁴³ The displacement is given in polar coordinates using the radial eccentricity ε and the corresponding orientation angle δ (Figure 5). For a detailed determination of

Figure 7. Deformation of crankshaft $Def_{X_{rot}}$ and $Def_{Y_{rot}}$ (ideal system).

Figure 8. Difference of $Def_{X_{rot}}$ between S_1 – S_5 and the ideal system.

Figure 9. Displacement of con-rod ε and δ (ideal system).

the hydrodynamic displacement, see Stuppy and Meerkamm's study.⁴⁴ The time-dependent displacement (ε and δ) of the ideal mechanism as well as the deviations of the eccentricity of the five nonideal samples are shown in Figures 9 and 10.

Meta-modeling

Several meta-modeling techniques can be used to generate the necessary meta-models of the operation-depending crankshaft's deformation and the con-rod's displacement. In the context of interactions between a mechanism's deviations, the considered techniques are the widely used RSM,⁶ ANNs and the SVR. However, previous investigations show that SVR-based meta-models are insufficient (low prediction qualities).⁴⁵ In contrast, the use of ANNs in tolerance analyses is recommended by Andolfatto et al.³⁴ It is stated that the prediction qualities of ANNs "can be seen as

Figure 10. Difference of ε between S_1 – S_5 and the ideal system.

satisfactory for many tolerancing applications."³⁴ Moreover, Simpson et al.⁶ recommend ANNs for "repeated, nonlinear and deterministic applications," even as statistical tolerance analyses take into account time-dependent systematic operation-depending deviations and the corresponding interactions toward these deviations. Consequently, the meta-modeling techniques applied in this article are the RSM and ANNs. A second-order response surface approximates the mathematical relation between the varying operation parameters and manufacturing deviations x_i and the four parameters of the operation-depending deviations y , using a quadratic approximation (β_i : regression coefficients)³⁷

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j=2}^k \sum \beta_{ij} x_i x_j + \vartheta \quad (1)$$

Table 2. COPs of the determined meta-models ($n = 100$).

Deviation	RSM	ANN
$\text{Def}_{x_{\text{glob}}}$	0.031	0.985
$\text{Def}_{y_{\text{glob}}}$	0.038	0.990
ε	0.001	0.999
δ	0.941	0.999

RSM: response surface methodology; ANN: artificial neural network.

However, there still remains an approximation error ϑ . Therefore, additional samples (taken from the oversampling) are available to determine this error based on the method of least squares.

An ANN is a mathematical model, which contains several artificial neurons and connections among them. Each neuron receives an input vector and delivers a certain output, depending on the synaptic weight w_i and a transfer function f . Usually, the ANNs use sigmoid functions for approximation purposes.⁴⁶ After the training, the network can calculate the correspondent output y to a specified combination of input values x_i .³⁴

$$y = f\left(\sum_i w_i \cdot x_i\right) \quad (2)$$

Evaluation of the meta-model's prediction quality

Based on the determined data set, four meta-models need to be determined (deformation: $\text{Def}_{x_{\text{rot}}}$ and $\text{Def}_{y_{\text{rot}}}$; displacement: ε and δ) to be integrated into the functional relation. Therefore, a repeated random subsampling validation is used. In contrast to the split validation, this validation method splits the data set n times, whereby the allocation of the samples to the training-set or to the test-set is random, with a split ratio of 90:10.⁴⁷ Therefore, for each of the four parameters, $n = 100$ meta-models are generated/trained and evaluated.

The remaining 10 (test) samples of each validation run are used to evaluate the prediction quality of the meta-models, according to the COP. Therefore, the standard deviations σ of the sample distributions Y_{test} and Y_{train} as well as the mean E are needed. The COP ranges between 0 and 1, and a COP of 0.5 is equal to 50% prediction quality³⁹

$$\text{COP} = \left(\frac{E(Y_{\text{test}} \cdot Y_{\text{train}})}{\sigma_{Y_{\text{test}}} \cdot \sigma_{Y_{\text{train}}}} \right)^2 \quad (3)$$

The highest COPs of the response surfaces and the ANNs (two hidden layers with 10 neurons each) are listed in Table 2. Due to their exceptionally high prediction qualities, the ANNs should be integrated into the functional relation.

Because the functional relation of the crank mechanism could be modified, the product developer can

proceed to the following steps of the “tolerance analysis of systems in motion” methodology: the application of the tolerance analysis method and the representation and interpretation of the results.

Conclusion

This article focused on the extension of the existing “integrated tolerance analysis of systems in motion” methodology.^{3,4} Because deviations appear with a difference in time during the product's life cycle, they cause both effects on a mechanism's FKCs and effects among themselves, caused by the so-called interactions. These interactions lead to additional variation of the time-dependent FKCs. Consequently, the interactions between deviations need to be taken into account during statistical tolerance analyses; especially, when the system's components underlie many deviations.

