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Citation: *AIP Conf. Proc.* **1363**, 227 (2011); doi: 10.1063/1.3630185

View online: <http://dx.doi.org/10.1063/1.3630185>

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Concentrating the phase of a coherent state by means of probabilistic amplification

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Abstract. We discuss the recent implementation of phase concentration of an optical coherent state by use of a probabilistic noiseless amplifier. The operation of the amplifier is described pictorially with phase space diagrams, and the experimental results are outlined.

Keywords: Phase variance, Noiseless amplification, Phase concentration

PACS: 42.50.Dv, 42.50.Lc, 42.65.Yj

INTRODUCTION

Phase estimation of a coherent state is at the heart of many metrological applications as well as communication protocols where information is encoded into the phase. The estimation process is however hampered by fundamental quantum noise of the coherent state: The noise gives rise to a phase uncertainty that plays an increasingly devastating role for the estimation of the phase as the excitation of the coherent state decreases as illustrated in Fig.1a. In order to concentrate the phase information, the state must be amplified in such a way that its phase variance is reduced as compared to the input state. This means that the amplified state should lie within the dashed lines in Fig.1b, where a particular noiseless amplification is illustrated.

But how can a state be amplified noiselessly? Actually noiseless, phase insensitive and deterministic amplification of an unknown state is a forbidden operation according to the fundamental rules of quantum mechanics [1, 2]. However, by relaxing some of all these constraints it is possible to reduce the noise in amplification and in some operations getting arbitrarily close to noise-free operation.

FIGURE 1. Effect of a) attenuation and b) noiseless amplification scheme on the phase variance.

First of all, by reducing and knowing the alphabet of possible states that undergo amplification, it is possible to set the amplifier to minimize the added noise in amplification. The added noise is generally connected with the efficiency in discriminating the possible input states: the smaller the error in discriminating the input states, the lesser the added noise in amplification.

Another way of reducing the added noise, which will be described in this manuscript, is by allowing for a probabilistic operation: Sometimes the amplification operation does not work noiselessly in which case the output will be discarded, and sometimes it works in a nearly noiseless way. Ideal noiseless but probabilistic amplification can be implemented using either a highly complicated interferometric setup including single photon sources [3, 4, 5], using a sequence of photon addition and photon subtraction [6, 8] or using a very strong cross-Kerr nonlinearity [9]. However, in this manuscript we will discuss a radically different noiseless amplifier that it is capable of amplifying the phase information noiselessly without the use of any non-classical resources or any strong parametric interactions. Remarkably, the source of energy is a thermal light source, so the noise-free amplifier is based on the addition of noise! This type of amplifier has been recently proposed [6] and experimentally realized [7]. In the present manuscript we elaborate further on the function of this amplifier.

PROBABILISTIC PHASE AMPLIFICATION

Ralph and Lund [3] have recently proposed a scheme for probabilistic amplification based on the quantum scissors proposal of Pegg, Phillips and Barnett [11]. The quantum scissors protocol was implemented by Lvovsky et al. [12] and the probabilistic amplifier was recently implemented by two groups simultaneously [4, 5]. The complete amplifier scheme consists of a large interferometer with non-Gaussian operations in each interferometer arm.

An alternative approach was recently put forward in Ref.[6]. This method was based on a sequence of photon addition and photon subtraction operations, and has recently been experimentally realized [13]. In this approach, by adding photons coherently to coherent state and subsequently subtracting them again, the state is amplified noiselessly.

Interestingly, it has been realized that by adding the photons incoherently (instead of coherent addition) and subsequently subtracting them again, it is also possible to amplify the state nearly noiselessly [7]. In other words, by substituting the highly sophisticated coherent photon addition operation with a thermal source, it is still possible to enable noiseless amplification to a certain extent. However, we should also note at this point that such an incoherent operation will never yield perfect noiseless amplification and cannot be used for e.g. entanglement distillation of continuous variable two-mode squeezing.

Our scheme is illustrated in Fig.2. The first operation is the addition of Gaussian noise to a coherent state which produces a displaced thermal state as shown in the phase space diagram. Next, we perform a weak measurement of the photon number by tapping off a small fraction of the beam in an asymmetric beam splitter and measuring the reflected part with a photon number resolving detector. We subsequently herald the state based on the measurement outcomes: If say M photons or more are detected the state is kept,

FIGURE 2. Schematic of the Noiseless Amplification.

otherwise it is discarded. This effectively corresponds to the subtraction of M (or more) photons from the displaced thermal state, and thus the action will increase the amplitude, conserve the mean value of the phase and reduce the phase variance. Because the mean value of the phase is unchanged and the noise is reduced, the signal to noise ratio is increased.

