**Friedrich-Alexander-Universität
Erlangen-Nürnberg**

Novel Aspects of Pulse Propagation in Photonic-Crystal Fibers

Neue Aspekte bei der Ausbreitung von Licht in Photonischen Kristallfasern

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Zusammenfassung

Die nichtlineare Ausbreitung von Licht in photonischen Kristallfasern (englisch: photonic-crystal fibers, PCFs) erlaubt aufgrund der einzigartigen Lichtwellenleitungseigenschaften der Fasern das Auftreten neuartiger Anwendungen. Die transversale Ausdehnung des Lichts ist auf die μm -große Kernregion beschränkt und führt zu einer starken Wechselwirkung des elektrischen Feldes mit den Quarzglas Molekülen. Als Folge treten nichtlineare Effekte merklich in Erscheinung und ebneten letztlich die weitverbreitete Nutzung von PCFs in unterschiedlichen Anwendungsgebieten. Der Bedarf an faserbasierten Lichtquellen steigt rasant an, und gleichzeitig auch die Anforderungen an die spektralen und zeitlichen Eigenschaften des Lichts. Dies führt zu der Aufgabenstellung dieser Arbeit: Die Beeinflussung der Lichteigenschaften in PCFs mit neuartigen Dispersionslandschaften. Speziell geformte Dispersionsprofile wurden dabei mittels der Tapering-Methode hergestellt. Zusätzlich wurde das Verhalten von Licht in den Fasern anhand numerischer Simulationen analysiert und mit dem Experiment verglichen.

Der erste Teil dieser Arbeit handelt über die Ausbreitung von Licht in PCFs, deren Dispersionsprofile mittels einer besonderen Kern- und Mantelgeometrie eingestellt werden. Die grosse Einstellbarkeit der Modendispersion kann zur effektiven Weisslichterzeugung, Pulsformung und Wellenlängenumwandlung ausgenutzt werden. Zu Beginn demonstrieren wir das Auftreten neuartiger zeitlicher Modulationen in Fasern von sehr kurzer Länge. Wir nutzen nichtlineare PCFs, da aufgrund des großen nichtlinearen Faktors die Bildung von Solitonen anfangs unterdrückt wird. Stattdessen formt sich ein Zug aus ultrakurzen Pulsen. Der zeitliche Abstand zwischen den Pulsen ist im fs-Bereich und der Zug entspricht demnach THz Strahlung. Wir demonstrieren einen Zug aus 50 fs Pulsen die mit einer Wiederholungsrate von 14 THz auftreten. Wir diskutieren die Abhängigkeit des Pulszuges von den Faserparametern und zeigen dass sub 10 fs Pulszüge in nur mm-langen Fasern geformt werden können.

Wenn die Größe des Faserkerns im Bereich der Lichtwellenlänge ist, wird die nichtlineare Antwort

der Moleküle auf den Puls maximiert. Die Dispersion hat zusätzlich einen schmalen anomalen Bereich im sichtbaren. Als Ergebnis zeigen wir, dass kurze Faserstücke zur Optimierung des Lichtspektrums ausgenutzt werden können. Dabei werden nur sehr geringe Pumpleistungen benötigt. Die spektrale Leuchtdichte lässt sich unter anderem mit der Eingangswellenlänge beeinflussen. Diese Fasern sind ausgezeichnete Kandidaten zur Erzeugung von Weisslichtquellen mit einigen GHz Wiederholungsraten. Weiterhin haben Fasern mit einem sub- μm großem Kern eine ungewöhnlich starke Wellenlängenabhängigkeit des nichtlinearen Faktors. In dieser Arbeit führen wir vor, dass daraus ein neuartiges Pulsverhalten ergibt. Für bestimmte Eingangsleistungen zerbricht der Puls in ein ko-propagierendes Solitonpaar. Die besondere Dispersionskurve erlaubt vielfache und starke Solitorkollisionen und führt letztlich zu einem starken Energieübertrag in lineare Strahlung.

Wird der Einfluss des mikrostrukturierten Mantels auf die Gesamtdispersion optimal ausbalanciert, kann es zum Auftreten von drei Nullstellen in der Dispersionskurve kommen. Diese Eigenschaft zeigt ein ungewöhnliches Verhalten — sowohl für quasi-CW Licht als auch für ultrakurze Pulse. Wir zeigen numerisch, dass Fasern dieser Art ein höchst unkonventionelles Verhalten in der Phasenanpassungskurve für Vier-Wellen Mischung (4WM) und resonanter Strahlung aufweisen. Zum Beispiel findet man in beiden Fällen die Existenz von Bandabstossungseffekten. Der 4WM Prozess ermöglicht die Erzeugung von sechs Seitenbändern, was sich als nützlich für quantenoptischen Studien herausstellen könnte. Zusätzlich können optische Solitonen Energie in drei resonanten Strahlungsbänder übertragen. Die Präsenz von drei Nulldispersionen erlaubt eine beispiellose Freiheit in der Positionierung dieser Strahlungsbänder.

Ein weiterer Teil dieser Arbeit bestand darin, Ausbreitungsprozesse von Licht in axial-variierten Fasern zu untersuchen. Die kontinuierliche Änderung der Dispersion und der Nichtlinearität führt zu spektralen Umgestaltungsprozessen in diesen Strukturen. Zum Beispiel kann das Weisslichtspektrum stark verbreitert werden. Insbesondere ist eine deutliche Erweiterung der blauen Weisslichtkante bis in das UV möglich. Dieser Prozess endet, wenn die zweite, langwellige Nulldispersion in die Nähe der Solitonen kommt. Die Dispersionslandschaft bewirkt einen unerwarteten Effekt auf diese Solitonen. Sie beginnen eine starke Blauverschiebung ihrer Zentralwellenlänge und wirken somit der Raman Rotverschiebung entgegen. In dieser Arbeit zeigen wir eine Verringerung der Solitonwellenlänge um mehr als 400 nm. Dies ist vielversprechend für die Herstellung neuer faserbasierter Lichtquellen, die ultrakurze Pulse im gesamten Transmissionsspektrum des Fasermaterials emittieren.

Abstract

The study of nonlinear pulse propagation in photonic-crystal fibers (PCFs) produced new opportunities in fiber-optical research, mainly due to the unique guiding properties of the PCFs. The light is strongly confined to a small area of glass, thus enhancing the interaction of the light with the silica molecules. This caused a widespread distribution of the PCFs for various applications. Still, the demand for fiber-based light sources, having tailored light properties, grows steadily. This gives the motivation for thesis: The study of light traversing PCFs with engineered and unconventional dispersion landscapes. For this work, a large range of different shapes was realized by postprocessing several stock PCFs. Additionally, numerical modeling of light propagation in the devices assisted to understand the governing physical mechanisms.

One part of this thesis shows that the influence of the core and cladding geometry on the modal properties can be adjusted to achieve efficient supercontinuum generation, novel pulse shaping mechanisms or nonlinear wavelength conversion. At the beginning we report on novel temporal and spectral modulations in short pieces of highly nonlinear PCFs. In optical fibers the evolution of the propagating light is usually governed by the interplay between the nonlinearity and dispersion of the fiber. In highly nonlinear PCFs this balance can be strongly biased in favor of the nonlinearity, and soliton formation is initially suppressed. Instead, a train of ultrashort pulses develops, having repetition rates in the THz region. We demonstrate the generation of a train of 50 fs pulses with a repetition rate as high as 14 THz; the length of the PCF was only a few centimeters. We discuss the tunability of the pulse-train parameters by the laser and fiber parameters.

PCFs can be fabricated to have a submicron core region, which maximizes the nonlinear coefficient. Short pieces of these devices can be used to control the spectrum of laser pulses, for low threshold pump powers. The spectral brightness can be adjusted by the pump wavelength. These fibers are promising candidates for white-light sources with repetition rates in the GHz range.

Additionally, the unique dispersion shape and the pronounced frequency dependence of the non-

linearity of submicron-core PCFs result in unexpected soliton dynamics. Light launched at the maximum dispersion point leads to surprisingly strong and multiple soliton interactions. The proximity of a zero-dispersion point increases the energy transfer into linear radiation by more than 25%, destroying afterwards the existence of one of the solitons.

In the next step, the influence of the microstructured cladding on the overall dispersion is balanced to create three zero-dispersion points close to each other, while still being endlessly single-mode. This scenario results in unexpected behavior — both for quasi-CW light and ultrashort pulses. It is shown numerically that fibers of this type offer highly unconventional phase-matching landscapes for four-wave mixing (FWM) and resonant radiation, for example featuring anti-crossings between the phase-matching curves. The FWM process allows the generation of six parametric sidebands, useful for studies in quantum optics. When ultrashort pulses are launched into the fibers up to three dispersive waves can be generated. The existence of a third zero-dispersion point introduces a great flexibility in the positioning of these waves, and lead to strong modifications of the spectro-temporal characteristics.

In the other part of this work, we study the behavior of light in axially-varying PCFs. The presence of a continuous variation of the dispersion and the nonlinearity raises unconventional processes in these devices. We demonstrate that the blue edge of the supercontinuum can be extended considerably, reaching far into the UV. This effect is maximized when the pump light lies in the normal dispersion region of the input facet. This interaction stops when the solitons approach the long-wavelength zero-dispersion wavelength. The changing dispersion landscape has then an unexpected influence on these solitons. They undergo a strong blue-shift, thus counteracting the well-known Raman-induced red-shift. So far we observed soliton blue-shifts of around 400 nm. This can be used to create novel fiber based sources of ultrashort pulses, covering the complete transmission window of the fiber material.

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Chapter 1

Introduction

Photonic-crystal fibers (PCFs) were first demonstrated in 1996 by P. Russell [1, 2]. The idea was to build a 2D-photonic crystal with a structural defect at the center of the device, which acts as a waveguide. This concept is based on earlier studies showing that photonic crystals can form complete stop-bands and pass-bands for photons [3,4]. Since then, many different designs of PCFs

Figure 1.1: *Different types of PCFs. (a) Endlessly single-mode PCF, (b) highly nonlinear PCF, (c) suspended core PCF, (d) hollow-core PCF.*

have been demonstrated, where the light is guided in a solid core or a hollow core. Fig. 1.1 shows several different PCF scanning electron micrographs (SEM) images, fabricated at the Max Planck Institute for the Science of Light. The unprecedented designability of solid-core PCFs offers a never-ending playground, featuring for example endlessly single-mode (ESM) fibers, birefringent

fibers, highly nonlinear fibers, fibers with a suspended core or even fibers with a planar web as guiding area [5].

Fig. 1.2a explains typical properties of PCFs by means of an endlessly single-mode fiber design. The cladding region consists of regularly arranged hollow channels. The center-to-center spacing between the nearest air holes is referred to as the pitch Λ . The air-hole diameter is d , and is often given in units of the pitch, i.e. d/Λ . Light can be confined in the core region where the

Figure 1.2: (a) Endlessly single-mode PCF design and the fundamental core mode, simulated by the fixed-frequency plane wave method. The inter-hole spacing Λ was $3.05 \mu\text{m}$ and the relative hole diameter was $d/\Lambda = 0.4$. (b) Wavevector diagram describing the TIR in PCFs. Medium 1 denotes the core region with refractive index n_{co} and medium 2 is the cladding region with n_{clad} . The thickened circle segment shows the range of allowed wavevectors for total-internal reflection.

refractive index n_{co} is larger than the refractive index n_{clad} in the cladding region. The existence of guided modes in the core is a natural consequence of phase-matching [6]. When plane waves impinge on the core-cladding boundary, the momentum along the interface is conserved. The allowed wavevector of the guided mode at a fixed frequency can be constructed by a wavevector diagram in the reciprocal space (Fig. 1.2b). For isotropic media the diagram consists of two concentric circles, each of radius $k_0 n_j$. Here, $k_0 = \omega/c$ is the vacuum wavevector; n_{co} and n_{clad} are the refractive indices of the core and the cladding region. The axial wavevector component β is the projection of the light's wavevector kn_{co} in the core onto a line parallel to the interface. A construction line can be drawn and is demonstrated in Fig. 1.2b. This line can intersect the two circles $k_0 n_{co}$ and $k_0 n_{clad}$ at several points. In Fig. 1.2b we show the case when refraction

at the interface is absent. The construction line does not intersect the inner circle. Instead, the construction line intersects the outer circle, corresponding to the existence of a reflected beam. This describes total-internal reflection (TIR) and light is confined to the core [5, 7]. The following condition for the existence of guided modes holds

$$k_0 n_{clad} < \beta < k_0 n_{co}. \quad (1.1)$$

Here $n_{clad} = n_{max}$ is the maximum axial refractive index of the cladding. Its value varies strongly with frequency, resulting from the microstructuring of the cladding [5]. The condition $\beta > k_0 n_{clad}$ ensures that the wave is evanescent in the cladding. However, in contrast to the TIR occurring in step-index fibers, the light in an ESM PCF is strongly influenced by the discontinuous nature of the core boundary-sections. In the long wavelength limit $k \rightarrow 0$ the bound mode extends considerably into the cladding, which then acts as a homogeneous material with a mean refractive index lower than the core index. As the wavelength of the light falls, the light is concentrated more in the glass, and is strongly modulated by the presence of air holes. For pitch values and hole diameters

Figure 1.3: *Supercontinuum generation in an ESM PCF. The core diameter of the fiber was around $4.4\mu\text{m}$, and $d/\Lambda \sim 0.5$. The pump laser was a ps-fiber laser with repetition rate of 1 MHz. The average power was 1 W.*

satisfying $d/\Lambda < 0.43$ [8], this unusual behavior leads to monomode waveguiding over the transmission window of silica — the fiber becomes endlessly single-mode. The mode cutoff at shorter wavelength is prevented¹, because for $\lambda \rightarrow 0$ the light is guided primarily in the glass region and

¹The mode cutoff frequency decreases for falling index difference, defined as $\sqrt{n_{co}^2 - n_{clad}^2}$ [9].

the effective index tends to the index of the glass. The mode pattern reflects the avoidance of the air holes, and is exemplified in Fig. 1.2b. The unique guiding properties of PCFs come hand in hand with a large tunability of the fiber parameters, seeding finally a wide-spread distribution of PCF-based applications [5].

