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Iterative identification of hysteresis in Maxwell's equations

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Abstract

Purpose – In a model resulting from Maxwell's equations with a constitutive law using Preisach operators for incorporating magnetization hysteresis, this paper aims at identifying the hysteresis operator, i.e. the Preisach weight function, from indirect measurements.

Design/methodology/approach – Dealing with a nonlinear inverse problem, one has to apply iterative methods for its numerical solution. For this purpose several approaches are proposed based on fixed point or Newton type ideas. In the latter case, one has to take into account nondifferentiability of the hysteresis operator. This is done by using differentiable substitutes or quasi-Newton methods.

Findings – Numerical tests with synthetic data show that fixed point methods based on fitting after a full forward sweep (alternating iteration) and Newton type iterations using the hysteresis centerline or commutation curve exhibit a satisfactory convergence behavior, while fixed point iterations based on subdividing the time interval (Kaczmarz) suffer from instability problems and quasi Newton iterations (Broyden) are too slow in some cases.

Research limitations/implications – Application of the proposed methods to measured data will be the subject of future research work.

Practical implications – The proposed methodologies allow to determine material parameters in hysteresis models from indirect measurements.

Originality/value – Taking into account the full PDE model, one can expect to get accurate and reliable results in this model identification problem. Especially the use of Newton type methods – taking into account nondifferentiability – is new in this context.

Keywords Iterative methods, Newton method

Paper type Research paper

1. Introduction

Hysteresis is a memory effect that appears in a wide range of time-dependent processes such as ferromagnetism, elastoplasticity, or porous media filtration, to name just a few examples. It is characterized by a lag behind in time of some output in dependence of the input sequence. Consider, e.g. the curve describing the (normalized) magnetization and demagnetization $m = M/M_{\max}$ of some magnetic material in dependence of the (normalized) applied magnetic field intensity $h = H/H_{\max}$: we write:

$$m(t) = \mathcal{P}[h](t) \quad (1)$$

with some hysteresis operator $\mathcal{P}$.

Probably the most simple example of hysteresis is the behavior of a relay (or hysteron, Della Torre, 1999), that is characterized by two threshold values $\alpha > \beta$. The output value m is either -1 or $+1$ and changes only if the input value h crosses one of the switching thresholds α, β . We formally define (neglecting for the moment the ambiguity in case of the input only touching but not crossing one of the thresholds) the relay operator $\mathcal{R}_{\beta, \alpha}$ by:

$$m(t) = \mathcal{R}_{\beta,\alpha}[h](t)$$

according to the description above.

A practically important phenomenological hysteresis model that was originally introduced in the context of magnetism but plays a role also in many other hysteretic processes, is given by the Preisach operator:

$$\mathcal{P}[h](t) = \int \int_{\beta \leq \alpha} p(\beta, \alpha) \mathcal{R}_{\beta,\alpha}[h](t) d(\alpha, \beta) \quad (2)$$

which is a weighted superposition of elementary relays with the Preisach weight function p . The initial values of the relays $\mathcal{R}_{\beta,\alpha}$ are set to some neutral (or virginal) initial state. Note that due to the integration with an integrable p , the ambiguity of the relay operator for the case of only touching thresholds does not contribute essentially (Brokate and Sprekels, 1996).

The domain $\{(\beta, \alpha) | \beta \leq \alpha\}$ of p is called the Preisach plane. Assuming that the weight function p is compactly supported and by a possible rescaling, we can restrict our attention to the Preisach unit triangle $\{(\beta, \alpha) | -1 \leq \beta \leq \alpha \leq 1\}$ within the Preisach plane.

Our task is to identify the Preisach operator $\mathcal{P}$, or equivalently, the Preisach weight function p in equation (2) from measurements of the magnetic flux Φ while the impressed current I exciting the magnetic assembly runs through a range of prescribed values. As will be stated in more detail below, there is a different bivariate function that – along with the so-called Preisach memory – also uniquely determines $\mathcal{P}$, namely the Everett function.

