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A theoretical derivation of the Cordier diagram for turbomachines

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Abstract: The design of high-efficiency fans is often based on the experience of the designer. In order to determine its main dimensions, fan designers use the Cordier diagram. For a given operating point (i.e. flowrate and pressure, and a rotating speed), the optimum diameter of high-efficiency fans can be found in the Cordier diagram. The Cordier diagram is an empirical diagram based on measurements. It delivers a relation between flowrate, pressure, rotating speed, and diameter. However, the Cordier diagram does not provide any information on the blade shape (i.e. the angles and the blade width). In order to fill this gap, there are design rules based on the experience of the designer and some analytical performance parameters in the literature. One very common performance parameter is the reaction, which is the ratio between the static and the total pressure rising from the impeller inlet to its outlet. These design rules and performance parameters are, however, of limited use. Therefore, the total-to-static ideal efficiency is introduced to yield, together with the speed and diameter numbers σ and δ , the essential parameters that distinguish the different turbomachines in the Cordier diagram. Based on the integral parameters of the flow and the geometry of turbomachines, a performance analysis of turbomachines is performed and the Cordier diagram is theoretically derived.

Keywords: Cordier diagram, turbomachinery, fans, blowers, performance analysis, computational fluid dynamics, inverse design method, integrated performance optimization, mean line analysis, radial impeller, axial impeller

1 INTRODUCTION AND AIM OF WORK

Fluid mechanics has developed methods that are generally applicable to fluid flows occurring in different fields of engineering. In many of these fields, specific considerations of fluid flow aspects have been performed, yielding relationships that somehow differ from those derived from the basic principles of fluid mechanics. Fluid mechanics is such a field where early results have been derived to show that there are similar parameters for turbomachines that permit axial, diagonal and radial fans, and blowers to be grouped.

In 1953, Otto Cordier [1] linked the optimum operating conditions (i.e. the volumetric flowrate Q_{opt} and

the specific head $Y = \Delta p_{\text{opt}}/\rho$), with the optimum diameter D and speed n , for one-stage machines operating at optimum efficiency, with the aid of speed and diameter numbers defined as

$$\sigma = 2\sqrt{\pi}n \frac{\sqrt{Q}}{(2Y)^{3/2}} \quad (\text{speed number}) \quad (1)$$

$$\delta = \frac{\sqrt{\pi}}{2} D \sqrt[4]{\frac{2Y}{Q^2}} \quad (\text{diameter number}) \quad (2)$$

where $Y = \Delta p_{\text{opt}}/\rho$ the specific head. Cordier plotted the optimum values σ_{opt} and δ_{opt} on a logarithmic graph, as shown, in the original representation of Cordier, in Fig. 1. In this way, for all classical impeller types one obtains a fairly well-defined curve known as the Cordier curve. Through experiments carried out for different types of fans, blowers, and pumps, Cordier could show that axial turbomachines possess high speed numbers σ and low diameter numbers δ , whereas radial turbomachines are characterized by

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Fig. 1 The original performance diagram of turbomachines as introduced by Cordier in 1953 [1]

low speed numbers σ and high diameter numbers δ . Diagonal machines are in the range of medium values of speed numbers σ and diameter numbers δ , see Fig. 1. Although this diagram does not show details of the blade shape for the design of turbomachines, it is definitely of help to make basic decisions on the kind of machine to be chosen for the particular operating point (i.e. for the head and flowrate to be achieved). Typically, if one has a given motor drive and hence a rotating speed and the prescribed operating point, using the Cordier diagram one can obtain the type of the machine (axial, diagonal or radial) and the diameter of the impeller which would meet this operating point at the best efficiency. On the other hand, if a certain operating point (i.e. flowrate and head), has to be achieved with an impeller of a given diameter, the Cordier diagram will provide the rotating speed to meet this operating point at the best efficiency. It is for this reason that the Cordier diagram is extensively used these days where layout considerations in turbomachines are performed. However, the Cordier diagram does not provide any information on the type of blade, whether single- or double-twisted, on the blade shape, on the blade angle, on the blade height, on the shape of the hub and shroud, and so on. A modern representation of the Cordier diagram is shown in Fig. 2.

There is somehow a lack of theoretical basis for the Cordier diagram. In connection with the Ph.D. research of P.E., the question came up on what kind of a diagram comes up for axial and radial turbomachines if basic fluid mechanics considerations would be applied to treat the overall performance of fans and blowers. What would be the resultant similarity

Fig. 2 The Cordier diagram

parameters and what interrelationship exists between them? Would the outcome be a diagram similar or equivalent to the Cordier diagram?

2 PERFORMANCE PARAMETERS

In the turbomachinery literature (e.g. Eck [2] or Bohl and Elmendorf [3]), it is traditional to treat the impellers from the blade inlet (1) to the blade outlet (2) as shown in Fig. 3. Actually, for radial impellers, the flow starts entering the impeller axially at position (0) and then, between position (0) and position (1), the flow turns around 90° to reach the blade inlet (1) radially. Therefore, if there is no prewhirl, accordingly (e.g. to Eck [2]), one has the so-called radial entry condition, which refers the so-called flow condition at the blade inlet at position (1). In the case of axial impellers, as can be seen (e.g. in Fig. 11), if there is no prewhirl, the flow enters the impeller and also reaches the blade inlet axially. The index (1) always refers to the blade inlet and not to the impeller inlet (0) and the index (2) refers to the blade outlet. The absolute flow velocity is c , the relative flow velocity is w and the peripheral velocity of the impeller is u . These velocities are related by the fundamental kinematic condition

$$c = w + u \quad (3)$$

The condition of radial entry in radial impellers and axial entry in axial impellers is equivalent to the condition of no prewhirl (i.e. the circumferential component of the absolute velocity at the impeller inlet $c_{u1} = 0$). As it is well known, the Euler turbomachinery equation

Fig. 3 Radial impeller

can be written as

$$Y = \frac{\Delta p_t}{\rho} = \frac{1}{2}[(c_2^2 - c_1^2) + (u_2^2 - u_1^2) + (w_1^2 - w_2^2)] \quad (4)$$

This form of the Euler pump and turbine equation is appropriate to distinguish between the different contributions to the pressure increase. It can be seen that the pressure increase is divided into three terms [2]. The first term

$$\Delta p_d = \frac{\rho}{2}(c_2^2 - c_1^2) \quad (5)$$

means an increase in kinetic energy. The term

$$\Delta p_{s\text{-centrifugal}} = \frac{\rho}{2}(u_2^2 - u_1^2) \quad (6)$$

means that a change in static pressure occurs in the impeller due to the centrifugal force acting on the fluid. The third term is a change in kinetic energy due to the change of the relative velocity in the impeller; this term represents a conversion of kinetic energy into static pressure within the impeller itself. This conversion into static pressure is given by Bernoulli's equation as

$$\Delta p_{s\text{-diffusion}} = \frac{\rho}{2}(w_1^2 - w_2^2) \quad (7)$$

As mentioned above, all these quantities are defined between the impeller inlet and outlet. Hence, one can in this first analysis interpret the Euler pump and turbine equation as follows

$$\Delta p_t = \underbrace{\frac{\rho}{2}(u_2^2 - u_1^2) + \frac{\rho}{2}(w_1^2 - w_2^2)}_{\text{static pressure}} + \underbrace{\frac{\rho}{2}(c_2^2 - c_1^2)}_{\text{dynamic pressure}} \quad (8)$$

