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Citation: [AIP Conf. Proc.](#) **1421**, 3 (2012); doi: 10.1063/1.3679581

View online: <http://dx.doi.org/10.1063/1.3679581>

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Equilibrium and Transport in a Strongly Coupled and Magnetized Ultra-Cold Plasma

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Abstract. Ultra-cold Plasmas obtained by ionization of atomic Rydberg states are qualified as classical and strongly coupled electron fluid. They are shown to share several common trends with ultra-cold electron flows used for ion beam cooling. They exhibit specific stopping behavior to charged particle beams, which may be used for diagnostics purposes. Ultra-cold plasmas are easily strongly magnetized. Then, one expects a strongly anisotropic behavior of low ion velocity slowing down when target electron cyclotron radius turns smaller than corresponding Debye length.

Keywords: Ultra-cold plasma, magnetized target, ion stopping, hydromodes

PACS: 52.40.Mj, 52.25.Xz, 52.25.Gj, 52.25.Fi

1. INTRODUCTION

We intend to review some conspicuous features of the ultra-cold plasma (UCP) observed in the quasi-reversible transformation of cold atomic Rydberg states into an expanding fluid of free electrons neutralized by much slower residual ions. Such a process is routinely observed, since the first unambiguous identification due to Vitrant et al. [1], as early as 1982. In the mean time, many accurate experiments [2–4], performed on both sides of the Atlantic, have definitively settled an unexpected territory in the temperature-density plane of current use by plasma physicists. However, it seems to us that the potentialities of cross fertilization afforded by a very unusual interplay of atomic physics with plasma collective phenomena have been largely underappreciated, up to now.

Such a view explains the organization of the present work.

In Sec. 2, we show how the inclusion of binary particle correlation leads to specific modellings of the expanding UCP. Sections 3 and 4 advocate charge particle stopping and energy loss in the UCP as a possible diagnostics method.

A first approach when no applied magnetic field is present ($\mathbf{B} = 0$), is presented in Sec. 3, while the strongly magnetized case ($\mathbf{B} \neq 0$ and $\omega_c \geq \omega_p$) is detailed in Sec. 4. Then, a specific attention is given to low ion velocity slowing down (LIVSD) in terms of the target plasma hydromodes in a hydrodynamic approach contrasted to a kinetic one including collisions. Conclusion and outlooks are offered in Sec. 5.

2. DENSE AND STRONGLY COUPLED ELECTRON FLUID

2.1. A bit of numerology

A priori, the very low particle densities observed in the expanding UCP could lead to a neglect of particle-particle correlations.

Such a view has been very recently stressed by Comparat et al. [5] through an intriguing analogy between expanding UCP and gravitational equilibrium of globular star clusters. These authors also advocate a time evolution monitored by a Fokker-Planck kinetic equation putting in parallel binary star formation and three-body recombination into Rydberg atoms. Such an approach might be well suited for the description of these atomic and local processes, where correlations are not fundamentally involved. However, to investigate genuine plasma physics implying collective effects, one needs to include particle correlations.

Here, we proceed to a standard analysis for qualifying the UCP phases in a density temperature framework. In view of the Coulomb and long ranged interaction, it is recommended to evaluate carefully binary correlations between particles within the UCP electron fluid. Such correlations are qualified for particles obeying classical Boltzmann statistics, by the dimensionless parameter

$$\Gamma = \frac{(Ze)^2}{aT_e} = 2.69 \times 10^{-5} Z^2 \left(\frac{n_e}{10^{12} \text{ cm}^{-3}} \right)^{1/3} \left(\frac{T_e}{10^6 \text{ K}} \right)^{-1}, \quad (1)$$

with $a = (4\pi n_e/3)^{-1/3}$ mean inter-electron distances and Z denotes the ion charge neutralizing the UCP.

UCP maintains such a low T_e than as soon as $n_e \geq 10^6 \text{ cm}^{-3}$, $\Gamma \geq 0.2$, a fiducial figure for most expanding UCP encountered in the literature. Therefore, it is appropriate to include correlation effects in any quantitative estimate of equilibrium or transport UCP properties.

Simultaneously, at the n_e values considered, the Wigner parameter

$$r_s = \frac{a}{r_B} \simeq \left[\frac{n_e}{1.6 \times 10^{24} \text{ cm}^{-3}} \right]^{1/3} \quad (2)$$

measuring a in numbers of Bohr radius r_B remains always much larger than one, which pertains to a classical UCP, fulfilling Boltzmann statistics. This feature is at variance with usual partially degenerate electron fluid [6] neutralizing and polarizing as well classical and strongly coupled ions with much larger n_e values.

This a priori paradoxical situation is also highlighted by the degeneracy parameter

$$\Theta = \frac{k_B T_e}{E_F} = \frac{2m_e k_B T_e}{\hbar^2 (3\pi^2 n_e)^{2/3}} = 2 \left(\frac{4}{9\pi} \right)^{2/3} \frac{r_s}{\Gamma} \quad (3)$$

also being much larger than one.

FIGURE 1. Effective electron–ion interaction [Eq. 4]