But how does it really work? The function of the amplifier is quite easy to understand by considering the phase space diagram illustrated in Fig.3. For this illustration we consider one specific coherent state (Fig.3a) from the phase randomized alphabet although it is important to note that the protocol is phase invariant and thus will work equally well for any state in that alphabet. The next step is to add Gaussian noise which means that the coherent state is randomly displaced in all directions thereby creating a Gaussian distribution of coherent states: Fig.3b. So far, the gain is zero and the phase variance is of course increased. Then the mixed state is asymmetrically split, creating a pair of thermal states, with classically correlated excitations of the individual coherent states.

FIGURE 3. Phase space diagram of the following states as seen by the HD detector (after the tap): a) the input state b) the displaced thermal state c) the displaced thermal state and the threshold that is imposed by the photon counting sitting on the other arm d) the heralded amplified output.

Now by counting the number of photons in the tap mode and keeping the remaining part of the state only when M photons are registered, we prepare a filtered state which is amplified compared to the original mixed state. The threshold of measuring M photons can be seen as measuring coherent states lying outside the circle in phase space illustrated in Fig.3c (red solid line). The circle represents the photon counting measurements of M photons, and only coherent states that are on the line or outside the circle is likely to produce M or more clicks in the detector. As these coherent states are classically correlated with the coherent states in the transmitted mode, we select the coherent states with the highest excitation and the right direction in phase space, resulting in an amplified state with reduced phase variance as can be seen in Fig.3d.

FIGURE 4. Normalized Holevo phase variance of the output state and phase space pictures associated with three different noise additions.

Rigorous theoretical calculations of the amplifier have also been performed and some of the results are shown in Fig.4. Here we plot the normalized phase uncertainty of the output state with respect to the input state as a function of the mean number of thermal photons added to the state. The dashed-dotted line represents the phase variance of the input state. In each inset we plotted for reference, the input state (small dotted circle), the thermal state (dashed circle) and the remaining states after postselecting on M photons. We note that the phase variance has a minimum for a certain amount of added Gaussian noise Fig.4a. When noise is added beyond this optimal value, the dashed circle becomes larger and more states are accepted (Fig.4b). The result is an increase of the phase variance in the output state. Any further increase of noise after the crossing point with the dashed dotted line ($V=1$), will result in a noisy amplification (Fig.4c).

EXPERIMENT

In the following, we describe the experimental implementation. The setup as it is realized in our laboratory is depicted in Fig.5. The output of a 809 nm diode laser is cleaned spatially by sending it through a single mode fiber. Subsequently the major part of the beam is split off and guided to a homodyne detector to serve as a local oscillator. The residual part of the beam is used to create the coherent signal state as well as the noise, which powers the concentration process. Both, signal and noise, are generated by a combination of two electro optical modulators and a half wave plate. The resulting signal state resides in a polarization mode orthogonal to the input. This allows us to separate the signal from the spare part of the beam simply via a polarizing beam splitter. Subsequently a small portion (20%) of the state is tapped off by an asymmetric beam

splitter and guided to an avalanche photo diode APD operating in an actively gated mode. The dead time of the APD (50ns) is much shorter than the pulse duration (800ns), which allows for multiple detection events.

FIGURE 5. Schematic of the experimental setup. A description can be found in the main text.

For small mean photon numbers the measured detection statistics proof a linear behavior, such that the APD can be used as a photon number resolving detector. The detected number of clicks in the APD is forwarded to a PC, where it is used for the heralding process. To verify the enhanced phase information of the heralded states, we perform a full tomography of the transmitted part of the state. For this the path length of the local oscillator is varied harmonically to allow for quadrature measurements at arbitrary phase angles. We reconstruct the states' density matrices from the measured tomograms via a maximum likelihood algorithm [14, 15]. From the reconstructed density matrices, we calculated the Wigner functions, which are shown in Fig.6a.

FIGURE 6. a) Contour lines of Wigner functions for the phase concentrated states. An increasing threshold parameter M enhances the phase information. b) The phase variance and the corresponding success rate for a coherent state with amplitude $\alpha = 0.431$ are shown for varying threshold parameter. A clear reduction of the phase variance is already achieved for the subtraction of a single photon.

We infer the density matrix of the coherent input state by considering the attenuation of the tap measurement. The phase variance of the inferred input is then compared to the phase variances of the heralded states. We find in Fig.6b that for increasing number of subtracted photons, the phase variance decreases drastically on the cost of lower success rate in the heralding process.

CONCLUSION

We have experimentally realized a phase insensitive device that successfully amplifies an initial coherent state and reduces its phase variance, utilizing only linear optics, thermal states and photon counting. The noiseless amplification takes advantage of noise addition, which at first sight seems counter-intuitive and has been pictorially explained in the present manuscript.

ACKNOWLEDGEMENTS

This work was supported by the EU project COMPAS, the BIOP Graduate school, the Lundbeck foundation and the DFG project LE 408/19-1. R.F. acknowledges support from projects No. MSM 6198959213 and No. LC06007 of the Czech Ministry of Education, the Grant 202/08/0224 of GA CR and the Alexander von Humboldt Foundation. P. M. acknowledges support from the Grant P205/10/P319 of GA CR.

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