As it is well known (e.g. Sigloch [4]), this equation can also be written as

$$\Delta p_t = \rho(u_2 c_{u2} - u_1 c_{u1}) \quad (9)$$

In general, there are no pre-swirl vanes and the flow always enters an axial impeller axially and a radial

impeller radially. In order to understand the radial entry condition for radial impellers, please see Fig. 3. The fluid enters a radial impeller usually axially, as shown at position 0 (i.e. with a velocity c_0).

Hence, for these most common cases

$$w_1^2 = u_1^2 + c_1^2 \quad (10)$$

and equations (8) and (9) reduce to (for example, see references [4] and [5])

$$\Delta p_t = \frac{\rho}{2}(u_2^2 + c_2^2 - w_2^2) \quad (11)$$

$$\Delta p_t = \rho u_2 c_{u2} \quad (12)$$

However, in order to develop the proper formulae to evaluate the performance of a fan, one has to look at the test rig (i.e. how the performance is evaluated at the test rig). In Fig. 4(a), a suction side test rig according to the DIN 24 163 norm is shown. In Fig. 4(b), the corresponding main components are schematically shown. Basically, the test rig is composed of:

- the inlet nozzle, where the flowrate can be adjusted with a throttle;
- the test rig chamber, where the flow velocity is reduced in such a way that the dynamic pressure is negligible (i.e. in the chamber, the static and total pressures are equal);
- after the chamber, at the suction side, the fan is connected to the test rig chamber.

In Fig. 4(c), the corresponding static and total pressures on the components are qualitatively shown. Since the air is sucked in from rest under atmospheric conditions, the static and total pressures in front of the nozzle are equal to the atmospheric pressure. Due to the acceleration of the flow entering the nozzle, the static pressure drops. In the nozzle, there are then friction losses leading to a further decrease in the static pressure and a corresponding decrease in the total pressure. Because of the sudden expansion at the test rig chamber entrance, there is a further decrease in the

Fig. 4 Test rig and measured pressures

total pressure. The static pressure tends to increase again due to the decrease in velocity at the chamber inlet because of the area increase, but it will not recuperate the pressure fully due to the losses of the sudden expansion (i.e. the Carnot shock losses), White [6], at the chamber entrance. In the chamber, since the cross-sectional area is large, the dynamic pressure is negligible, and therefore the static and total pressures can be considered equal to each other. At the end of the test rig chamber, close to the impeller inlet,

the flow accelerates due to the huge area reduction from the test rig section to the impeller inlet area. Here the static pressure decreases by $\rho c_1^2/2$, as shown in the detailed Fig. 4(d), but the total pressure remains constant since there is no energy transfer to the flow before the impeller and the losses are negligible. This is the first key issue that has to be considered when performing the design. In the classical literature, the term $\rho c_1^2/2$ is included in the static pressure increase from the impeller inlet to the impeller outlet, although it is

not associated with any energy transfer. This part of the static pressure arises only due to the acceleration of the flow at the inlet of the impeller, there is no energy gain here. Now the flow enters the impeller at the inlet 1 and leaves the impeller at the outlet 2. Between the inlet of the impeller 1 and the outlet of the impeller 2 power is introduced into the flow through the shaft: the static pressure increases by $\Delta p_s = p_{s,2} - p_{s,1}$ and the total pressure increases by $\Delta p_t = p_{t,2} - p_{t,1}$. It is important to note that at the exit of the impeller, the static pressure is atmospheric (i.e. $p_{s,2} = p_{atm}$), and the total pressure exceeds the static pressure by the dynamic pressure at the exit (i.e. $p_{t,2} = p_{atm} + \rho c_2^2/2$). This excess dynamic pressure at the impeller exit (i.e. $\rho c_2^2/2$), in fans and blowers normally is the term that is lost, unless it is partially reconverted into static pressure by a diffuser or guide passages, and not the term $(\rho/2)(c_2^2 - c_1^2)$ considered by Eck [2]. It is therefore very important to note that the pressure difference measured at the test rig is the total-to-static pressure, $\Delta p_{t-s} = p_{s,2} - p_{t,1}$, and not the static pressure increase $\Delta p_s = p_{s,2} - p_{s,1}$, although the later one is the one that is treated in the classical literature. Hence, the total-to-static pressure is the pressure difference to be used for a proper performance parameter definition.

In the classical literature (e.g. Eck [2]), the design performance parameter commonly used is the reaction. It is recommended that the ratio

$$r \equiv \frac{\Delta p_s}{\Delta p_t} = \frac{[(u_2^2 - u_1^2) + (w_1^2 - w_2^2)]}{[(c_2^2 - c_1^2) + (u_2^2 - u_1^2) + (w_1^2 - w_2^2)]} \quad (13)$$

should be kept as high as possible. This ratio r is called *degree of reaction*, *reaction effect*, or simply *reaction*. For radial entry, which is normally the case, using equation (10), the reaction reduces to

$$r = \frac{u_2^2 - w_2^2 + c_1^2}{u_2^2 - w_2^2 + c_2^2} = \frac{u_2^2 - w_2^2 + c_1^2}{2u_2 c_{u2}} \quad (14)$$

From this equation it follows immediately that

$$\left. \begin{array}{l} c_2 \geq c_1 \Rightarrow r \leq 1 \\ c_2 \leq c_1 \Rightarrow r \geq 1 \end{array} \right\} \quad (15)$$

This can be visualized in Fig. 4(d), from which one can also directly read out that

$$r = \frac{\Delta p_s}{\Delta p_t} = \frac{\Delta p_t - (\rho/2)(c_2^2 - c_1^2)}{\Delta p_t} = 1 - \frac{\rho}{2\Delta p_t}(c_2^2 - c_1^2) \quad (16)$$

This means that depending on whether the absolute velocity c_1 at the inlet is greater or smaller than the absolute velocity c_2 at the outlet, the reaction is smaller or greater than one. Hence, the reaction can be larger than one, which makes its use, as suggested

Fig. 5 Typical reaction for exit angles $\beta_2 < 90^\circ$ ($AR = A_2/A_1$)

in the literature (e.g. Eck [2] and Enßlinger [7]), actually impracticable as a performance parameter for design. In Fig. 5, the three typical reaction curves are shown, considering that $AR = A_2/A_1$ is the ratio of the impellers inlet and outlet areas:

- for $AR = 1$, the graph of the reaction against the flow coefficient is a straight line reaching the value of 1 at the maximum flow coefficient;
- for $AR < 1$, the graph of the reaction against the flow coefficient is a curve that reaches infinity at the maximum flow coefficient; for $AR > 1$, the graph of the reaction against the flow coefficient is a curve that reaches zero at the maximum flow coefficient and is never higher than one. For details, please see Epple [5].