2.2. Time evolution

Most of the significant physics encountered in the Rydberg-UCP transition has to do with the expansion of the initial ultra-cold atoms and their subsequent ionization. So, comprehending intricacies of the plasma phase time development appears as a basic challenge involving dynamical correlations between charged particles. Hopefully, the given UCP expansion may rather be convincingly modeled through adequate molecular dynamics (MD) numerical simulations [7], provided the ultra-violet divergence associated to the short ranged electron-ion interaction is carefully taken care of. An accurate albeit simple expression validated by many applications reads as [8]

$$V_{\alpha\beta}(r) = Z_{\alpha}Z_{\beta}e^2 \frac{1 - \exp(-\kappa_{\alpha\beta}r)}{r}, \quad V_{\alpha\beta}(0) = Z_{\alpha}Z_{\beta}e^2 \kappa_{\alpha\beta} \quad (4)$$

with $\kappa_{\alpha\beta} = 2\pi/\lambda_{\text{th}}(\mu_{\alpha\beta})$ for a hydrogenic pair with charges Z_{α} and Z_{β} , and thermal de Broglie wavelength $\lambda_{\text{th}}(\mu_{\alpha\beta}) = \hbar/\sqrt{2m\mu_{\alpha\beta}k_{\text{B}}T}$, where $\mu_{\alpha\beta} = m_{\alpha}m_{\beta}/(m_{\alpha} + m_{\beta})$. Interaction (4) is plotted on Fig. 1 as

$$\frac{V(r)}{V(0)} = \frac{1 - \exp(-\kappa r)}{\kappa r} \quad (5)$$

contrasted to the approximation

$$\frac{\bar{V}(r)}{\bar{V}(0)} = \begin{cases} \frac{(\kappa r)^2}{27} - \frac{\kappa r}{3} + 1, & \kappa r < 3 \\ \frac{1}{\kappa r}, & \kappa r \geq 3 \end{cases} \quad (6)$$

Time evolution per se, is displayed on Fig. 2 for potential energy U_{pot} and temperatures renormalized by the thermal temperature $T(=T_e = T_i)$ at two distinct Γ values. Running time t is given in number of $\tau_p = \omega_p^{-1}$ with $\omega_p^2 = 4\pi Z_{\alpha}^2 n_e e^2 / m_e$. The weak

FIGURE 2. Temporal evolution of particle temperatures and potential energy in (a) weakly ($\Gamma = 0.0825$) and (b) coupled ($\Gamma = 1.08$) plasmas under 3-body recombination.

coupling case (Fig. 2a) advocates steady $T_e \simeq T_i$ and $U_{\text{pot}} \geq 0$. The correlated situation (Fig. 2b) looks very different. The ion temperature remains constant as previously. However, the electrons temperature T_e steadily increases as a result of partial 3-body recombination into Rydberg atoms, while U_{pot} turns more and more negative.

At large t , one can expect that colder ions mimic the homogeneous and rigid neutralizing background in the one-component-plasma (OCP) models [9].

3. ION STOPPING IN ULTRA-COLD PLASMAS ($\mathbf{B} = 0$)

Up to now, only a few classical electron fluid have been identified experimentally. One of them which seems to share lot of common features with the UCP of the present interest, is the electron flow used for cooling of energetic ion beams (Fig. 3).

Very cold electrons stream parallel (or sometimes antiparallel) to the ion beam in

FIGURE 3. Heidelberg Test Storage Ring (TSR). Electron cooling device. Electron energies can be varied between 2 and 320 keV with a maximum design current of 10 A. The e-beam diameter is 5 cm and the length of the electron-ion interaction region 250 cm.

a given accelerating structure, to reduce the ion beam emittance (transverse entropy), while raising it in the electron beam.

The quantitative understanding of the cooling process is well documented by now. It essentially relies on a careful understanding of ion beam stopping and energy loss into the parallel electron fluid.

There are two conventional approaches to the stopping power, the dielectric linear response (DLR) formalism and the binary collision treatment. In the *dielectric* description the stopping power dE/ds is calculated as the force between the ion and the polarization cloud created by the ion and can be expressed in terms of the dielectric function $\epsilon(k, \omega)$

$$-\frac{dE}{ds} \frac{\lambda_D}{k_B T} = \frac{Z_p^2 \sqrt{3} \Gamma^{3/2}}{2\pi^2} \int_{k < k_m} d^3 \mathbf{k} \frac{\mathbf{k} \cdot \mathbf{v}}{k^2} \text{Im} \left[\frac{-1}{\epsilon(k, \mathbf{k} \cdot \mathbf{v})} \right], \quad (7)$$

where $\mathbf{v}$ = Projectile velocity.

For classical systems the dielectric function ϵ can be written in terms of the Fried-Conte plasma dispersion function and represents the limit $\hbar \rightarrow 0$ of the RPA dielectric function for any degeneracy of the electron target. The classical linear response description, however, cannot treat close ion-electron collisions. To correct for this, a cut-off k_m is introduced to incorporate a Bloch-correction for the stopping power which accounts for the contribution of close collision in an approximate manner for $|Z_p| \Gamma^{3/2} / [1 + (v/v_{th})^2]^{3/2} \ll 1$. For classical ion-electron collisions, this cutoff is given by

$$k_m \simeq \frac{1}{\langle b_0 \rangle} = \frac{m \langle v_r \rangle^2}{|Z_p| e^2} = \frac{1 + v^2/v_{th}^2}{\sqrt{3} |Z_p| \Gamma^{3/2} \lambda_D}.$$

λ_D is the usual target electron Debye length $\lambda_D = (k_B T / 4\pi n_e e^2)^{1/2}$.

In the *binary collision approach (BCA)* the stopping power is obtained as the average over the momentum transfer in isolated collisions between the ion and target electrons. In terms of the transport cross-section σ_{tr} for the ion-electron collisions, the stopping power dE/ds on heavy ions (i.e. in the limit of an infinite projectile mass) reads as

$$\frac{dE}{ds} \frac{\lambda_D}{k_B T} = \frac{\sqrt{3}}{12\pi\Gamma^{3/2}} \int \frac{d^3\mathbf{v}_r}{(2\pi)^{3/2}v_{th}^3} \exp\left(-\frac{(\mathbf{v}_r + \mathbf{v})^2}{2v_{th}^2}\right) \frac{v_r}{v_{th}} \frac{\hat{\mathbf{v}} \cdot \mathbf{v}_r}{v_{th}} \frac{\sigma_{tr}(v_r)}{\lambda_D^2} \quad (8)$$

for an electron target with a Maxwell velocity distribution. $\mathbf{v}_r$ denotes the relative velocity between projectile and target electron.