So far, PCFs have allowed the observation of many previously inaccessible phenomena. PCF-assisted sources can be used for the generation of light with a high spectral brightness and is applied in medicine, biology and chemistry [10–15]. In addition, the potential of PCFs as pulse shaping devices is still uncapped. Temporal pulse shaping is pivotal for applications such as the coherent control of quantum systems [16] and the generation of light in the deep UV [17]. A very effective way of tailoring the optical field stems from the interaction of light with matter. In this respect, PCFs offer unchallenged advantages. Besides the long interaction lengths they allow the confinement of the light to a small core area, enabling thus the onset of nonlinear effects. The first practical success of PCFs was the shift of the zero-dispersion wavelength to 1 μm and the subsequent demonstration of supercontinuum generation. Fig. 1.3 shows a supercontinuum spectrum generated during the propagation of a picosecond laser pulse in an endlessly single-mode photonic crystal fiber. This spectrum has a very high power density in the visible spectral region and can therefore be used for many detection or characterization measurements, where a high light intensity in the UV and visible is essential.

In this thesis, we explore the scope of the designability of PCFs. We study the behavior of light in dispersion-tailored PCFs and study the emergence of unusual pulse propagation dynamics. In the end, this can be used for next-generation nonlinear wavelength conversion, UV generation or the tailoring of temporal pulse shapes.

To begin with, we introduce the optical fiber in chapter 2. We show that a strong confinement of light in the core region gives rise to a pronounced interaction of light with matter. As a result, the fiber reacts by exerting linear and nonlinear responses to the guided light. This constitutes the background for chapter 3, summarizing the impact of the linear and nonlinear effects on propagating pulses. Besides the conventional spectral and temporal reshaping processes, the interplay of linear and nonlinear processes can lead to the generation of supercontinua. Chapter 4 introduces several experimental techniques; the laser sources and the experimental setup are presented. Additionally, the postprocessing method and the measurement of the group-velocity dispersion are discussed. Chapters 5 and 6 contain the main body of this work. We present new results describing unconventional propagation dynamics in various kinds of dispersion-tailored PCFs. Specifically,

in chapter 5 we make use of the large tunability of the fiber dispersion. Firstly, we focus on small-core fibers and use the large nonlinear contribution to manipulate the spectro-temporal properties of ultrashort pulses. Then we show that a homogeneous PCF layout can result in the appearance of several zero-dispersion wavelengths. We discuss the advantage when more than the usual one or two ZDWs are present. In chapter 6, we add a degree of freedom to the propagation dynamics and let the PCFs vary axially along the propagation length. We show the impact on the propagation dynamics of solitons, which are influenced by the continuous change of the GVD landscape. This can be used to extend the supercontinuum or to increase the conversion efficiency into the UV.

Chapter 2

Linear and nonlinear properties of solid-core fibers

In this chapter we introduce the concept of silica-based fibers as optical waveguides. The transmission window of silica spans from the UV to the IR and makes it the ideal material to host the light. In section 2.1 we derive the wave equation from Maxwell's equations. The linear version of the wave equation is used to show the existence of guided modes. Light is confined to the core region and responds to the presence of the fiber material. In this section we summarize the linear effects on the fundamental mode. However, the small dimension of the guiding area results in a nonlinear response of the medium to the light. In section 2.2 we discuss the origin of the nonlinearity in silica. Using the nonlinear wave equation, we are able to study the consequences created by the fiber nonlinearity on the characteristics of guided modes.

2.1 Linear properties of circular waveguides

PCFs are optical waveguides, where the electric field is bound to the core region and can be transmitted over a long distance. In this section, we derive and discuss the physical origin of wave-guiding, potentially occurring in structures with a spatially varying refractive index. This is done by means of a step-index fiber model, and we show that the propagation of light is influenced by the material and the waveguide structure.

2.1.1 The wave equation

All electric, magnetic and electromagnetic phenomena are governed by the fundamental laws of Maxwell's equations. In principle these laws can be used to study all optical phenomena, for example light propagation in PCFs. They link the electric $\mathbf{E}$ and the magnetic field vector $\mathbf{H}$ and are (in SI units) [18, 19]

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho & \nabla \times \mathbf{E} + \frac{\partial}{\partial t} \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{H} &= \frac{\partial}{\partial t} \mathbf{D} + \mathbf{J}\end{aligned}\quad (2.1)$$

where ρ is the free charge density. The field $\mathbf{J}$ is the free current density, produced by all charges other than those associated with polarization; for the course of this thesis the field $\mathbf{J}$ is neglected. The electric displacement $\mathbf{D}$ and the magnetic-flux density $\mathbf{B}$ are linked to the polarization $\mathbf{P}$ and magnetization $\mathbf{M}$ of the medium:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}, \quad (2.2)$$

where ϵ_0 and μ_0 are the electric permittivity and the magnetic permeability of vacuum (or free space). The magnetization term in Eq. (2.2) vanishes in non-magnetic media, for example optical fibers. An important consequence of Maxwell's equations is the wave equation, which is the governing equation for light propagation in optical fibers. By applying the curl operator on both sides of Ampère's and Faraday's law (right-hand side of Eq. (2.1)), $\mathbf{B}$ and $\mathbf{D}$ can be eliminated in favor of $\mathbf{E}$ and $\mathbf{P}$. By noting that the divergence of the electric displacement and the magnetic-flux density vanish (assuming no charged particles in the fiber), we obtain with help of Eq. (2.2),

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}, \quad (2.3)$$

where c is the speed of light in vacuum ($c=299792458$ m/s). To arrive at Eq. (2.3) we have used an identity from vector calculus, $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$. The first term vanishes due to Coulomb's law [18]. Deep physical insight into light propagation in optical fibers can be obtained from the nonlinear wave Eq. (2.3). We begin by decomposing the polarization term $\mathbf{P}$ into a linear and a nonlinear part,

$$\mathbf{P} = \mathbf{P}^{(1)} + \mathbf{P}^{(nl)}. \quad (2.4)$$

Then, by neglecting the nonlinear polarization $\mathbf{P}^{(nl)}$, we can determine the influence of the linear polarization $\mathbf{P}^{(1)}$ on the electric field in the waveguide. In general, an optical field $\mathbf{E}(t)$ forces a polarization response $\mathbf{P}(t)$ of the medium. Working in the Fourier domain, the spectral components $\tilde{\mathbf{E}}(\omega)$ of the electric field can be linked to the Fourier components of the linear polarization¹ $\tilde{\mathbf{P}}^{(1)}(\omega)$ [20],

$$\tilde{\mathbf{P}}^{(1)}(\omega) = \epsilon_0 \chi^{(1)}(\omega) \tilde{\mathbf{E}}(\omega), \quad (2.5)$$

where $\chi^{(1)}(\omega)$ is the linear susceptibility of the medium. Inserting Eq. (2.5) into the linear wave Eq. (2.3) yields

$$\nabla^2 \tilde{\mathbf{E}} + n^2(\omega) \frac{\omega^2}{c^2} \tilde{\mathbf{E}} = 0 \quad (2.6)$$

where $n(\omega)$ is related to the relative dielectric constant ϵ_r ,

$$\epsilon_r(\omega) = 1 + \chi^{(1)}(\omega) \quad (2.7)$$

$$n(\omega) = \sqrt{\epsilon_r} \quad (2.8)$$

In general $\epsilon_r = (n + \frac{i\alpha}{2k})^2$ where k is the wavevector. However, since we neglected the attenuation α , we can write the refractive index n as in Eq. (2.8).

2.1.2 Optical waveguides

2.1.2.1 Fiber modes

Optical waveguides are devices with a spatially varying refractive index $n(\mathbf{r}, \omega)$. Typical examples are the so-called slab, ridge and circular waveguides [21]. Waveguides in general and optical fibers in particular support a number of guided modes whose spatial distributions $\tilde{\mathbf{E}}(\mathbf{r}, \omega)$ are the solutions of the Helmholtz Eq. (2.6). The modes of propagation can be found from the wave equation in cylindrical coordinates (r, Φ, z) ,

$$\frac{\partial^2 \tilde{\mathbf{E}}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{\mathbf{E}}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \tilde{\mathbf{E}}}{\partial \Phi^2} + \frac{\partial^2 \tilde{\mathbf{E}}}{\partial z^2} + n^2(\omega) \frac{\omega^2}{c^2} \tilde{\mathbf{E}} = 0. \quad (2.9)$$

By considering the translation invariance along the z axis, the electric field can be factorized into the field distribution along the transverse direction and an oscillatory function along the propaga-

¹and the nonlinear polarization

tion direction z ,

$$\mathbf{E}(r, \Phi, z, t) = \mathbf{E}(r) e^{-il\Phi} e^{i\omega t - i\beta z} \quad (2.10)$$

where β is the z -component of the wavevector and is the propagation constant to be determined. $e^{-il\Phi}$ comes from the fact that $\mathbf{E}(r, \Phi, z, t)$ is periodic in the azimuth with period 2π . Insertion of Eq. (2.10) in Eq. (2.9) yields a Bessel-type differential equation. The propagation constant β is then determined by the following equation (see Ref. [9] for a detailed derivation)

$$\left(\frac{J'_l(U)}{U J_l(U)} + \frac{K'_l(W)}{W K_l(W)} \right) \left(\frac{J'_l(U)}{U J_l(U)} + \frac{n_{cl}^2}{n_{co}^2} \frac{K'_l(W)}{W K_l(W)} \right) = \left(\frac{l\beta}{kn_{co}} \right)^2 \left(\frac{V}{UW} \right)^4. \quad (2.11)$$

Here, $J(U)$ and $K(W)$ are the Bessel function of the first kind and the modified Bessel function of the second kind [9]. U and W are the modal parameters for the core (of radius a) and cladding,

$$U = a \sqrt{k^2 n_{co}^2 - \beta^2}$$

$$W = a \sqrt{\beta^2 - k^2 n_{cl}^2}.$$

The solutions of Eq. (2.11) can be separated into two classes², that are designated as HE and EH modes. The notion refers to the longitudinal components of the electric and magnetic field, which are generally not decoupled in circular fibers. In this notation, the fundamental mode is HE₁₁, and occurs for $l = 0$. This describes the absence of knots in the azimuthal or radial direction. Hence the fundamental mode is bell shaped, in contrast to higher-order modes which exhibit radial or azimuthal positions of zero intensity. In general, the propagation constant needs to be determined numerically [21]. When solving Eq. (2.11), one finds that the number of eigenvalues is finite, corresponding to a finite number of guided modes. Once the eigenvalues are found, the six field components for $\mathbf{E}$ and $\mathbf{H}$, constituting the mode, can be determined. The fundamental mode of an optical fiber is the mode with the highest effective refractive index n_{eff} , defined as $n_{\text{eff}} = \beta \frac{c}{\omega}$.

Mode field diameter

The field distribution of a waveguide mode can be characterized by the mode field diameter w and the effective mode area A_{eff} [22–24]. The intensity of the light, propagating in the waveguide, is

²because Eq. 2.11 is quadratic in $\frac{J'_l(U)}{U J_l(U)}$

proportional to the Poynting vector S_z ,

$$S_z = (\mathbf{E} \times \mathbf{H}^*)_z \quad (2.12)$$

and can be used to define the mode field diameter (MFD) and the effective mode area as [23]

$$w = 2\sqrt{2} \sqrt{\frac{\int S_z r^2 d^2\mathbf{r}}{\int S_z d^2\mathbf{r}}} \quad (2.13)$$

$$A_{eff} = \frac{(\int S_z d^2\mathbf{r})^2}{\int S_z^2 d^2\mathbf{r}} \quad (2.14)$$

where the integrals are evaluated over the transverse profile. For a gaussian field distribution, w corresponds to the 1/e-width of the field. The effective mode area defines the fiber region filled by the symmetric modal distribution and is an important quantity; it determines the magnitude of the nonlinear coefficient of the fiber, which is inversely proportional to the effective area. We introduce the nonlinear coefficient in chapter 3. In general, a reduction of the core size of the fiber results in a decrease of the effective area. However, when the core size falls below certain values, the mode tends to spread into the cladding and the effective area increases. This behavior is summarized in Fig. 2.1, showing the dependence of the MFD on the radius of a circular silica strand embedded in air. The mode size reaches a minimum value for around 460 nm. When the radius is further reduced, the light cannot resolve the discontinuity in the spatial distribution of the refractive index and the MFD increases rapidly.

2.1.2.2 Group-velocity dispersion

Guided modes can form in optical fibers, each with a characteristic propagation constant β . The precise magnitude of β depends on the spatial refractive-index distribution. This is given by the composing material and the layout of the waveguide and varies with frequency. The frequency dependence of β is the fiber dispersion. It leads to varying fiber transit times for light at different frequencies.