2. Preisach hysteresis operators

In this section we will shortly outline some important features of Preisach operators. These can be set into the more general framework of rate independent operators, in the sense that the values attained by the output are independent of the speed of the input. As a consequence, given a piecewise monotone continuous input h , the output is (up to the speed in which it is traversed) uniquely determined by the local extrema of the input only. Thus, to determine the action $\mathcal{P}[h](t)$ of such a hysteresis operator at some time instance t , it suffices to store and update a string $s = (h_0, \dots, h_N)$ of finitely many input values, namely its local minima and maxima before time t :

$$\mathcal{P}[h](t) = \mathcal{P}_f(h_0, \dots, h_N), \quad (3)$$

where $\mathcal{P}_f$ is called the final value mapping. This string can be further reduced by applying the following deletion rules describing the so-called Preisach memory:

- *Monotone deletion rule.* Only the local maxima and minima of the input are relevant.
- *Madelung's (third) rule.* Inner minor loops are forgotten.
- *Wipe out.* Previous absolutely smaller local maxima (minima) are erased from memory by subsequent absolutely larger local maxima (minima).

- *Initial deletion.* A maximum (minimum) is also forgotten if it is followed by a minimum (maximum) with sufficiently large modulus.

When computing the hysteretic evolution of some output function by a time stepping scheme, we have to update the input string and apply deletion in each time step. In doing so, we augment an irreducible input string from the previous time step by one value (the input value at the current time step). It can be shown that the new string can be made irreducible according to the rules 1-4 above by a simple and efficient method (Kaltenbacher and Kaltenbacher, 2005).

As soon as we have reduced our input string to an irreducible one, we can make use of the so-called Everett function (or shape function, Everett, 1955) defined by:

$$e(h_{N-1}, h_N) = 2 \operatorname{sign}(h_N - h_{N-1}) \int \int_{\Delta(h_{N-1}, h_N)} p(\beta, \alpha) d(\alpha, \beta), \quad (4)$$

where:

$$\Delta(h_{N-1}, h_N) = \{(\beta, \alpha) | \min\{h_{N-1}, h_N\} \leq \beta \leq \alpha \leq \max\{h_{N-1}, h_N\}\}.$$

Therewith, and with the function e_0 describing the action of $\mathcal{P}_f$ on strings of length one:

$$e_0(h_0) = \mathcal{P}_f(h_0), \quad h_0 \in \mathbf{R},$$

we obtain:

$$\mathcal{P}_f(s) = e_0(h_0) + \sum_{k=1}^N e(h_{k-1}, h_k). \quad (5)$$

Note that by inversion of the integral relation (4), the weight function can be computed from the Everett function by differentiation:

$$p(\beta, \alpha) = -\frac{1}{2} \partial_1 \partial_2 e(\beta, \alpha). \quad (6)$$

Remark 1. Since, numerical differentiation is an ill-posed problem (Engl *et al.*, 1996), evaluation of equation (6) brings an instability aspect into the problem of identifying p , so that regularization has to be applied. We do so by sufficiently coarse discretization as well as by early stopping.

The Everett sum (5) provides us with a very efficient method for evaluating Preisach operators. This is especially important in view of the fact that when incorporating hysteresis in PDEs as we will do below, one has to take into account the history of each single space point (or in a discretized setting, of each finite element, Bastos and Sadowski, 2003).

To model not only scalar, but also vectorial hysteresis, as it is relevant, e.g. in magnetics, the Vector Preisach model of Mayergoyz (1991), can be used, which is given by:

$$\mathcal{M}[\mathbf{h}](t) = \int_{\{|\theta|=1\}} \theta \mathcal{P}^\theta[\theta \cdot \mathbf{h}](t) d\theta \quad (7)$$

where $\{|\theta|=1\}$ is the unit sphere in $\mathbf{R}^3$, i.e. the integral goes over all unit directions θ , and $(\mathcal{P}^\theta)_{\theta \in \{|\theta|=1\}}$ is a family of scalar Preisach operators (Della Torre, 1999, for different vectorial extensions of the Preisach model).