Figure 4 exhibit stopping profiles according to Eqs. 7 and 8 contrasted to Molecular Dynamic (MD) and Vlasov simulations, in terms of projectile velocity divided by target electron thermal velocity. The agreement between the four considered approaches deteriorate some what with increasing Γ value, especially for moderate or small projectile velocity. *Clearly, the linear response (7) exhibits a marked correlation hole, not confirmed by other estimates [10, 11].*

On the other hand, the Bethelike high v behavior remains correct at any Γ . An encouraging feature of these calculations is the good agreement for all Γ and v values of simulation results obtained either through MD or particle-in-cell (PIC) simulations. This allows to benchmark quantitatively their given stopping profiles before confrontation with experimental measurements.

In the limit of low velocities ($v \ll v_{th}$), the stopping power behaves as

$$-\frac{dE}{ds} = R_1 v + R_3 v^3 + O(v^5),$$

where the friction coefficient R_1 in the binary collision approximation is given by

$$R_1 = \frac{Z_p^2 \Gamma^3}{3\pi} \int_0^\infty dx e^{-x} \ln\left(1 + \frac{4x^2}{3Z_p^2 \Gamma^3}\right). \quad (9)$$

In the dielectric linear response formalism, it is given by

$$R_1 = -\frac{Z_p^2 \Gamma^3}{3(2\pi)^{1/2}} \left[\ln(k_m^2 + 1) - \frac{k_m^2}{k_m^2 + 1} \right],$$

$$k_m = \frac{2}{\sqrt{3} |Z_p| \Gamma^{3/2}}. \quad (10)$$

Corresponding friction coefficients $-dE/vds$ are graphed in terms of coupling parameter $Z_p \Gamma^{3/2}$, on Fig. 5, altogether with MD simulation results and pure Z_p^2 law curve (straight dotted line).

Again BCA profile fits more closely the trustable simulation results than DLR ones.

As a straightforward application of present reasoning, let us consider the simultaneous expansion of ion cloud surrounded by free electrons [12], with

$$\frac{V_{\text{expanding heavyion}}}{V_{\text{thermalized free electron}}} \simeq 1.2 \times 10^{-4}.$$

FIGURE 4. Normalized stopping power $dE/ds/Z_p^2$ in units of $3^{1/2}\Gamma^{3/2}k_B T/\lambda_D$ as function of the ion velocity v in $v_{th} = (k_B T/m)^{1/2}$ for an ion of charge $Z_p = 10$ in electron plasmas with $\Gamma = 0.11$ (top) 0.34 (centre) and 1.08 (bottom): MD simulations (Δ), with typical size of error bars as indicated right top in each case, Vlasov simulations (*), the linear response description, Eq. (7) (dashed curve) and the binary collision treatment, Eq. (8) (dash-dotted).

So, we can safely state that no friction is expected between expanding ions and expanding free electron fluid, which seems to confirm simulation results [12] for the expansion process.

FIGURE 5. Friction coefficients $-dE/ds/v$ as a function of the coupling parameter $Z_p \Gamma^{3/2}$. Energies E and velocities v are in thermal units; and distances are in Landau length $e^2/k_B T_e$ (see Fig. 1). MD simulation results for $\Gamma = 1.08$ (Δ), $\Gamma = 0.34$ ($\diamond$), and $\Gamma = 0.11$ ($\circ$), with error bars compared with the binary collision approximation (dash-dotted curve, Eq. (9)) and the linear dielectric response treatment (solid curve, Eq. (10)). The dashed line is a pure Z_p^2 law curve.

4. LOW ION VELOCITY SLOWING IN A STRONGLY MAGNETIZED ULTRA-COLD PLASMAS

It is straightforwardly observed that it is rather easy to magnetize, even strongly, an expanding UCP. This is achieved as soon as electron cyclotron frequency fulfills $\omega_b \gg \omega_p$ with

$$B [\text{G}] \gg 3.203 \times 10^{-3} [n_e (\text{cm}^{-3})]^{1/2} \quad (11)$$

while corresponding heavy ion (Rb for instance) gyro-radius $\omega_{b,i}$ is obviously fulfilling

$$r_{ci} = v_{\text{thi}}/\omega_{b,i} = 1.02 \times 10^2 \mu^{1/2} Z^{-1} T_i^{1/2} B^{-1} \text{ cm} \gg r_L \quad (12)$$

with $\mu = m_i/m$ proton, T_i in eV and B in Gauss. r_L denotes electron Larmor radius.

The combination of Eqs. (11) and (12) highlights a strongly magnetized electron UCP neutralized by an ion fluid hardly affected by magnetic field B .

Ion beam stopping in a dense plasma submitted to an arbitrary large and steady magnetic field $\mathbf{B}$, is a recurrent topic encompassing a huge range of practical situations of very high interest. Here we focus attention on ultracold plasmas (UCP) and cold electron setups used for ion beam cooling.

These interaction geometries also highlight low ion velocity slowing down (LIVSD) as playing a fundamental role in asserting the confining capabilities and thermonuclear burn efficiency in dense and strongly magnetized media.

Our present goal is to demonstrate that transverse and parallel LIVSD to $\mathbf{B}$, may be given analytic expressions through a derivation free from ambiguities usually plaguing the most sophisticated combination of binary collision approximation and dielectric response [13]. We thus implement a radically novel approach [14] to LIVSD when projectile velocity v remains smaller than target electron thermal v_{the} . We thus consider ion stopping

$$S(v) \equiv \frac{dE_b}{dx}(v), \quad (13)$$

near $v = 0$. The ratio $S(v)/v$ usually monitors a linear stopping profile, up to 100 keV/a.m.u in cold matter. Similar trends are also reported in highly ionized plasma with $B = 0$ or $B \neq 0$ [13].

4.1. Hydrodynamic approach

From now on, we intend to make use of a very powerful connection between very low velocity ion stopping and particle diffusion through Einstein characterization of ion mobility associated to thermal electron fluctuations in target, around the slow ion projectile visualized as an impurity immersed in a dense and homogeneous electron fluid.

