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Citation: [Physics of Fluids \(1994-present\)](#) **26**, 053303 (2014); doi: 10.1063/1.4874604

View online: <http://dx.doi.org/10.1063/1.4874604>

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How do neighbors affect incipient particle motion in laminar shear flow?

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(Received 24 July 2013; accepted 11 April 2014; published online 9 May 2014)

We experimentally study how neighboring particles affect the incipient motion of particles on regular substrates and exposed to a laminar shear flow. To this end, we determine the critical Shields number and determine whether the particle rolls or slides. The substrates consist of a monolayer of fixed spheres of uniform size that are regularly arranged in triangular and quadratic configurations. Neighboring particles influence the incipient motion by shielding to the shear flow and may inhibit continuous motion once they are in direct contact with the particle. At the low particle Reynolds numbers studied, neighboring spheres on the monolayer only affect the incipient particle motion if they are closer than about 3 particle diameters. Direct contact inhibits continuous motion and results in a strong increase of the critical Shields number. For identical beads, we found two different regimes for the onset of continuous motion. Depending on the substrate geometry, the upstream particle may start to roll like a single particle passing the downstream neighbor or it may push its downstream neighbor forward. In the latter case, the downstream sphere rolls while the upstream bead slides in contact with the downstream neighbor. Both regimes yield about the same critical Shields number although the critical Shields number for single particle motion differs by about 50%. If particle contact is avoided by a sudden jump in the Shields number, the critical Shields number for onset of continuous particle motion can be reduced considerably. Finally, the lowest critical Shields numbers for dislodging buried beads in the configurations studied coincides with the critical Shields number for incipient motion of irregular granular beds.

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I. INTRODUCTION

Incipient particle motion has been studied intensively in hydrology, geology, and civil engineering. It is encountered in several environmental systems like sediment transport in rivers, bed erosion, or dune formation.^{1–4} Onset of particle transport induced by shear flows also finds applications in industrial operations such as pneumatic conveying, filtration, or cleaning of surfaces in food and pharmaceutical industries. Since typical bed-load in rivers or pneumatic transport applications involve turbulent conditions and heterogeneous particle size distributions, most of the studies in these fields have been carried out under these conditions.^{5–7} In industrial operations such as heavy oil transportation in pipes,⁸ the internal mechanism involved in filtration processes⁹ or microfluidics, particle motion is also encountered in laminar flows. The template-assembly of microparticles,¹⁰ the assembly of coupled microdevices in confined structured geometries¹¹ or the intrinsic particle-induced lateral transport in microchannels,¹² are examples of applications at low Reynolds numbers. Even in natural phenomena as in the last phase of outburst floods, the process is characterized by a viscous regime and dominated by channel bed friction.¹³ Therefore, several authors have focused recently on the initiation of particle transport under laminar flow conditions.^{14–16} The critical Shields

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number, θ_C , is in the range between 0.02 and 0.04 for the incipient motion of a single sphere on top of an irregular sediment bed¹⁷ and it is about 0.12 for saturated sediment beds.¹⁸ Accordingly, theoretical models^{19,20} and direct numerical simulations²¹ have been developed for low Reynolds number flows.

The importance of the local grain arrangement on the incipient motion has been considered in models for incipient motion under turbulent conditions. Some authors addressed it by geometrical parameters such as the angle of repose,^{22,23} exposure to the stream flow,²⁴ relative grain protrusion,²⁵ or slope.²⁶ While other authors suggested modeling the angle of repose, which coincides with the pivot angle for non-cohesive single spheres,²⁷ and exposure as independent parameters,^{28,29} Martino *et al.* showed that it is possible to combine them into one for describing their experiments on the incipient motion of a cylinder on varying bottoms.³⁰ Under laminar conditions, the studies of Charru *et al.* for a single bead on an irregular layer of wall-fixed spheres pointed to the significant role of the bed geometry on the fluctuating particle motion.¹⁷ In later simulations of disordered assemblies of uniformly sized spheres attached to a flat bottom, Derksen and Larsen showed that the mean drag and lift forces on the beads sensitively depend on the mean distance between the spheres.³¹ In recent experiments, we corroborated the strong impact of the substrate geometry on the incipient motion of a single bead.³² Depending solely on the substrate's geometrical arrangement, we obtained critical Shields numbers ranging between 0.015 and 0.06 in laminar shear flows. These values for incipient motion of a single particle resulted considerably lower than those of irregularly arranged granular beds. This may indicate that the burial degree in these beds is quite high and that interaction with neighboring particles may play a significant role.

The role of different mechanisms taking place in incipient particle motion is mostly discussed in literature for turbulent conditions,^{33–35} being different for individual or multiple particles. While for a single exposed particle in near critical conditions, where shielding effects are minor, initial motion is supposed to occur by rolling,^{35,36} for individual particles that are hidden or almost completely shielded by neighbors, lifting seems to be the most appropriate mechanism.³⁶ Many authors therefore considered rolling^{5,24,37} or lifting^{36–38} as the initial particle motion in their models. Besides a particle at rest, a spherical particle rotating in place is also considered as initial configuration before dislodgment.²³ On the other hand, Paintal used the sliding probability in his model.³⁹ For laminar conditions, rolling and sliding are regarded as the main mechanisms during the onset of motion,¹⁵ yet lift forces are usually considered to be too small to initiate lifting directly in granular beds of mobile beads.^{31,37,40} It still remains an open question, however, how the particle bed geometry or interactions between neighboring beads affect the mechanism on the onset of particle motion.