3. The PDE model

The hysteresis relation (1) in a generalized vectorial form will be included as a constitutive equation into our model of magnetic behavior, that consists of (part of) Maxwell's equations as balance laws. Ampère's law in the quasistatic case and Faraday's law combined with constitutive equations:

$$\mathbf{J} = \gamma \mathbf{E} + \mathbf{J}^{\text{imp}} \quad (8)$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \quad (9)$$

where γ is the electric conductivity, μ_0 the magnetic permeability in air, $\mathbf{J}$ ($\mathbf{J}^{\text{imp}}$) the (impressed) current density, $\mathbf{E}$ the solenoidal electric field, $\mathbf{B}$ the magnetic induction, $\mathbf{H}$ the magnetic field intensity, and the magnetization $\mathbf{M}$ is defined via a (vectorial) hysteresis operator, i.e.:

$$\mathbf{M} = \mathcal{M}[\mathbf{H}], \quad (10)$$

yields:

$$\mu_0 \mathbf{H}_t + (\mathcal{M}[\mathbf{H}])_t + \nabla \times \left(\frac{1}{\gamma} \nabla \times \mathbf{H} \right) = \nabla \times \left(\frac{1}{\gamma} \mathbf{J}^{\text{imp}} \right) \quad \text{in } \Omega. \quad (11)$$

Additionally, we stipulate boundary conditions:

$$\mathbf{H} \cdot \mathbf{n} = 0 \quad \text{on } \partial\Omega. \quad (12)$$

where $\mathbf{n}$ denotes the outward unit normal vector on the Lipschitz domain Ω and an initial condition:

$$\mathbf{H}(t=0) = \mathbf{H}_0. \quad (13)$$

Alternatively, for a magnetic vector potential $\mathbf{A}$ with $\mathbf{B} = \nabla \times \mathbf{A}$ one can use the formulation:

$$\gamma \mathbf{A}_t + \nabla \times ((\mu_0 \mathbf{I} + \mathcal{M})^{-1} [\nabla \times \mathbf{A}]) = \mathbf{J}^{\text{imp}} \quad \text{in } \Omega. \quad (14)$$

where $\mathbf{I}$ is the unit matrix.

To model the experimental measurement setup, the region $\Omega \subseteq \mathbf{R}^3$ includes the coil, the probe as well as the surrounding air, and:

$$\mathbf{J}^{\text{imp}}(x, t) = \begin{cases} \frac{I(t)}{|\Gamma_w|} \mathbf{e}_f(x) & \text{for } x \text{ in the coil region } \Omega_c \subset \Omega \\ 0 & \text{else} \end{cases}$$

The measured magnetic flux can be expressed as:

$$\Phi = \int_{\Gamma_c} \mathbf{B} \cdot \mathbf{n} d\Gamma = \oint_{\gamma_c} \mathbf{A} \cdot d\boldsymbol{\sigma} \quad (15)$$

by Stokes' theorem. Here, Γ_w denotes the cross sectional area of the coil wire, Γ_c the cross sectional area of the coil, γ_c its boundary curve, I the impressed current, and $\mathbf{e}_j$ the unit vector in current direction. For a numerical solution method for the initial boundary value problem (IBVP) equations (11)-(13), using a finite element space discretization within a combination of a trapezoidal scheme with an iterative resolution of the hysteresis nonlinearity in each time step, we refer to Kaltenbacher (2006), (Bastos and Sadowski, 2003).