Technically, we are then led to use the recently proposed and exact Dufty-Berkovsky relationship [14, 15]

$$\lim_{v \rightarrow 0} \frac{S(v)}{v} = k_B T_e D^{-1}, \quad (14)$$

connecting the ratio of stopping to v in the zero velocity limit with the ion diffusion coefficient in target.

In a magnetized plasma D can be readily expressed in terms of Green-Kubo integrands (GKI) involving field fluctuations in the target electron fluid, under the form

$$D = \frac{c^2}{B^2} \int_0^\infty d\tau \langle \mathbf{E}(\tau) \cdot \mathbf{E}(0) \rangle \quad (15)$$

in terms of an equilibrium canonical average of the two-point autocorrelation function for fluctuating electric fields [16–18].

At this juncture we need to frame the GKI in suitable magnetized one component plasma (OCP) models [16, 17] for the transverse and parallel geometry, respectively. This procedure implies that the slowly incoming ions are evolving against a background of faster fluctuating target electrons ($v < v_{\text{the}}$) providing the OCP rigid neutralizing background thus validating the OCP assumption.

Moreover, restricting to proton projectiles impacting an electron-proton plasma [19], we immediately perceive the pertinence of the diffusion-based LIVSD as phrased by Eq. (14).

First, the proton beam can easily self-diffuse amongst its target homologues, while the same mechanism experienced by target electrons allow them to drag ambipolarly the incoming proton projectiles [19].

So, the transverse electron LIVSD can be either monitored by the well know classical diffusion $D_{\perp} \sim B^{-2}$, or by the Bohmlike hydrodynamic one with $D_{\perp} \sim B^{-1}$. In the first case, momentum conservation at the level of the electron-ion pair implies that the ions will diffuse with the same coefficient as the electrons. On the other hand, the hydro Bohm diffusion across $\mathbf{B}$ is operated through clumps [20] with a large number of particles involved in this collective process.

Transverse $D_{\perp}$ and parallel $D_{\parallel}$ diffusion coefficient have already been discussed at length by Marchetti et al. [16] and Cohen-Sutton [17]. Their derivation is based on the specific features of four finite frequency and propagating hydromodes in a strongly magnetized OCP with the ratio of plasma to cyclotron frequencies, $\omega_p/\omega_b < 1$.

First, two high frequency modes generalizes first Bernstein modes ($B = 0$) and two finite frequency modes extend the $B = 0$ shear modes.

So, exploring first the $\omega_b \geq \omega_p$ domain, one can explicit the parallel and B -independent diffusion [16],

$$D_{\parallel}^{(0)} = \frac{3\sqrt{\pi}v_{\text{thi}}^2}{v_c} \sim O(\omega_b^0), \quad (16)$$

yielding back readily the unmagnetized ($B = 0$) LIVSD [16], where $v_{\text{thi}}^2 = k_B T/M_i$, and $v_c = \omega_p \epsilon_p \ln(1/\epsilon_p)$ in terms of the redefined dimensionless plasma parameter $\epsilon_p = 1/n_e \lambda_D^3$, and λ_D , the Debye length, in a beam-plasma system taken as globally neutral with $v_c/\omega_b \ll 1$.

At the same level of approximation transverse diffusion reads as [16]

$$D_{\perp}^{(0)} = \frac{r_L^2 v_c}{3\sqrt{\pi}} \sim O(\omega_b^{-2}), \quad (17)$$

in terms of Larmor radius $r_L = v_{\text{thi}}/\omega_b$.

With higher B values ($\omega_b \gg \omega_p$) one reaches the transverse hydro Bohm regime featuring [16]

$$D_{\perp}^{(0)} = D_{\perp}^0 + \frac{0.5v_{\text{thi}}^2}{\omega_b} \epsilon_p^2 [\ln(1/\epsilon_p)]^{3/2}, \quad (18)$$

while parallel diffusion retains a ω_b -dependence through [17]

$$D_{\parallel}^{-1} = \frac{\Gamma^{5/2}}{\omega_p a^2} \left(\frac{3}{\pi} \right)^{1/2} \left[0.5 \ln(1 + X^2) - 0.3 + \frac{0.0235}{r^2} \right], \quad (19)$$

where $\Gamma = a^2/3\lambda_D^2$ with $a = (3/4\pi n_e)^{1/3}$, $r = \omega_p/\omega_b$ and $X = 1/\sqrt{3}\Gamma^{3/2} < 1$ encompasses, most if not all, situations of practical interest.

When electron diffusion is considered, v_{the} should be used in Eq. (18), and the above ambipolar process has to be implemented.

The $D_{\perp}$ and $D_{\parallel}$ expressions introduced in Eq. (14) are expected to document a strong anisotropy between transverse and parallel slowing down. However, in both cases, B -dependence is obviously increasing with B^2 (classical) or B (Bohmlike). The temperature behavior is much more intriguing, as respectively displayed on Figs. 6–7 for

FIGURE 6. Proton transverse LIVSD in a cold plasma ($n = 3.5 \times 10^7 \text{ e-cm}^{-3}$, $10 \leq T \text{ (}^\circ\text{K)} \leq 10^5$ and $B = 10^4 \text{ G}$) in terms of $T \text{ (K)}$. a) electron stopping $D_\perp \simeq B^{-2}$; b) electron stopping $D_\perp \simeq B^{-1}$; c) ion stopping

FIGURE 7. Proton parallel LIVSD in a cold plasma ($n = 3.5 \times 10^7 \text{ e-cm}^{-3}$, $B = 10^4 \text{ G}$), $10 \leq T \text{ (}^\circ\text{K)} \leq 10^5$). a) target ion slowing down ($B \neq 0$); b) target ion slowing down ($B = 0$); c) target electron slowing down ($B \neq 0$); d) target electron slowing ($B = 0$). (b) and (c) stand in a $\log_{10}(43)$ ratio.

transverse and parallel LIVSD in a UCP considered for ion beam cooling (see Fig. 3). One then witnesses a monotonous increase for transverse stopping (Fig. 6) contrasted to a monotonous decay for the parallel counterpart (Fig. 7).