In the present article, we study the influence of geometrical bed properties on the onset of motion of multiple uniformly sized beads. We vary the angle of repose by using different substrates. In each of them the exposure degree, defined as the ratio of the cross-sectional area exposed to the incident flow to the total cross-sectional area of the particle,^{28,29} can be zero by placing neighboring beads upstream. We show how neighboring particles affect incipient bead motion by altering the flow field and by direct contact between adjacent particles. Beyond a critical Shields number, beads leave their initial positions on the substrate. If they encounter neighbors downstream, these neighbors may hinder the bead from moving once in contact. Then the particle may stay in a position of dynamic equilibrium where the angle of repose is significantly higher than that of static equilibrium on the substrate. We characterize the incipient motion, distinguishing between particle rolling and sliding for different bed configurations. In Sec. II, we describe the experimental setup. The results are reported in Sec. III and discussed in Sec. IV. The conclusions are summarized in Sec. V.

II. EXPERIMENTAL SET-UP

We use regular substrates with quadratic and triangular arrangements of soda-lime glass beads of $(405.9 \pm 8.7) \mu\text{m}$ diameter from The Technical Glass Company. The triangular substrate consists of densely arranged beads. In the quadratic arrangement, we used two substrates with different spacing between the particles. The narrow one has a spacing of $14 \mu\text{m}$ and for the wider one it is $109 \mu\text{m}$. The spacing was determined using sieves of different mesh sizes. Their geometrical properties are given in Table I. Microscopic pictures of the bed geometries are shown in Figure 1. In

TABLE I. Substrate properties.

Configuration	Particle spacing (μm)
Triangular	$0 + 4$
Quadratic	14 ± 12
Quadratic	109 ± 20

this article, the flow was always along the horizontal axis of the pictures in Figure 1. Thus, placing in front of the mobile beads neighboring particles of same size, results in an exposure degree of zero in each of the substrates studied, while the angle of repose remains different. Further details on the substrate preparation are found in Ref. 32.

Like in Ref. 32, the substrates are fixed on supports and placed on the bottom of a circular container with an inner diameter of 176 mm and with 25 mm high sidewalls made from Plexiglas. The container is placed in an MCR 301 rotational rheometer from Anton Paar. The experiments are carried out in a parallel-disk configuration, where the gap width, h , defined as the distance from the top of the substrate spheres to the rotating plate, and the angular frequency, Ω , are controlled with the rheometer. All experiments were carried out at a gap width of 2 mm.

We used substrates of two different sizes: $15 \times 15 \text{ mm}^2$ and $15 \times 70 \text{ mm}^2$. The smaller ones are identical to those used for studies on a single particle.³² In this case, the beads are located in the middle of the substrate. The substrates themselves are placed off-centered into the container such that the mobile beads are at a distance, r , of 21 mm from the rheometer's turning axis. In this case, we used a glass plate of 65 mm diameter as the top plate. As shown in Ref. 32, the chosen parameter range is independent from any boundary effects within the range of particle Reynolds numbers considered. The larger substrates, together with a Plexiglas or glass plate of 150 mm diameter, are used for studies of several mobile spheres. In these configurations, the mobile beads are placed at a distance of about 35 mm from the substrate's upstream edge and the substrate itself is placed off-centered into the container such that the mobile beads are at a radial distance of 50 mm from the turning axis.

Fluids and particles are chosen to change the relative density difference between particle and liquid, $\rho_S/\rho - 1$, where ρ_S and ρ are particle and liquid densities, respectively, as well as the range of Reynolds number, Re , and of the corresponding particle Reynolds number, Re_P , which remains below 1 for all experiments. These dimensionless numbers are defined as

$$\text{Re} = \frac{\Omega r h}{\nu}, \quad \text{Re}_P = \text{Re} \left(\frac{D_P}{h} \right)^2, \quad (1)$$

where ν is the kinematic viscosity and D_P is the particle diameter. As liquids we used two silicone oils from Basildon Chemicals with dynamic viscosities of $(9.95 \pm 0.30) \text{ mPa s}$ and $(103.0 \pm 3.3) \text{ mPa s}$ and densities of $(0.935 \pm 0.005) \text{ g/cm}^3$ and $(0.965 \pm 0.005) \text{ g/cm}^3$ at a temperature of $(295.16 \pm 0.5) \text{ K}$, respectively. The operating temperature remains fixed at this value, controlled with a P-PTD 200 Peltier element connected to the rheometer and measured with an external thermometer. Particle density was varied by using beads of four different materials: PMMA spheres from Microparticles

FIG. 1. Microscopic top view of the substrates of identical and regularly arranged spherical glass beads of $(405.9 \pm 8.7) \mu\text{m}$ diameter. Triangular configuration (a) and quadratic configuration with a particle spacing of $14 \mu\text{m}$ (b).