4. Hysteresis identification

4.1 Identification from direct measurements

If input h and output m of a Preisach operator $\mathcal{P}$ are directly available, then the problem of identifying the weight function p amounts to a linear integral equation of the first kind:

$$\int \int_{\beta \leq \alpha} p(\beta, \alpha) \mathcal{R}_{\beta, \alpha}[h](t) d(\alpha, \beta) = m(t) \quad t \in [0, T]. \quad (16)$$

Since, the data h, m are given only on a one-dimensional time interval $[0, T]$ and the searched for function p depends on two variables, it cannot be expected to be uniquely determined. For some considerations concerning partial identifiability we refer to Kaltenbacher (2006).

Using a discretization of the Preisach operator as a linear combination of elementary hysteresis operators $\mathcal{P}_\lambda$:

$$\mathcal{P} = \sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda \quad (17)$$

and evaluating the output at n_T discrete time instances $0 \leq t_1 < t_2 < \dots < t_{n_T} \leq T$, we approximate the solution of equation (16) by solving a linear least squares problem for the coefficients $\mathbf{a} = (a_\lambda)_{\lambda \in \Lambda}$:

$$\min_{\mathbf{a}} \sum_{i=1}^{n_T} \left(\sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda[h](t_i) - m(t_i) \right)^2. \quad (18)$$

see (Hoffmann and Meyer, 1989). In equation (17), $\mathcal{P}_\lambda$ may be chosen as simple relays:

$$\mathcal{P}_\lambda = \mathcal{R}_{\beta_i, \alpha_j},$$

with $-1 \leq \beta_i \leq \alpha_j \leq 1$, $i = 1, \dots, n_\beta$, $j = 1, \dots, n_\alpha$ which corresponds to a piecewise constant approximation of the weight function $a_\lambda \approx p(\beta_i, \alpha_j)$. In that case, obviously the set Λ consists of index pairs $\lambda = (\beta_i, \alpha_j)$, corresponding to different up- and down-switching thresholds. Possible smoother basis functions for the

discretization of p lead to smoothly shaped elementary hysteresis operators $\mathcal{P}_\lambda$ in place of $\mathcal{R}_{\beta_i, \alpha_j}$.

In case of vectorial hysteresis modeled, e.g. by equation (7), the dimensionality of the identification problem becomes still higher and uniqueness is therefore an open problem. Still, one can discretize the problem and solve it in a least squares sense. The *ansatz*:

$$\mathcal{M} = \sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda \quad (19)$$

then has to account additionally for the angular dependence of the scalar Preisach operators $\mathcal{P}^\theta$ in equation (7), which in case of piecewise constant discretization of p and of the unit sphere $\{|\theta| = 1\} = \{(\sin \phi \cos \psi, \sin \phi \sin \psi, \cos \phi)^T | \phi \in [0, \pi], \psi \in [0, 2\pi]\}$ leads to an index set Λ consisting of quadruples $\lambda = (\beta_i, \alpha_j, \phi_k, \psi_l)$ and to elementary hysteresis operators:

$$\mathcal{P}_\lambda[\mathbf{h}] = \mathcal{R}_{\beta_i, \alpha_j}[\theta_{kl} \cdot \mathbf{h}] \theta_{kl},$$

where $\theta_{kl} = (\sin \phi_k \cos \psi_l, \sin \phi_k \sin \psi_l, \cos \phi_k)^T$.

4.2 Identification in PDEs

Since, we cannot measure the input $\mathbf{H}$ and the output $\mathbf{M}$ in the hysteretic relation (10) directly, we have to make use of the magnetic field equation (11) (or equation (14)) to recover the hysteresis operator in an iterative manner.