We thus implemented the very simple LIVSD expression (14) to the a priori very involved ion beam-arbitrarily magnetized plasma interaction. We used transverse and parallel diffusion coefficients [16, 17] in suitably framed magnetized one-component-plasma (OCP) with target electrons building up the corresponding neutralizing background. Thus, we reached analytic LIVSD transverse and parallel expressions advocat-

ing contrasting temperature behavior. These quantities are of obvious significance in asserting the confinement capabilities ultracold ones at high B values.

4.2. Connections with kinetic theory results

We consider

$$S = \frac{Z^2 e^2}{2\pi^2 \nu} \int d\mathbf{k} \frac{\mathbf{k} \cdot \mathbf{v}}{k^2} \text{Im} \frac{-1}{\epsilon_M(\mathbf{k}, \mathbf{k} \cdot \mathbf{v}, \gamma)}. \quad (20)$$

where the collision frequency γ is measure of damping of excitations in the plasma and $\epsilon_M(\mathbf{k}, \omega, \gamma)$ is the collision-inclusive longitudinal dielectric function of the magnetized plasma [21].

Thus from Eq. (20) we obtain the usual (linear with respect to ν) friction law

$$S(\vartheta) \simeq \frac{2Z^2 e^2}{\sqrt{2\pi} \lambda_D^2} \frac{\nu}{v_{\text{th}}} R(\vartheta), \quad (21)$$

where $R(\vartheta)$ is the dimensionless friction coefficient,

$$R(\vartheta) = \int_0^\kappa \frac{k^3 dk}{(k^2 + 1)^2} \left[\psi_1(k) \cos^2 \vartheta + \frac{1}{2} \psi_2(k) \sin^2 \vartheta \right]. \quad (22)$$

Here $\kappa = k_{\text{max}} \lambda_D$ and ϑ is the angle between $\mathbf{v}$ and $\mathbf{B}$. In Eq. (22) we have introduced a cutoff parameter $k_{\text{max}} = 1/r_{\text{min}}$ (where r_{min} is the effective minimum impact parameter) in order to avoid the logarithmic divergence at large k . This divergence corresponds to the incapability of the linearized kinetic theory to treat close encounters between the projectile ion and the plasma electrons properly. For r_{min} we thus use the effective minimum impact parameter of classical binary Coulomb collisions which at low-velocities of the ion reads $r_{\text{min}} = |Z|e^2/mv_{\text{th}}^2$. It is seen that the parameter $\kappa = 4\pi n_e \lambda_D^3/|Z| \gg 1$. Also the other quantities in Eq. (22)

$$\psi_n(k) = \frac{1}{2} \int_0^\infty \exp \left[-\frac{2k^2}{\zeta^2} \sin^2(\zeta t) - 2\nu t \right] \Phi_n(kQ(t)) \frac{dt}{t\Upsilon(\zeta t)} \quad (23)$$

with $n = 1, 2$, $\nu = \gamma/\omega_p$, $\zeta = \lambda_D/a_B = \omega_b/\omega_p$, $Q(t) = \sqrt{2}t\Upsilon(\zeta t)$, $\Upsilon^2(t) = 1 - (\sin t/t)^2$, $\Phi_1(x) = x^{-2}\Phi(x)$, $\Phi_2(x) = 2\text{erf}(x) - x^{-2}\Phi(x)$,

$$\Phi(x) = \frac{4}{\sqrt{\pi}} \int_0^\infty e^{-t^2} t^2 dt = \text{erf}(x) - \frac{2}{\sqrt{\pi}} x e^{-x^2},$$

and $a_B = v_{\text{th}}/\omega_b$, electron gyro-radius.

In many experimental situations, the ions move in a plasma with random orientations of ϑ with respect to the magnetic field direction $\mathbf{b}$. The friction coefficient appropriate to this situation may be obtained by carrying out a spherical average over ϑ of $R(\vartheta)$ in Eq. (22). We find

$$\langle R(\vartheta) \rangle = \frac{1}{3} \int_0^\kappa \frac{k^3 dk}{(k^2 + 1)^2} [\psi_1(k) + \psi_2(k)]. \quad (24)$$

Let us analyze the general expression (22) for some particular cases. For instance, at vanishing magnetic field ($\zeta \rightarrow 0$) using the relation $Q(t) \simeq \sqrt{2/3}\zeta t^2$ at $\zeta \rightarrow 0$, one gets

$$\psi_1(k) = \frac{1}{2}\psi_2(k) = \frac{1}{3}A\left(\frac{v}{\sqrt{2}k}\right), \quad (25)$$

where $A(z) = e^{z^2} \text{erfc}(z)$, $\text{erfc}(z)$ is the complementary error function. In this case the friction coefficient is isotropic and becomes

$$R_0(\vartheta) = \frac{1}{3} \int_{p_0}^{\infty} \frac{A(vk) dk}{k(2k^2 + 1)^2}. \quad (26)$$

Here $1/p_0 = \sqrt{2}\kappa$. In addition at vanishing damping, i.e. at $v \rightarrow 0$, $A(vk) \rightarrow 1$ and we recover the usual low-velocity stopping power in an unmagnetized collisionless plasma with a friction coefficient (see, e.g., [22])

$$R_0(\vartheta) = \frac{1}{6}U(\kappa) \equiv \frac{1}{6} \left[\ln(1 + \kappa^2) - \frac{\kappa^2}{\kappa^2 + 1} \right]. \quad (27)$$