In this respect, one needs to discern between the phase velocity and the group velocity. The propagation of a single plane wave is described by the *phase velocity* v_p — the speed of the points of constant phase, $v_p = \omega/k$. The *group velocity* v_g , on the other hand, denotes the speed of a pulse envelope along an optical fiber and is the velocity in the limit of a small frequency difference,

Figure 2.1: *Mode-field diameter (MFD) as a function of the silica-strand diameter, for a wavelength of 800 nm. The minimum occurs for a strand diameter of 460 nm, and corresponds to the maximum of the nonlinearity.*

$$v_g = d\omega/dk.$$

The group velocity of a pulse is inversely proportional to the group-delay time per unit length of waveguide, $\beta_1 = 1/v_g = T/L$, where T is the mean transit time for a waveguide of length L [25].

The group delay can be determined from the frequency dependence of the propagation constant,

$$\frac{1}{v_g} = \beta_1 = \frac{d\beta(\omega)}{d\omega} = -\frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda}. \quad (2.15)$$

The frequency dependence of the group velocity is the group-velocity dispersion (GVD), meaning that different frequency components of an optical pulse travel with different velocities. It is determined by two contributions, the waveguide and the material dispersion. The waveguide dispersion describes the light-guiding influence of the fiber structure on the propagation constant, and can be found from Eq. (2.11) by assuming that the refractive index n is independent of ω . The material dispersion results from the frequency dependence of refractive index of the material and can be described with interpolation formulas, for example Sellmeier's equation

$$n^2(\omega) = 1 + \sum_{i=1}^m \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2}, \quad (2.16)$$

where A_i denotes the resonance strengths and λ_i refers to the pertaining wavelengths of the dominant transitions [25]. The GVD is given in terms of the quantity $\beta_2 = d^2k/d\omega^2$ or by the wave-

length derivative of β_1 ,

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2. \quad (2.17)$$

When the parameter D is positive, the fiber has anomalous dispersion and the blue parts of the light will travel faster through the medium than the red parts. A negative value of D is defined as the normal dispersion regime of a fiber. In general, the GVD is dependent on the frequency and both dispersion regimes exist in the fiber. This necessitates a wavelength where $D(\lambda = \lambda_{\text{ZDW}})=0$, and λ_{ZDW} is called the zero-dispersion wavelength (ZDW). Although the second-order dispersion β_2 is zero, higher-order dispersion terms ($\beta_n = \partial_\omega^n \beta$, with $n > 2$) can still exist. PCFs take a special place in this respect — the microstructuring accounts for an unprecedented tuning range of dispersion profiles, with ZDW between 500 nm and 2000 nm. The strong waveguide contribution of PCFs can dominate the overall dispersion, allowing the fabrication of fibers with unconventional dispersion curves [26,27]. The frequency variation of the dispersion curve can be represented by higher-order dispersion terms and are obtained from a Taylor expansion

$$\beta_2(\omega) = \beta_2(\omega_0) + \beta_3(\omega_0)(\omega - \omega_0) + \frac{1}{2}\beta_4(\omega_0)(\omega - \omega_0)^2 + \dots \quad (2.18)$$

Modelling of the fiber dispersion

The GVD and the ZDW are important characteristics of a fiber. In addition to the experimental measurement of the dispersion curve (see section 4.3), it is very helpful to calculate the GVD, which can be done by various methods, varying by their speed and accuracy. The dispersion curves of this work have been evaluated by the fixed-frequency plane-wave (FFPW) method and the silica-strand model (see section 2.1.2.1).

In the FFPW method [28] the optical fields and the dielectric functions are expanded in the form of plane waves, which are functions of the reciprocal lattice vector of the geometry. These equations are then solved in the frequency domain, and an implicit time dependence in the form $\exp(-i\omega t)$ is assumed. Then one fixes the frequency and computes a set of modes characterized by their wavevectors. The FFPW method requires a periodic dielectric function. A fiber with a single defect at the center can be modelled by employing a supercell approximation (for detailed information, see [29]).

2.1.2.3 Sources of light attenuation in PCFs

The great success of the PCF stems partly from the fact, that the light can be guided with very low attenuation. The loss of a fiber is one of its most crucial properties and determines its value for possible applications. Fiber losses have their origin in various mechanisms. The dominating effects are scattering, material resonances and absorption by OH^- ions. Fig. 2.2 shows a sketch of the resulting loss spectrum of an optical fiber [25].

Figure 2.2: Schematic of the attenuation in silica fibers. Several sources contribute to the overall loss. Reproduced after [25].

- An important source for scattering losses stems from density fluctuations on a scale smaller than the wavelength. This leads to Rayleigh scattering and is proportional to $\frac{1}{\lambda^4}$ [30, 31]. In addition, scattering can also occur at the core-cladding interface, where the surface roughness leads to a loss due to the imperfection [7].
- Fiber impurities are mainly introduced by OH^- ions and cause attenuation peaks near 950 nm, 1.25 μm and 1.38 μm . Those ions penetrate the glass material during the preform fabrication and the fiber drawing process [32].
- Additional sources of attenuation are given by electronic resonances in the UV region and by vibrational resonances in the IR spectral range beyond 2 μm .

Confinement loss

The photonic cladding design gives rise to an additional loss mechanism, the confinement loss. The finite number of air holes in the fiber cladding causes the guided mode to be leaky. Generally,

many photonic crystal layers are required for a substantial reduction of the confinement loss. The confinement loss is higher for longer wavelengths [33] where the fundamental core-mode extends further into the fiber cladding [34], and leads to a sharp cutoff in the loss spectrum. The cutoff wavelength depends on the core size and in small-core fibers the confinement loss can impact the propagating light. A quantitative model for the cutoff wavelength is derived in [35], leading to

$$\lambda_c \simeq 2\pi \rho_{co} \sqrt{\frac{\ln(\rho_{cl}/\rho_{co})}{2} f \cdot (n_{co}^2 - 1)}, \quad (2.19)$$

where ρ_{co} is the effective radius of the core, ρ_{cl} is the radius of the inner cladding and f is the air-filling fraction, given by $f = \pi \frac{\sqrt{3}}{6} (d/\Lambda)^2$ for a cladding with a hexagonal array of air holes.

2.1.2.4 Polarization effects

The guided modes can be distinguished whether the x or y component of the electric field dominates. These are the x - and y -polarized modes of propagation, and can be assigned to the existence of two orthogonal fiber axes, named the fast and the slow axis. Each axis has a certain propagation constant, β_x and β_y . In a fiber with an ideal circular symmetry no axis is preferred and two independent states (i.e. orthogonal states) of polarization exist, with $\beta_x = \beta_y$. When the two propagation constants are not equal, the fiber exhibits birefringence. Birefringence is introduced during the fabrication process, either by using a non-circular shape as the guiding center of the fiber or by introducing a nonuniform hole-size distribution in the fiber cladding. However, even fibers with a circular core can exhibit a small birefringence, due to slight imperfections introduced during the drawing process. During propagation, orthogonal modes accumulate a different amount of phase change, dependent on the difference between β_x and β_y . Whenever the difference equals an integer multiple of 2π , the modes are in phase, thus defining the beat length L_B ,

$$L_B = \frac{2\pi}{|\beta_x - \beta_y|}. \quad (2.20)$$

The modal birefringence B is used to define the birefringence of a fiber, and depends on the wavelength λ

$$B = \frac{\lambda}{L_B} = \frac{\lambda}{2\pi} (|\beta_x - \beta_y|). \quad (2.21)$$

The time delay Δt of two orthogonal modes during propagation in a fiber of length L can then be expressed as,

$$\Delta t = \frac{L}{c} B. \quad (2.22)$$

Generally, when light propagates in both axes, the different phase-change between the two modes gives rise to a change of the polarization state. This can be eliminated when the light is strictly coupled into one axis, which is an important task for accurate experimental observations of scalar effects.

2.2 Nonlinear properties of silica fibers

2.2.1 Nonlinear susceptibility of silica glass

Up to now we have described the linear properties of optical fibers by using the linear polarization $\mathbf{P}^{(1)}$ in Eq. (2.3). We have seen that light can be confined to the core region, creating a guided mode. The physical origin of the polarization term in Eq. (2.3) stems from the displacement x of the atomic electric charges due to the applied field. When the nonlinear polarization is neglected, the restoring force is a linear function of x ; the atomic electrons feel a harmonic potential well. When the light intensity in the fiber core is high enough, the restoring force is not linear, and the potential energy acquires an anharmonic correction term. This induces an electric dipole moment, acting as a source to emit secondary electromagnetic waves. We account for the non-harmonic potential by expanding the polarization $\mathbf{P}(\mathbf{t})$ into a Taylor series [20],

$$\mathbf{P}(\mathbf{t}) = \mathbf{P}^{(1)}(\mathbf{t}) + \mathbf{P}^{(2)}(\mathbf{t}) + \mathbf{P}^{(3)}(\mathbf{t}) + \dots + \mathbf{P}^{(n)}(\mathbf{t}), \quad (2.23)$$

valid far from medium resonances. Glass is a material with a statistical structure and is isotropic on average [18]. Therefore glass has an inversion symmetry, and the potential energy is symmetric under the operation $x \rightarrow -x$. As a consequence the second order contribution vanishes, $\mathbf{P}^{(2)} = 0$. The first nonvanishing higher-order term in the series expansion is then $\mathbf{P}^{(3)}$. In the same way as in Eq. (2.5) we connect the Fourier components $\tilde{\mathbf{P}}^{(3)}(\omega)$ of the third-order polarization with the Fourier components $\tilde{\mathbf{E}}(\omega)$ of the applied field $\mathbf{E}(\mathbf{t})$,

$$\tilde{\mathbf{P}}^{(3)}(\omega) = \epsilon_0 \chi^{(3)}(\omega_1, \omega_2, \omega_3) \tilde{\mathbf{E}}(\omega_1) \tilde{\mathbf{E}}(\omega_2) \tilde{\mathbf{E}}(\omega_3), \quad (2.24)$$

where $\chi^{(3)}$ is the third-order susceptibility. Note that the third-order susceptibility is a fourth-order tensor, having 81 elements. Thus above equation reads then in matrix expression [20]

$$P_i^{(3)}(\omega) = \epsilon_0 \sum_{jkl} \chi_{ijkl}^{(3)}(\omega_1, \omega_2, \omega_3) E_j(\omega_1) E_k(\omega_2) E_l(\omega_3). \quad (2.25)$$

2.2.2 Intensity-dependent refractive index in optical fibers

In section 2.1.1 we saw that the linear susceptibility $\chi^{(1)}$ gives rise to a relative dielectric constant ϵ_r , describing the response of a linear medium to an external field by means of the refractive index. Now, we are going to deal with the nonlinear response in the same way, and start by writing the total polarization as

$$\mathbf{P}(\omega) = \mathbf{P}^{(1)}(\omega) + \mathbf{P}^{(3)}(\omega). \quad (2.26)$$

Putting Eqs. (2.5) and (2.24) into Eq. (2.26) yields a generalized expression for the relative dielectric constant of a third-order nonlinear medium:

$$\epsilon_r(\omega) = 1 + \chi^{(1)}(\omega) + \chi_{xxxx}^{(3)}(\omega) \cdot |E_0(\omega)|^2 \quad (2.27)$$

Here, we used that the only involved nonzero element of the third-order susceptibility tensor in isotropic media is $\chi_{xxxx}^{(3)}$ [18, 20] and is called χ_3 thereafter. Treating the nonlinear contribution as a perturbation, the generalized expression of the refractive index is then

$$n(\omega) = \sqrt{\text{Re}\{\epsilon_r\}} = n_0(\omega) + \frac{1}{2n_0(\omega)} \text{Re}\{\chi^{(3)}\} \cdot |E_0(\omega)|^2. \quad (2.28)$$

The physical meaning of Eq. (2.28) is stunning: an intense optical field induces a change of the refractive index proportional to the intensity of the applied field, mediated by the so-called nonlinear refractive index n_2 of the medium. The change of the refractive index is given by

$$\Delta n(\omega) = n - n_0 = n_2 \cdot I \quad (2.29)$$

$$n_2 = \frac{3}{4n_0^2 \epsilon_0 \cdot c} \text{Re}\{\chi^{(3)}\} \quad (2.30)$$

where I is the time-averaged intensity of the optical field, $I = 2n_0\epsilon_0 c |\tilde{E}(\omega)|^2$. The factor 3 in Eq. (2.30) stems from the third-order polarization at frequency ω . The nonlinear refractive index

of silica is $n_2=2.6\times 10^{-20} \text{ m}^2/\text{W}$, and is strongly wavelength dependent below 400 nm [36].

2.2.3 Ramifications of the third-order susceptibility

The interaction of light with the silica-glass core leads to a nonlinear response, generated by the quasi-instantaneous response of the electron cloud. However, an additional delayed response exists due to the molecular vibrations of the glass, which occur over a time scale of 70 fs (see Fig. 2.3a). In the following the impact of the nonlinearity of a fiber on the light is discussed.

