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Incipient motion of a single particle on regular substrates in laminar shear flow

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We study experimentally the critical conditions for incipient motion of a single spherical particle deposited on a regular substrate under laminar flow conditions. The substrates are triangular and quadratic arrangements of identical glass spheres. For the latter configuration, the distance between the substrate spheres is varied, resulting in different partial shielding of the deposited particle to the shear flow. For the studied particle Reynolds numbers range between 3×10^{-4} and 3, the critical Shields number is independent from the particle density and from the particle Reynolds number but it depends significantly on the geometry of the substrate. Depending on the spacing between the substrate beads and thus on the exposure of the particle to the flow, we have observed an increase of about 50 percent in the critical Shields number. Studying the onset of particle motion as a function of the orientation of the substrate to the flow direction we find that the critical Shields number changes by up to a factor of 2, which is mainly due to the fact that the particle travels through the troughs of the substrate and hence the shear force in travel direction diminishes if not in line with the flow direction. Besides the critical Shields number we study the initial stage of particle motion by detecting the minimum time that is necessary for maintaining a certain Shields number to change the position of a single particle on the regular substrates. In the range studied, the initial stage of motion on the scale of the substrate's periodicity is mainly governed by the equilibrium particle motion. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4753941>]

I. INTRODUCTION

Flow induced removal of solid particles from a substrate is encountered in a wide variety of industrial operations including cleaning of surfaces, e.g., filtration and production facilities in food and pharmaceutical industries, oil extraction, and microfluidics.^{1,2} It is also the initial process of several natural situations, i.e., sediment transport in rivers and coastal flows, granular beds erosion, or dune formation.^{3–5} Consequently, over the past decades, numerous studies have been dedicated to study the basics of incipient motion. Leighton and Acrivos^{6,7} and later Muthana *et al.*,⁸ for instance, studied the onset of motion of single particles in direct contact with a plain wall. These authors focused on single particles of 0.1–2 mm diameter at planar surfaces. Similarly, Peters and Meister performed experiments to provide the wall shear stress needed to set into motion sand particles of 0.2–1.4 mm from a flat surface.⁹ Others studied directly the onset of motion of granular beds, focused on the grain size-dependent transport of fine particles in turbulent open channel flow.^{10–13}

While some authors focused the sediment transportation induced by turbulent flows,^{14,15} Charru *et al.*,¹⁶ Ouriemi *et al.*,¹⁷ Lobkovsky *et al.*,¹⁸ and Derksen,¹⁹ studied the onset of particle motion in laminar shear flows using an irregularly arranged granular bed. Charru *et al.* observed an increase of the compactness of the granular bed due to the local arrangement of the particles and characterized

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this phenomenon as bed armouring, which yields an increase of the threshold for particle motion.¹⁶ Independent from the particle Reynolds number, Ouriemi *et al.*¹⁷ observed in their experimental study a constant critical Shields number for incipient motion of about 0.12 in laminar flow at about the same value as Charru *et al.*¹⁶ for a saturated compact bed.

Studying a single particle on an irregularly arranged bed of identical particles, Charru *et al.* observed an influence of the local substrate geometry on the onset of particle motion and the traveling trajectory.²⁰ Accordingly, they provided a critical Shields number range around 0.02–0.04 for onset of single particle motion. Likewise, the authors showed that the fluctuating motion of the particle was due to the irregular bed geometry. Other authors have also recognized the dependence of the incipient motion on the geometrical substrate properties.^{21,22} Martino *et al.*, for instance, studied experimentally the influence of the burial degree on the onset of motion of an isolated cylinder partially exposed to a laminar shear flow and provided a model for the resistance to particle motion due to burial.²²

In the present article, we study the geometrical impact of the substrate configuration on the incipient motion in laminar shear flow. Therefore, we analyze the motion of single particles on regular substrates of identical spherical beads. Section II describes the experimental setup. For particle Reynolds numbers ranging from creeping flow conditions up to about 3, we determine the critical Shields number in function of the substrate configuration, the gap distance between the substrate spheres and the angle of flow orientation with respect to the particle bed. Besides studying the critical parameters for the onset of motion, we characterize the initial stage of particle motion by measuring the time necessary for a single particle to change one position on the substrate as function of the Shields number. The results obtained are reported in Sec. III and discussed in Sec. IV. Finally, the conclusions are summarized in Sec. V.