GmbH with a density of $(1.190 \pm 0.002) \text{ g/cm}^3$ and a diameter of $(406.0 \pm 9.5) \mu\text{m}$, soda lime glass spheres from the Technical Glass Company with a density of $(2.530 \pm 0.025) \text{ g/cm}^3$ and a diameter of $(405.9 \pm 8.7) \mu\text{m}$, steel spheres from Nanoball GmbH with a density of $(7.73 \pm 0.02) \text{ g/cm}^3$ and a diameter of $(400 \pm 1) \mu\text{m}$ and tungsten-carbide/cobalt (94:6) spheres with a density of $(14.95 \pm 0.03) \text{ g/cm}^3$ and a diameter of $(400 \pm 20) \mu\text{m}$ from Goodfellow.

The incipient motion is characterized by the critical Shields number, which compares the characteristic shear stress acting on the particle to the resistant specific particle weight that retains it in place. For our configuration, the Shields number reads:³²

$$\theta = \frac{\nu \Omega r}{(\rho_s/\rho - 1)hgD_p}, \quad (2)$$

where g is the acceleration of gravity.

The onset of particle motion is usually determined by increasing the speed of the rotating plate in small steps of less than 0.5% until the beads move to an adjacent neighboring position. In some experiments, the beads are submitted to a sudden shear-rate jump with a step width of about 0.02 s from subcritical conditions to a Shields number that is kept constant during 20 s. This time exceeds considerably the time interval needed for particles to move to the next equilibrium position.³²

The particles are illuminated and detected through the rotating disk. Therefore, we use a digital camera with a chip of 1280×1024 pixels and equipped with a macro objective that incorporates a tilted mirror. The particle location is tracked optically and evaluated with image processing software. For further details we refer to Ref. 32. Some particles are marked to analyze the mechanism of motion during displacement, i.e., rolling or sliding. Therefore, Plexiglas and glass spheres are sprayed using a black paint after passing through a fine sieve. Steel and tungsten-carbide/cobalt spheres are manually marked. We remark that using these marked spheres, we obtained the same critical Shields number as for beads without marks within measurement uncertainty.

III. EXPERIMENTAL RESULTS

Particles may affect the incipient motion of their mobile neighbors by altering the flow field or via direct contact. First we address these effects by studying the impact of immobile particles on the critical Shields number of beads of same diameter. Then we report the results obtained for two identical beads, where we also analyze the different mechanisms of motion (i.e., rolling and sliding). Thereafter incipient motion of multiple identical particles is studied. Finally we describe the results for the incipient motion of completely buried beads.

One of the immediate effects of neighboring particles is shielding to shear flow. Reduction in exposure to shear flow results in an increase of the critical Shields number. To focus on the impact of altering the flow field, we place tungsten-carbide/cobalt spheres as stationary neighboring particles on the substrate. Due to their high density, the tungsten-carbide/cobalt particles start moving at much higher shear rates than the PMMA, glass, and steel particles used. This allows studying independently shielding by upstream and downstream placed neighboring particles.

Figure 2 shows the critical Shields number as a function of the distance between immobile tungsten-carbide/cobalt and mobile bead for different substrate geometries. Negative values for the position indicate that the immobile particles are located downstream of the mobile bead (see inset of Figure 2). Like in all diagrams presented here, the experiments have been repeated five times and the error bars represent the standard deviation. Immobile particles affect the onset of motion upstream as well as downstream of the free beads. Their influence decreases with distance. Beyond about three particle diameters we do not find any shielding within measurement uncertainty. This holds for all substrates studied. Furthermore, the critical Shields number remains independent from the mobile bead density and from the particle Reynolds number within the range studied.

If the heavy particle is placed at the adjacent downstream position of the mobile bead (i.e., position -1), the immobile particle serves as an obstacle. It stops the moving sphere at contact and hinders it to travel to another equilibrium position on the substrate. Only beyond a, in many cases, considerably higher Shields number the mobile bead passes its immobile neighbor.

FIG. 2. Critical Shields number as a function of the distance between the mobile bead and its immobile neighbor. Triangles, circles, and rhomboids: triangular substrate and quadratic substrates with narrow and wider spacing, respectively. Solid and open symbols: mobile beads made from glass and steel, respectively. Large (red) symbols at position -1 : initial bead motion (bead stops once touching its neighbor). Experiments performed with higher viscous oil. Dotted lines indicate the critical Shields numbers for a single bead and are drawn to guide the eye. The configuration is sketched in the inset. Here white, grey and black circles indicate substrate, mobile, and immobile beads, respectively.

Therefore, we distinguish between initial bead motion and continuous motion. The former denotes leaving the initial position on the substrate while the latter leads to a full displacement to other steady equilibrium positions. In the triangular configuration, mobile bead and immobile particle are in direct contact. Therefore, there is no distinction between initial and continuous motion. We note that for the quadratic substrate with narrow spacing the critical Shields number for initial motion of the upstream bead is slightly lower than that of the downstream bead (see position -1 and 1 in Figure 2).