Spectral broadening due to self-phase modulation

The third-order nonlinear susceptibility of silica leads to the formation of an intensity-dependent refractive index. When the applied optical field varies in time, the induced refractive-index change Δn acquires a time dependence. This is related to the Doppler effect and results in the generation of new spectral components. For example, when a bell-shaped optical pulse of intensity $I(t)$ propagates in a third-order nonlinear optical medium, a phase is accumulated, which is given by

$$\Phi_{nl}(t) = -n_2 I(t) \omega_0 \frac{L}{c}. \quad (2.31)$$

The temporal distribution of the phase factor results in a frequency change $\Delta\omega$, determined by the instantaneous frequency $\omega_c(t)$

$$\Delta\omega(t) = \omega_c(t) - \omega_0 = -\frac{d}{dt}\Phi_{nl}(t) = -n_2 \frac{\omega_0 L}{c} \frac{\partial}{\partial t} I(t). \quad (2.32)$$

This self-induced change of the temporal phase of the optical pulse is called self-phase modulation (SPM). In the temporal domain, the emergence of new frequency components can be understood by an argument related to the Doppler effect. The rising edge of the leading part is responsible for an increase of the refractive index. This corresponds to a continuous increase of the optical density and slows down the leading edge of the pulse. According to the Doppler effect, red-shifted light is created at the leading edge. The same analogy holds for the trailing edge, acquiring blue-detuned spectral components. The newly generated frequencies are separated in time and the pulse exhibits a so-called up-chirp. At the center of the bell-shaped pulse the frequency-shift is zero.

Raman effect

The molecular structure of glass gives rise to a second nonlinear contribution in silica, due to Raman scattering. The light beam is scattered on temporal modulations of the molecular polarizability, created by the intramolecular (vibrational) motion. The new (Stokes) field is downshifted by a frequency determined by the vibrational modes of the medium. The growth of the Stokes wave is determined by the Raman gain. The Raman-gain coefficient $g_R(\Omega)$ depends on the frequency difference $\Omega = \omega_0 - \omega_S$ between pump (ω_0) and Stokes (ω_S) waves. We show the Raman gain of fused silica in Fig. 2.3. Due to the amorphous structure of silica, the molecular vibrational frequen-

Figure 2.3: (a) Raman response function of fused silica. The response time is 76 fs. (b) Raman gain curve for fused silica. This curve is normalized to 1.0 at 13.2 THz. The peak gain at 13.2 THz for a pump wavelength of 532 nm is $1.86 \cdot 10^{-11}$ cm/W and varies with pump wavelength λ_0 as $1/\lambda_0$ [37].

cies spread out and the gain spectrum forms a continuum with a maximum at 13.2 THz [38, 39]. For an input pump wavelength of 1 μm , the peak gain coefficient is around 10^{-11} cm/W [37].

Raman scattering can give rise to a second scattering component, upshifted by Ω , and is called the anti-Stokes wave. The Stokes and anti-Stokes components in the optical spectrum are located symmetrically around the pump wavelength. However, the Stokes component has higher intensity because the vibrational levels are occupied according to Boltzmann distribution, leaving the lowest levels with the highest population probabilities. For a dominating anti-Stokes wave, the medium must be prepared by means of population inversion.

Parametric wave-mixing in silica glass

A nonlinear polarization component in matter is responsible for a higher-order electric dipole moment, and can emit photons at new frequencies. In a third-order nonlinear medium, this results in the nonlinear coupling of four coherent monochromatic waves. A physical picture of the wave mixing process can be obtained by regarding matter and radiation as combined and quantized systems. The interaction of the radiation field with matter forces quantum transitions of the pertaining molecules. This interaction is accompanied by the change of the photon distribution among their different modes. The interaction can be picturized by the concept of an intermediate state: this describes a state where the molecule is in a superposition of all possible excited states, each having a certain population probability. The intermediate states are then represented by virtual states in the energy level diagram. The two most common parametric wave-mixing effects are displayed

Figure 2.4: *Energy level description of parametric wave-mixing in silica. (a) Third-harmonic generation and (b) four-wave mixing. After [20].*

in Fig. 2.4. Fig. 2.4a shows third-harmonic generation (THG), where a single monochromatic optical wave with frequency ν results in the emission of a new coherent wave with frequency 3ν . In the energy level description this is described as the annihilation of three photons with energy $h\nu$, exciting the molecule into an intermediate state. Then the molecule returns to the ground state by emitting radiation at 3ν . The parametric nature of this process necessitates the conservation of the energy ($\nu_2 = 3\nu_1$) and the momentum ($\mathbf{k}_2 = 3\mathbf{k}_1$), where $\mathbf{k}$ are the wavevectors of the waves. With $k = \frac{2\pi}{\lambda}n$ we obtain the phase-matching condition, which needs to be satisfied for efficient generation of THG,

$$n(3\nu_1) = n(\nu_1) \quad (2.33)$$

In contrast to THG, four-wave mixing (FWM) describes the annihilation of two photons ν_1 and ν_2 of a two-color incident optical field, see Fig. 2.4b. The molecule responds to this perturbation by emitting two photons at ν_3 and ν_4 , the process of which is most efficient when phase-matched,

$$\nu_3 + \nu_4 = \nu_1 + \nu_2 \quad (2.34)$$

$$\mathbf{k}_3 + \mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2.$$

2.2.4 The nonlinear Schrödinger equation

The linear and nonlinear response of the fiber material can modify the light propagating in the fiber. In principle the propagation dynamics can be described by the nonlinear wave Eq. (2.3), where the z -dependence is linked to the evolution of the electric field along the fiber. The solution of the nonlinear wave equation involves heavy numerical efforts and the equation does not account for a deep physical understanding of the propagation mechanism. In this respect, it is advantageous to use the nonlinear Schrödinger equation, which can be analytically solved and has been used to understand the fundamentals of optical pulse propagation.

Nonlinear Schrödinger equation – The first derivation of an envelope equation for the propagation in fibers was found as early as 1968 by Karpman and Krushkal [40] and was used by Hasegawa and Tappert for the discovery of distortion-free pulse transmission in 1973 [41]. The dimensionless form is [25,42]

$$i \frac{\partial u}{\partial \zeta} \pm \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0 \quad (2.35)$$

and describes the dynamical evolution of the normalized envelope $u(\zeta, \tau)$ in a frame of reference copropagating with the group velocity v_g at the reference frequency. In this equation ζ and τ are the normalized distance and time and the normalized variables are related to real-world variables by

$$\begin{aligned} \tau &= \frac{t - z/v_g}{t_c} \\ \zeta &= \frac{|\beta_2|}{t_c^2} z \\ u &= t_c \left(\frac{\gamma}{|\beta_2|} \right)^{1/2} \cdot A \end{aligned}$$

with the dispersion parameter β_2 , the nonlinearity γ and the pulse amplitude A . The free parameter t_c corresponds to the pulse duration. The sign of Eq. (2.35) denotes the pertaining dispersion region: anomalous (+) and normal (−) group velocity dispersion. Eq. (2.35) is called the nonlinear Schrödinger equation (NLSE) and is similar to the well-known Schrödinger equation of quantum mechanics. The nonlinear term $|u|^2 u$ is analogous to a negative potential energy, allowing the possibility of self-trapped pulse solutions (called solitons). The NLSE can be solved analytically and has a continuous-wave (CW) and a solitonic solution [25, 43, 44].

- The CW solution can be found by setting

$$u = (\sqrt{u_0} + a)e^{iu_0\zeta}. \quad (2.36)$$

A small perturbation a added to this solution can be unstable in the anomalous dispersion region and grows exponentially, leading to the modulation instability.

- Another solution is the soliton solution and exists for normal (dark soliton) or anomalous (bright soliton) dispersion. The canonical form is given by

$$u = \text{sech}(\tau) \cdot e^{i\zeta/2} \quad (\text{bright}), \quad (2.37)$$

$$u = \tanh(\tau) \cdot e^{i\zeta/2} \quad (\text{dark}), \quad (2.38)$$

and describes non-dispersing wave packets, which have a constant pulse shape and are stable against perturbations. [41]

Generalized nonlinear Schrödinger equation – The solitonic solution of the NLSE Eq. (2.35) was first observed in 1980 by L. Mollenauer [45]. Since then, a great variety of soliton-related effects were discovered and described with help of the NLSE [46], such as the soliton self-frequency shift [47, 48] and soliton collisions [49, 50]. However, the off-the-shelf availability of sub-50fs pulses and the advent of supercontinuum generation in optical fibers [51] required the use of more sophisticated equations. One of the major advantages of the nonlinear Schrödinger equation is that it can be made more precise by adding new terms, each describing a certain contribution of the material to the pulse dynamics. This gave rise to the so-called generalized nonlinear Schrödinger equation, and is described in detail in the next chapter.

Chapter 3

Nonlinear light propagation in photonic-crystal fibers

In the previous chapter we saw that light can be confined to the core region of an optical fiber. The optical field induces a linear and a nonlinear material response which can then modify the propagation of the field. This dynamical evolution is described by the nonlinear Schrödinger equation, which unveiled several unexpected properties, as the existence of a soliton. In this chapter, we introduce the generalized nonlinear Schrödinger equation (GNLSE). This equation can be solved by the split-step Fourier method (SSFM), which will be explained in the appendix A. A SSFM solver was implemented in MATLAB¹ and will be used to explain several optical effects, resulting in temporal or spectral modifications of the input pulse. This leads ultimately to the existence of octave-spanning supercontinua in PCFs.

3.1 The generalized nonlinear Schrödinger equation

The derivation of the GNLSE is extensively discussed in literature [43, 44, 52]. The final scalar form reads,

$$\frac{\partial A(z, \tau)}{\partial z} = \hat{D}A(z, \tau) - \frac{\alpha}{2}A(z, \tau) + i\left(\gamma(\omega_0) + i\gamma_1 \frac{\partial}{\partial \tau}\right) \times \left(A(z, \tau) \int R(t') |A(z, \tau - t')|^2 dt'\right). \quad (3.1)$$

¹Version 2009a; The MathWorks, Inc.

This equation is a scalar equation and the first two terms on the right-hand side incorporate the linear response of the waveguide on the envelope $A(z, \tau)$. The operator $\hat{D}$ accounts for the frequency dependence of the propagation constant and the parameter α introduces the fiber loss. The nonlinear properties of the fiber are given by the third term, where $\gamma(\omega_0)$ is the nonlinear coefficient at a reference frequency. The nonlinear coefficient of the PCF depends on the intensity distribution of the fiber mode and can be calculated by [23]

$$\gamma = \frac{\omega}{c} \frac{\int n_2 S_z^2 d^2 \mathbf{r}}{\left(\int S_z d^2 \mathbf{r} \right)}, \quad (3.2)$$

where n_2 is the nonlinear index of refraction of Eq. (2.30) and the integrals are evaluated over the transverse profile. With the effective area A_{eff} , as defined in section 2.1.2.1, the nonlinear coefficient is then given by

$$\gamma_0 = \frac{n_2(\omega_0)\omega_0}{cA_{\text{eff}}(\omega_0)}. \quad (3.3)$$

The time derivative in Eq. (3.1) accounts for self-steepening (section 3.2.5) where γ_1 corresponds to the frequency dependence of the nonlinearity. The response function $R(t)$ introduces the instantaneous electronic (Kerr effect) and the delayed vibrational (Raman effect) contributions and reads

$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t), \quad (3.4)$$

where $f_R = 0.18$ is the fractional contribution of the delayed Raman response to nonlinear polarization. The Raman response function $h_R(t)$ has been experimentally determined by Stolen et al. [39] and is shown in Fig. 2.3. One characteristic feature of the GNLSE is the presence of a reference frequency ω_0 , normally chosen as the carrier frequency of the initial pulse. The reference frequency is a free parameter and the obtained results must be independent of its concrete choice. Several approximations have been used to arrive at Eq. (3.1), the most important of which are:

- The GNLSE has been derived with the slowly-varying envelope approximation. This requires that the spectral content $\Delta\omega$ of the electric field is narrower than the carrier frequency ω_0 , leading to the condition $|\partial_z A| \leq \beta_0 |A|$ [44, 53]. This approximation removes any existing second-order spatial derivatives from the wave equation.
- The GNLSE is an envelope equation, lacking the fast carrier-oscillations of the optical field. Therefore Eq. (3.1) does not account for the generation of third harmonics, see section

(2.2.3). Several ways have been devised to implement third-harmonic generation, which can additionally shape the pulse spectrum. The most advanced method is the uni-directional propagation equation, described in [54, 55] .

- The GNLSE is a scalar equation and does not account for polarization effects. A vectorial model can be derived by treating the input field as a two-dimensional Jones vector in a linear polarization base, see [43, 56]. In general, the third-order susceptibility of a fiber is a tensor, thus coupling the orthogonal field amplitudes.

3.2 Nonlinear optics in optical fibers

This section gives an overview of the linear and nonlinear optical effects occurring in optical fibers. The fundamental properties of the third-order nonlinearity were introduced in section 2.2.3. Here, we investigate the consequences in more detail and use the GNLSE to visualize characteristic pulse propagation phenomena.

3.2.1 Strictly linear and nonlinear propagation

The propagation in an optical fiber can be predominantly linear or nonlinear, determined by the parameters of the fiber and the light pulse. Two length scales can be defined which are useful to allocate the governing regime.

- The (second-order) dispersion length L_{GVD} is the length scale over which the dispersive effects become important. After propagating the distance L_{GVD} , an unchirped gaussian-shaped pulse with amplitude²,

$$A(z, \tau) = \sqrt{P_0} e^{-\frac{\tau^2}{2\tau_0^2}} \quad (3.5)$$

broadens by a factor of $\sqrt{2}$.

$$L_{\text{GVD}} = \frac{\tau_0^2}{|\beta_2(\omega_0)|} \quad (3.6)$$

- The nonlinear length L_{nl} denotes the length scale after which the self-phase modulation

²The connection to the real-valued electric field is given in appendix A.2

starts to reshape the optical pulse. It is defined as

$$L_{\text{nl}} = \frac{1}{\gamma \cdot P_0}. \quad (3.7)$$

More precisely, it corresponds to the propagation distance at which the pulse center acquires a phase shift of $\phi = 1$, and will be introduced in Eq. (3.15).