II. EXPERIMENTAL SETUP

The experiments are performed using an MCR 301 rotational rheometer from Anton Paar. A laminar shear flow is produced using a parallel-disk configuration with a rotating glass plate of 65 mm diameter. The substrates are built from spherical soda-lime glass beads of (405.9 ± 8.7) μm diameter from The Technical Glass Company. To build different beds of quadratically arranged identical particles, the glass spheres are deposited and glued on stainless steel wire sieves of different mesh sizes and wire diameters from Bückmann GmbH & Co., which are produced for industrial screening. Since the sieves are hidden underneath the substrate beads, the influence of their precise geometry on the incipient motion is supposed to be negligible. The configurations together with the sieves used are given in Table I. Mesh size and wire diameter are data provided by the provider; the gap size has been measured with a microscope. Small variations in the gap are supposed to be mainly due to deviations in the sphere's diameter and the steel wire meshes (see Table I). The triangular substrate was built by fixing the spherical beads on an epoxy layer after the natural arrangement of the particles in a quadratic box had been submitted to small vibrations. Finally, the substrates are fixed on brass square islands of $15 \times 15 \text{ mm}^2$. Figure 1 shows microscopic pictures of different bed geometries.

The substrate is placed into a circular container with transparent Plexiglas sidewalls. The container is fixed concentrically to the rheometer. Its components and dimensions are provided in Figure 2. The non-concentric circular bottom plate, on which the substrate is located, permits to

TABLE I. Substrate properties, mesh size, wire diameter, and corresponding gap between particles.

Configuration	Mesh size (μm)	Wire diameter (μm)	Gap between particles (μm)
Triangular	...	...	0 ± 4
Quadratic	260 ± 15	160	14 ± 12
Quadratic	300 ± 17	200	94 ± 17
Quadratic	315 ± 18	200	109 ± 20

FIG. 1. Microscopic top view of the substrates of identical and regularly arranged spherical glass beads of $(405.9 \pm 8.7) \mu\text{m}$ diameter. Triangular configuration (a) and quadratric configuration on a mesh, gap between particles: $14 \mu\text{m}$ (b), and $109 \mu\text{m}$ (c).

change the orientation angle of the flow direction with respect to the symmetry axis of the substrate, β . Additionally, the substrate can be displaced in radial direction of the rheometer's rotating disk to analyze the onset of motion in the range of radius, r , between 16 and 27 mm. The gap width, h , defined as the distance from the top of the substrate spheres to the rotating plate, and the angular frequency, Ω , are controlled with the rheometer.

The container is filled with a liquid. Two silicone oils from Basildon Chemicals have been used with viscosities of (9.95 ± 0.30) mPas and (103.0 ± 3.3) mPas and densities of (0.935 ± 0.005) g/cm³ and (0.965 ± 0.005) g/cm³ at the temperature of (295.16 ± 0.5) K, respectively. In our experiments, the temperature is fixed at this value and it is controlled with a P-PTD 200 Peltier element connected to the rheometer and measured with an external thermometer. The viscosity was measured with a capillary viscometer, the density was determined with a Mohr balance.