The glass bead passes the facing tungsten-carbide/cobalt particle occasionally on either side but not in main flow direction. The critical Shields numbers remain independent from the particle material. On the quadratic substrate with wider spacing, the tungsten-carbide/cobalt sphere is buried deeper and the local resistance to continuous motion offered by the immobile particle is hence considerably lower than at narrow substrate spacing. Nevertheless, the critical Shields number for initial motion is higher than for the narrow substrate spacing. We observe that adding laterally placed particles increases the critical Shields number slightly further.

We now study the influence of bead material and substrate geometry on the incipient motion of two identical mobile beads and focus on the threshold for continuous motion. Figure 3 shows the critical Shields number as a function of the relative density difference for the two quadratic configurations. The particle Reynolds number ranges from creeping flow conditions up to about 1 . In the quasi-stationary experiments, the critical Shields number for both configurations is about the same. Taking all measurements for the two configurations it results in 0.114 ± 0.006 . Compared to a single bead, this is an increase by a factor of about $2-3$.

The fact that the critical Shields number is hardly affected by the substrate spacing is apparently due to the different mechanisms of motion involved in the two configurations, which are shown in Figure 4: On the substrate with wider spacing, the beads are buried deeper in the substrate than on the substrate with narrow spacing. The bead placed upstream moves already at a Shields numbers of about 0.075 (see Figure 2) and touches the adjacent free sphere buried considerably in the substrate (picture 2 in Figure 4(b)). At the critical Shields number, it then passes the neighboring bead laterally (picture 3 in Figure 4(b)). As it moves away, it leaves the bead that initially was placed downstream exposed to the shear flow. This one then also starts moving. In this configuration the two beads tend to move separately (picture 4 in Figure 4(b)). On the other hand, for a quadratic arrangement within narrow spacing, the downstream bead is only slightly buried and both spheres are situated approximately at the same level after touching each other beyond a Shields number of about 0.053 (picture 2 in Figure 4(a)). In this case, the upstream bead does not pass its downstream neighbor

FIG. 3. Critical Shields number for a single and for two identical beads as a function of the relative density difference. Triangles and rhomboids: single bead on a quadratic substrate with narrow and wider spacing, respectively. Circles and squares: two identical beads on a quadratic substrate with narrow and wider spacing, respectively. Open and solid symbols: less and higher viscous oils, respectively. Large (red) symbols: experiments performed exposing the beads to a shear-rate jump.

but pushes it forward beyond the critical Shields number, resulting in a joined motion of the beads (pictures 3 and 4 in Figure 4(a)).

During the joined motion, the beads have to slide over the substrate or against each other at their contact point. To clarify this point, we analyzed the rotation angle of the spheres by following marks on the beads and compared it to the motion of single beads. Figure 5 depicts the angle of rotation covered from the initial position to the adjacent equilibrium position as a function of the trajectory ΔL along the substrate (see inset of Figure 5) for a single bead (a), and for two identical spheres (b). The experimental results are compared with the angle for a pure rolling motion over the substrate, $\gamma_T = 2L/D_T$. Here, L is the distance travelled by the bead along the curved path between two equilibrium positions and $D_T/2$ is the turning radius defined as the distance from the particle center to the rotation axis over the substrate (see inset of Figure 5(a)). L can be defined as $L = 2\varphi D_T$, where φ is the pivot angle, which, in turn, is a function of the substrate spacing. For the quadratic substrates with narrow and wider spacing the angle of rotation $\gamma_T = 4\varphi$ is about 140° and 210° , respectively. The experimental results collapse fairly well with the angle of rotation for a purely rolling motion, which is indicated by a dotted line in Figure 5(a). This holds for all studied bead materials and substrates. We remark that in all other configurations studied where a single sphere moves independently the bead carried out a rolling motion.

For the joined motion of two identical beads on the quadratic substrate with narrow spacing, the angle of rotation is shown in Figure 5(b). The experiments are performed with glass and steel beads

FIG. 4. Sequence of pictures showing a top view of two steel particles during the incipient motion on the quadratic substrate with narrow (a) and wider (b) spacing, respectively. Flow is from left to right.

FIG. 5. Angle of rotation normalized by the theoretical rolling angle, γ_T , for a single bead (a) and for two identical beads (b) on a quadratic substrate. Circles, rhomboids, and triangles: glass, steel, and tungsten-carbide/cobalt spheres, respectively. Solid and open symbols in (a): experiments performed on the quadratic substrate with narrow and wider spacing, respectively. Open and solid symbols in (b) indicate the upstream and downstream bead, respectively. Experiments in (b) are performed using the quadratic substrate with narrow spacing. Dotted lines indicate the angle of rotation for a pure rolling motion. The experiments are performed with the higher viscous oil.

and the error bars indicate representative values for the range of uncertainty. While the downstream bead seems to carry out a pure rolling motion, the upstream one moves over the substrate by both, rotation and translation. It rolls until it touches the downstream sphere (not shown). Once the downstream bead starts moving, the upstream one seems to slide and, in the case of glass beads, even to slightly recede. Once the beads have overcome the main resistance to motion, being therefore more exposed to the flow, they tend to slightly separate on the way downward to the substrate indentations and the upstream bead rolls again until they reach the new ascent. We always observed that the upstream bead slides over the substrate during the first phase of joined motion. In some experimental runs with glass as well as with steel spheres, we observed pure sliding motion of the upstream sphere even along the entire path.