In Fig. 4, a test rig according to the DIN 24 163 norm was presented and it was explained that the most commonly measured pressure is neither the static nor the total pressure differences, Δp_s and Δp_t , but the total-to-static pressure (i.e. Δp_{t-s})

$$\begin{aligned} \Delta p_{t-s} &= \Delta p_{s12} - \frac{\rho}{2} c_1^2 = \Delta p_{t12} - \frac{\rho}{2} c_2^2 \\ &= \frac{\rho}{2} [(u_2^2 - u_1^2) + (w_1^2 - w_2^2)] - \frac{\rho}{2} c_1^2 \end{aligned} \quad (17)$$

Due to the radial entry $w_1^2 = c_1^2 + u_1^2$ and hence

$$\Delta p_{t-s} = \frac{\rho}{2} (u_2^2 - w_2^2) \quad (18)$$

Therefore, it is necessary to define a corresponding efficiency in the design stage as being the total-to-static hydraulic power divided by the shaft power

$$\begin{aligned} \eta_{t-s} &= \text{total-to-static efficiency} \\ &= \frac{P_{t-s}}{P_{\text{shaft}}} = \frac{Q \Delta p_{t-s}}{Q \Delta p_t} = \frac{\Delta p_{t-s}}{\Delta p_t} \end{aligned} \quad (19)$$

Referring to Fig. 4, the total-to-static pressure can be written as

$$\Delta p_{t-s} = p_{\text{atm}} - p_{t1} = \Delta p_s - \frac{\rho}{2} c_1^2 = \Delta p_t - \frac{\rho}{2} c_2^2 \quad (20)$$

and therefore the total-to-static efficiency is equal to

$$\eta_{t-s} = \frac{\Delta p_s - (\rho/2) c_1^2}{\Delta p_t} = \frac{\Delta p_t - (\rho/2) c_2^2}{\Delta p_t} \quad (21)$$

$$\eta_{t-s} = r - \frac{\rho}{2} \frac{c_1^2}{\Delta p_t} = 1 - \frac{\rho}{2} \frac{c_2^2}{\Delta p_t} \quad (22)$$

With this equation, it is clear that the reaction r is not a measure of efficiency and it is always higher than the total-to-static efficiency. Besides, from the second form, it is clear that the total-to-static efficiency is always less than one. Considering the Euler equation (12) for radial entry, this equation can be further simplified as

$$\eta_{t-s,i} = r - \frac{1}{2} \frac{c_1^2}{u_2 c_{u2}} = 1 - \frac{1}{2} \frac{c_2^2}{u_2 c_{u2}} \quad (23)$$

This is an ideal total-to-static efficiency. It is a central equation to the design method for radial impellers presented in this work. These results are summarized in Table 1.

The flow coefficient and the head coefficient are given as (e.g. Lewis [8])

$$\varphi = \frac{c_{m2}}{u_2} \quad (24)$$

$$\psi = \frac{c_{u2}}{u_2} \quad (25)$$

This is a consistent way to define the head coefficient, which proves to have many formal advantages.

Nevertheless, in the literature (e.g. Bohl and Elmen-dorf [3], Cordier [1], and Eck [2]), the head coefficient is defined as $\psi_C = 2c_{u2}/u_2 = 2Y/u_2^2$ (i.e. $\psi_C = 2\psi$), which is the way most authors write it. Therefore, this has to be taken into account when comparing the theoretical results developed here with experimental data in the literature. Hence, the efficiency can be written as

$$\begin{aligned} \eta_{t-s,i} &= 1 - \frac{1}{2} \frac{c_2^2}{u_2 c_{u2}} = 1 - \frac{1}{2} \frac{c_{m2}^2 + c_{u2}^2}{u_2 c_{u2}} \\ &= 1 - \frac{1}{2} \frac{c_{m2}^2}{u_2 c_{u2}} - \frac{1}{2} \frac{c_{u2}^2}{u_2 c_{u2}} = 1 - \frac{1}{2} \frac{c_{m2}}{u_2} \frac{c_{m2}}{c_{u2}} - \frac{1}{2} \frac{c_{u2}}{u_2} \\ &= 1 - \frac{1}{2} \varphi \frac{\varphi}{\psi} - \frac{1}{2} \psi \end{aligned} \quad (26)$$

and finally

$$\eta_{t-s,i} = 1 - \frac{1}{2\psi} (\varphi^2 + \psi^2) \quad (27)$$

Although the algebraic equation (27) might seem trivial at first glance, it depicts the maximum achievable efficiency (i.e. the ideal total-to-static efficiency). It represents the maximum useful hydraulic power that can be generated by a radial impeller for a given shaft power: it is a theoretical upper limit. It is similar in its function to the equation of Betz for the maximum power which can be extracted from the air with a wind machine (see also [9]) or the equation of Froude for the ideal efficiency of an airscrew (for example, see reference [10]). Although the equations of Betz and Froude are about 100 years old and one could expect that a similar equation for radial impellers for fans to already exist, which seems not to be the case even after an extensive bibliographic research. This equation is essential for understanding the Cordier diagram and the design procedure of radial impellers.

Table 1 Impeller and test rig formulae

Impeller – classical literature	Test rig
(a) Total pressure $\Delta p_t = \frac{\rho}{2} [(c_2^2 - c_1^2) + (u_2^2 - u_1^2) + (w_1^2 - w_2^2)]$	(a) Total pressure $\Delta p_t = \frac{\rho}{2} [(c_2^2 - c_1^2) + (u_2^2 - u_1^2) + (w_1^2 - w_2^2)]$ For radial (radial fan) or axial (axial fan) entry $\Delta p_t = \frac{\rho}{2} (c_2^2 + u_2^2 - w_2^2)$
(b) Static pressure increase $\Delta p_s = \frac{\rho}{2} [(u_2^2 - u_1^2) + (w_1^2 - w_2^2)]$	(b) Total-to-static pressure $\Delta p_{t-s} = \frac{\rho}{2} [(u_2^2 - u_1^2) + (w_1^2 - w_2^2) - c_1^2]$ For radial (radial fan) or axial (axial fan) entry $\Delta p_{t-s} = \frac{\rho}{2} (u_2^2 - w_2^2)$
(c) Dynamic pressure increase $\Delta p_d = \frac{\rho}{2} (c_2^2 - c_1^2)$	(c) Dynamic pressure at the exit (in general this term is lost) $p_{d-2} = \frac{\rho}{2} c_2^2$
(d) Reaction (for radial entry) $r = \frac{\Delta p_s}{\Delta p_t} = \frac{u_2^2 - w_2^2 + c_1^2}{u_2^2 - w_2^2 + c_2^2} = \frac{u_2^2 - w_2^2 + c_1^2}{2 u_2 c_{u2}}$	(d) Total-to-static efficiency (for radial entry) $\eta_{t-s} = \frac{\Delta p_{t-s}}{\Delta p_t} = \frac{\Delta p_t - p_{d-2}}{\Delta p_t} = \frac{\Delta p_s - \rho c_1^2/2}{\Delta p_t} = \frac{\Delta p_t - \rho c_2^2/2}{\Delta p_t}$