For the single beads we use four different materials. The bead properties are given in Table II. According to the providers, the surface roughness average corresponds to $0.05 \mu\text{m}$ for glass and tungsten-carbide beads, and to $0.025 \mu\text{m}$ for the steel spheres. This surface roughness is defined as all those irregularities which form the surface relief but are not deviations of form or waviness. Visual inspection with an optical microscope of the PMMA particles did not show any defects. The particles are illuminated through the rotating disk with an LED light source. Their incipient motion

FIG. 2. Sketch of the container and the rotating rheometer disk. The particular geometry allows the rotation of the substrate with respect to the flow direction. The shadow regions are inlets of same height as the bottom that can be removed to vary the radial location of the substrate.

TABLE II. Particle properties of single beads according to providers.

Material	Density ρ_s (g cm ⁻³)	Diameter D_p (μ m)	Provider
PMMA	1.190 ± 0.002	406.0 ± 9.5	Microparticles GmbH
Soda lime glass	2.530 ± 0.025	405.9 ± 8.7	Technical Glass Company
Steel	7.73 ± 0.02	400 ± 1	Nanoball GmbH
Tungsten-carbide/cobalt (94:6)	14.95 ± 0.03	400 ± 20	Goodfellow

is detected through the rotating disk with a digital camera of 1280×1024 pixels equipped with a macro objective that incorporates a tilted mirror.

The position of the camera is regulated with a micro linear slide coupled to a stepper motor with a step width of 1.8° , permitting displacements with an accuracy of a few micrometers. The incipient motion is detected by increasing the speed on the rotating plate in small steps of less than 0.5% until the particle starts to move crossing the separatrix to the neighboring equilibrium position. The particle location is tracked optically and evaluated with image processing software. The complete setup is sketched in Figure 3.

III. EXPERIMENTAL RESULTS

In literature, different criteria may be found to specify incipient motion in granular beds.²³ Some authors consider the motion of the first few particles as incipient condition.^{24–26} Others take into account the motion of a significant portion of the bed surface.²⁷ Including laminar flow conditions, the scatter of data in literature is supposed to be due to different criteria for onset of motion concept.¹⁷ In the present article, we consider the displacement of the particle to a neighboring equilibrium position on the substrate as the threshold condition. We characterize the incipient motion by the critical Shields number, which compares the characteristic shear stress acting on the particle to the resistant specific particle weight that retains it in place.

The fluid motion is created by the moving top plate. For the parallel disks geometry with an angular velocity Ω and a gap width h , the shear rate $\dot{\gamma}$ at the radial distance of the particle from the turning axis r is given by $\dot{\gamma} = \Omega r/h$. Hence the shear stress results in $\tau = \mu \Omega r/h$ where μ is the

FIG. 3. Sketch of the experimental setup.

dynamic viscosity. Thus, for our setup the Shields number is given by

$$\theta = \frac{\nu \Omega r}{(\rho_s / \rho - 1) h g D_p}, \quad (1)$$

where $\nu = \mu / \rho$ is the kinematic viscosity, ρ_s and ρ are particle and liquid densities, respectively, g is the acceleration of gravity and D_p is the particle diameter. Accordingly, the Reynolds number for the shear flow and the particle Reynolds numbers are given by

$$\text{Re} = \frac{\Omega r h}{\nu}, \quad \text{Re}_p = \text{Re} \left(\frac{D_p}{h} \right)^2. \quad (2)$$

To focus on the influence of the substrate geometry, we first identify a parameter range that is independent from any boundary effects. Therefore, we study the influence of distance from the rotation axis, finite-size effects of the substrate and the gap height on the onset of particle motion.

Figure 4 shows the critical Shields number for a quadratic substrate with a gap between the beads of $14 \mu\text{m}$. Diagram (a) depicts the data for the radius range between 16 mm and 27 mm at a

FIG. 4. Critical Shields number as a function of the distance between the particle position and the turning axis of the rotating disk (a), the distance between the particle position and the upstream border of the substrate (b), and the gap width (c). The experiments were performed with single glass particles on a quadratic substrate with a gap between the beads of $14 \mu\text{m}$. Circles and triangles: critical Shields numbers and Reynolds numbers, respectively. Open and solid symbols indicate less and higher viscous oils, respectively. Gap height in (a) and (b): 2 mm, distance from upstream step of the substrate island in (a) and (c): 7.5 mm, distance from turning axis in (b) and (c): 21 mm.