Figure 5(b) shows that direct contact between the neighboring beads hinders the motion of the downstream particle and changes the motion of the upstream sphere. Therefore, one may expect that avoiding particle contact considerably decreases the critical Shields number for continuous motion. This may be achieved by exposing the beads to a sudden shear-rate jump to set both beads into simultaneous motion. On the quadratic substrate with narrow spacing, both particles may set into motion individually at Shields numbers beyond 0.055 (see Figure 2).

We indeed found simultaneous continuous motion at Shields numbers well below the critical ones in quasi-steady experiments. The contact between particles is minimized and, although very small phases of sliding, rolling is the main mechanism of motion. Larger (red) symbols in Figure 3

FIG. 6. Dependence of the critical Shields number for the first particle to move on the number of beads in a single row. Triangles and squares: triangular and quadratic substrate with narrow spacing, respectively. Open and solid symbols: experiments with steel and glass beads, respectively. The experiments are performed using the higher viscous oil.

show the critical Shields numbers in shear-rate jump experiments. Unlike quasi-stationary experiments, it differs between the two substrates. At the critical Shields number both beads successfully cover about 20% of the span between two equilibrium positions on the regular substrate. This distance corresponds to overcome most of the local resistance offered by the substrate.

The incipient motion of multiple identical beads deposited in a row was studied for two different configurations: the triangular substrate, and the quadratic one with narrow spacing. On the quadratic substrate the upstream bead, which is most exposed to the flow, moves first until it touches its nearest neighbor. On the triangular configuration the two upstream beads are in contact with each other from the beginning. The mechanism of continuous motion is the same for both substrates: At a certain critical Shields number, which is about 0.03 for the triangular configuration and about 0.06 for the quadratic one, the last downstream bead moves. Maintaining the Shields number during a prolonged period of time, all the beads successively travel along the substrate except for the two upstream ones. These beads start travelling at a critical Shields number of around 0.06 and 0.12 for the triangular and the quadratic configuration, respectively. In rare instances, we observe groups of more than two beads on the triangular substrate that hinder each other and move at a critical Shields number of about 0.06. Figure 6 depicts the critical Shields number for the initial change of static equilibrium position for the two substrates. The critical Shields number remains independent from the bead material. Experiments with compact layers of three rows yield similar results. The critical Shields number for the last particles to move increases only slightly. Due to the fact that the beads are in close proximity or already in contact from the beginning, we observe clusters of more than two beads that move simultaneously.

We now consider the limiting case of entirely hidden beads, i.e., beads that are completely surrounded by immobile particles. This is studied for the quadratic substrates. As shown in Figure 7, complete shielding results in critical Shields numbers for initial motion within the range of 0.120 ± 0.02 irrespective of the substrate spacing. The initial motion ends once the bead touches its neighbor in front. For the substrate with wider spacing, the bead escapes from its cavity at a slightly higher Shields number. Once it moves to the side, it passes over the trough between the downstream neighbors. On the substrate with narrow spacing, however, the bead leaves the cage only beyond a Shields number of about 0.3. Experiments performed with marked beads show rolling as the only mechanism for initial motion.

We note that adding further immobile or mobile beads did not have any effect on the critical Shields numbers within measurement uncertainty. If two mobile beads are buried, both leave the cavity formed by the immobile particles at the same Shields number. Irrespective of the initial motion, the upstream bead leaves the cage first on both substrates. Once the upstream bead moves to the side, it passes over the trough between its mobile and immobile neighbors in front. The mobile downstream neighbor follows shortly afterwards.

FIG. 7. Critical Shields number for one and two entirely hidden mobile beads. Experiments are performed with mobile glass beads using the higher viscous oil. Open and solid symbols: critical Shields number for initial motion and for full displacement, respectively.

IV. DISCUSSION

The impact of the substrate geometry on incipient motion is usually described by the pivot angle, which characterizes the local bed resistance to the bead motion,^{22,23} and by the exposure degree, E , of the considered particle to the flow.^{28–30} The pivot angle depends on the local substrate arrangement on the downstream side of the particle while the exposure degree depends on the upstream side and can be altered independently by the presence of neighboring particles. Martino *et al.*, combined these two parameters in the burial degree, $\tan \phi/E$, with which they could describe the main dependency of their data on the critical conditions for incipient motion. Their two-dimensional theoretical model, however, overestimates the experimental data for large burial degrees.³⁰ We studied the effect of neighboring particles on the incipient motion on three substrates with different pivot angles. Exposure is varied by placing particles in the neighborhood. Using different fluid viscosities and beads of different densities, the particle Reynolds number was varied up to about 1. As our data shows, inertia does not play any significant role in this regime.