4.2.1 Fixed point iterations. An alternating iteration. The probably most straightforward approach for recovering $\mathcal{M}$ is the following: Given a preliminary approximation to $\mathcal{M}$, we use the IBVP equations (11)-(13), i.e. with a discretization equation (19):

$$\begin{aligned} \mu_0 \mathbf{H}_t + \sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda[\mathbf{H}]_t + \nabla \times \frac{1}{\gamma} \nabla \times \mathbf{H} &= \nabla \times \frac{1}{\gamma} \mathbf{J}^{\text{imp}} \quad \text{in } \Omega \times [0, T] \\ \mathbf{H} \cdot \mathbf{n} &= 0 \quad \text{on } \partial\Omega \times [0, T] \quad \mathbf{H}(\cdot, 0) = \mathbf{H}_0 \quad \text{in } \Omega. \end{aligned} \quad (20)$$

to compute $\mathbf{H}$. Then we insert $\mathbf{H}$ into the relation:

$$\sum_{\lambda \in \Lambda} a_\lambda \int_{\Gamma_c} \mathcal{P}_\lambda[\mathbf{H}](t_i) \cdot \mathbf{n} d\Gamma = \Phi(t_i) - \int_{\Gamma_c} \mu_0 \mathbf{H}(t_i) \cdot \mathbf{n} d\Gamma \quad i = 1, \dots, n_T, \quad (21)$$

that results from equations (9), (10) and (15), to identify an improved version of $\mathbf{a}$ and therewith of $\mathcal{M}$.

Algorithm 1. Set $\mathbf{a}^0$ To An Initial Guess For $\mathbf{a}$.

For $n = 1, 2, 3, \dots$

Solve The Hysteretic IBVP equation (20) for $\mathbf{H}^n = \mathbf{H}$.

Compute $\mathbf{a}^n$ From The equation (21) With $\mathbf{H} = \mathbf{H}^n$.

Each step of the method consists of:

- Solution of an initial-boundary value problem for a PDE with hysteresis.

- Computation of the coefficients $\mathbf{a}^n = (a_\lambda^n)_{\lambda \in \Lambda}$ (and therewith, of the hysteresis operator $\mathcal{M}^n$) from an input-output hysteresis model. This can be treated as a least squares problem for a linear system of equations, see subsection 4.1 above.

A *Kaczmarz iteration*. Instead of doing the forward solution over the whole time interval before fitting the Preisach operator, one can also split $[0, T]$ into smaller subintervals and fit the Preisach operator after each of these time horizons. If applying an incremental triangle excitation, a natural choice of the time horizons is given by the quasi periods of the input.

Algorithm 2. Set $\mathbf{a}^0$ To An Initial Guess For $\mathbf{a}$.

Choose $0 = T_0 < T_1 < \dots < T_{n_{\text{hor}}} = T$

For $n = 1, 2, 3, \dots$

Set $\mathbf{a}_0^n = \mathbf{a}_{n_{\text{hor}}}^{n-1}, \mathbf{H}_0^n \equiv \mathbf{H}_0$.

For $m = 1, \dots, n_{\text{hor}}$

Solve The Hysteretic IBVP equation (20) With $\mathbf{a} = \mathbf{a}_{m-1}^n$
on $[T_{m-1}, T_m]$ in place of $[0, T]$ with $\mathbf{H}_{m-1}^n(T_{m-1})$ in place of $\mathbf{H}_0$

For $\mathbf{H}_m^n = \mathbf{H}$

Compute $\mathbf{a}_m^n$ From The equation (21) With $\mathbf{H} = \mathbf{H}_m^n$

and $n_{T_m} := \max\{i \in \{1, \dots, n_T\} | t_i \leq T_m\}$ in place of n_T

4.2.2 Newton type methods. To prove contractivity and therewith convergence of the fixed point methods described above, one has to assume that the input is not too large. However, due to the fact that hysteresis occurs mainly for high inputs, this can constitute a severe restriction. Newton type methods are known to always converge at least locally. Moreover, they are usually faster than fixed point methods. Considering the nonlinear system of equations:

$$F_i(\mathbf{a}) = \int_{\Gamma_c} \left(\mu_0 \mathbf{H}^{\mathbf{a}}(t_i) + \sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda[\mathbf{H}^{\mathbf{a}}](t_i) \right) \cdot \mathbf{n} \, d\Gamma - \Phi(t_i) = 0 \quad (22)$$