gap height of 2 mm. The critical Shields number remains constant within a narrow band around a mean value of 0.039. The standard deviation is about ± 0.002 for both oils and all radii. There is no dependence on the radial location neither on the particle Reynolds number, which ranges between 0.004 and 0.4 for the data in Figure 4. In what follows we always place the single bead at a radius of 21 mm, which corresponds to a dimensionless distance from the axis r/D_p of 52.

To study the effect of the finite size of our substrate, we measured the critical Shields number as a function of the distance from the upstream border of the island. Figure 4(b) depicts the data. Within the range of uncertainty, there is a plateau in an interval between around 8 and 30, corresponding to distances from the upstream border of 3.5 mm and 12.5 mm, respectively. At the edges of the island we find lower critical Shields numbers. Thus, at the Reynolds numbers considered here, i.e., $Re < 40$, in the central part of the substrate, inflow and outflow effects due to the substrate edge are negligible. The average critical Shields number collapses again in 0.039 within a standard deviation of about ± 0.003 . We remark that while the experiments in Figure 4(a) were performed always at the same location on the substrate, in Figure 4(b) the Shields numbers are analyzed at different positions along the substrate. Hence, substrate variations result in additional scatter of the data. For the following experiments, we place the particle at the center of the substrate.

The influence of the gap width on the onset of particle motion is shown in Figure 4(c). While for the experiments performed using the higher viscous oil, the Shields number remains roughly constant in the entire gap height interval, for the less viscous oil, we observe a steady Shields number increase from normalized gap heights of around 10. Thus, at Reynolds numbers below 50, constant values of Shields number of 0.040 within a standard deviation of ± 0.002 were obtained. Note that while the Reynolds number depends strongly on the gap height, the particle Reynolds number remains almost constant at around 0.4 for the less viscous oil and about 0.004 for the more viscous liquid. For the following studies, we keep the gap width at 2 mm, which corresponds to a dimensionless value of 5.

We now study the influence of the particle density and the substrate geometry on the critical Shields number. Figure 5 shows the critical Shields number as a function of the relative density of the single particle for five different geometrical configurations at Reynolds numbers ranging from creeping flow conditions to about 70 and particle Reynolds numbers up to about 3. At the studied Reynolds numbers, the critical Shields number seems to be independent from relative density and viscosity. We remark that using tungsten-carbide beads as substrate spheres in contact to the single

FIG. 5. Dependence of the critical Shields number on the relative density difference for different geometrical arrangements and distances between the substrate spheres. The square and cross symbols show the results obtained using the two symmetrical triangular configurations. Circles, diamonds, and triangles depict the results obtained with the quadratic substrates with gaps between substrate particles of 14, 94, and 109 μm , respectively. Open and solid symbols represent experiments performed with less viscous and higher viscous oils, respectively. The dotted lines are drawn to guide the eye. Gap height: 2 mm, distance from upstream step of the substrate island: 7.5 mm, radius: 21 mm.

FIG. 6. Dependence of the critical Shields number on the angle of orientation for a single glass particle deposited on a triangular (a) and quadratic (b) substrate configurations. Circles and triangles: gap between beads of $14\ \mu\text{m}$ and gap between beads of $109\ \mu\text{m}$, respectively. Open and solid symbols: experiments performed with less and higher viscous oil, respectively. The solid line in (a) indicates the function $\theta_c(\beta) = \theta_{c\min}/\cos(\pi/3 - |\beta|)$. In (b) the solid lines shows the minimum of $\theta_c(\beta) = \theta_{c\min}/\cos\beta$ and $\theta_c(\beta) = \theta_{c\min}/\cos(\pi/2 - |\beta|)$. Radius: $21\ \text{mm}$, distance from upstream step of the substrate island: $7.5\ \text{mm}$, gap height: $2\ \text{mm}$.

particle we obtained the same critical Shields numbers as for the glass substrate within measurement uncertainty.