We first focused on the effect of shielding on the initial particle motion. As shows Figure 2, the critical Shields number is affected by both, upstream and downstream immobile neighbors. For the quadratic substrate with narrow spacing, we observe roughly a symmetrical dependence of the critical Shields number for initial motion on the distance between mobile bead and its immobile neighbor. Compared to a single bead, the presence of a single neighbor yields an increment of the critical Shields number for initial motion of up to about 40% at closest distance. This increment is about the same for all substrates studied. Shielding strongly diminishes with distance between the beads. At distances about three times the bead diameter, shielding by neighbors decreases to noise level.

Laterally and downstream placed neighbors increase the critical Shields number for the initial motion by perturbing the flow around the exposed bead. For beads completely surrounded by neighbors on the quadratic substrate with narrow spacing (Figure 7), we observed that the critical Shields number for initial motion is tripled compared to a single bead. For these caged beads, we find that the angle of repose is of less relevance for the initial motion: Figure 7 shows that initial motion sets in at almost the same critical Shields number, irrespective of the substrate configuration.

In their numerical study on random assemblies of wall-attached spheres in low-Reynolds-number shear flows, Derksen and Larsen observed that the average drag force on spheres in the top layer of a double-layer bed decreases as the surface occupancy σ , defined as $n\pi D_p^2/4$, being n the number of spheres per unit surface area, of the top layer increases.³¹ They related this to an enhanced mean mutual shielding of the particles to the flow. This is in good agreement with the increment of the critical Shields number for initial motion as shielding increases in our experiments: Considering spheres on top of a bottom layer with an occupancy of 0.7, Derksen and Larsen obtained a reduction

FIG. 8. Sketch for the bypass rolling motion (a), and of the forces acting on the bead in main flow direction (b) and in direction of sideways motion (c). In (a) the red dashed area is perpendicular to the axis of rotation between A and B and contains the center of the mobile bead. φ_L is the pivot angle with the projection of the vertical axis in this area.

of the average drag force by a factor of about 3.5 as the occupancy of the top layer increases from below 0.05, which corresponds practically to a lack of interaction with neighbors, up to about 0.5. At higher occupancy, the mean drag force remains constant. In our experiments, the occupancy for the quadratic substrates with narrow and wider spacing and thus for the caged beads in Figure 7 is 0.76 and 0.61, respectively. Despite the differences on the substrate arrangements, the drag force reduction obtained by Derksen and Larsen is close to the increment of the critical Shields number for initial motion in our experiments for narrow spacing, where the surface occupancy is close to 0.7.

Besides altering the flow field, particles in adjacent downstream positions hinder beads from moving once they are in contact. If there is only one immobile downstream neighbor as in Figure 2, the mobile bead may pass it sideways. If other immobile neighbors are placed laterally like in Figure 7, the mobile bead moves over the trough between the downstream neighbors. In this case, the mobile bead detaches from the substrate and the critical Shields number is somewhat higher than with a single obstructing particle in front.

In contact with their downstream neighbors the mobile beads stay in dynamic equilibrium until they pass them. Compared to the initial location, this position is generally characterized by a higher exposure but also by a higher pivot angle. In main flow direction, the pivot angle in contact with the downstream neighbor is 88.6° and 76.3° for narrow and wider spacing while that at the initial position on the substrate is 37.2° and 55.1° , respectively.³² Without any neighbor upstream, as in position -1 of Figure 2, the exposure degree augments from 0.87 and 0.75³² to 0.90 and 0.96 for narrow and wider spacing, respectively. With an additional neighbor upstream as in Figure 7, the exposure degree, which is zero at the initial position, reaches 0.03 and 0.32 for narrow and wider spacing, respectively.

In our experiments the bead always passes its immobile neighbor sideways instead of travelling over it in main flow direction. To focus on the lateral rolling motion, we consider the axis of rotation that connects the contact points of the mobile bead with a substrate particle downstream (A) and with the downstream neighbor (B), see Figure 8(a). In the plain perpendicular to this axis that contains the center of mass of the mobile bead, the pivot angle, φ_L , formed with the projection of gravity into this plain, is lower than in that in main flow direction. For narrow and wider particle spacing it results in 55.2° and 52.4° , respectively.

To see why the lateral rolling motion is favored over that across the immobile bead in flow direction, we consider the momentum balance at the critical Shields numbers for these two cases. Neglecting lift forces as can be assumed for particle Reynolds numbers below one,^{23,31,40} the

necessary drag force, F_D , for incipient motion in main flow direction can be written as

$$F_D = (F_G - F_B) \frac{L_G}{L_D} \tan \varphi, \quad (3)$$

where F_G is gravity and F_B buoyancy force. L_D and L_G are the lever arms of drag, and of gravity and buoyancy force components that point into the direction of motion, respectively, see Figure 8(b). Considering the lateral movement, the momentum balance yields accordingly

$$F_{DL} = (F_{GL} - F_{BL}) \frac{L_{GL}}{L_{DL}} \tan \varphi_L, \quad (4)$$

where the subscript L indicates lateral motion. The axis of rotation $\overline{AB}$ is inclined by the angle α with respect to the horizontal, see Figure 8(c). Thus, the resulting vertical forces perpendicular to the axis of rotation are reduced by $\cos \alpha$. Furthermore, the horizontal projection of the lateral bead trajectory has an angle β with respect to the main flow direction. Yet, the drag force in that direction of motion is reduced by $\cos \beta$, where β is 19.3° and 26.0° for narrow and wider spacing, respectively. Hence, in this case the drag force in main flow direction F_{Dmf} is

$$F_{Dmf} = (F_G - F_B) \frac{\cos \alpha}{\cos \beta} \frac{L_{GL}}{L_{DL}} \tan \varphi_L. \quad (5)$$