3.2.1.1 Linear propagation regime

Linear propagation prevails when

$$\frac{L_{\text{GVD}}}{L_{\text{nl}}} \ll 1. \quad (3.8)$$

In this case, the nonlinear response can be neglected. The propagation equation Eq. (3.1) can then be simplified and transformed into the Fourier domain, reading

$$\frac{\partial \tilde{A}}{\partial z} = i(\beta(\omega) - \beta_1\omega - \beta_0) \cdot \tilde{A}(z, \omega). \quad (3.9)$$

Propagation in the linear propagation regime leads to a change of the pulse duration. Non-zero dispersion $\beta_2(\omega)$ requires that the group velocity³ varies across the pulse spectrum. The Fourier components of the pulse travel with different group velocities and the pulse disperses. A rigorous derivation can be obtained with the spectral transfer function of a pulse traveling in a purely dispersive medium. The spectral transfer function is

$$\Psi = (\beta(\omega) - \beta_1\omega - \beta_0) \cdot z$$

and describes the accumulation of spectral phase. The phase factor for each frequency component is given by $\exp(-i\Psi) = \exp(-i(\beta(\omega) - \beta_1\omega - \beta_0) \cdot z)$. The temporal profile can be derived from a Fourier transform of the spectral field at position z ,

$$A(z, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(0, \omega) e^{i\omega_0\tau - i(\beta(\omega) - \beta_1\omega - \beta_0)z} e^{i\omega\tau} d\omega \quad (3.10)$$

³given by $\beta_1 = \int \beta_2(\omega) d\omega$

The propagation constant β can be expanded around ω_0 [see Eq. (2.18)] and we keep all orders up to order two. Then the solution of Eq. (3.10) for a gaussian input pulse defined in Eq. (3.5) reads

$$A(z, \tau) = \frac{1}{\sqrt{1 + i\beta_2 z / \tau_0^2}} e^{-\frac{\tau^2}{2\tau_0^2 + 2\beta_2^2 z^2 / \tau_0^2}} e^{i\frac{\beta_2 z \tau^2}{2\tau_0^4 + 2\beta_2^2 z^2}} \quad (3.11)$$

After propagation of a distance z the pulse duration τ_0 changes by

$$\tau(z) = \tau_0 \sqrt{1 + \left(\frac{\beta_2 z}{\tau_0^2}\right)^2}. \quad (3.12)$$

The temporal duration is obtained from the real-valued phase factor in the exponential function of Eq. (3.11). In addition, the frequency chirp, expressed in terms of the instantaneous frequency ω_c , changes continuously. The chirp is given by the imaginary part of the overall phase of Eq. (3.11), and we find

$$\omega_c(z, \tau) = \omega_0 + 4 \frac{\beta_2}{(2\tau_0^2)^2 + 4\beta_2^2 z^2} \cdot z\tau \quad (3.13)$$

The Taylor expansion up to order two is valid for pulse durations with $T_0 > 1$ ps. For shorter pulses, the associated spectral width increases and the GVD can vary considerably across the pulse bandwidth. In this case, higher-order dispersion terms need to be included. Fig. 3.1 illustrates the temporal changes of the pulses when propagating in a medium with constant $\beta_2(\omega) = \beta_2(\omega_0)$ and with a linearly varying dispersion, $\beta_2(\omega) = \beta_2(\omega_0) + \beta_3(\omega_0) \cdot \omega$.

Figure 3.1: Temporal envelopes of a 50fs pulse after 5m propagation in different dispersion landscapes. In (a) the dispersion is independent of frequency, with $\beta_2(\omega_0) = 1 \times 10^{-27} \text{ s}^2/\text{m}$. In (b) the dispersion varies linearly with frequency and $\beta_3(\omega_0) = 35 \times 10^{-42} \text{ s}^3/\text{m}$. The dotted lines refer to the input pulse.

3.2.1.2 Nonlinear propagation regime

Conventional step-index fibers have core diameters ranging from around 4 to 500 μm and the associated nonlinear coefficient is very small. The propagation dynamics can therefore be attributed to the linear propagation regime. PCFs, in contrast, can be fabricated with sub-micron cores, and the nonlinear coefficient can be very large [see Eq. (3.3)].

When the nonlinear length L_{nl} is smaller than the dispersion length L_{GVD} , the dispersion operator can be neglected from the GNLSE. By disregarding additionally the Raman effect, self-steepening and loss, the GNLSE simplifies to

$$\frac{\partial A(z, \tau)}{\partial z} = i\gamma(\omega_0) |A(z, \tau)|^2 A(z, \tau). \quad (3.14)$$

This equation can be solved directly in the time domain by using

$$A(z, \tau) = u(z, \tau) \cdot e^{i\Phi_{\text{nl}}(z, \tau)} \quad (3.15)$$

Inserting Eq. (3.15) into Eq. (3.14), and equating the real and imaginary parts, one finds that the amplitude $u(z, \tau)$ stays constant; however, the pulse acquires a phase change during propagation, proportional to

$$\Phi_{\text{nl}}(z, \tau) = |A(0, \tau)|^2 \gamma z. \quad (3.16)$$

This modulation of the temporal phase leads to modulations in the spectral domain and is called self-phase modulation (SPM). It describes the generation of new frequency components which broaden the pulse spectrum. The instantaneous frequency can be calculated from Eq. (3.16) and is in the case of the gaussian pulse Eq. (3.5)

$$\Delta\omega(z, \tau) = -\frac{\partial\Phi_{\text{nl}}}{\partial\tau} = \gamma LP_0 \frac{2\tau}{\tau_0^2} \cdot e^{-\frac{\tau^2}{\tau_0^2}} \quad (3.17)$$

The maximum frequency shift $\Delta\omega_{\text{max}}$ can be obtained from Eq. (3.17) and is inversely proportional to the initial pulse duration τ_0 :

$$\delta\omega_{\text{max}}(z) = \gamma LP_0 \frac{1.21}{\tau_0}. \quad (3.18)$$

In Fig. 3.2 we illustrate the SPM induced broadening for a gaussian-shaped pulse with 35 fs duration. After 25 cm propagation the pulse width has broadened considerably. The spectral broaden-

ing manifests itself as a pronounced oscillatory structure, where the outermost peaks are the most intense ones. This structure is caused by the nonlinear chirp function Eq. (3.17), which generates equal frequencies at two different temporal delays. This leads to pronounced beat signals in the spectral envelope.

Figure 3.2: *Spectral broadening by self-phase modulation. The dashed line refers to the 35 fs input pulse with peak power 1 kW. The solid line shows the SPM broadened spectrum after 25 cm propagation.*

3.2.2 Soliton formation

So far we have studied the impact of the linear and the nonlinear term of the GNLSE on optical pulses separately. When the dispersion length falls in the same order as the nonlinear length, both effects are equally important. This section focuses on pulses propagating in the anomalous dispersion region. For propagation in the normal dispersion regime, refer to [43]. In section 2.2.4 we mentioned the existence of solitonic solutions as steady-state solutions of the equation

$$\frac{\partial A}{\partial z} = \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial \tau^2} - \gamma |A|^2 A. \quad (3.19)$$

The general form of the soliton is

$$A_{sol}(z, \tau) = N \cdot \text{sech}\left(N \frac{\tau - v_g z}{T_0}\right) e^{-iv_g \tau + i(N^2 - v_g^2)z/2} \quad (3.20)$$

and is called a bright soliton, where v_g is the velocity of the soliton and T_0 is the pulse duration. The parameter N is called the soliton order. The existence of the soliton is a result of the interplay between SPM and GVD. SPM creates new frequency components during propagation; the red-shifted (frequency-decreased) light is near the leading edge, and the blue-shifted can be found in the trailing edge of the pulse. Due to the anomalous dispersion, the red-shifted light travels slower and falls back into the pulse center. At the same time, the blue-shifted components advance towards the pulse center. Both components can then overlap and are annihilated by SPM in favor of two pump photons.

Note that the phase term in Eq. (3.20) is independent of the time τ (besides the self-evident carrier oscillations) and the soliton is completely non-dispersive. The soliton order N is determined by the relative contribution of the dispersion and the nonlinear length, and reads

$$N = \sqrt{\frac{L_{\text{GVD}}}{L_{\text{nl}}}} = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} \quad (3.21)$$

For example, $N = 1$ corresponds to a fundamental soliton. Another very useful quantity, obtained from Eq. (3.19), is the soliton pulse energy W_{sol}

$$W_{\text{sol}} = P_0 \int_{-\infty}^{\infty} \text{sech}^2(t/T_0) dt = 2P_0 T_0 \quad (3.22)$$

This relation states the inverse relationship between the peak power P_0 and the width T_0 of a soliton — the shorter the soliton duration, the higher its peak power, for a given fiber. When the soliton order N is larger than one, the soliton is referred to as higher-order soliton. In contrast to fundamental solitons, higher-order solitons evolve periodically along the fiber length, with a periodicity $L_{\text{sol}} = \frac{\pi}{2} L_{\text{GVD}}$. Note that the period of this breathing behavior is independent of the soliton order. In Fig. 3.3 we show the temporal and spectral evolution of a $N = 3$ soliton. Due to their unconventional characteristics, solitons are often described as particles [42, 57]. Solitons interact with each other and the time of soliton overlap is known as a collision. It has been shown that soliton interactions depend on the relative phase and on the amplitude ratio [25, 58]. For example, when equal-amplitude solitons are in phase they attract each other, because their fields constructively interfere in the overlap region between them. This leads to a periodic collision along the fiber length. When the solitons are out of phase they repel each other and increase their separation, due to destructive interference of their fields in the overlap region.

Figure 3.3: (a) Temporal evolution of a $N = 3$ soliton in a fiber with constant dispersion, $\beta_2(\omega_0) = -8.76 \times 10^{-27} \text{ s}^2/\text{m}$. The pulse duration was 110 fs (FWHM), leading to a dispersion length of $L_{\text{GVD}} = 44.6 \text{ cm}$ and a soliton length of $L_{\text{sol}} = 70 \text{ cm}$, after which the soliton cycle acquires its original state. The color scale refers to the power, normalized to the input power. (b) Spectral evolution. The color scale is in dB.

3.2.3 Intra-pulse Raman scattering

In section 2.2.2 we mentioned that the molecular nature of silica gives rise to the Raman effect. Raman scattering leads to the generation of new spectral components, which are seeded by noise photons that are amplified by the Raman gain. When a pump and a suitably detuned probe beam copropagate in an optical fiber, stimulated Raman scattering (SRS) can be observed [20, 43]. The probe beam is amplified at the expense of the pump beam. The magnitude of the effect is strongly limited by the group-velocity mismatch⁴, and near-CW pulses are often used for experimental realizations [43]. However, the Raman gain influences also the propagation of ultrashort pulses and induces the so-called intrapulse Raman scattering [48]. Red spectral components of the propagating soliton provide a seed for Raman amplification, which are pumped by the blue components. The growth of the red spectral components at the expense of the blue components leads to a red-shift of the ultrashort pulse. In case of a soliton this effect is called soliton self-frequency shift (SSFS). It has been shown in [47] that the shift rate of the soliton center frequency is inversely

⁴The group-velocity dispersion, self-phase modulation and cross-phase modulation add additional constraints to the efficiency of the Raman process by altering the temporal and spectral pulse shapes of the pump and probe beam.

proportional to the fourth power of the pulse duration, and is

$$\Omega_p(z) = -\frac{8T_R|\beta_2|}{15T_0^4}z, \quad (3.23)$$

where $T_R = \int_0^\infty tR(t)dt$ is the first moment of the nonlinear response function $R(t)$ in Eq. (3.4). Strictly speaking, Eq. (3.23) is only valid for fibers with constant dispersion. In all other cases, the soliton permanently adjusts its parameters to the fiber parameters, which change as the soliton frequency changes. This leads to a deviation from a frequency shift linear in z . In Fig. 3.4 we investigate the influence of a positive and a negative third-order dispersion (TOD) contribution. The dispersion was approximated as $\beta_2(\omega) = \beta_2(\omega_0) \pm \beta_3(\omega_0) \cdot \omega$. We show the change of the center wavelength (Fig. 3.4a), pulse duration (Fig. 3.4b) and peak power (Fig. 3.4c) in case of a positive and negative TOD. We see that all three parameters change nonlinearly. A positive

Figure 3.4: *Soliton self-frequency shift in fibers with positive or negative slope of the dispersion curve. β_2 was $20 \text{ ps}^2/\text{km}$, and $\beta_3 = +8 \times 10^{-2} \text{ ps}^3/\text{km}$ and $\beta_3 = -1.6 \times 10^{-2} \text{ ps}^3/\text{km}$. The initial pulse duration was 20 fs (FWHM) and the peak power was 1900 W . (a) Soliton wavelength, (b) soliton duration, (c) peak power.*

TOD decelerates the frequency shift and is accompanied by an increase of the pulse duration and decrease of the soliton peak power. The soliton adapts its parameter according to the prevailing fiber parameters at the soliton wavelength. In contrast, a negative TOD contribution leads to an acceleration of the soliton shift, combined with a reduction of the soliton duration. Note that a negative TOD in an anomalous GVD region causes a zero-dispersion wavelength which is red-detuned to the soliton. When the wavelength of the soliton is close to this ZDW, the SSFS is canceled [59]. The stabilization of the soliton is accompanied by a strong emission of dispersive waves.