Although the critical Shields number is independent from inertia, it strongly depends on the substrate configuration. As depicted in Figure 5, with increased spacing between the quadratically arranged beads, the critical Shields number augments by 50%. For triangularly arranged substrates, the critical Shields numbers are lower than for quadratic ones. Nevertheless, depending on the orientation of the substrate to the flow direction, the critical Shields number changes by a factor of 2.

As shown in Figure 5, the critical Shields number depends strongly on the flow direction with respect to the triangular configuration. For a substrate with triangular particle arrangement, Figure 6(a) depicts the variation of the critical Shields number with respect to the angle between flow direction and symmetry axis of the substrate. The angle dependence reflects the symmetry of the substrate. At angles around $\pm\pi/3$ the critical Shields number is minimum and hardly changes in an interval of about $\pm\pi/12$. Beyond this range, there is a slight increase of the critical Shields number with a small buckle. Approaching the angle 0 yields a continuous increase of the critical

FIG. 7. Time interval for incipient particle motion, Δt , as a function of the Shields number for quadratic substrate configurations. Open and solid circles: experiments with a single glass particle on a substrate with a gap between beads of $14\ \mu\text{m}$ with less and higher viscous oil, respectively. Squares: experiments with a single glass particle on a substrate with a gap between beads of $109\ \mu\text{m}$ and higher viscous oil. Open and solid triangles: experiments with a single Plexiglas particle on a substrate with a gap between beads of $14\ \mu\text{m}$ and less viscous oil, and on a substrate with a gap of $109\ \mu\text{m}$ and higher viscous oil, respectively.

Shields number. We remark that the bead always travels through the substrate troughs. At an angle of zero, the loose particle passes the facing substrate bead occasionally on either side indicating that centrifugal forces due to the spinning rheometer disk have negligible impact on the incipient motion.

The influence of the orientation angle on the critical Shields number for quadratic configurations is shown in Figure 6(b). Again, it reflects the symmetry of the substrate and the lack of dependence on the particle Reynolds number. Like for the triangular substrate, the critical Shields number hardly changes around its minimum values in an angle interval of about $\pi/6$. Also, it is maximum where a sphere is directly in front of the single bead. The peak for the maximum critical Shields number, however, is less pronounced as compared to the triangular configuration. The increment of the critical Shields number between its minimum and maximum values increases with the gap between particles. For a gap of $14\ \mu\text{m}$, the relative Shields number increase, defined as $(\theta_{\max} - \theta_{\min})/\theta_{\min}$, is $(34 \pm 7)\%$ and for a gap of $109\ \mu\text{m}$ it is $(52 \pm 8)\%$.

Finally, we study the minimum time necessary for maintaining a certain Shields number to change the position of a single particle on the regular substrates. Therefore, the rheometer is first set up at a Shields number that is about 30% below the critical one. After a shear rate jump with a step width of about 0.03 s to a supercritical Shields number, we detect the time necessary for the single particle to reach the watershed that separates its initial position from the neighboring substrate valley. Figure 7 shows the time interval Δt , as a function of the Shields number θ . The data are average values of 3 runs. The time interval decreases continuously with the Shields number. Furthermore, the lower the viscosity and the higher the particle density the shorter is the time interval at the same Shields number.