Comparing the two drag forces necessary for incipient motion in main flow and lateral direction yields

$$\frac{F_{Dmf}}{F_D} = \frac{\cos \alpha}{\cos \beta} \frac{L_{GL}}{L_{DL}} \frac{L_D}{L_G} \frac{\tan \varphi_L}{\tan \varphi}. \quad (6)$$

Since the lever arms for the drag force are unknown, we consider two limiting cases: First, the drag force acts on the center of the mobile bead and second it acts at the upmost point of the bead. For narrow and wider spacing, the ratio is 0.03 and 0.34 for the first case and 0.46 and 0.62 for the latter, respectively. In either case, the drag force for incipient lateral motion is smaller than for traveling across the immobile neighbor. Hence, lateral motion takes place at lower Shields numbers.

As Figure 2 shows, the critical Shields number for passing the immobile neighbor is slightly higher for the substrate with narrow spacing than for the one with wider spacing. This can be traced back to the fact that the lateral pivot angle is slightly larger and the exposure degree at dynamic equilibrium slightly smaller on the substrate with narrow spacing, contrary to the situation for a sole single bead.³²

Using two identical mobile beads, we find initially the same situation as with an immobile bead in front of a mobile one: The exposed upstream bead initially moves until it touches the downstream bead and stops at a new position of dynamic equilibrium. This is due to the fact that the critical Shields number for initial motion of the upstream bead is smaller than that of the downstream bead (see Figure 2). Once both beads are in contact, however, the downstream bead does not move at Shields numbers beyond the critical one for their initial motion depicted in Figure 2. We favor three effects that may be responsible for this: Shielding by the upstream neighbor has increased due to its higher position of dynamic equilibrium. Second, the line of action of the contact force between the mobile beads passes through the center of mass and the lever arm for rolling over the substrate is rather low. Furthermore, if the difference in the pivot angles of the downstream and upstream bead in dynamic equilibrium is rather low, as on the substrate with wider spacing, the upstream bead mainly presses down onto the downstream bead. Finally, another important effect seems to be hindrance by the upstream neighbor due to friction.

On the substrate with wider spacing, continuous motion sets in the same way as with an immobile downstream bead, see Figure 4(b). On the substrate with narrow spacing, the critical Shields number is about the same, see Figure 3, but the mechanism is different, see Figure 4(a). The joined motion cannot happen with two rolling beads in contact. As shows Figure 5(b), the downstream bead rolls, thus maintaining stiction with the substrate, while the upstream bead initially slides along the substrate. Apparently this is due to the rather high difference in the pivot angles between the two beads, which results in a large contribution of the contact force to the torque of the downstream

bead. Although the two beads slide along each other, we could not identify major differences in the critical Shields number between the bead materials.

The resulting critical Shields numbers for their continuous motion are considerably higher than that for the initial motion of either the single mobile upstream and downstream bead. As shown in Figure 3, avoiding or delaying direct contact between beads during the first phase of motion drastically reduces the critical conditions for the onset of continuous motion. Different from the quasi-stationary runs, the critical Shields number in this case varies with the substrate spacing. This is apparently due to the fact that dislodging beads that are not in contact with each other depends on the substrate configuration (see Figure 2). Furthermore, a comparison with Figure 2 shows that the critical Shields number for shear-rate jumps is higher than that for the initial motion of upstream and downstream beads for the respective substrates.

The incipient motion of multiple beads in our configurations can be traced back to that of two-particle interaction. First, the exposed upstream bead touches its next neighbor. As Figure 6 shows, both beads hinder each other from moving while the beads further downstream move one after the other starting with the last one. For these beads, the critical Shields number remains constant. Comparison with Figure 2 shows that it is about the same as that for two adjacent beads. This suggests that other than the nearest neighbor has hardly any effect on shielding. The two remaining upstream particles in contact finally move at the corresponding critical Shields number for two identical beads (compare Figure 3). The critical Shields numbers for neighboring lines of beads are slightly higher than for a single line showing that the impact of laterally placed beads is small. Direct contact between identical beads apparently plays a limiting role in the incipient motion of multiple beads in our experiments.

The critical Shields number for completely embedded beads to escape the cage of surrounding immobile particles strongly depends on the substrate as shows Figure 7. Once the single mobile bead moves to the side on the substrate with wider spacing, it keeps on moving and escapes the cage by passing between the downstream neighbors. The trough between the neighbors apparently does not stop the bead. The critical Shields number for continuous motion on the substrate with narrow spacing is about 2.5 times higher than that for the substrate with wider spacing. This is apparently to be due to lower exposure degree and higher pivot angle at the final position of dynamic equilibrium on this substrate. Note that in this position the exposure degree is 0.08 compared to 0.32 with wider spacing.