$$i = 1, \dots, n_T,$$

where $\mathbf{H}^{\mathbf{a}}$ solves equation (20), Newton's method:

$$\mathbf{a}^{n+1} = \mathbf{a}^n - F'(\mathbf{a}^n)^{-1} F(\mathbf{a}^n) \quad (23)$$

would involve the derivative of $\mathbf{H}^{\mathbf{a}}$ with respect to $\mathbf{a}$. However, hysteresis operators are only Lipschitz continuous but not differentiable. Hence, Newton's method is not applicable directly. Possible remedies for the nondifferentiability are the replacement of $\sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda$ by an appropriate differentiable operator, or the use of derivative free approximations to the Jacobian $F'(\mathbf{a}^n)$.

Differentiable substitutes. To approximate a scalar hysteresis operator $\mathcal{P}$ by a differentiable operator, we choose $\mathcal{S}$ to be the superposition operator:

$$\mathcal{S}[h](t) = c(h(t))$$

with c being a real function, e.g. the centerline of the main hysteresis loop, (equations (3) and (5)):

$$c(h_0) = \frac{\mathcal{P}_f(1, h_0) + \mathcal{P}_f(1, -1, h_0)}{2} = \frac{\mathcal{P}_f(-1) + e(-1, h_0) + \mathcal{P}_f(1) + e(1, h_0)}{2}, \quad (24)$$

or alternatively the commutation curve. Therewith, the derivative of $\mathcal{S}$ is the multiplication operator defined by:

$$(\mathcal{S}'[h]d\mathbf{h})(t) = c'(h(t)) \cdot d\mathbf{h}(t).$$

Substitutes $\mathcal{S}^\theta$ for $\mathcal{P}^\theta$ in the vectorial hysteresis operator $\mathcal{M}$ according to equation (7) can be defined analogously by using centerlines or commutation curves for each fixed direction θ . Therewith, we arrive at IBVPs:

$$\left. \begin{aligned} \mu_0 \mathbf{Z}_{\lambda t} + \sum_{\tilde{\lambda} \in \Lambda} a_{\tilde{\lambda}}^n (\mathcal{S}'_{\tilde{\lambda}}[\mathbf{H}^{\mathbf{a}^n}] \mathbf{Z}_{\tilde{\lambda}})_t + \nabla \times \frac{1}{\gamma} \nabla \times \mathbf{Z}_{\lambda} &= -\mathcal{P}_{\lambda}[\mathbf{H}^{\mathbf{a}^n}]_t \text{ in } \Omega \times [0, T] \\ \mathbf{Z}_{\lambda} \cdot \mathbf{n} &= 0 \quad \text{on } \partial\Omega \times [0, T] \\ \mathbf{Z}_{\lambda}(\cdot, 0) &= 0 \quad \text{in } \Omega \end{aligned} \right\} \lambda \in \Lambda \quad (25)$$

Algorithm 3. Set $\mathbf{a}^0$ To An Initial Guess For $\mathbf{a}$.

For $n = 0, 1, 2, 3, \dots$

Solve The Hysteretic IBVP equation (20) For $\mathbf{H}^{\mathbf{a}^n} = \mathbf{H}$.

Solve The Linear IBVPs equation (25) For $(\mathbf{Z}_{\lambda})_{\lambda \in \Lambda}$.