3.2.4 Four-wave mixing

Section 2.2.2 introduced the four-wave mixing (FWM) as the parametric interaction between four photons. The FWM involves the annihilation of two incident photons at frequency ω_1 and ω_2 and the simultaneous creation of two photons at frequencies ω_3 and ω_4 , called signal and idler waves. Since the quantum states of the molecules are the same at the beginning and at the end, the conservation of energy and of momentum should hold among the four involved photons [18, 20]. This results in the following phase-matching conditions

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 \quad (3.24)$$

$$\beta(\omega_1) + \beta(\omega_2) = \beta(\omega_3) + \beta(\omega_4). \quad (3.25)$$

When the two incident photons can have the same frequency, $\omega_1 = \omega_2 \equiv \omega_0$, the process is called degenerate FWM. In this case, FWM can be observed for a single-frequency pump beam. An example will be shown later in section 3.3.2. In degenerate FWM, the generated photons ω_3 and ω_4 have equal detuning Ω from the pump wave at ω_0 . The energy conservation simplifies to

$$2\omega_0 = (\omega_0 + \Omega) + (\omega_0 - \Omega) \quad (3.26)$$

The length evolution of the three field amplitudes $\tilde{A}_0(z, \omega_0)$, $\tilde{A}_1(z, (\omega_0 + \Omega))$ and $\tilde{A}_2(z, (\omega_0 - \Omega))$ can be decoupled, and are then described by the following set of coupled equations [43]

$$\begin{aligned} \frac{\partial A_0(z, \tau)}{\partial z} + \beta_1(\omega_0) \frac{\partial A_0}{\partial \tau} + \sum_{n=2}^4 \beta_n(\omega_0) \frac{i^{n-1}}{n!} \frac{\partial^n}{\partial \tau^n} A_0 + \frac{1}{2} \alpha(\omega_0) \\ = i\gamma(|A_0|^2 + 2(|A_1|^2 + |A_2|^2))A_0 + 2i\gamma A_0^* A_1 A_2 e^{i\Delta K z} \\ \frac{\partial A_1(z, \tau)}{\partial z} + \beta_1(\omega_1) \frac{\partial A_1}{\partial \tau} + \sum_{n=2}^4 \beta_n(\omega_1) \frac{i^{n-1}}{n!} \frac{\partial^n}{\partial \tau^n} A_1 + \frac{1}{2} \alpha(\omega_1) \\ = i\gamma(|A_1|^2 + 2(|A_2|^2 + |A_0|^2))A_1 + 2i\gamma A_0^2 A_2^* e^{-i\Delta K z} \\ \frac{\partial A_2(z, \tau)}{\partial z} + \beta_1(\omega_2) \frac{\partial A_2}{\partial \tau} + \sum_{n=2}^4 \beta_n(\omega_2) \frac{i^{n-1}}{n!} \frac{\partial^n}{\partial \tau^n} A_2 + \frac{1}{2} \alpha(\omega_2) \\ = i\gamma(|A_2|^2 + 2(|A_1|^2 + |A_0|^2))A_2 + 2i\gamma A_0^2 A_1^* e^{i\Delta K z} \end{aligned} \quad (3.27)$$

These equations can be obtained by decomposing the optical field into three slowly-varying envelopes, centered around the three involved frequencies. Refer to [18,20] for a detailed derivation. The parameter

$$\Delta k = 2\beta(\omega_0) - \beta(\omega_0 + \Omega) - \beta(\omega_0 - \Omega) + 2\gamma P \quad (3.28)$$

describes the wavevector mismatch of the three coupled light fields. A constructive growth of the signal and idler waves requires phase-matching, $\Delta k = 0$. Eqs. (3.27) are very general and include the effects of GVD (section 3.2.1.1), SPM (section 3.2.1.2), and cross-phase modulation. Cross-phase modulation describes the coupling of two optical fields without an energy exchange. One optical field induces a modification of the effective refractive index; the second field experiences the index change and acquires an additional phase-factor [43,60].

In general, Eq. (3.27) can only be solved by numerical schemes. Considerable insight into the physics of FWM is gained for CW fields and an undepleted pump pulse. In this limit, the dispersive effects vanish and the solutions for the signal and idler fields read [18,20,43]

$$\begin{aligned} B_2(z) &= (a_3 e^{gz} + b_3 e^{-gz}) e^{-i\kappa z/2} \\ B_3^*(z) &= (a_4 e^{gz} + b_4 e^{-gz}) e^{i\kappa z/2} \end{aligned} \quad (3.29)$$

where B_j is related to the field amplitudes by $B_j = A_j e^{-4i\gamma P_0 z}$. The parameter γP_0 accounts for the phase shift of the optical phase introduced by the nonlinearity, see chapter 3.2.1.2. The parametric

Figure 3.5: (a) Density plot of the parametric gain as a function of the pump wavelength. The dispersion curve used is shown in Eq. 3.7 and was used for nanosecond pulse propagation simulations in chapter 3.3.2. The pump power was 2 kW. The dashed line refers to the ZDW at 991 nm. The density scale is in m^{-1} . (b) Gain spectrum taken from (a) at $\lambda_p = 1064 \text{ nm}$.

gain $g = \sqrt{(2\gamma P_0)^2 - (\kappa/2)^2}$ determines the growth rate of the sidebands and depends on the pump power and on the frequency. The parametric gain and therefore the growth of the sidebands has its maximum for zero phase mismatch, occurring at the detuning Ω . The relevant expression for the gain (in units per meter) is

$$g(\Omega) = 2\text{Im}\left(\sqrt{Q \cdot (Q + 2\gamma P_0)}\right) \quad (3.30)$$

$$\Omega = \frac{1}{2}\beta_2\Omega^2 + \frac{1}{24}\beta_4\Omega^4 + \frac{1}{720}\beta_6\Omega^6 + \dots$$

Here, the β_i are higher-order dispersion coefficients. Fig. 3.5a plots the dependence of the gain on the pump wavelength and sideband detuning. The gain is distributed symmetrically around the pump frequency. In the anomalous dispersion region the gain spectrum is broadband and lies close to the pump frequency. In Fig. 3.5b we exemplified the gain spectrum for a pump wavelength of 1064 nm. In contrast, for pump frequencies falling into the normal dispersion region, the gain bandwidth shrinks and the detuning starts to grow considerably [43].

3.2.5 Self-steepening

When the duration of the propagating pulse is below 100 fs, higher-order nonlinear effects become important, described by the time derivative in the nonlinear term of Eq. (3.1). This induces pronounced temporal and spectral changes and is called self-steepening of optical pulses. Self-steepening results in the formation of an optical shock on the trailing edge of the pulse. The Kerr effect (see section 2.2.2) of the ultrashort pulse depends on the delay within the pulse, $n(I) = n_0 + n_2 I(t) > n_0$, and leads to an intensity dependence of the group velocity. The peak of the pulse experiences an optically denser material than the pulse wings. The pulse maximum is therefore slowed down and the pulse develops a steepened trailing edge. We visualize this dramatic effect in Fig. 3.6, and show the temporal and spectral field of a 35 fs pulse after 20 cm propagation. In the temporal domain, Fig. 3.6a, the optical shock creates an asymmetrical pulse shape with a discontinuity at the trailing edge. This goes hand in hand with a spectral asymmetry (Fig. 3.6b), where the blue side extends farther than the red side [43]. In combination with self-phase modulation, the self-steepening increases continuously the spectral bandwidth. This leads finally to the arrest of the shock formation. The GVD of the medium disperses the large frequency content of the pulse. The resulting increase of the pulse duration removes the steep edge and the decrease of

Figure 3.6: *Self-steepening of an optical pulse in a 20 cm long PCF. (a) shows the temporal profile and (b) the pulse spectrum. The pulse and fiber parameters correspond to the SPM case in Fig. 3.2. The dotted lines refer to the input pulse.*

the intensity stops the SPM broadening.

3.3 Supercontinuum generation in photonic-crystal fibers

A supercontinuum (SC) denotes an optical spectrum that extends over one or more octaves. Supercontinua have been generated in various materials, including condensed matter [52, 61, 62], water [63, 64] and gases [65–67]. Since the advent of the PCF, the availability of supercontinua increased dramatically. The PCFs allowed the fabrication of compact and cheap white-light sources. The spectral composition of the SC can be tailored to suit different experimental demands, for example in biology, medicine or laser spectroscopy. In addition, the small core sizes of PCFs reduced the energy threshold for supercontinuum generation (SCG).

The mechanism of the pulse broadening process and the resulting spectrum depends on the fiber and the pulse parameters. Here, we discuss the supercontinuum generation (SCG) for two different cases: SCG by femtosecond pump pulses at 800 nm and SCG by picosecond pump pulses at 1064 nm. In both cases the pump pulse is launched into the anomalous dispersion region. The femtosecond pulse is launched at 800 nm and the GVD curve is shown in Fig. 3.7a. A SEM image can be found in Fig. 1.1. In contrast, the SCG for a near-CW pump pulse is studied in a large-core ESM fiber, and the dispersion curve Fig. 3.7b belongs to the fiber shown in Fig. 1.1. In-depths reviews of SCG can be found for example in Refs. [27, 52].

Figure 3.7: *GVD profiles used for SCG. (a) GVD profile of a small-core PCF. The ZDW is at 711 nm. The pump wavelength of the ultrashort pulses is at 800 nm. (b) Dispersion curve of an ESM fiber. The ZDW is at 991 nm. The pump wavelength of the near-CW pulses is at 1064 nm.*

3.3.1 Supercontinuum generation with ultrashort light pulses

Many studies of femtosecond SCG have involved the usage of a Ti:Sapphire laser, centered at 800 nm with pulse durations around $T_{\text{FWHM}} = 110$ fs (see section 4.1.1). Therefore, we discuss the formation of an octave-spanning spectrum by means of such an input pulse. The fiber dispersion curve we use for the following SCG simulation is shown in Fig. 3.7a, and has a ZDW at 711 nm.

Length dependence

Fig. 3.8 displays the spectral and temporal evolution of the pulse in a 10 cm long PCF. The nonlinear coefficient was $100 \text{ W}^{-1} \text{ km}^{-1}$. The dispersion length in the fiber was 20.6 cm, and the nonlinear length was around 3 mm.

- From Fig. 3.8a one finds that the initial broadening is governed by SPM. At the activation or fission length, which is around 2 cm for the pulse parameters used, a rapid extension of the spectral width appears. This is accompanied by strong temporal compression of the pulse [27, 68] and is demonstrated in Fig. 3.8b. The compression results from higher-order soliton dynamics (section 3.2.2). For Fig. 3.3 the GVD was approximated by a constant β_2 . However, in the presence of higher-order dispersion, as considered here, the soliton propagation is modified. In fact, the soliton reacts to the higher-order perturbations by emitting linear radiation. This results in the onset of the strong radiation band in the visible spectral range, which is centered at 517 nm in Fig. 3.8a. This process is called the dispersive-wave

Figure 3.8: (a) Density plot of the spectral evolution in a 10 cm long PCF. The dispersion curve is shown in Fig. 3.7a. The pulse duration was 110 fs. The input peak power was 4 kW. The figure on the right-hand side shows the conversion efficiency into the 500 to 600 nm spectral range. The dashed line is the ZDW at 711 nm. The color scale is in dB. (b) Temporal profile of the pulse after several propagation distances. The slanted lines in the top figure point to several solitons.

(DW) generation, also known as resonant radiation or Cherenkov radiation (see for example [69, 70]). The position of the DW can be found from a phase-matching argument, requiring the equality of the soliton and radiation propagation constants [70–74]. The resonance frequency $\omega = \omega_r$ can be found from

$$\beta(\omega) - \beta(\omega_0) - \beta_1(\omega_0)(\omega - \omega_0) = \gamma P_0/2 \quad (3.31)$$

The intensity of the DW is proportional to the power P_0 of the soliton. In general, the energy transfer into the dispersive wave increases as the soliton frequency ω_0 and the ZDW converge. The group-velocity difference between the DW at 517 nm and the remaining pump wave at 800 nm separates them quickly and the walk-off is visible after 3 cm in Fig. 3.8b.

- The higher-order soliton at the fission length breaks up into several fundamental solitons [75–78]. This process is called soliton fission. Kodama and Hasegawa [78] derived explicit expressions for the peak power P_j and temporal width T_j of the j -th fundamental soliton

created in the fission process [78]

$$P_j = P_0 \frac{(2N - 2j + 1)^2}{N^2}, \quad 1 \leq j \leq N \quad (3.32)$$

$$T_j = \frac{T_0}{2N - 2j + 1}, \quad 1 \leq j \leq N \quad (3.33)$$

where T_0 is the temporal width and P_0 is the peak power of the initial laser pulse. As usual N refers to the soliton order, introduced in section 3.2.2. Eq. (3.32) describes the generation of a series of N fundamental solitons, decreasing their peak power and increasing the duration as j increases. However, the validity of Eqs. (3.32) and (3.33) is limited. It has been shown that for $N > 8$ an anomalous pulse breakup occurs and bound pairs of solitons are ejected [79].

- During further propagation, each soliton undergoes a red-shift due to the soliton self-frequency shift, introduced in section 3.2.3. This process increases the spectral extent into the near-IR spectral range. The red-shift of the ($j = 1$) soliton is the strongest, because it has the shortest temporal width according to Eq. (3.33). The remaining solitons experience a smaller shift and overlap spectrally during propagation. This produces the spectral fringes at around 800 nm. The SSFS separates the solitons spectrally and temporally. This causes the

Figure 3.9: XFROG spectrogram of a 4 kW pulse after traversing a 10 cm long PCF. The dashed line indicates the position of the ZDW. The A (N) denotes the anomalous (normal) dispersion region. The slanted arrows point to bright solitons. The dashed line refers to the ZDW.

presence of the four solitons, visible after 10 cm propagation in Fig. 3.8b. The distribution

of the soliton peak power and soliton duration is according to Eq. 3.32. The first soliton has the highest power, the shortest duration and has separated considerably from the remaining solitons.