To first have a better understanding of equation (27) one can start solving equation (27) for the head coefficient to obtain

$$\psi = (1 - \eta) \pm \sqrt{(1 - \eta)^2 - \varphi^2} \quad (28)$$

This equation can be rearranged as

$$\varphi^2 + [\psi - (1 - \eta)]^2 = (1 - \eta)^2 \quad (29)$$

which is the equation for circles of radius $(1 - \eta)$ centred at $[0; 1 - \eta]$ (Fig. 6). The optimum duty line is given by $\psi = \varphi$ (for details on the optimum duty line, see reference [5]). To operate an impeller on the optimum duty line means to have the maximum efficiency for a given flow coefficient: this is possible only for one unique head coefficient. Operating at any lower flow coefficient at the same efficiency means either a lower head coefficient (for axial impellers) or a higher head coefficient (for radial impellers).

Head and flow coefficients are not independent. They are connected through the velocity triangle (Fig. 7)

$$c_{u2} = u_2 - \frac{c_{m2}}{\tan \beta_2} \quad (30)$$

which is another way to express Euler's turbomachinery equation. Hence, one can derive a relation between flow coefficient, outlet angle, and flow coefficient

$$\psi = 1 - \frac{\varphi}{\tan \beta_2} \quad (31)$$

and

$$\varphi = (1 - \psi) \tan \beta_2 \quad (32)$$

Fig. 6 $\varphi - \psi$ diagram with efficiency contours

Fig. 7 Velocity triangle at outlet

Substituting equations (31) and (32) into equation (27) the ideal total-to-static efficiency can be expressed as

$$\eta_{t-s,i} = 1 - \frac{1}{2\psi} [(1 - \psi)^2 \tan^2 \beta_2 + \psi^2] \quad (33)$$

$$\eta_{t-s,i} = 1 - \frac{1}{2(1 - (\varphi / \tan \beta_2))} \left[\varphi^2 + \left(1 - \frac{\varphi}{\tan \beta_2} \right)^2 \right] \quad (34)$$

These two expressions represent a family of parametric curves for the efficiency. In equation (33), the parameter is the head coefficient ψ and in equation (34) the parameter is the flow coefficient φ . It can easily be shown [11] that by differentiating these equations on both sides with respect to the parameters ψ and φ , respectively, and by solving these variables and substituting the result in the original equations, thus eliminating the parameters that for both families, the involutes are given by

$$\eta_{t-s,max} = \frac{1}{1 + \sin(\beta_2)} \quad (35)$$

The involute is the limiting curve for efficiency. Hence, it gives the maximum efficiency reachable at each exit angle β_2 . This is the upper limit for the efficiency at each exit angle value. It can be proved, see Epple [5], that for exit angles $\beta_2 > 90^\circ$ the maximum total-to-static efficiency is always $\eta_{t-s,max} = 0.5$. These results are shown in Fig. 8.

2.1 The ideal total-to-static efficiency and the Cordier diagram

As already mentioned, Cordier [1] linked the optimum operating conditions (i.e. the flowrate Q_{opt} and the specific head $Y = \Delta p_{opt} / \rho$), with the optimum diameter D and speed n , for one-stage machines operating at the optimum efficiency, with the aid of speed and diameter numbers, equations (1) and (2). In order to correlate the speed and diameter numbers with the head and pressure coefficients, one has to bear in mind that Cordier defined the head and pressure coefficients

Fig. 8 Maximum theoretical total-to-static efficiency

directly in terms of the diameter D , flowrate Q , speed n , and specific work $Y = \Delta p / \rho$

$$\varphi = \frac{c_m}{u} = \frac{Q}{Au} = \frac{4Q}{D^3 \pi^2 n} \quad (\text{flow coefficient}) \quad (36)$$

$$\psi_C = \frac{2Y}{u^2} = \frac{2Y}{D^2 n^2 \pi^2} \quad (\text{head coefficient}) \quad (37)$$

instead of, as defined by equations (24) and (25), as a function of the meridian and circumferential velocities divided by the peripheral velocity. The basic difference, when comparing these equations with equations (24) and (25), is the factor 2 in equation (37), and therefore in equation (37) the head coefficient has an index C, indicating that it is the definition used by Cordier [1].

It can be readily shown from equations (1), (2), (36), and (37) that

$$\varphi = \frac{1}{\sigma \delta^3} \quad (38)$$

$$\psi = \frac{1}{2\sigma^2 \delta^2} \quad (39)$$

with $\psi = \psi_C / 2$, where ψ_C is the head coefficient as defined by Cordier and ψ is the head coefficient as defined in this work, equation (25). Substituting equations (38) and (39) into equation (27) one obtains

$$\eta_{t-s,i} = 1 - \frac{1}{4\delta^2} \left[\frac{4}{\delta^2} + \frac{1}{\sigma^2} \right] \quad (40)$$

This equation can be rearranged to deliver a parametric equation for the speed number as a function of the diameter number, having the ideal total-to-static efficiency as a parameter

$$\sigma = \frac{1}{2} \frac{\delta}{\sqrt{\delta^4 (1 - \eta_{t-s,i}) - 1}} \quad (41)$$

Assigning values to the total-to-static efficiency $\eta_{t-s,i}$, it is now possible to plot this equation in the

Fig. 9 The Cordier diagram and the ideal total-to-static efficiency

Cordier diagram, Fig. 9. One can see that the resulting curves fit very well into the Cordier diagram. The Cordier line lies between the curves of total-to-static ideal efficiency of 60% and 80% for values of the speed coefficients $\sigma_{\text{opt}} < 0.4$ (i.e. for all radial machines).