At strong magnetic fields ($\zeta \rightarrow \infty$) the plasma becomes highly anisotropic and the friction coefficient depends essentially on the angle ϑ . For an evaluation of the functions $\psi_1(k)$ and $\psi_2(k)$ we note that $Q(t) \rightarrow \sqrt{2}t$ and $Y(\zeta t) \rightarrow 1$ as $\zeta \rightarrow \infty$. Then substituting these relations into Eq. (23) and after integration by parts one obtains

$$\psi_1(k) = \frac{1}{2}A(a) + a^2B(a) - \frac{a}{\sqrt{\pi}}, \quad (28)$$

$$\psi_2(k) = (1 - a^2)B(a) - \frac{1}{2}A(a) + \frac{a}{\sqrt{\pi}} \quad (29)$$

with $a = v/\sqrt{2}k$, and

$$B(z) = \int_z^{\infty} \frac{dt}{t} A(t) = \frac{2z}{\sqrt{\pi}} \int_0^{\infty} \ln(t + \sqrt{t^2 + 1}) e^{-z^2 t^2} dt. \quad (30)$$

Then the friction coefficient at infinitely strong magnetic field reads

$$R_{\infty}(\vartheta) = \frac{1}{2} \int_{p_0}^{\infty} \frac{dk}{k(2k^2 + 1)^2} \left\{ \sin^2 \vartheta \left[(1 - v^2 k^2) B(vk) - \frac{1}{2} \left(A(vk) - \frac{2vk}{\sqrt{\pi}} \right) \right] \right. \\ \left. + \cos^2 \vartheta \left[A(vk) - \frac{2}{\sqrt{\pi}} vk + 2v^2 k^2 B(vk) \right] \right\}. \quad (31)$$

Similarly for the angular averaged friction coefficient we obtain

$$\langle R_{\infty}(\vartheta) \rangle = \frac{1}{3} \int_{p_0}^{\infty} \frac{B(vk) dk}{k(2k^2 + 1)^2}. \quad (32)$$

The function $B(z)$ involved in Eq. (31) at small z behaves as $B(z) \simeq \ln(1/z) - C/2$, where $C = 0.5772$ is the Euler's constant, and diverges logarithmically when $z \rightarrow 0$. Using asymptotic behavior of this function, it is straightforward to calculate from Eq. (31) the friction coefficient at vanishing γ . In this limit and in the leading order we obtain

$$R_\infty(\vartheta) = \frac{1}{4} \left\{ \sin^2 \vartheta \left[\left(\ln \frac{\sqrt{2}\omega_p}{\gamma} - \frac{C+1}{2} \right) U(\kappa) + U_1(\kappa) \right] + U(\kappa) \cos^2 \vartheta \right\}, \quad (33)$$

where $U(\kappa)$ is given by Eq. (27), and

$$U_1(\kappa) = U(\kappa) \ln \kappa - \frac{1}{4} [\ln^2(\kappa^2 + 1) - 2 \ln(\kappa^2 + 1)] - \frac{1}{2} \text{Li}_2 \left(\frac{\kappa^2}{\kappa^2 + 1} \right). \quad (34)$$

Here $\text{Li}_2(z)$ is the dilogarithm function. Note that at large $\kappa \gg 1$, which is a requirement of weak ion-plasma coupling, the functions $U_1(\kappa)$ and $U(\kappa)$ can be approximated by $U_1(\kappa) \simeq \ln^2 \kappa - \pi^2/12$ and $U(\kappa) \simeq 2 \ln \kappa - 1$, respectively. It is seen that the first term in Eq. (33) diverges logarithmically at vanishing γ . It can be shown that the general expression (22) with (23) for the friction coefficient derived for arbitrary but finite magnetic field behaves similarly. This is due to the magnetic field since the field-free result (26) remains finite as $\gamma \rightarrow 0$, see, e.g., Eq. (27). The divergent term in Eq. (33) vanishes, however, when the ion moves along the magnetic field ($\vartheta \rightarrow 0$). Then the friction coefficient is solely given by the last term of Eq. (33). In addition, the friction coefficient Eq. (33) for strong magnetic fields shows an enhancement for ions moving transverse ($\vartheta = \pi/2$) to the magnetic field compared to the case of the longitudinal motion ($\vartheta \rightarrow 0$). This effect is in agreement with particle-in-cell simulation results [10].

At vanishing damping ($\gamma \rightarrow 0$), it can be shown that $D_\parallel$ obtained from Eqs. (21)-(23) and for $\vartheta \rightarrow 0$ coincides with the result of Cohen and Suttrop [17]. In particular, at $\gamma \rightarrow 0$, it is found from Eqs. (27) and (33) that $R_0(0)/R_\infty(0) = D_{\infty,\parallel}/D_{0,\parallel} = 2/3$ in agreement with Sec. 4.1. Here $D_{\infty,\parallel}$ and $D_{0,\parallel}$ are the parallel self-diffusion coefficients at infinite and vanishing magnetic field, respectively. However at finite γ and for $\vartheta = 0$, comparing Eq. (22) with (31), we conclude that the simple relation cited above is not obeyed in general, due to damping.

As an example, we show in Fig. 8 plots of the dimensionless friction coefficient $R(\vartheta)$ given by Eq. (22), as a function of the scaled magnetic field ω_b/ω_p for model parameter γ ($\gamma/\omega_p = 0.1$) (left panel). The right panel of Fig. 8 shows $R(\vartheta)$ as a function of the scaled damping parameter γ/ω_p for $\omega_b = \omega_p$ (i.e. for a given magnetic field). It is seen that the low-velocity SP increases with an increase in the angle ϑ and also with the magnetic field. In the latter case the SP asymptotically tends to the value given by expression (31). In the opposite limit of a weak magnetic field the friction coefficient tends to the value given by Eq. (26), which is independent of the angle ϑ . Also Fig. 8 shows that the friction coefficient decreases with damping.