A methodology was presented that supports the product developer in systematically determining appropriate mathematical models, which represent the affected deviations, as well as the effects of the appearing interactions. These meta-models can be integrated into the tolerance analysis process by modifying the functional relation. In order to show the methodology's practical use, the interactions between random manufacturing-caused and systematic operation-depending deviations of a nonideal crank mechanism were analyzed and by means of ANNs integrated into the statistical tolerance analysis. The modified tolerance analysis methodology enables the product developer to gain information about the effects of deviations on a FKCs of a system in motion, as well as the effects of the deviations among themselves. Furthermore, the use of meta-models results in a significant reduction in time and computational expense of statistical tolerance analyses. However, it is obvious that the variation of the FKCs, caused by the appearing interactions between deviations, is usually smaller than the variation due to the deviations itself. Nevertheless, the higher the given requirements concerning the system's precision and quality as well as the number of deviations, the more interactions between deviations should be considered during a statistical tolerance analysis of a system in motion.

Funding

The research project ME1029/16-1 “Functional product validation and optimization of technical systems in motion as a part of product's lifecycle oriented tolerance management” was supported by German Research Foundation (DFG).

Acknowledgements

The authors would like to thank the anonymous referees for their helpful comments, which helped to improve the article.

References

- Walter M, Breitsprecher T, Gruber G, et al. Simulation based generation of an initial design taking into account geometric deviations and deformations. In: *Proceedings of the 18th international conference on engineering design*, Copenhagen, Denmark, 15–18 August 2011, pp.78–90.
- Armillotta A and Semeraro Q. Critical operating conditions for assemblies with parameter-dependent dimensions. In: *Proceedings of the 12th CIRP conference on computer aided tolerancing*, Huddersfield, UK, 18–19 April 2012, paper no. 70.
- Stuppy J and Meerkamm H. Tolerance analysis of a crank mechanism by taking into account different kinds of deviations. In: *Proceedings of the 11th CIRP international seminar on computer aided tolerancing*, Annecy, France, 26–27 March 2009, paper no. A4-1.
- Wartzack S, Meerkamm H, Stockinger A, et al. Lebenszyklusorientierte Toleranzsimulation zur funktionalen und ästhetischen Produktsicherung. *Konstruktion* 2011; 6: 63–67.
- Shah JJ, Ameta G, Shen Z, et al. Navigating the tolerance analysis maze. *Comput-Aided Design Appl* 2007; 5: 705–718.
- Simpson TW, Peplinski JD, Koch PN, et al. Metamodels for computer-based engineering design: survey and recommendations. *Eng Comput* 2001; 17: 129–150.
- Morrison SJ. The study of variability in engineering design. *J Roy Stat Soc C: App* 1957; 2: 133–138.
- Hasenkamp T, Arvidsson M and Gremyr I. A review of practices for robust design methodology. *J Eng Design* 2009; 6: 645–657.
- Adabi ME, Farkhondeh S and Farkhondeh F. Tolerance analysis of mechanisms by using a fictitious slider. *Eur J Sci Res* 2010; 3: 385–398.
- Choi DH, Wickert JA and Yoo HH. Effect of manufacturing tolerances on dynamic equilibria of multibody systems undergoing prescribed rotational motion. *J Mech Sci Technol* 2008; 22: 1747–1756.
- Germer C. Statistische Toleranzanalyse von beweglichen Karosserie-Anbauteilen. *Konstruktion* 2010; 7: 44–47.
- Bruyère J, Dantan JY, Bigot R, et al. Statistical tolerance analysis of bevel gear by tooth contact analysis and Monte Carlo simulation. *Mech Mach Theory* 2007; 42: 1326–1351.
- Sacks E and Joskowicz L. Parametric kinematic tolerance analysis of general planar systems. *Comput Aided Design* 1998; 30: 707–714.
- Muvengi O, Kihui J and Ikua B. Dynamic analysis of multi-body mechanical systems with imperfect kinematic joints: a literature survey and review. *Sustain Res Innov Proc* 2011; 3.
- Dupac M and Beale DG. Dynamic analysis of a flexible linkage mechanism with cracks and clearance. *Mech Mach Theory* 2010; 45: 1909–1923.
- Imani BM and Pour M. Tolerance analysis of flexible kinematic mechanism using DLM method. *Mech Mach Theory* 2009; 44: 445–456.
- Guowei L, Weimin Z and Can C. Tolerance analysis considering the impact of working conditions on product performance. In: *Proceedings of the 2nd international conference on industrial mechatronics and automation*, Wuhan, China, 25–27 June 2010, pp.47–51.
- Hanzaki AR, Rao PVM and Saha SK. Kinematic and sensitivity analysis and optimization of planar rack-on-pinion steering linkages. *Mech Mach Theory* 2009; 44: 42–56.
- Kharoufeh JP and Chandra MJ. Statistical tolerance analysis for non-normal or correlated normal component characteristics. *Int J Prod Res* 2002; 40: 337–352.
- Huang X and Zhang Y. Robust tolerance design for function generation mechanisms with joint clearances. *Mech Mach Theory* 2010; 45: 1286–1297.
- Hong SY and Chang TC. A comprehensive review of tolerancing research. *Int J Prod Res* 2002; 11: 2425–2459.
- Dantan JY, Gayton N, Dumas A, et al. Mathematical issues in mechanical tolerance analysis. In: *Proceedings of the 13th Colloque National AIP PRIMEC*, Le Mont Dore, France, 27–30 March 2012, pp.1–12.
- Wynn H and Ograjensek I. Is statistics becoming the number one discipline for business and industry? *Qual Reliab Eng Int* 2010; 8: 789–793.
- Kim JK and Cho RC. The use of response surface designs in the selection of optimum tolerance allocation. *Qual Eng* 2000; 1: 35–42.
- Sehirlioglu AK and Özler C. The use of mixture experiments in tolerance allocation problems. *Int J Adv Manuf Tech* 2008; 7–8: 769–777.
- Chen MC. Tolerance synthesis by neural learning and nonlinear programming. *Int J Prod Econ* 2001; 1: 55–65.
- Jayaprakash G, Sivakumar K and Thilak M. Parametric tolerance analysis of mechanical assembly by developing direct constraint model in CAD and cost competent tolerance synthesis. *Intell Contr Autom* 2010; 1: 1–14.
- Wang H, Yu TX, Pang H, et al. Mechanism reliability simulation analysis for multi-support axis seizure considered assembly tolerance. In: *Proceedings of the international conference on quality, reliability, risk, maintenance, and safety engineering*, Xi'an, China, 17–19 June 2011, pp.35–41.
- Schleich B, Stockinger A and Wartzack S. On the impact of geometric deviations on structural performance. In: *Proceedings of the 12th CIRP conference on computer aided tolerancing*, Huddersfield, UK, 18–19 April 2012.
- Lepadatu D, Kobi A, Baguenard X, et al. Springback of stamping process optimization using response surface methodology and interval computation. *Qual Technol Quant Manag* 2009; 4: 409–421.
- Watrin S, Binz H and Van de Lindeloof B. Methodology for requirement-driven tolerance specification of bevel gears. In: *Proceedings of the 12th CIRP conference on computer aided tolerancing*, Huddersfield, UK, 18–19 April 2012.
- Huele AF and Engel J. A response surface approach to tolerance design. *Stat Neerl* 2006; 3: 379–395.
- Kim YK, Kim SI, Rhyu SH, et al. Tolerance allocation in electric machine based on stochastic response surface methodology. In: *Proceedings of the XIX international conference on electrical machines*, Rome, Italy, 6–8 September 2010, pp.1–4.
- Andolfatto L, Thiébaud F, Douilly M, et al. On neural networks' ability to approximate geometrical variation propagation in assembly. In: *Proceedings of the 12th CIRP conference on computer aided tolerancing*, Huddersfield, UK, 18–19 April 2012, paper no. 27.