Compute $\mathbf{b} := (b_{\lambda})_{\lambda \in \Lambda}$ From The Equation:

$$\begin{aligned} \sum_{\lambda \in \Lambda} b_{\lambda} \int_{\Gamma_c} \left(\mu_0 \mathbf{Z}_{\lambda} + \mathcal{P}_{\lambda}[\mathbf{H}^{\mathbf{a}^n}] + \sum_{\tilde{\lambda} \in \Lambda} a_{\tilde{\lambda}}^n \mathcal{S}'_{\tilde{\lambda}}[\mathbf{H}^{\mathbf{a}^n}] \mathbf{Z}_{\tilde{\lambda}} \right) \cdot \mathbf{n} \, d\Gamma \\ = \Phi - \int_{\Gamma_c} \left(\mu_0 \mathbf{H}^{\mathbf{a}^n} + \sum_{\lambda \in \Lambda} a_{\lambda}^n \mathcal{P}_{\lambda}[\mathbf{H}^{\mathbf{a}^n}] \right) \cdot \mathbf{n} \, d\Gamma \end{aligned}$$

And Set $\mathbf{a}^{n+1} := \mathbf{a}^n + \mathbf{b}$.

Alternatively, to avoid solving card (Λ) linear boundary value problems, an inner iterative coupling of the equations obtained from setting $\mathbf{Z} = \sum_{\lambda \in \Lambda} b_{\lambda} \mathbf{Z}_{\lambda}$ can be implemented (Kaltenbacher, 2006).

Quasi Newton methods. Appropriate low rank update formulas are able to provide approximations of the Jacobian that lead to superlinearly convergent variants of Newton's method. Among these quasi Newton methods, Broyden's method is the certainly most well-known and most widely used one. It is based on the update formula:

$$B^n = B^{n-1} + \frac{1}{(\mathbf{a}^n - \mathbf{a}^{n-1})^T (\mathbf{a}^n - \mathbf{a}^{n-1})} (F(\mathbf{a}^n) - F(\mathbf{a}^{n-1})) (\mathbf{a}^n - \mathbf{a}^{n-1})^T \quad (26)$$

where $\mathbf{a}^n - \mathbf{a}^{n-1}$ is assumed to be reordered in a column vector. Therewith, B^n is an approximation of the Jacobian in the sense that it satisfies the so-called secant condition:

$$B^n (\mathbf{a}^n - \mathbf{a}^{n-1}) = F(\mathbf{a}^n) - F(\mathbf{a}^{n-1})$$

and in place of equation (23) the next iterate can be defined by:

$$\mathbf{a}^{n+1} = \mathbf{a}^n - (B^n)^{-1} F(\mathbf{a}^n). \quad (27)$$

An initial guess B^0 can be chosen using a superposition operator $\mathcal{S}$ (e.g. $\boldsymbol{\mu} \cdot \mathbf{I}$) in place of $\sum_{\tilde{\lambda} \in \Lambda} a_{\tilde{\lambda}} \mathcal{P}'_{\tilde{\lambda}}[\mathbf{H}^{\mathbf{a}}]$. The analysis in Griewank (1987) provides a basis of applicability

of Broyden's method also to problems that are – as in our case — only Lipschitz continuous but not differentiable.

Application of Broyden's method for hysteresis identification in equation (11) leads to the following algorithm:

Algorithm 4. Set $\mathbf{a}^0$ To An Initial Guess For $\mathbf{a}$, And Initialize B^0 .

For $n = 0, 1, 2, 3, \dots$

Solve The Hysteretic IBVP equation (20) For $\mathbf{H}^{\mathbf{a}^n} = \mathbf{H}$.

Evaluate The Integrals:

$$F_i(\mathbf{a}^n) = \int_{\Gamma_c} \left(\mu_0 \mathbf{H}^{\mathbf{a}^n}(t_i) + \sum_{\lambda \in \Lambda} a_\lambda \mathcal{P}_\lambda[\mathbf{H}^{\mathbf{a}^n}](t_i) \right) \cdot \mathbf{n} \, d\Gamma - \Phi(t_i),$$

$$i = 1, \dots, n_T.$$

Update B^n According To equation (26).

Solve The System $B^n \mathbf{b} = -F(\mathbf{a}^n)$ For $\mathbf{b}$ And Set $\mathbf{a}^{n+1} = \mathbf{a}^n + \mathbf{b}$.