A decrease of $R(\vartheta)$ with γ in the present case of a classical plasma is not attributable to the applied magnetic field because the field-free friction coefficient given by (26) shows a similar behavior (not shown in Fig. 8). In a degenerate plasma an enhancement of the low-velocity SP with γ is a quantum effect which is absent in our present study. For a DEG the domain of plasmon excitations is shifted towards smaller ion velocities

FIGURE 8. The friction coefficient $R(\vartheta)$ vs, left panel, the scaled magnetic field ω_b/ω_p and, right panel, damping parameter γ/ω_p for solid line, $\vartheta = 0$; dashed line, $\vartheta = \pi/4$; and dotted line, $\vartheta = \pi/2$, $\kappa = 10$; left panel, $\gamma/\omega_p = 0.1$; right panel $\omega_b = \omega_p$. The line with symbols corresponds to $\langle R(\vartheta) \rangle$.

[21], this increases the SP in this velocity regime. But in the present case the domain of collective excitations is shifted towards higher velocities and the friction coefficient decreases with γ .

In order to document the LIVSD physics highlighted by the relation (18), we first pay attention to the unmagnetized $B = 0$ limit. We consider it through the small $\varepsilon = 1/(4\pi n_e \lambda_D^3) \ll 1$ plasma parameter approximation for the self-diffusion. Employing Eqs. (21), (27) and (14), an inspection shows that at vanishing damping ($\gamma = 0$) the self-diffusion coefficient obtained from these formulas coincides with the ionic charge number square Z^2 in Eq. (21) replaced by the quantity $Z^2 \rightarrow P(Z)$, where [18]

$$P(Z) = \left(Z + \frac{1}{\sqrt{2}} \right) \frac{32Z^2 + 75\sqrt{2}Z + 50}{104Z^2 + 111\sqrt{2}Z + 59}. \quad (35)$$

For a proton projectile with $Z = 1$ this factor is $P(1) = 1.003$ and the agreement between both approaches is almost perfect. The factor (35) which is nonlinear with respect to Z accounts for the nonlinear coupling ion and the surrounding plasma [18]. However for highly charged ions with $Z \gg 1$ this factor increases linearly with Z , $P(Z) = (4/13)Z$, while more rigorous treatment shows that at strong ion-plasma coupling the energy loss of an ion scales with its charge approximately like $Z^{1.5}$.

Now we turn to the case of a magnetized and collisional plasma. For simplicity we consider electron-proton plasma and a proton as a projectile particle. Exploring first the moderately magnetized domain, $\omega_b \geq \omega_p$ one can explicit the field-free parallel and B -dependent transverse diffusion (16), (17).

The collision frequency ν_c is related to the e-e collisional relaxation rate ν_{ee} as $\nu_{ee} = \sqrt{2/9\pi} \nu_c$, (see Sec. 4). The transverse diffusion coefficient given by Eqs. (16) and (17) corresponds to a classical region, where $D_\perp \sim B^{-2}$ and is valid for $\nu_c < \omega_b < 0.4\omega_p Y(\varepsilon)$ with $Y(\varepsilon) = [\varepsilon^2 \ln(1/\varepsilon)]^{-1/2}$, as explained in Ref. [16].

With higher magnetic field values one reaches the transverse hydro-Bohm regime with $\omega_b/\omega_p > 4Y(\varepsilon)$ featuring $D_\perp \sim B^{-1}$.

FIGURE 9. Proton LIVSD in a plasma with $n_e = 1.064 \times 10^{16} \text{ cm}^{-3}$, $T_e = 1 \text{ eV}$ ($\varepsilon = 0.02$) in term of ω_b/ω_p . The line with symbols in the left and right panels represent parallel and transverse LIVSD, respectively. The solid line were obtained from Eqs. (21)–(23) with left $\vartheta = 0$ and right $\vartheta = \pi/2$.

To the intermediate plateau regime with $D_{\perp} \sim B^0$ between transverse diffusion coefficient given by Eqs. (16)–(18) corresponds the diffusion coefficient [20] valid at $0.4Y(\varepsilon) < \omega_b/\omega_p < Y(\varepsilon)$,

$$D'_{\perp} = D_{\perp}^{(0)} \left[1 + \frac{0.6\varepsilon v_c}{\omega_p} \zeta^2 \right] \quad (36)$$

with $\zeta = \omega_b/\omega_p$. When electron diffusion is considered, the electron thermal velocity v_{th} should be used in Eqs. (16)–(36) instead of v_p . It is also important to stress that the quantitative predictions (16)–(36) of the mode coupling theory developed in Ref. [16] are strongly dependent on the values of the hydrodynamic cutoffs which, in contrast to the kinetic theory, are introduced linearly. A reliable estimate of the magnetic field and plasma parameter dependence of the cutoffs have also been obtained.

The friction coefficients $S(v)/v$ (at $v \rightarrow 0$) calculated with the help of Eqs. (18) and (16)–(36) are shown in Fig. 9 as the lines with symbols. In this figure the solid lines without symbols demonstrate the friction coefficient calculated from Eqs. (21)–(23) with $\vartheta = 0$ (left panel) and $\vartheta = \pi/2$ (right panel), assuming, for consistency, the same collision frequency $\nu = \nu_c$ as in Eqs. (16), (17) and (21)–(23). There are no fundamental contradiction between kinetic (Eqs. (21)–(23)) and hydrodynamic (Eq. (18)) and the relation (16) approaches for the parallel case, see Fig. 9 left panel. However, there are differences between the two approaches.

Indeed assuming a vanishing damping ($\gamma = 0$) and magnetic field ($B = 0$) from Eqs. (21) and (27) at $\varepsilon \ll 1$ the ratio of the low-velocity SPs of both approaches is $S_{\parallel,kin}/S_{\parallel,hyd} \simeq \sqrt{2}$. As discussed above the numerical coefficients in Eqs. (16)–(36) are not precisely known. Thus, including the numerical factor $\sqrt{2}$ into the denominator of the parallel diffusion coefficient in Eqs. (16) and (17) the agreement between both approaches becomes complete. Note that this is equivalent to the redefinition $D_{\parallel} \rightarrow D_{\parallel}^* = v_p^2/\nu_{ee}$, where $\nu_{ee} = \sqrt{2/9\pi}\nu_c$.

