Shape analysis of counts maps

M. A. Klatt, D. Göring, C. Stegmann, and K. Mecke

Citation: *AIP Conf. Proc.* **1505**, 737 (2012); doi: 10.1063/1.4772365

View online: <http://dx.doi.org/10.1063/1.4772365>

View Table of Contents: <http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1505&Issue=1>

Published by the [American Institute of Physics](#).

Additional information on AIP Conf. Proc.

Journal Homepage: <http://proceedings.aip.org/>

Journal Information: http://proceedings.aip.org/about/about_the_proceedings

Top downloads: http://proceedings.aip.org/dbt/most_downloaded.jsp?KEY=APCPCS

Information for Authors: http://proceedings.aip.org/authors/information_for_authors

ADVERTISEMENT

Submit Now

Explore AIP's new open-access journal

- Article-level metrics
now available
- Join the conversation!
Rate & comment on articles

Shape analysis of counts maps

M. A. Klatt^{*,†}, D. Göring[†], C. Stegmann^{**} and K. Mecke^{*,†}

**Institut für Theoretische Physik, Universität Erlangen-Nürnberg,
Staudtstr. 7, D 91058 Erlangen*

*†Erlangen Center for Astroparticle Physics, Universität Erlangen-Nürnberg,
Erwin-Rommel-Str. 1, D 91058 Erlangen*

***DESY, Platanenallee 6, D 15738 Zeuthen*

Abstract. A novel approach for source detection via structural deviations from the typical features of a random background counts map is presented. Minkowski functionals, powerful tools from integral geometry, quantify the shape of level sets of a counts map. Compared to standard techniques, which use the total number of counts only, additional morphometric information is incorporated without the need for any prior knowledge about the source. Minkowski sky maps quantify local structural deviations; they localize and visualize potential sources.

Keywords: Morphology, Data analysis, Source detection, Shape measures, Minkowski functionals
PACS: 95.75.Mn

Introduction. The standard analysis technique for source detection in γ -ray astronomy is based on the total number of counts [1]. If the excess of counts in the observed on-region is significant compared to the off-region, the detection of a source is claimed. While such statistics of the total number of counts have proven sufficient for the detection of point sources, the additional structural or morphological information contained in the counts maps can be beneficial to the detection of extended galactic γ -ray sources [e.g. 2, 3].

The novel approach of source detection presented here incorporates the morphology without the need for any prior information about the source. The shape of the counts map itself is characterized and compared to the expected features of a random background counts map, i.e. the background shape. Instead of investigating and looking for an explicit shape of a source, any significant structural deviation is quantified. A detailed introduction to this new shape analysis in γ -ray astronomy can be found in Refs. [4, 5].

Shape quantification. The shape analysis based on the Minkowski functionals area A , perimeter P and Euler characteristic χ has been successfully applied in statistical physics [6–9], for pattern analysis [10–12] and in astronomy for point processes in cosmology [13–15] and the cosmic microwave background [16]. A fundamental theorem by Hadwiger ensures robustness and comprehensiveness of a morphology analysis based on Minkowski functionals, in the following sense [17]: any functional which is defined on unions of convex sets and which is motion invariant, additive and at least continuous on convex sets is a linear combination of Minkowski functionals; black and white pixel images can for example be represented as the union of quadratic bins, i.e. convex sets.

In order to quantify the shape of a counts map, which is a gray scale image, thresholding procedure turns it into a black and white image, i.e. a bin is set to black, if its number

FIGURE 1. Minkowski functionals quantify the shape of level sets of a counts map: (a) The number of counts k are simulated following a Poisson distribution. (b) Thresholding turns the counts map into a black and white image. (b-d) The Minkowski functionals are evaluated: area A , perimeter P and Euler characteristic χ , which is here given by the number of components minus the number of holes.

of counts k is larger or equal than a chosen threshold ρ – see Fig. 1 (a,b). For the black bins the Minkowski functionals are evaluated. For planar images these are the area A , the perimeter P and the Euler characteristic χ – a topological constant. They characterize the shape as a function of the threshold ρ [10]. The outside of the observation window is here considered to be white. Minkowski functionals are robust against noise and can be computed by linear time algorithms [8, 12].

Global null hypothesis. A homogeneous Poisson random field is chosen as a background model. This is justified by the fact that in very high energy γ -ray astronomy the bulk part of the background is caused by hadrons which arrive at earth in a uniform flux from every direction. The number of counts in each bin of the map is a random number following a Poisson distribution with intensity λ . At a threshold ρ , a bin is set to black, if the number of counts $k \geq \rho$. The probability for a black bin is thus given by $p_\rho = \sum_{i=\rho}^{\infty} \frac{\lambda^i}{i!} e^{-\lambda}$.

The probability distribution $\mathcal{P}$ for a Minkowski valuation $X \in \{A, P, \chi\}$ has to be determined for each threshold ρ or each probability p_ρ , respectively. These probability distributions derived for a purely random Poisson process describe the background shape – see Fig. 2 (a). The null hypothesis is that there are only background signals. Following Ref. [18] a most efficient hypothesis test can be constructed: the *compatibility* $\mathcal{C}$ of a

FIGURE 2. (a) Probability distribution $\mathcal{P}(\chi)$ for a value of the Euler characteristic χ to appear in a Poisson random field is plotted for different probabilities p_ρ that a bin is black. Accordingly, (b) depicts the compatibility following Eq. (1) and (c) the deviation strength $\mathcal{D} := -\log_{10}(\mathcal{C})$.