This means that the slope of the theoretically derived curves, with the ideal total-to-static efficiency as a parameter, fits the Cordier diagram very well for speed coefficients $\sigma_{\text{opt}} < 0.4$. For speed coefficients, $\sigma_{\text{opt}} > 0.4$, it seems that the theoretical curves turn around faster than the Cordier line and finally cross it. Hence, it seems that they do not really fit in the axial range of the Cordier line. However, in the original Cordier publication, there is no Cordier line in the axial range; instead, Cordier measured the efficiency level curves in the axial region. This original Cordier diagram can be seen in Fig. 10. The straight lines of slope -1 of constant head coefficient are a direct consequence of equation (39). In the same way, constant values of the flow coefficient result in straight lines of slope -3

$$\log \sigma = -\frac{1}{2} \log(2\psi) - \log \delta \quad (42)$$

$$\log \sigma = -\log \varphi - 3 \log \delta \quad (43)$$

In Cordier's publication, one can see optimal $\sigma_{\text{opt}} / \delta_{\text{opt}}$ points for fans, blowers, and centrifugal pumps. Further, for the range of speed coefficients, $\sigma_{\text{opt}} > 0.4$, contour plots of iso-efficiency lines for axial and diagonal impellers are shown.

Here, it can now be seen that the ideal total-to-static efficiency $\eta_{t-s,i}$ also matches the original Cordier diagram for values of speed number $\sigma_{\text{opt}} > 0.4$.

For the sake of clarity, it should be mentioned that in Cordier's publication actually there is no explicit

Fig. 10 Original Cordier diagram with $\eta_{t-s,i}$ contours

definition of the efficiency considered in the measurements. However, since it is mentioned that measurements were made for axial fans (number not specified) and blowers for the region $\sigma_{\text{opt}} > 0.4$ and for 120 radial fans and blowers and also for centrifugal pumps for the region $\sigma_{\text{opt}} < 0.6$, it is reasonable to presume that it was the total-to-static efficiency, since this is the efficiency that disregards the dynamic pressure at the exit and evaluates the performance using the useful hydraulic power divided by the shaft power. This is how the performance of fans and blowers is usually evaluated in industry. But even if the total efficiency is meant, one has to bear in mind that the total efficiency is computed adding the dynamic pressure at the exit of the fan to the total-to-static pressure. The dynamic pressure $\rho c_2^2/2$ is computed estimating the total velocity at the exit c_2 as the ratio of the flowrate and the exit area of the fan (e.g. of the diameter of the exit pipe of the spiral casing). Computing the total pressure in this way would not differ much from the total-to-static pressure. Finally, as shown above, it fits the Cordier diagram fairly well.

For radial machines, it is clear from the original Cordier diagram, Fig. 10, that the line of optimum efficiency (i.e. the Cordier line for $\sigma_{\text{opt}} < 0.4$), is closer to the line of constant head coefficient $\psi = \psi_C/2 = 0.5$ ($\psi_C = 1$).

After the derivation of the total-to-static efficiency and the location in the Cordier diagram, some remarks on these results might be useful before proceeding with the derivation for axial machines.

The present theoretical derivation is based on the non-viscous and incompressible mean line theory, as it is usual for fans, blowers, and pumps (e.g. Bohl and Elmendorf [3], Eck [2], and Sigloch [4]). Hence,

in this first analysis, the effects due to viscosity, such as viscous losses, were neglected. This is admissible, because the flows treated here are high Reynolds flows. The Reynolds number, which is the ratio between the inertial and the viscous forces (e.g. Durst [12] or White [6]), can be written for an impeller as

$$\text{Re}_{\text{impeller}} = \frac{w D_{\text{hyd}}}{\nu} \quad (44)$$

where w is the relative velocity in the impeller blade channel, D_{hyd} is a typical hydraulic diameter for the blade passage and ν is the kinematic viscosity of the fluid. Therefore, in order to capture the basic properties of the flow in the impeller a non-viscous treatment is sufficient. This conforms to the usual aerodynamic and hydrodynamic design procedure for fluid machines, since here one is interested to obtain workable formulae in order to perform the design. The theory that results is not exact, as it is usually the case for design purposes, but it does capture the essential physics of the process, failing only to lead to the exact value. This is what is needed for design, in order to set the main dimensions of an impeller. This is also the purpose of the Cordier diagram (i.e. deliver information for the main dimensioning of turbomachines). In the design practice, in the final step of the design process, these designs are then validated by measurements or computational fluid dynamics (CFD) simulations. Since the Cordier diagram itself is based on measurements it directly delivers a validation to the present theory. A second way to validate the theory is performing CFD computations. How well the design predictions fit to the CFD computations has been already done by the authors and presented in Epple [5] and Epple *et al.* [13, 14]. Here, it was confirmed for several practical cases that the design process and hence these simplified formulae, which do not take into account the viscosity, indeed capture the essential physics of the process, failing to lead to the exact value.

A final but fundamental remark has to be made on efficiency. A large number of efficiency definitions are present in the literature on turbomachines and most workers in this field would agree that there are too many, as mentioned by Dixon [15]. Therefore, one has to use those considered to be important and useful. In the present case, one has to restrict it even more: the efficiency definition has to be useful for design. Having taken into consideration, the ideal total-to-static efficiency, equation (23) or, in order to compare it with the total-to-total efficiency, equation (21), has been developed. The overall or total-to-total efficiency of the fan or pump is defined as the ratio of the total hydraulic power divided by the shaft power. If no viscous and three-dimensional losses are considered, the shaft power and the total hydraulic power are equal and all the mechanical powers of the shaft is

transferred to the fluid

$$\eta_{t-t} = \frac{P_{\text{hyd},t}}{P_{\text{shaft}}} = \frac{Q\Delta p_t}{Q\Delta p_t} = 1 \quad (45)$$

That means that from a non-viscous ideal point of view, the total-to-total efficiency is always one. It is less than one only when considering viscous losses. Although this is valid for the total-to-total efficiency, for the total-to-static efficiency considered here it is different, since here the major amount of losses is not due to viscous losses but due to pure mechanical losses. To understand this issue better, one can rewrite equation (21) as follows

$$\eta_{t-s} = \frac{Q\Delta p_t - Q(\rho/2)c_2^2}{Q\Delta p_t} = \eta_{t-t} - \frac{\rho}{2} \frac{c_2^2}{\Delta p_t} \quad (46)$$

The term that makes the difference to the total-to-total efficiency, which is less than one only when also considering the viscous losses, is the last term on the right-hand side of equation (46): $(\rho/2)(c_2^2/\Delta p_t)$. This is independent of the viscosity: it is a purely mechanical loss that has to be considered when computing the efficiency of fans and pumps. Hence, the total-to-static efficiency is always less than one, even when the viscous losses are neglected. This mechanical loss term, combined with the fact that high Reynolds flows are being treated and hence the viscous losses are relatively low, is the reason why this theory works well without considering the viscous dissipation effects.

2.2 Axial machines

Equation (27) was derived for radial impellers. The same equation is also valid for a section of an axial impeller. This will be shown in this section.