Remark 2. Considering the IBVP equation (20) and the measurements equation (21) simultaneously for each time step as a larger system of equations instead of equation (22), one would arrive at “all at once” versions of the Newton type iterations proposed here.

5. Numerical tests

Using Matlab, we numerically tested the proposed iterative identification methods on a 1-d scalar model problem:

$$\begin{cases} \text{PDE :} & \tilde{\gamma} \tilde{A}_t - (\tilde{\mathcal{P}}[\tilde{A}_x])_x = 0 \quad x \in [0, 1], \quad t \in [0, \tau] \\ \text{boundary conditions :} & \tilde{A}(0, t) = 0 \quad A(1, t) = \frac{1}{B_{\max} |\Gamma_c|} \Phi(t) \\ \text{initial conditions :} & \tilde{A}(x, 0) = \tilde{A}_0(x) \end{cases}$$

$$\text{additional data :} \quad (\tilde{\mathcal{P}}[\tilde{A}_x])(1, t) = \frac{n_w}{H_{\max} l_{\text{typ}}} I(t)$$

which is a normalized version of the magnetic vector potential formulation (14) with:

$$\tilde{\gamma} = \frac{\gamma}{B_{\max} H_{\max} T / \tau}, \quad \tilde{\mathcal{P}}[\tilde{B}] = \frac{1}{H_{\max}} (\mu_0 \mathbf{I} + \mathcal{M})^{-1} [B_{\max} \tilde{B}],$$

where $B_{\max}, H_{\max}$ are the maximal amplitudes of the magnetic flux density and magnetic field intensity, respectively, n_w is the number of coil windings, l_{typ} the typical length of a

	Altern.Iter. CPU/Its/resid	Kaczmarz CPU/Its/resid	Newton(c) CPU/Its/resid	Broyden CPU/Its/resid
10.58 sec 2 0.0295	11.48 sec 1 0.0406	10.89 sec 1 0.0297	8.26 sec 1 0.0324
13.60 sec 2 0.0287	-	16.45 sec 1 0.0300	689.94 sec 23 0.0460
10.47 sec 2 0.0250	11.78 sec 1 0.0373	11.03 sec 1 0.0246	8.15 sec 1 0.0298

Figure 1.
CPU times and iteration
numbers for data with
1 percent noise, and
stopping criterion:
 $\text{residual} \leq 1.5 \times \text{noise}$
level

magnetic field line, T the measurement time, and we used $\tau = 16\pi$. To generate synthetic data from this PDE model, we consider three different examples of hysteresis curves, as symbolized by their main hysteresis loops in the Figure 1: a hysteresis operator with saturation, a relatively narrow, and a relatively broad hysteresis operator. In each of our tests, we started with a constant weight function. The Kaczmarz iteration was carried out with eight time horizons corresponding to the eight quasi periods of the excitation signal, as well as to the discretization of the weight and Everett functions by eight rows and columns of pixels.

Figure 1 shows the computation times and numbers of iterations for synthetic data with 1 percent randomly generated noise. Here, we used a smoothed version of the usual incremental triangles excitation (see Kaltenbacher, 2006 for results with different excitation signals). As a stopping criterion, we used the discrepancy principle known from regularization theory to account for the instability induced by numerical differentiation equation (6), that is contained implicitly in our reconstruction of p .

6. Conclusions and remarks

In this paper, we discussed the problem of identifying Preisach hysteresis operators in the magnetic field equations and propose several iterative methods for solving this inverse problem. For further details and additional numerical results we refer to Kaltenbacher (2006).

Note that the methods proposed here are sufficiently general to be also used for the identification of parameters in other hysteresis models such as the Stoner-Wohlfarth model, the Jiles-Atherton model, or the Landau-Lifshitz equation. Moreover, they can also be applied for the identification of hysteresis in other phenomena modeled by PDEs such as piezoelectricity (Kaltenbacher and Kaltenbacher, 2005).

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Further reading

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