- Much insight into the broadening mechanism can be obtained from cross-correlation frequency resolved optical gating (XFROG) [80, 81]; the resulting spectrogram visualizes the spectro-temporal dynamics of the pulse. The spectrogram at distance z can be obtained by

$$S(z, \tau', \omega) = \left| \int_{-\infty}^{\infty} A(z, \tau) A_{ref}(z, \tau - \tau') e^{i\omega\tau} d\tau \right|^2 \quad (3.34)$$

Here, $A_{ref}(z, \tau - \tau')$ is the amplitude of the reference pulse, which is gating the probe pulse for different delays τ' . Fig. 3.9 shows the spectrogram after 10 cm long propagation. One observes the formation of four solitons in the anomalous dispersion region. The shortest soliton has the shortest duration and exhibits the strongest SSFS. In contrast, the normal dispersion region is filled by the DWs. The notion "dispersive wave" becomes obvious, because they obey the linear dispersion relation, disperse strongly and acquire thus the typical down-chirp. The U-shape of the spectrogram is peculiar to the femtosecond SC mechanism and imprints the parabola-like group velocity of the fiber.

Power dependence

The shape of the generated SC depends on the input peak power. In Fig. 3.10 we show the spectro-temporal change of the pulse for varying input peak powers, generated in a 10 cm long PCF. Initially the spectral broadening is governed by SPM. Then, for a peak power of approx. 1 kW, dispersive radiation appears at 566 nm. At this power level, the fission length corresponds to the fiber length and DW generation can take place. As the power raises, the blue and red spectral edge of the SC start to expand.

The shift of the blue edge follows from Eq. (3.31). The wavelength of the DW depends on the input power, and is lowering with an increase of the pump power. Additionally, the spectral intensity of the DW increases with pump power (see the conversion efficiency curve in Fig. 3.10a). The dispersive radiation can also be observed in the temporal domain Fig. 3.10b, and is trailing behind the solitons. For an input power of 1.7 kW, a second radiation band appears in the trailing edge. In Fig. 3.10b it can be ascribed to the DW generated by a second soliton.

Figure 3.10: (a) Power dependence of the spectral broadening in a 10 cm long PCF. The graph on the top shows the conversion efficiency (CE) in the 500 to 600 nm spectral region. The dashed lines are the ZDW at 711 nm. (b) Temporal evolution of the SC for varying input power. The arrows show the emergence of fundamental solitons. The color scale is in dB and applies to (a) as well.

The growth of the red spectral edge stems from the SSFS of the involved solitons. The rising input power increases the soliton order N [see Eq. (3.21)]. According to Eq. (3.32), a large value of N reduces the temporal duration of the emerging solitons. This is clearly visible by the brightest soliton in Fig. 3.10a: the spectral width of this soliton grows due to the decrease of the pulse duration. A shorter pulse duration leads then to a stronger SSFS.

A second contribution to the increase of the red shift stems from the reduction of the fission length. The created solitons are subjected to the SSFS for a longer propagation distance. However, this effect can be neglected for large power values as the activation length is known to saturate [68].

3.3.2 Spectral broadening with quasi-CW light

An important development in the nonlinear propagation of light in PCFs was the advent of white-light sources, pumped by cheap laser systems [51]. Commonly employed laser sources included compact Q-switched lasers with pulse energies up to 6 μ J. The wavelength is at 1064 nm and is used to generate a bright white-light SC in endlessly single-mode PCFs with ZDWs around 1 μ m. In Fig. 3.7b we show the dispersion curve of an ESM fiber, having a zero-dispersion wavelength at 991 nm. The duration of the pump pulse is in the near-CW range and the supercontinuum generation process involves different mechanisms when compared to the femtosecond case. The SCG

Figure 3.11: *Spectral (a) and temporal (b) evolution of a 50 ps pulse in an ESM fiber with a ZDW at 991 nm.*

process in the near-CW regime is exemplified in Fig. 3.11. It shows the simulated propagation of a 50 ps pulse with pulse energy 0.1 μJ . This leads to a dispersion length as long as $L_{\text{GVD}} = 94 \text{ km}$. The nonlinear coefficient of the fiber was $10 \text{ W}^{-1}\text{km}^{-1}$, resulting in a nonlinear length $L_{\text{nl}} = 5 \text{ cm}$. The broadening mechanism is based on FWM (section 3.2.4). Fig. 3.11 plots the spectral and the temporal evolution for several positions in a 5 m long PCF. The simulation required a very broad temporal window (150 ps width) to enclose the pulse, as well as a very fine temporal grid size (1.1 fs) to be able to resolve ultrafast temporal fluctuations. Therefore we used 2^{17} points, increasing the simulation time considerably. The broadening steps are as follows.

- After around 1 m propagation in the fiber, Fig. 3.11a shows the emergence of symmetrically detuned sidebands, at 1030 nm and 1099 nm. They have been created by degenerate FWM (section 3.2.4), and the frequency detuning of $\Omega = 60 \text{ THz}$ from the pump corresponds nicely to the result obtained from the phase-matching Eq. (3.24), yielding $\Omega = 64.5 \text{ THz}$. We show the parametric gain in Fig. 3.5. In the temporal domain (Fig. 3.11b), FWM created ultrafast fluctuations on the pulse envelope. The explanation can be obtained from the XFROG spectrogram of this pulse (Fig. 3.12). It displays the formation of symmetric sidebands around

the pump wave. They accumulate around the pump maximum because the parametric gain g is proportional to the product γP_0 . In addition, the spectrogram shows the emergence of several higher-order FWM sidebands, which are then responsible for the subsequent broadening.

- After 2 m propagation the spectrum has already developed into a continuous band. This is due to cascaded FWM processes, because the FWM products can interact with the pulse and generate new spectral components. Additionally, Raman scattering transfers energy from the pump light to the red spectral side. The cascaded FWM processes increased the temporal fluctuations on the pulse envelope; one finds ultrashort spikes of radiation, having magnitudes up to 12 kW.

Figure 3.12: XFROG spectrograms of a 50 ps at different propagation distances. (a) After 1 m, pronounced FWM peaks appear on either side of the input light wave. Higher-order FWM bands are visible. (b) After 5 m, a smooth SC formed, ranging from 500 nm to beyond 2 μm . The anomalous dispersion region features a so-called soliton sea.

- After 3 m propagation, the red side has extended considerably, which can be traced back to the generation of solitons, formed from the ultrafast temporal fluctuations. The subsequent SSFS of the solitons increases continuously the red spectral edge. We note that the additional increase of the blue spectral side is due to so-called radiation trapping of the solitons [82–86]. The formation of solitons influences the temporal profile of the optical field, because the the SSFS starts to shift the solitons into the trailing part of the temporal domain (see temporal profile at 3 m, and as well at 4 and 5 m).

- Further propagation results in a continuous increase of the red spectral edge, due to the SSFS. The strong absorption in the IR region will at some point limit the spectral extent. However, longer propagation is desirable because it results in a flat spectrum spanning from 500 nm to beyond 2 μm . In Fig. 3.12b we summarize the supercontinuum generation process by the XFROG trace at the end of the fiber. The red spectral side is dominated by a sea of solitons, shifting towards the trailing edge of the pulse due to SSFS. In addition, each soliton is accompanied by linear waves in the blue-green spectral range, which are a reminder from the initial FWM process.

Chapter 4

Experimental techniques

This chapter discusses several experimental techniques we used to study ultrashort pulse dynamics in dispersion-tailored PCFs. Section 4.1 summarizes the main properties of the experimental setup. We give a short overview over the laser sources as well as the diagnostic tools, which are essential for a qualitative study of pulse broadening. Section 4.2 outlines the pull-and draw technique, allowing the fabrication of tapered PCF devices. At the end, in section 4.3, we present the dispersion measurement rig.

4.1 Experimental setup

4.1.1 Ti:Sa oscillator

The experimental setup used a commercially available mode-locked titanium:sapphire ($\text{Ti}:\text{Al}_2\text{O}_3$) laser system¹. The gain medium is a sapphire crystal doped with Ti^{3+} ions. The luminescence spectrum of the crystal spans from 680 nm to 1100 nm [87] and is suitable for mode-locking [88]. However, filtering elements in the cavity place restrictions on the usable bandwidth. Typically, the Ti:Sa laser is operated at 800 nm. The crystal is pumped by diode-pumped solid-state lasers, operating at 532 nm. Due to Kerr-lens mode-locking [88], the output of the cavity is an ultrashort pulse with duration $T_{\text{FWHM}} = 110$ fs. The repetition rate of 76 MHz corresponds to the 13.2 ns round-trip time in the cavity. The spectro-temporal characteristics of the output light can be analyzed by a SH-FROG² spectrogram [81], and is shown in Fig. 4.1a. The spectrogram allows the retrieval of

¹Mira Optima 900-D from Coherent Inc.

²Short for second-harmonic frequency-resolved optical gating, introduced later in this section

the temporal profile and temporal phase. Both are presented in Fig. 4.1b, and the duration of the sech-shaped pulse was 133 fs.

Figure 4.1: (a) SH-FROG spectrogram of the MirA pulse. (b) Retrieved temporal shape and phase of the pulse. The duration was 133 fs. The spectrogram was measured without an optical isolator in the beam path, which would broaden the pulse to around 180 fs.

4.1.2 Non-collinear optical parametric amplification

The output beam of the Ti:Sa laser can be amplified and used to operate a non-collinear optical parametric amplifier (NOPA). The NOPA consists of three functional blocks.

- Firstly, a continuum is generated in a sapphire plate.
- Secondly, a certain spectral window of the continuum is used as seed light and is amplified by an intense 400 nm pump beam.
- Thirdly, the highly-chirped signal pulse is compressed by a prism compressor stage, yielding a Fourier-limited ultrashort pulse in the visible [89].

A schematic of the setup is shown in Fig. 4.2a, highlighting the non-collinear arrangement of the pump and seed beam. The non-collinear layout results in an angle between the signal and idler beam. The angle can be chosen to satisfy group-velocity matching of the signal and the idler wave. This prevents a pulse lengthening of the amplified signal. The signal is provided by a broad continuum. The continuum was generated by focusing an intense 800 nm beam onto a thin sapphire disk and has a visible shoulder ranging from 460 to 700 nm (see spectrum in Fig. 4.2a) [90]. The

Figure 4.2: (a) Principle of the non-collinear optical parametric amplification. The seed continuum was generated from a sapphire plate [90]. (b) SH-FROG spectrogram of the amplified and compressed signal beam. The wavelength was at around 575 nm. (c) Retrieved temporal profile of the pulse shown in (b). The duration corresponds to 25 fs. An uncompensated third-order dispersion contribution resulted in the slightly asymmetric pulse.

continuum pulse is chirped and different frequency components have different delays. The parametric interaction takes place in a BBO crystal with a 400 nm pump pulse.

The wavelength of the amplified light is determined by the delay between pump pulse and the continuum pulse. The output wavelength of the NOPA can be tuned by changing the delay of the pump pulse. The power of the signal beam is optimized by adjusting the angle of the BBO crystal to achieve phase-matching. The amplified signal pulses are chirped and the pulse duration is approx. 150 fs.

A prism pulse-compressor is used to compress the signal pulses to their Fourier limit. The compressor stage consists of two SF-10 prisms and are separated by 20 cm. The pulse duration can be compressed to as short as 25 fs. Figs. 4.2b&c show a FROG trace and the retrieved temporal profile of a NOPA pulse after the compressor stage. With an optimized alignment the NOPA emits tunable pulses from 490 to around 650 nm with energies up to 250 nJ.

4.1.3 Pulse characterization

A widely used technique to estimate the pulse duration is the intensity autocorrelation (AC) [81, 91, 92]. In this type of interferometer the pulse is splitted into two replicas $I(t)$ and $I(t - \tau)$ which are then recombined with a time delay τ between them. The temporal overlap between the replicas is measured by a nonlinear process, generating a detection signal proportional to the intensity product of the two pulses. The detection signal is measured as a function of the time delay τ and yields the intensity autocorrelation function $S_{\text{int AC}}(\tau)$

$$S_{\text{int AC}}(\tau) = \int_{-\infty}^{\infty} I(t)I(t - \tau) dt. \quad (4.1)$$

The autocorrelation function is symmetric and its width $\Delta t_{\text{int AC}}$ is proportional to the duration of the pulse Δt [81]. The limitations of this technique are that an initial pulse shape has to be assumed, and that an infinite number of symmetric and asymmetric pulse shapes lead to very similar autocorrelation traces. However, the method is widely used for gaussian and hyperbolic-secant (sech) shapes, and the relation of the AC to the FWHM pulse width are given by

$$\begin{aligned} \frac{\Delta t_{\text{int AC}}}{\Delta t} &= 1.414 \quad \text{gaussian} \\ \frac{\Delta t_{\text{int AC}}}{\Delta t} &= 1.543 \quad \text{sech} \end{aligned}$$

The information given by the autocorrelation is not sufficient to characterize optical pulses entirely by means of their temporal and spectral amplitude and phase functions. The phase information can be obtained by adding a spectrum analyzer into the autocorrelator setup. The decomposition of the intensity correlation into its frequency components is known as frequency-resolved optical gating (FROG) [81, 91]. When the intensity product of the temporal overlap is measured by second-harmonic (SH) generation, the device is called SH-FROG. In Fig. 4.3 the principle of a FROG system is presented. The output recorded during a FROG measurement is called a spectrogram

Figure 4.3: *Principle of SH FROG. The delay line separates the two pulse replicas by a delay τ . When the spectrum analyzer is removed, the setup corresponds to an intensity autocorrelator.*

and describes the temporal distribution of the frequency content of the pulse. It is given by

$$I_{\text{SH-FROG}}(\omega, \tau) = \left| \int_{-\infty}^{\infty} E_c(t) E_c(t - \tau) e^{-i\omega t} dt \right|^2, \quad (4.2)$$

where E_c is the optical field under investigation. The SH-FROG yields symmetric spectrograms but has a much higher sensitivity than, for example, third-harmonic FROG or self-diffraction FROG [81, 93]. The on-axis³ components are the frequency components contained in the pulse, and the cross-correlation signals indicate the relative delay between them. Having measured a spectrogram, the complex electric field can be obtained from the FROG trace $I_{\text{SH-FROG}}(\omega, \tau)$ by commercially available retrieval programs [94–96].