Considering the velocity triangles of axial fans, Fig. 11, one can apply Euler's general equation (4)

$$\Delta p_t = \frac{\rho}{2} [(c_2^2 - c_1^2) + (u_2^2 - u_1^2) + (w_1^2 - w_2^2)] \quad (47)$$

Since axial fans $u_1 = u_2 = u$, this equation reduces to

$$\Delta p_t = \frac{\rho}{2} [(c_2^2 - c_1^2) + (w_1^2 - w_2^2)] \quad (48)$$

which, by Pythagoras's theorem, can also be written as

$$\Delta p_t = \frac{\rho}{2} [(c_{m2}^2 + c_{u2}^2 - c_{m1}^2 - c_{u1}^2) + (w_{m1}^2 + w_{u1}^2 - w_{m2}^2 - w_{u2}^2)] \quad (49)$$

Since, by mass flow conservation, $c_{m1} = c_{m2} = w_{m1} = w_{m2}$, this equation reduces to

$$\Delta p_t = \frac{\rho}{2} [(c_{u2}^2 - c_{u1}^2) + (w_{u1}^2 - w_{u2}^2)] \quad (50)$$

Fig. 11 Velocity triangle for axial fans

Because of axial entry (no pre-swirl), $c_{u1} = 0$ and $w_{u1} = u$. Hence

$$\Delta p_t = \frac{\rho}{2} [c_{u2}^2 + u^2 - w_{u2}^2] \quad (51)$$

Since, from the velocity triangle of Fig. 11, $w_{u2} = u - c_{u2}$, this equation reduces to

$$\Delta p_t = \rho u c_{u2} \quad (52)$$

From equation (20), which is valid for any kind of turbomachinery (i.e. radial, diagonal, and axial), the total-to-static pressure can be expressed as

$$\Delta p_{t-s} = \Delta p_t - \frac{\rho}{2} c_2^2 \quad (53)$$

Hence, the ideal total-to-static efficiency for axial impellers can be written as

$$\begin{aligned} \eta_{t-s} &= \frac{\Delta p_{t-s}}{\Delta p_t} = \frac{\Delta p_t - (\rho/2)c_2^2}{\Delta p_t} = 1 - \frac{1}{2} \frac{c_2^2}{u_2 c_{u2}} \\ &= 1 - \frac{1}{2} \frac{c_{2m}^2 + c_{2u}^2}{u_2 c_{u2}} = 1 - \frac{1}{2} \frac{c_{2m}^2}{u_2 c_{u2}} - \frac{1}{2} \frac{c_{2u}^2}{u_2 c_{u2}} \\ &= 1 - \frac{1}{2} \frac{c_{2m}}{u_2} \frac{c_{2m}}{c_{u2}} - \frac{1}{2} \frac{c_{2u}}{u_2} = 1 - \frac{1}{2} \varphi \frac{\varphi}{\psi} - \frac{1}{2} \psi \end{aligned} \quad (54)$$

and finally one obtains the same result as for radial impellers, equation (27)

$$\eta_{t-s,i} = 1 - \frac{1}{2\psi} (\varphi^2 + \psi^2) \quad (55)$$

As mentioned at the beginning of this section, this formula is valid at one radius of an axial machine. In order to understand how a full axial machine behaves

and where it is located in the Cordier diagram, as compared to radial fans, one has to integrate this expression over the full fan taking the area average. This will be done for a simple case in the next section.

2.3 Axial and radial machines in the Cordier diagram

In order to understand the fundamental differences between axial and radial fans, a simplified analysis can be performed. A detailed analysis is being prepared by the main author for a further publication. For the purpose of this paper, however, a simplified analysis is adequate to explain the fundamental differences between axial and radial fans, as for example their location in the Cordier diagram.

From the velocity triangle, Fig. 7, the relative velocity at the exit of the impeller, w_2 , can be written as

$$w_2 = \frac{c_{m2}}{\sin \beta_2} \quad (56)$$

Combining equations (18) and (56) the expression for the total-to-static pressure is obtained

$$\Delta p_{t-s} = \frac{\rho}{2} \left[u_2^2 - \left(\frac{c_{m2}}{\sin \beta_2} \right)^2 \right] \quad (57)$$

This equation is basically valid for radial as well as for axial fans. One has to observe, however, that for radial fans the equation is valid for the whole impeller, whereas for axial fans it is valid for one section (i.e. one radius), only. In order to obtain the expression for a whole axial impeller, equation (57) has to be integrated for all radii between hub and tip, taking the area average (see Fig. 12)

$$\begin{aligned} \Delta p_{t-s, \text{axial fan}} &= \frac{1}{\pi(R_{\text{tip}}^2 - R_{\text{hub}}^2)} \int_{R_{\text{hub}}}^{R_{\text{tip}}} \Delta p_{t-s} 2\pi R dR \\ &= \frac{1}{\pi(R_{\text{tip}}^2 - R_{\text{hub}}^2)} \int_{R_{\text{hub}}}^{R_{\text{tip}}} \frac{\rho}{2} \left[u_2^2 - \left(\frac{c_{m2}}{\sin \beta_2} \right)^2 \right] 2\pi R dR \end{aligned} \quad (58)$$

For axial fans, the integration of equation (58) can be very complicated, since β_2 and c_{m2} can be functions of the radius R , and furthermore require the coupled solution of the equation of radial equilibrium (for example, see reference [16]), which takes into account the mass transfer in the radial direction in the impeller and corrects the value of $c_m(R)$ accordingly. This full solution will be shown in a separate paper by the main author. Therefore, here, a simplified computation will be performed.

The simplified procedure works as follows. First, one has to find the maximum flowrate and the maximum total-to-static-pressure of a axial impeller at

Fig. 12 Full axial fan integration of the total-to-static pressure

a given section (i.e. radius). Then these values are integrated over the whole impeller, taking the area average, in order to obtain these values for the axial fan. Afterwards, a correction is applied to obtain the approximate values of flowrate Q_0 at the best operating point and the corresponding pressure $\Delta p_{t-s,0}$. With these values, the corresponding flow and head coefficients and afterwards the speed and diameters will be computed. The later ones will be plotted in the Cordier diagram and it will be shown that the location of axial fans in the Cordier diagram is at high speed numbers and low diameter numbers and that the location of radial fans in the Cordier diagram is at low speed numbers and high diameter numbers. This will now be performed step-by-step.

2.3.1 Computation of the maximum total-to-static pressure and the maximum flowrate for one section

In Fig. 13, the total pressure according to equation (11) and the total-to-static pressures according to equation (18) are shown qualitatively against the flowrate. The total pressure is represented by the straight line and the total-to-static pressure by the parabola.