IV. DISCUSSION

We have identified an interval of parameters independent from the radius of curvature of the shear flow generated in the parallel-disk geometry of the rheometer, the gap height, and the finite size of the substrate. Centrifugal forces due to the rotating rheometer disk have negligible impact on the incipient motion: The symmetry of the substrate is maintained in the dependence of the critical Shields number on the orientation angle between mean flow and substrate as depicted in Figure 6. This also holds for the motion of the particle: The bead always travels through the substrate troughs. If the loose particle faces directly a substrate bead downstream, it passes the substrate

FIG. 8. Dependence of the critical Shields number on the burial degree. Squares, circles, rhomboids, and triangles represent experiments performed with PMMA, glass, steel, and tungsten-carbide particles, respectively. Open and solid symbols represent experiments performed with less and higher viscous oils, respectively. Radius: 21 mm, distance from upstream step of the substrate island: 7.5 mm, gap height: 2 mm.

bead occasionally on either side indicating that radial forces have negligible impact on the incipient motion.

The experimental results provide reproducible data for the onset of single particle motion. Using the quadratic substrate with a gap between beads of $14 \mu\text{m}$, for instance, we obtain a critical Shields number around 0.040 ± 0.002 . The standard deviation is determined taking into account all individual measurements of Figure 5, i.e., five different runs for each material combination. A similar value for the standard deviation is found for other configurations demonstrating the reproducibility of the experimental results (see Figure 8).

In the parameter range studied, the critical Shields number strongly depends on the substrate geometry but not on inertia nor on particle density as depicted in Figure 5. As shown in Figure 5, the larger the gap between substrate beads and hence the less the loose particle is exposed to the flow, the higher is the critical Shields parameter. The effect of the level of exposure on the incipient motion of a partly hidden cylinder in two-dimensional laminar flow has been studied by Martino *et al.*²² They demonstrated the influence of two parameters related to the bed geometry on the onset of motion: the pivot angle, which coincides with the angle of repose, φ (i.e., the critical angle at which motion occurs²⁸) and the exposure to the incident flow, E , defined as the ratio between the cross-sectional area effectively exposed to the incident flow to the total cross-sectional area of the particle. With these two parameters, Martino *et al.* defined a burial degree, ζ , as $\tan \varphi / E$ and proposed a model that relates the particle Reynolds number to the Galileo number and to the burial degree according to the following equation:²²

$$\text{Re}_p = \left(\frac{\pi}{2\alpha^2 C_D} \right)^{0.5} Ga^{0.5} \zeta^{0.5}, \quad (3)$$

where α is a proportionality constant less than one, C_D is the drag coefficient, and Ga , the Galileo number defined as $(\rho(\rho_s - \rho)gD_p)/\mu^2$.

Figure 8 shows the critical Shields number as a function of the burial degree for each of our configurations. The angle of repose for the triangular configuration is obtained from literature.²⁸ For the quadratic substrate arrangements, this parameter is calculated accordingly to

$$\tan \varphi = \frac{D_p + a}{2\sqrt{D_p^2 - \frac{(D_p + a)^2}{2}}}, \quad (4)$$

where φ is the angle of repose, D_p is the particle diameter, and a is the gap distance between the substrate beads. The exposure to the incident flow and the angle of repose are geometrically coupled

FIG. 9. Dependence of the particle Reynolds number on the burial degree. $Re_{p \max}$ represents the maximum particle Reynolds number reached in our experiments for each of the viscous oils, i.e., the Re_p corresponding to the experiments performed with the denser particle over the quadrangular substrate with the larger gap between beads for the less and high oil viscosity, respectively. Squares, circles, rhomboids and triangles: PMMA, glass, steel and tungsten-carbide particles, respectively. Open and solid symbols: less and higher viscous oils, respectively. The solid lines represent the power law fits with exponent 0.5 according to Ref. 22. Radius: 21 mm, distance from upstream step of the substrate island: 7.5 mm, gap height: 2 mm.

in our quadratically arranged substrates and the burial degree is calculated in the same way as in Ref. 22 as $\tan \varphi/E$. As appears in Figure 8, the critical Shields number increases as a function of the burial degree showing the strong influence of partially shielding the particle to the shear flow. This includes comparing triangular to quadratic substrate configurations.