35. Kopardekar P and Anand S. Tolerance allocation using neural networks. *Int J Adv Manuf Tech* 1995; 4: 269–276.
36. Barbato G, Panciani GD, Ricci F, et al. Form tolerance verification using the Kriging method. In: *Proceedings of the ENBIS-DEINDE 2011 spring conference*, Torino, Italy, 16–18 March 2011.
37. Myers R. *The response surface methodology—process and product optimization using designed experiments*. 3rd ed. NJ: John Wiley & Sons, 2009.
38. Cameron AC and Windmeijer FAG. An R-squared measure of goodness of fit for some common nonlinear regression models. *J Econometrics* 1997; 77: 329–342.
39. Most T and Will J. Meta-model of optimized prognosis—an automatic approach for variable reduction and optimal meta-model selection. In: *Weimar optimization and stochastic days 5.0*, Weimar, Germany, 12–13 November 2008.
40. Stockinger A, Wokusch AL, Walter M, et al. Impact of probability distributions and covariances on numerical simulation results in engineering design. In: *Proceedings of the 16th international conference on industrial engineering—theory, applications and practice*, Stuttgart, Germany, 20–23 September 2011.
41. McKay MD, Beckman RJ and Conover WJ. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics* 1970; 21: 239–245.
42. Walter M and Wartzack S. Analysis of the effects of manufacturing-caused deviations and varying operation parameters on operation-depending deviations of systems in motion. In: *Proceedings of the 12th CIRP conference on computer aided tolerancing*, Huddersfield, UK, 18–19 April 2012, paper no. 55.
43. Holland J. *Beitrag zur Erfassung der Schmierverhältnisse in Verbrennungsmaschinen*. PhD Thesis, Verein Deutscher Ingenieure, Germany, 1959.
44. Stuppy J and Meerkamm H. Tolerance analysis of mechanisms taking into account joints with clearance and elastic deformations. In: *Proceedings of the 17th international conference on engineering design*, Stanford, CA, 24–27 August 2009, pp.488–500.
45. Walter M, Sprügel T and Wartzack S. Tolerance analysis of mechanism taking into account the interactions between deviations using meta-models. In: *Proceedings of the 9th Nord-design*, Aalborg, Denmark, 22–24 August 2012, pp.587–594.
46. Kleijnen JP. Kriging metamodeling in simulation: a review. *Eur J Oper Res* 2009; 3: 707–716.
47. Efron B. *The Jackknife, the Bootstrap, and other resampling plans*. 1st ed. Philadelphia, PA: Society for Industrial and Applied Mathematics, 1982.