As can be seen in Fig. 13, the total-to-static head-flow-rate-characteristic can be well characterized by the maximum pressure at zero flowrate, setting the flowrate or the meridian velocity c_m in equation (57) equal to zero and solving for the maximum total-to-static pressure at zero flowrate

$$\Delta p_{t-s, \text{max}} = \frac{\rho}{2} u_2^2 \quad (59)$$

Fig. 13 Total and total-to-static pressure

and the maximum flowrate at zero pressure, considering that

$$c_{m2} = \frac{Q}{A_2} \quad (60)$$

where A_2 is the exit area of the impeller, whether radial or axial, and considering also equation (57)

$$Q_{t-s,max} = u_2 A_2 \sin \beta_2 \quad (61)$$

2.3.2 Integration of the maximum total-to-static pressure and the maximum flowrate for the whole fan

The maximal pressure for radial fans is simply given by equation (59)

$$\Delta p_{t-s,max,radial} = \frac{\rho}{2} u_2^2 \quad (62)$$

In the case of axial fans, one has to integrate equation (59) from hub to tip taking the area average, as illustrated in Fig. 12

$$\begin{aligned} \Delta p_{t-s,max,axial} &= \frac{\rho}{2} \frac{1}{A_2} \int_{R_{hub}}^{R_{tip}} u^2(R) dA \\ &= \frac{\rho}{2} \frac{1}{\pi (R_{tip}^2 - R_{hub}^2)} \int_{R_{hub}}^{R_{tip}} (2\pi R n)^2 2\pi R dR \end{aligned} \quad (63)$$

$$\begin{aligned} \Delta p_{t-s,max,axial} &= \frac{\rho}{2} \frac{8\pi^3 n^2}{\pi (R_{tip}^2 - R_{hub}^2)} \int_{R_{hub}}^{R_{tip}} R^3 dR \\ &= \frac{\rho}{2} \frac{8\pi^3 n^2}{\pi (R_{tip}^2 - R_{hub}^2)} \frac{1}{4} (R_{tip}^4 - R_{hub}^4) \end{aligned} \quad (64)$$

Defining the ratio of the hub-to-tip radii as

$$m \equiv \frac{R_{hub}}{R_{tip}} \quad (65)$$

equation (64) can be rearranged as

$$\begin{aligned} \Delta p_{t-s,max,axial} &= \frac{\rho}{2} \frac{8\pi^3 n^2}{\pi} \frac{1}{4} R_{tip}^2 (1 + m^2) \\ &= \frac{\rho}{2} \frac{1}{2} u_{tip}^2 (1 + m^2) \end{aligned} \quad (66)$$

$$\Delta p_{t-s,max,axial} = \frac{\rho}{2} \frac{u_{tip}^2}{2} (1 + m^2) \quad (67)$$

For radial impellers, equation (59) is valid

$$\Delta p_{t-s,max,radial} = \frac{\rho}{2} u_2^2 \quad (68)$$

Hence, the maximum pressure of an axial fan lies between half of the maximum pressure of a radial fan and reaches the maximum pressure of a radial fan in the case of a radii ratio $m = 1$, which is rather a theoretical limit, since, as will be shown below, in this limiting case the maximum flowrate of the axial fan is zero. In general, the maximum pressure of an axial fan is less than the maximum pressure of a radial fan.

For radial fans, the maximum flowrate is directly given by equation (61)

$$Q_{max,radial} = u_2 A_2 \sin \beta_2 \quad (69)$$

For axial fans, one has to take the area average of equation (61) integrating from hub to tip

$$\begin{aligned} Q_{max,axial} &= \frac{1}{A_2} \int_{r_{hub}}^{r_{tip}} u_2 A_2 \sin \beta_2 dA \\ &= \int_{r_{hub}}^{r_{tip}} (2\pi n R) \sin \beta(R) 2\pi R dR \end{aligned} \quad (70)$$

Strictly speaking, the exit angle $\beta_2(r)$ has to be kept in the integral, since this value may vary from hub to tip and from impeller to impeller. For this general and simplified analysis, however, it is enough to assume some mean value $\bar{\beta}_2$, that is

$$\begin{aligned} Q_{max,axial} &= 4\pi^2 n \sin \bar{\beta}_2 \int_{r_{hub}}^{r_{tip}} R^2 dR \\ &= 4\pi^2 n \sin \bar{\beta}_2 \frac{1}{3} (R_{tip}^3 - R_{hub}^3) \\ &= 4\pi^2 n \sin \bar{\beta}_2 \frac{R_{tip}^3}{3} (1 - m^3) \end{aligned} \quad (71)$$

which can also be rearranged as

$$\begin{aligned} Q_{max,axial} &= \frac{2}{3} \pi R_{tip}^2 u_{tip}^2 \sin \bar{\beta}_2 (1 - m^3) \\ &= \frac{(1 - m^3)}{6} \pi D_{tip}^2 u_{tip}^2 \sin \bar{\beta}_2 \end{aligned} \quad (72)$$

Equations (69) and (72) have to be compared with some care, since for the case of the radial impeller

$A_{2,\text{radial}} = \pi b_2 D_2$, where b_2 is the exit height of the radial impeller. Since in most radial impellers usually $b_2 \ll D_2$ the maximum flowrate reached by a radial impeller is much lower than the maximum flowrate reached by an axial impeller.

2.4 Mean flowrate and pressure

Referring to Fig. 13 and to equations (57), (59), and (61), one can describe the total-to-static head characteristic by

$$\Delta p_{t-s} = \Delta p_{t-s,\text{max}} \left[1 - \left(\frac{Q}{Q_{\text{max}}} \right)^2 \right] \quad (73)$$

Since it is a parabolic equation, one can choose as a good estimate of the flowrate in the operating point (see Fig. 13)

$$Q_0 = \frac{1}{2} Q_{\text{max}} \quad (74)$$

Substituting the estimate given by equation (74) into equation (73) one can find an estimate of the total-to-static pressure in the operating point

$$\Delta p_{t-s,0} = \frac{3}{4} \Delta p_{t-s,\text{max}} \quad (75)$$

Substituting equations (74) and (75) into equations (67) to (69), and (72), one finally obtains

$$\Delta p_{t-s,0,\text{axial}} = \frac{\rho}{2} \frac{3}{8} u_{\text{tip}}^2 (1 + m^2) \quad (76)$$

$$\Delta p_{t-s,0,\text{radial}} = \frac{\rho}{2} \frac{3}{4} u_2^2 \quad (77)$$

$$Q_{0,\text{axial}} = \frac{(1 - m^3)}{12} \pi D_{\text{tip}}^2 u_{\text{tip}}^2 \sin \bar{\beta}_2 \quad (78)$$

$$Q_{0,\text{radial}} = \frac{1}{2} u_2 A_2 \sin \bar{\beta}_2 \quad (79)$$

Equations (76) to (79) are the equations to be used in order to locate radial and axial fans in the Cordier diagram. As mentioned before, the axial fan equations are simplified equations.