An apparently large discrepancy is documented for the transverse situation (Fig. 9, right panel) where typically $S_{\perp,\text{kin}}/S_{\perp,\text{hyd}} \sim [\epsilon \ln(1/\epsilon)]^\alpha \ll 1$ with $\epsilon \ll 1$ and α varies between $2 \leq \alpha \leq 4$ depending on the strength of B . The kinetic regime seems to be restricted to $0.08 = v_c/\omega_p < \omega_b/\omega_p < 0.4Y(\epsilon) = 10.1$ as explained in Ref. [21]. The discrepancy in the orthogonal case might be due to a different treatment of cutoffs in kinetic and hydrodynamic theories, i.e logarithmic vs linear. Actually, the different hydromodes are normalized to distinct cutoffs. Upper hybrid ones are normalized to $1/a_B$, a_B being electron Larmor radius while low frequency modes are normalized to inverse mean free path $1/\ell$ with $\ell = v_{\text{th}}/v_c$. On the other hand in the extreme limit ω_b/ω_p the only reasonable transverse cutoff should be $1/a_B$ which, for instance, in the kinetic treatment is included as $\ln(1/a_B)$. The basic physics involved in this orthogonal geometry pertains to kinetic theory when we rely on a collisional time while in the $B \rightarrow \infty$ limit leading to hydrodynamics, we incorporate the Larmor rotation of the charged particles, as well.

5. CONCLUSION AND OUTLOOKS

Ultracold plasmas (UCP) are thus seen as offering unique opportunities for exploring the specific static and dynamic properties of strongly coupled but mostly classical electron fluids. Efficient experimental, numerical and theoretical methodologies are presently available to converge efficiently on the completion of these goals. Interplay between laser cooled UCP's and ion beam cooling experiments is likely to provide an interesting cross-fertilization process.

Expanding classical electron fluid could be specifically diagnosed through low velocity ion stopping.

UCP can be easily magnetized. This affords an unique opportunity for probing ion stopping in the rather extreme location of the target space parameters, with electron cyclotron radius much smaller than corresponding Debye length. Then one expects the OCP hydromodes to play a dominant role (Sec. 4). Other topics not explicated here include the recombination-ionization mechanisms under a strong imposed magnetic field.

ACKNOWLEDGMENTS

Part of this work has been supported by the Euratom-CEA contract n° 3599.001.

REFERENCES

1. G. Vitrant, J. M. Raimond, M. Gross, and S. J. Haroche, *J. Phys. B* **15**, L49–L56 (1982).
2. T. C. Killian, S. Kulin, S. D. Bergeson, L. A. Orozco, and S. L. Rolston, *Phys. Rev. Lett.* **83**, 4776–4779 (1999).
3. M. P. Robinson, N. Laburthe Tolra, M. W. Noel, T. F. Gallagher, and P. Pillet, *Phys. Rev. Lett.* **85**, 4466–4469 (2000).
4. T. C. Killian, V. S. Ashoka, P. Gupta, S. Caha, S. B. Nagel, C. E. Simien, S. Kulin, S. L. Rolston, and S. D. Bergeson, *J. Phys. A* **36**, 6077–6085 (2003).

5. D. Comparat, T. Vogt, N. Nahzam, M. Mudrich, and P. Pillet, *Mont. Not. Roy. Astr. Soc.* **361**, 1227–1247 (2005).
6. C. Gouedard and C. Deutsch, *J. Math. Phys.* **19**, 32–38 (1978).
7. G. Zwicknagel, Ph. D. Thesis., *Institute of Theor. Phys. II*, Univ. Erlangen, (1994)
8. C. Deutsch, *Phys. Lett. A* **60**, 317–318 (1977); C. Deutsch, Y. Furutani, and M. M. Gombert, *Phys. Rep.* **69**, 85–165 (1981).
9. G. S. Stringfellow, H. E. DeWitt, and W. L. Slattery, *Phys. Rev. A* **41**, 1105–1111 (1990).
10. G. Zwicknagel, C. Toepffer, and P. G. Reinhard, *Phys. Rep.* **309**, 117–208 (1999); Erratum, *Phys. Rep.* **314**, 671 (1999).
11. G. Zwicknagel, *Nucl. Instrum. Meth. in Phys. Res. B* **197**, 22–38 (2001).
12. F. Robicheaux and J. D. Hanson, *Phys. Rev. Lett.* **88**, 5502–5505 (2002).
13. H. B. Nersisyan, C. Toepffer, and G. Zwicknagel, *Interaction Between Charged Particles in a Magnetic Field*, Springer-Verlag, Berlin, 2007.
14. J. W. Dufty and M. Berkovsky, *Nucl. Instrum. Meth. in Phys. Res. B* **96**, 626–632 (1995).
15. A fundamental restriction on the relations (14) arises a priori from the strong inequality $M_p/m \gg 1$, between ion projectile and electron mass. However, Dufty-Berkovsky [14] demonstrates how it can be considerably relaxed.
16. M. C. Marchetti, T. R. Kikpatrick, and J. R. Dorfman, *J. Stat. Phys.* **46**, 679–708 (1987); M. C. Marchetti, T. R. Kikpatrick, and J. R. Dorfman, *Phys. Rev. A* **29**, 2960–2962 (1984).
17. J. S. Cohen and L. G. Suttrop, *Physica A* **126**, 308–327 (1984).
18. C. Deutsch and R. Popoff, *Phys. Rev. E* **78**, 056405 (2008).
19. The extension to a more general ion beam-plasma system requires us to replace the OCP by a binary ionic mixture (BIM) modelization.
20. D. Montgomery, C. S. Liu, and G. Vahala, *Phys. Fluids* **15**, 815–821 (1972).
21. H. B. Nersisyan, C. Deutsch, and A. K. Das, *Phys. Rev. E* **83**, 036403 (2011).
22. C. Deutsch, *Ann. Phys.* **1** (Paris), 1–111 (1986).