In their study, Martino *et al.* found that the particle Reynolds number at onset depends on the burial degree according to a power law with an exponent of 0.5.²² Figure 9 shows the critical particle Reynolds numbers in our experiments as a function of the burial degree together with fits according to the law proposed by Martino *et al.*,²² indicating that the power law reflects quite well the general trend of our data.

The burial degree takes into account the exposure of the single particle to the shear flow and thus the effect of the substrate's gap width. In the quadratically arranged substrates it may affect the dependence of the critical Shields number on the orientation angle between substrate geometry and flow direction, yet the exposure varies with the orientation angle. However, the main effect is apparently due to the fact that the particle always travels through the troughs of the substrate and not in the main flow direction. This is especially apparent for different orientations of the triangular configuration where we have about the same burial degree but find a change by a factor 2 in the critical Shields number. Fitting the angle dependence of the critical Shields number by $\theta_C(\beta) = \theta_{C\min}/\cos(\pi/3 - |\beta|)$, which corresponds to a reduction of the shear force created by the flow in the particle's travel direction and which is displayed in Figure 6(a), recovers the main tendency of the experimental data. Only a small buckle at angles of about $\pm\pi/6$, which seems to be related to small variations in the burial degree, deviates slightly from this curve. The solid lines in Figure 6(b) represent again the effect of shear-force reduction with respect to the travel direction. While this seems to describe the angle dependent critical Shields number quite well for the substrate with $14 \mu\text{m}$ gap, we observe larger deviations for the quadratically arranged substrates with $109 \mu\text{m}$ gap. This may indicate that the variation in the exposure also affects the angle dependence of the critical Shields number.

Our values of the critical Shields numbers for narrow or vanishing gaps between substrate beads are in the range of the threshold Shields number of 0.02–0.04 found by Charru *et al.* for a single particle placed on a randomly arranged substrate of identical spheres with a typical packing fraction of 0.77 ± 0.02 .²⁰ The packing coefficient of our substrates, defined as the fraction of an area filled with circles, corresponds to the optimal density in the plane for the triangular arrangement, 0.91 and around 0.76 for the quadratic substrate with a gap of $14 \mu\text{m}$ between particles.

Our data can also be viewed under the aspect of bed armouring, as it has been considered, for instance by Charru *et al.* for granular beds with uniform particle size.¹⁶ Bed armouring produces an increase in compactness of arbitrarily arranged, loosely packed granular beds by moving exposed particles, which have a low critical Shields number, until they may be trapped in deeper locations of the granular bed, from where they can be moved only at higher Shields numbers. This rearrangement yields a temporal increase of the threshold for the onset of granular flow due to an increase of the burial degree. Our data confirm this view showing an increase of the critical Shields number with the burial degree by up to a factor of about 4.

We remark that the critical Shields numbers detected in our study and also in that by Charru *et al.*²⁰ are considerably lower than those around 0.12 provided by Ouriemi *et al.*,¹⁷ Charru *et al.*²⁰ and Derksen¹⁹ for the onset of particle motion in irregularly arranged granular beds. In view of our results, this can be understood due to the strong dependence of the critical Shields number on the burial degree as well as on the angle between flow and substrate orientation. Nevertheless, there is still a considerable discrepancy between our highest Shields number and those reported for irregularly arranged granular beds. This may indicate that the burial degree in these beds is quite high and thus interaction between neighboring particles of comparable exposure may play a significant role.

Finally, we consider the initial stage of particle motion. As shows Figure 7, the time interval strongly depends on particle density and viscosity. This can be taken into account by scaling the time interval by the settling time $\Delta s/U_S$, where $\Delta s = (D_p + a)/2$ is the horizontal distance the particle travelled during the time interval and $U_S = (\rho_s/\rho - 1)gD_p^2/(18\nu)$ is the settling velocity.