2.5 Mean speed and diameter numbers and the Cordier diagram

The definition of the head and flow coefficients as given in the literature (e.g. Bohl and Elmendorf [3]), already mentioned in equations (36) and (37), considering that the specific work $Y = \Delta p_{t-s}/\rho$, since the total-to-static and the total pressure, as measured in the praxis of fan measurements, as mentioned above,

do not differ substantially, are given by

$$\varphi = \frac{4Q}{D_2^3 \pi^2 n} \quad (80)$$

$$\psi = \frac{2\Delta p_{t-s}}{\rho \pi^2 D_2^2 n^2} \quad (81)$$

Furthermore, the speed and diameter numbers are computed, solving from equations (38) and (39), as (for example, see reference [3])

$$\sigma = \frac{\varphi^{1/2}}{\psi^{3/4}} \quad (82)$$

$$\delta = \frac{\psi^{1/4}}{\varphi^{1/2}} \quad (83)$$

Substituting equations (76) to (79) into equations (80) to (83), one obtains the results summarized in Table 2.

The equations in Table 2 allow a direct comparison of the head and flow coefficients of axial and radial fans in the operating point

$$\frac{\varphi_{0,\text{axial}}}{\varphi_{0,\text{radial}}} = \frac{1 - m^3}{6(b_2/D_2)} \quad (84)$$

$$\frac{\psi_{0,\text{axial}}}{\psi_{0,\text{radial}}} = \frac{1}{2} (1 + m^2) \quad (85)$$

Equations (84) and (85) are shown in Figs 14 and 15, respectively. The ratio b_2/D_2 in equation (84) is the ratio of the height to the diameter of a radial fan. Usually, for a radial impeller, this ratio does not exceed 0.1 (i.e. b_2 does not usually exceed 10% of the outer diameter D_2). Furthermore, in the case of axial fans, the ratio of the hub-to-tip radii m does not usually exceed 0.5. Hence, from Fig. 14, it is clear that axial fans, in general, have a higher flow coefficient as radial fans.

Table 2 Summary of equations for axial and radial fans and pumps

	Radial	Axial ($m = R_1/R_2$)
$\Delta p_{t-s,0}$	$\frac{\rho}{2} \frac{3}{4} u_2^2$	$\frac{\rho}{2} \frac{3}{8} (1 + m^2) u_2^2$
Q_0	$\frac{1}{2} u_2 \pi D_2 b_2 \sin \beta_2$	$\frac{1}{12} (1 - m^3) u_2 \pi D_2^2 \sin \beta_2$
φ_0	$2 \frac{b_2}{D_2} \sin \beta_2$	$\frac{2}{6} (1 - m^3) \sin \beta_2$
ψ_0	$\frac{3}{4}$	$\frac{3}{8} (1 + m^2)$
$\sigma_0 = \frac{\varphi_0^{1/2}}{\psi_0^{3/4}}$	$\sqrt{2} \left(\frac{4}{3} \right)^{3/4} \sqrt{\left(\frac{b_2}{D_2} \right) \sin \beta_2}$	$\frac{\sqrt{1/3(1 - m^2) \sin \beta_2}}{[3/8(1 + m^2)]^{3/4}}$
$\delta_0 = \frac{\psi_0^{1/4}}{\varphi_0^{1/2}}$	$\frac{\sqrt{2}}{2} \left(\frac{3}{4} \right)^{1/4} \frac{1}{\sqrt{(b_2/D_2) \sin \beta_2}}$	$\frac{[3/8(1 + m^2)]^{1/4}}{\sqrt{1/3(1 - m^2) \sin \beta_2}}$

Fig. 14 Ratio of flow coefficients of axial and radial fans

Fig. 15 Ratio of head coefficients of axial and radial fans

Looking now to equation (85), which is plotted in Fig. 15, one can see that the head coefficient of axial fans is always lower than the one of radial fans, except in the limiting case when $m = 1$, but this is a singular point since here the axial fan has no flowrate and hence has no practical relevance. Since usually the ratio of the hub-to-tip radii m does not usually exceed 0.5, the head coefficient of an axial fan is about half or a little bit above half of the head coefficient of a radial fan.

In Figs 16 and 17, the speed against diameter curves for axial and radial fans according to Table 2 were plotted. In this example, the exit angle β_2 of the axial fan was chosen to be 40° and of the radial fan 15° , since these are typical values and usually the exit angle of radial fans is higher than the one of axial fans. One has to mention, however, that also choosing the same exit angle for the axial and radial fan, these plots does not change qualitatively. One can see in Figs 16 and 17 that the curve of the axial fans lies in the region of high speed numbers and low diameter numbers and that the curve of the radial fans lies in the region of low

Fig. 16 Axial and radial fans in the Cordier diagram

Fig. 17 Axial and radial fans in the original Cordier diagram

speed numbers and high diameter numbers, as it is predicted in the Cordier diagram. One can see that the lower the radii ratio m , the more 'axial' are the axial machines and, on the other side, the smaller the ratio b_2/D_2 , the more 'radial' are the radial machines.

3 CONCLUSIONS

It was shown that by total-to-static efficiency and geometrical considerations it is possible to analytically derive the Cordier curve and to obtain good agreement with the original Cordier curves. Furthermore, it was shown analytically that, on the one hand side, axial

fans have high speed and low diameter numbers and, on the other hand, side radial fans have low speed and high diameter numbers. It was also shown that axial fans, already from an analytical point of view, usually have lower head coefficients and higher flow coefficients than radial impellers. Hence, one can conclude that the experimental information contained in the Cordier diagram can also be derived analytically by energy and geometrical considerations.

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APPENDIX

Notation

A	area (m ²)
AR	ratio of the impellers inlet and outlet areas (–)
b	blade height (m)
c	absolute velocity (m/s)
D	diameter (m)
m	radius ratio $R_{\text{hub}}/R_{\text{tip}}$ (–)
n	speed (min ^{–1})
p	pressure (Pa)
P	power (W)
Q	flowrate (m ³ /s)
r	reaction (–)
R	radius (m)
Re	Reynolds number (–)
s	blade thickness (m)
u	peripheral velocity (m/s)
w	relative velocity (m/s)
Y	specific work (J/kg)
α	Absolute velocity angle (rad)
β	impeller blade angle (rad)
γ	flow angle (rad)
δ	diameter number (–)
Δ	variation of a quantity (–)
η	efficiency (–)
ν	kinematic viscosity of air (m ² /s)
ρ	density (kg/m)
σ	speed number (–)
φ	flow coefficient (–)
ψ	head coefficient (–)
ω	angular velocity (rad/min)

Subscripts and superscripts

1	at the impeller inlet
2	at the impeller outlet
axial	referring to axial fans
C	As an index referring to Cordier
d	dynamic
dif	diffuser
hyd	hydraulic
imp	impeller
m	meridional
max	referring to the maximum value of a quantity
o	at operating point
radial	referring to radial fans
rig	referring to test rig
s	static
t	total
t–s	total-to-static
u	circumferential