For both substrates the critical Shields number for two completely buried mobile beads is about the same as for a single buried one (see Figure 7). For the substrate with narrow spacing, the downstream bead is cramped between its immobile and mobile neighbors and cannot move. The dynamic equilibrium position of the upstream bead is almost the same as for a single buried bead. Yet, it rolls over the downstream one and escapes first, being the critical Shields number about the same as for a single buried bead. On the substrate with wider spacing, the mechanism of motion is the same as for a single buried bed. Therefore, we do not observe any difference on the critical Shields number as compared to a single buried bead.

In models for incipient motion different critical Shields numbers are obtained depending on the type of motion considered.³⁷ In our study we found rolling as the only mechanism of motion for a sole single bead deposited on the different substrates (see Figure 5(a)). This confirms the assumption of numerous models that consider rolling as the incipient movement of a spherical particle.^{5,15,24,35,38} Even shielded and caged single mobile beads, starting from a position of dynamic equilibrium with high angles of repose like in Figure 7, pass their immobile neighbors by rolling. Sliding is observed only during the joined bead motion (see Figure 5(b)), which occurs at high angles of repose and a high exposure degree of the upstream bead.

In their study on incipient granular bed motion of spherical particles in laminar flow, Charru *et al.* showed that the number of moving particles decreases to a saturated value within finite time.¹⁴ The minimum Shields number to obtain a non-zero saturated value was found to be 0.12. Similar values were reported in other studies.^{15,18,21} The slow decrease of the number of moving beads at lower Shields numbers was related to particles getting trapped in surface cavities, and results in an armored bed.¹⁴ In our experiments on regular substrates, beads without contact (Figures 2 and 3), at downstream edges and on the triangular substrate (Figure 6) already move at these lower values.

They may be stopped by downstream neighbors or become stuck in cavities. At a Shields number of about 0.12, two beads in contact on quadratic configurations (Figures 3 and 6) become unstable. In our experiments, this is the most stable configuration for beads moving on the same level. Yet, on an irregular substrate a traffic jam of particles in contact, which originally might be released from, say, local triangular configurations (Figure 6), can be resolved by backing off of downstream beads (Figure 4(a)) or by bypassing downstream neighbors (Figure 4(b)). At about the same Shields number the first configuration of buried beads (Figure 7) becomes unstable, i.e., beads move upwards to the uppermost level and travel over their neighbors. In this case the troughs formed by downstream neighbors do not obstruct the bead's motion. This mimics the onset of motion of an armored bed: If the surrounding beads do not move, arriving beads cannot hide anymore in the cavities. On the other hand, if they rearrange locally, they become more exposed to the flow making motion easier.

The fact that the critical Shields number for continuous motion is close to that for initial motion on the substrate with wider spacing lets us hypothesize that the critical Shields number obtained on that substrate is indeed close to the minimum for completely caged beads. Yet, at narrower spacing the critical Shields number for continuous motion increases due to a higher contact angle and a lower exposure degree at the position of dynamic equilibrium, while at even wider spacing the onset of continuous motion is supposed to be governed solely by the initial motion: Once the bead leaves the substrate indentation it keeps on moving, passing its neighbors downstream.

V. CONCLUSIONS

We have studied how neighbors affect incipient particle motion on regular substrates in laminar shear flow at particle Reynolds numbers below one. In our experiments, the critical Shields numbers result independent from bead material and inertia. In stream-wise direction, only spheres that are closer than about 3 particle diameters influence incipient motion. Direct contact with a neighbor inhibits continuous motion. The bead stops and remains in a position of dynamic equilibrium. It usually starts moving again at a considerably higher Shields number. This is apparently due to the higher contact angle at the position of dynamic equilibrium compared to the initial location on the substrate. Before critical conditions for motion into main flow direction are reached, the bead passes obstructing immobile neighbors sideways.

If the downstream neighbor is identical to the upstream bead, the downstream bead may recede resulting in a joined motion of the two beads. This happens if the contact angle between the beads is rather large. In this case we observed a rolling motion of the downstream bead while the upstream bead slides in contact with its neighbor. If the contact angle is lower, the downstream bead starts moving after the upstream bead has passed, resulting in a reversal of the particle order. Although in both cases the mechanism of motion is different, both regimes have about the same critical Shields number, irrespectively of the fact that the critical Shields number for single particle motion differs by about 50%. In compact layers the two-particle contact plays the limiting role for incipient motion. If initial contact between identical neighbors is avoided by a sudden jump in the Shields number, the critical Shields number for continuous particle motion is considerably lower and differs depending on the substrate configuration. In this case, particle order is maintained.

Except for the joined motion of beads in contact, we always observed initial rolling motion even for buried beads. The lowest critical Shields number for dislodging buried beads in the configurations studied coincides with the critical Shields number for incipient motion of irregular granular beds.

ACKNOWLEDGMENTS

The authors are thankful to Mrs. J. Schwendner and Mr. M. Kobylko for collaborating in setting up the experiment. The support from Deutsche Forschungsgemeinschaft through WI 2672/4-1 is gratefully acknowledged.

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