FIGURE 3. Shape analysis: (a) The counts map k is turned into a b/w image for each threshold ρ . If $k \geq \rho$, the bin is black, otherwise white. (b) Minkowski functionals characterize the shape as a function of threshold – here w.r.t. area of black bins (top). Eq. (1) provides the deviation strength $\mathcal{D} = -\log_{10}(C)$ (bottom). As $\mathcal{D} > 6.2$ for some thresholds ρ in the highlighted regime, the null hypothesis is rejected.

given counts map with the background shape is defined as

$$C(X) = \sum_{\mathcal{P}(X_i) \leq \mathcal{P}(X)} \mathcal{P}(X_i), \quad (1)$$

which is the probability to find an at least as unlikely shape in a purely Poisson random map – see also Fig. 2 (b).

The null hypothesis is rejected, if the compatibility is less than $0.6 \cdot 10^{-6}$, which is the compatibility of a normal distribution at a deviation of 5σ . Relating the deviation of a normally distributed random number to its compatibility according to Eq. (1), allows for a conversion between compatibility C and standard deviation σ . For convenience, the deviation strength $\mathcal{D} := -\log_{10}(C)$ is defined – see also Fig. 2 (c). It quantifies the lack of trust in the null hypothesis for each threshold ρ . The whole shape analysis is summarized and visualized in Fig. 3.

Local Minkowski sky maps. Minkowski functionals represent averages over the entire image and allow a global null hypothesis test. The Minkowski sky map is defined to quantify local shape deviations. It hence localizes and visualizes a potential source. The main idea is to restrict the analysis to a small sliding window. The maximum deviation strength for all thresholds ρ is assigned to the central bin of the sliding window – see Fig. 4.

Taking the maximum increases the probability of a background fluctuation; a trial factor has to be introduced. If N^2 is the number of bins, a conservative estimate is to assume N^2 independent trials. The compatibility is multiplied with N^2 , thus $-2\log_{10}N$ is added to the deviation strength. Figure 4 allows only a qualitative comparison between Minkowski sky map (c) and significance map (d), which is constructed following Refs. [1, 19]. A more detailed discussion is given in Ref. [5].

The Minkowski sky map is as useful as a significance map and in qualitative good agreement. The information about the area is comparable to that about the total number of counts. The additional shape information can be incorporated via a multivariate probability distribution of the three functionals.

FIGURE 4. Local Minkowski sky maps: (a) Simulated counts map. The shape analysis is restricted to a small sliding window. (b) Black and white image: the deviation strength $\mathcal{D}$ is computed for the sliding window as a function of the threshold. (c) The maximum $\mathcal{D}$ is assigned to the central bin of the sliding window; iterating over all bins provides the Minkowski sky map. (d) Significance map.

Outlook. Minkowski functionals quantify the shape of a counts map and incorporate morphometric information without any prior information about the source. Structural deviations from the expected background shape enable the detection of a source. Detector effects and nonuniform background components can be corrected for in a uniform manner [4]. A more detailed introduction to the Minkowski functional shape analysis [5] and the possible gain in sensitivity goes beyond the scope of this paper and will follow in future publications.

Acknowledgments. We thank the German science foundation (DFG) for the grant ME1361/11 as part of the group “Geometry and Physics of Spatial Random Systems”.

REFERENCES

1. T.-P. Li, and Y.-Q. Ma, *Astrophys. J.* **272**, 317–324 (1983).
2. F. Aharonian, et al., *Astron. Astrophys.* **449**, 223–242 (2006).
3. F. Aharonian, et al., *Astrophys. J.* **661**, 236 (2007).
4. D. Göring, *Gamma-Ray Astronomy Data Analysis Framework based on the Quantification of Background Morphologies using Minkowski Tensors*, Ph.D. thesis, Universität Erlangen-Nürnberg (2012).
5. D. Göring, M. A. Klatt, C. Stegmann, and K. Mecke, Morphometric analysis in gamma ray astronomy using Minkowski functionals (2012), in preparation for *Astron. Astrophys.*
6. K. Mecke, *Int. J. Mod. Phys. B* **12**, 861–899 (1998).
7. K. Mecke, and D. Stoyan, *Statistical Physics and Spatial Statistics*, vol. 554 of *Lecture Notes in Physics*, Springer, 2000, 1st edn.
8. G. E. Schröder-Turk, S. C. Kapfer, B. Breidenbach, et al., *J. Microsc.* **238**, 57–74 (2010).
9. G. E. Schröder-Turk, W. Mickel, S. C. Kapfer, et al., *Adv. Mater.* **23**, 2535–2553 (2011).
10. K. Mecke, *Phys. Rev. E* **53**, 4794–4800 (1996).
11. J. Becker, G. Grun, R. Seemann, et al., *Nat. Mater.* **2**, 59–63 (2003).
12. H. Mantz, K. Jacobs, and K. Mecke, *J. Stat. Mech.* **12**, 12015 (2008).
13. K. Mecke, T. Buchert, and H. Wagner, *Astron. Astrophys.* **288**, 697 (1994).
14. M. Kerscher, K. Mecke, and P. Schücker, *Astron. Astrophys.* **377**, 1 (2001).
15. M. Kerscher, K. Mecke, J. Schmalzing, et al., *Astron. Astrophys.* **373**, 1–11 (2001).
16. J. Schmalzing, T. Buchert, A. L. Melott, et al., *Astrophys. J.* **526**, 568 (1999).
17. H. Hadwiger, *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie*, Springer, 1957.
18. J. Neyman, and E. S. Pearson, *Philos. Trans. R. Soc. London, Ser. A* **231**, 289–337 (1933).
19. F. Aharonian, et al., *Astron. Astrophys.* **457**, 899–915 (2006).