The scaling of the inertial terms for particle motion is $(\rho_s/\rho)\text{Re}_p$.²⁹ Comparing the measured time intervals to the inertial time $(\rho_s/\rho)\text{Re}_p/\dot{\gamma}$ yields ratios between 1.5×10^{-4} and 6.5, showing that inertia plays only a minor role. This implies that the particles obtain their local equilibrium velocity in an early stage of the measured time interval and that the time interval is governed rather by the local equilibrium particle motion. In what follows, we therefore compare the time interval with data reported by Charru *et al.* for the travelling velocity of a single particle on an irregular bed of identical particles. They found that the particle's mean velocity can be described properly as a linear function of the Shields number. Deviations from this linear dependency at higher Shields numbers could be traced back to detachment of the particle from the substrate.²⁰

In line with Bagnold³⁰ who considered the mean horizontal force as a superposition of a fluid force and a friction force due to interactions with the substrate, Charru *et al.* described the mean travelling velocity by

$$\frac{U}{U_S} = 18u_{eff}\theta - \frac{\mu_F}{C_D}, \quad (5)$$

where U and U_S are the travelling and the settling velocity and u_{eff} , μ_F , and C_D are an effective dimensionless velocity, an effective friction coefficient, and the drag coefficient, respectively. In particular, Charru *et al.* obtained for their irregular bed of identical particles and for Shields numbers up to 0.17 the following values: $18u_{eff} = 6.67$, $\mu_F = 0.38$, and $C_D = 2.4$.²⁰

The last term in (5) is a threshold value that takes into account the resistance to motion due to the uneven substrate and thus depends on the critical Shields number. Extending the model by Charru *et al.* to account for different substrates, we assume the resistance to be proportional to the critical Shields number of the substrates. Then (5) may be written as follows:

$$\Delta t \frac{U_S}{\Delta s} = [A(\theta - B\theta_c)]^{-1}, \quad (6)$$

where A and B are constants, which for the data from Ref. 20 are $A = 6.67$ and $B = 0.59$. Assuming the same value for B and fitting our data for $\Delta t/\Delta s \cdot U_S$ to $\theta - B\theta_c$ yields an exponent of -1.1 instead of -1 . Assuming an exponent of -1 yields $B = 0.69$, Figure 10 shows the latter fit, indicating that in our study the initial stage of particle motion on the scale of the bead diameter can be described by neglecting inertia and accounting for the resistance of different substrates by the critical Shields number.

FIG. 10. Dimensionless time interval for initial particle motion as a function of $\theta - 0.69\theta_c$. Open and solid circles: experiments with a single glass particle on a substrate with a gap between beads of $14\ \mu\text{m}$ with less and higher viscous oil, respectively. Squares: experiments with a single glass particle on a substrate with a gap between beads of $109\ \mu\text{m}$ and higher viscous oil. Open and solid triangles: experiments with a single Plexiglas particle on a substrate with a gap between beads of $14\ \mu\text{m}$ and less viscous oil, and on a substrate with a gap of $109\ \mu\text{m}$ and higher viscous oil, respectively.

V. CONCLUSIONS

We have studied the influence of the substrate geometry on the incipient motion of a single particle in laminar shear flow. The particle's incipient motion is characterized by the Shields number. It is independent from the particle density and from inertia within the range of 0.007 – 70 , and the corresponding particle Reynolds number range of 0.0003 – 3 . We found a strong influence of the substrate's geometrical arrangement and the gap distance between substrate beads on the onset of motion showing the strong impact of exposure to the flow. Similarly, the orientation angle of the substrate with respect to the flow direction can have a significant influence on the critical Shields number of up to a factor of 2. It is mainly due to the fact that the particle travels through the troughs of the substrate and hence the shear force in travel direction diminishes if the travel direction is not in line with the flow direction. Finally, we studied the minimum time necessary for maintaining a certain Shields number to change the position of a single particle on regular substrates. In the range studied, we find that even the initial stage of particle motion on the scale of the substrate's periodicity is mainly governed by the equilibrium particle motion and propose that the effect of different substrates can be accounted for by the critical Shields number.

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