³Defined by the temporal center of symmetry

4.1.4 Details of the experimental setup

A schematic of the experimental setup is shown in Fig. 4.4. The input laser beam can either be the Mira or the NOPA output, both presented earlier in this section. The telescope is used to adjust

Figure 4.4: *Layout of the experimental setup. The power control is achieved by a neutral-density attenuator or a Glan-Taylor polarizer.*

the beam waist diameter. The pulse power was changed with help of a $\lambda/2$ waveplate and a Glan-Taylor polarizer. The mirrors A and B can be used to increase the incoupling further by means of beam-walk, allowing a fine adjustment of the beam alignment into the incoupling lens. The following $\lambda/2$ waveplate allows polarization control of the input beam. This is important since small-core PCFs can exhibit a slight birefringence (section 2.1.2.4). Matching the polarization of the light to one fiber axis prevents unwanted cross-phase modulation between the axes. The axes can be determined by recording the spectral interference pattern after the fiber for several input polarizations. When light is propagating simultaneously in both axes, their temporal delay results in a spectral beat signal. The fringe contrast is maximal when the analyzer is parallel to the fiber axes. When the input polarization aligns with one of the axes no interference occurs.

The incoupling lens of the experimental setup is matched to the numerical aperture of the fiber core. Working with tapered or small-core fibers requires stable incoupling conditions and care was taken to optimize the stability of the launching efficiency.

Several devices are used to characterize the output light from the fiber. The time-frequency information can be obtained from the SH-FROG, and the light spectrum is measured by a scanning-type optical spectrum analyzer (ANDO AQ 6315 E).

4.2 Post-processing of photonic-crystal fibers

Post-processing of PCFs is a powerful tool to transform stock PCFs into devices like mode converters, spectral filters or nonlinear elements [34, 97, 98]. The tapering process detaches one from the use of a fiber drawing tower to obtain various kinds of PCF geometries. The fibers can be manipulated on length scales inaccessible during fiber fabrication. Post-processing enables the fabrication of a single taper waist or multiple waists, with lengths in the tens of centimeters, usually sufficient for nonlinear applications. The diameter of the tapered part can be as small as several hundreds of nanometers.

The tapering process can be divided into two parts: heating of a part of the fiber and pulling the ends in opposite directions. The heating device is a small flame delivered by a small opening and fueled by a butane-oxygen mixture. The increase of the temperature reduces the viscosity of the glass and leads to a flow of the glass. Due to the stress applied to the fiber tips, the heated section is shrinking in diameter for each time the flame passes this section. Fig. 4.5 shows a schematic of

Figure 4.5: *Schematics of the taper process. A single flame brush reduces the diameter of the heated zone from d_n to d_{n+1} . The additional elongation $\mathbf{AB}(n+1) - \mathbf{AB}(n)$ is accumulated on both sides of the flame, labeled as x and y .*

the tapering process. A flame sweeps with velocity u over a distance L , which is called hot-zone. During the flame brush, the diameter of the fiber reduces from d_n to d_{n+1} . The distance $\mathbf{AB}$ has now increased by $(x + y)$. The diameter of the taper waist can be evaluated by using the mass and volume conservation laws; the knowledge of fluid mechanics is not necessary. However, it is assumed that the temperature is uniform over the heated part of the fiber.

During the movement of the flame, the volume V of the glass between $\mathbf{AB}$ has to be conserved,

$V_{n+1} = V_n$. From Fig. 4.5 the volume between the points **A** and **B** is

$$\begin{aligned} \text{before sweep} \quad V_n &= \pi L \left(\frac{d_n}{2} \right)^2, \\ \text{after sweep} \quad V_{n+1} &= \pi(L + x + y) \left(\frac{d_{n+1}}{2} \right)^2. \end{aligned} \quad (4.3)$$

The elongation y is gained by the pull at the right-hand side of the flame and can be calculated by the total heating time, $t = \frac{L}{u}$ and the pulling speed v :

$$y = \frac{L}{u} \cdot \frac{v}{2}. \quad (4.4)$$

The other part of the final elongation, x , is acquired by the pulling force at the left-hand side of the flame. Note that at this position the diameter has already reduced to d_{n+1} . Therefore the length of x is longer than y , determined by the relative cross-section,

$$x = \frac{L}{u} \frac{v}{2} \cdot \left(\frac{d_n}{d_{n+1}} \right)^2, \quad (4.5)$$

By putting Eqs. (4.4) and (4.5) in Eq. (4.3), the diameter reduction for a single pass is

$$d_{n+1} = d_n \sqrt{\frac{1 - \frac{v}{2u}}{1 + \frac{v}{2u}}}. \quad (4.6)$$

With help of Eq. (4.6) the necessary number of sweeps to reach any target waist diameter can be evaluated. In addition to the final waist diameter, it is important to control the length of the waist and the shape of the transition. The final waist length equals the hot-zone length of the final sweep. The length of the hot-zone can vary during the tapering process, and the variation determines the shape of the taper transition. A concise derivation of the dependence of the transition on the hot-zone variation is given in [97].

Adiabaticity

The taper transition can be used to insert light into the fiber waist. The insertion loss of is then minimized when the transition is adiabatic. In an adiabatic transition the fundamental core mode can match its distribution to the variation of the fundamental local mode whose shape is determined

by the local taper cross section. Otherwise power loss occurs due to coupling of the fundamental mode with propagation constant β_{co} to cladding modes with propagation constant β_{cl} [34, 99]. The loss is minimized when the slope of the transition satisfies the adiabatic criterion,

$$\left| \frac{dr}{dz} \right| \leq \frac{r(\beta_{co} - \beta_{cl})}{2\pi} \quad (4.7)$$

which gives an upper bound on the local taper angle $\left| \frac{dr}{dz} \right|$, defined by the length variation of the core radius r .

Figure 4.6: *Schematic of the taper setup. Two motorized stages are used to pull the fiber apart. The flame is delivered from a small opening and is mounted on a stage to ensure an adjustment of the flame height relative to the PCF.*

Setup

We conclude this section by outlining briefly the layout of the taper rig. A schematic is shown in Fig 4.6. The flame is supported by a small needle and was fueled by a mixture of butane and oxygen. The hot-zone part of the fiber is stripped and meticulously cleaned with isopropanol, preventing the formation of inhomogeneities in the temperature variation across the fiber. The cleaned fiber is attached to mounting stages, which move outwards and deliver the pull. In the course of this thesis, the PCFs were tapered "fast and cold", which is necessary to preserve the cross-sectional structure [98]. Therefore a good fixture is very important to avoid an unwanted slipping of the fiber during the tapering process. When tapering PCF, the height of the flame is crucial in order to avoid a collapse of the cladding holes.

Figure 4.7: Sketch of the dispersion measurement setup. The Mach-Zehnder interferometer consists of a delay line and contains the PCF with unknown dispersion. The beam splitters BS1 and BS2 mark the entrance and exit of the MZI. The interference signal is measured by the photodiode PD. MC is the monochromator.

4.3 Measurement of the group-velocity dispersion

It is well-known that the propagation dynamics of pulses are governed by the dispersion properties of an optical fiber [27]. PCFs can have regions of normal and anomalous dispersion (section 2.1.2.2), and the position of the ZDW is most crucial to nonlinear experiments (see section 3.3). Therefore the accurate characterization of the GVD profile is most advantageous.

Principle and schematic of the setup

The dispersion of a fiber can be derived by measuring the group velocity at different wavelengths, by help of a white-light interferometer. The layout of this device is sketched in Fig. 4.7. The principle of operation is based on a Mach-Zehnder interferometer (MZI). The MZI has two arms, labeled **A** and **B** in in Fig. 4.7. The white light has been generated by SCG in an endlessly single-mode fiber, pumped by a near-CW laser source (section 3.3.2). At the beam splitter BS1 the SC is splitted into two replicas. One part is sent into the MZI arm **A** and is traversing the PCF. The other part of the light is sent into the reference arm **B**, which contains a delay line. Note that arm **B** has two objectives, used to compensate the dispersion introduced by the objectives in arm **A**⁴. Polarization optics before and after the MZI is used to control the polarization state of the light. This is important when the fiber exhibits birefringence (section 2.1.2.4). The interference signal can then be unambiguously ascribed to one fiber axis.

The light in both arms combine at the second beam splitter BS2, and are then sent into an endlessly

⁴All objectives have been obtained from the same fabrication batch

Figure 4.8: *Interference signal as a function of the delay. (a) The wavelength was 850 nm. The top graph shows the fringe envelope and a fitted curve. (b) The wavelength was 493 nm, at the edge of the white-light source. The determination of the fringe envelope allows an accurate measurement of the fringe position.*

single-mode PCF (labeled ESM in Fig. 4.7). This fiber is used to enhance the overlap of the two beams — both are identical replicas of each other and can interfere. The maximum interference signal will be obtained when both parts overlap in time: the delay ΔT_A introduced by the PCF in arm **A** is compensated by the delay line in arm **B**. The photodiode is monitoring the interference signal as a function of the delay line position x . The photodiode is processed by a lock-in amplifier (LIA) and allows the detection of very weak light signals, for example at the visible edge of the supercontinuum, which is close to 490 nm. The LIA is triggered by a chopper. The monochromator is used to resolve the spectral distribution of the interference signal.

In Fig. 4.8 we show the recorded fringes, obtained during measurement of a highly nonlinear PCF. Fig. 4.8a is measured at 850 nm. The peak of the fringe is extracted from the fringe envelope. The exact position of the fringe can then be retrieved by fitting a Gaussian function to the envelope. This is especially helpful for weak signal, as it is shown in Fig. 4.8b, which is at a wavelength of 493 nm. The group-velocity dispersion can be retrieved from the wavelength dependence of the fringe position. The two pulse replicas arrive at the beam splitter BS2 at times ΔT_A and ΔT_B ,

which are given by

$$\begin{aligned}\Delta T_A &= \frac{L}{v_{group}(\lambda)} \\ \Delta T_B &= -\frac{2x}{c},\end{aligned}\tag{4.8}$$

relative to the path length of both arms. Here, x is the delay line position and c the speed of light. The position x_{max} of the maximum of the interference signal indicates temporal overlap. For $x = x_{max}$ we have $\Delta T_A = \Delta T_B$, leading to

$$\beta_1 = \frac{1}{v_{group}} = -\frac{2x_{max}}{cL}\tag{4.9}$$

Note that the measurement gives the group velocity relative to an arbitrary reference point. The wavelength dependence of x_{max} leads to a wavelength dependence of β_1 , and by differentiating β_1 with respect to λ gives the group-velocity dispersion of the fiber, $D = \frac{d\beta_1(\lambda)}{d\lambda}$. An example is given in Fig 4.9a, showing an experimental $\beta_1(\lambda)$ curve, obtained from a large-core fiber with a high air-filling fraction. These data points are differentiated by using a quadratic splines. Fig 4.9b presents the resulting GVD data points, obtained from the group velocity in Fig 4.9a. In addition we show a polynomial fit through the data points.

Figure 4.9: Measured inverse group velocity of a large-core fiber. The PCF structure is shown in the inset. The dots correspond to the measured data points and the solid line is a guide for the eyes. (b) The group-velocity dispersion curve, obtained from (a) by differentiation with quadratic splines. The solid line is a polynomial fit to the data points.

Chapter 5

Nonlinear pulse dynamics in dispersion-tailored PCFs

5.1 Introduction

In chapters 2 and 3 it was shown that photonic-crystal fibers (PCFs) are useful devices to manipulate the spectral and temporal characteristics of ultrashort pulses. As a result, PCFs entered many research fields, with applications such as supercontinuum generation, useful in biological studies and optical coherence tomography [13, 15, 100]. Besides, PCFs proved to be valuable devices for the compression and shaping of pulses [101, 102] and were employed in quantum optics [103, 104]. Recently, PCFs assisted in the observation of unconventional objects like the rogue waves [105, 106] and the Peregrine soliton [107]. However, the demand for more sophisticated light sources grows, pushing forward the development of PCFs with novel linear or nonlinear characteristics.

In this chapter, we discuss new ideas stemming from PCFs with engineered dispersion profiles or unusually large nonlinear coefficients. This chapter is sectioned in two parts. In section 5.2 we study the propagation dynamics of ultrashort light pulses in small-core PCFs. Small-core fibers have large nonlinear coefficients and can offer unconventional dispersion profiles, strongly governed by the core size. This allows one to take advantage of different nonlinear effects, leading to novel modifications on the temporal and spectral properties. In this section, we show novel pulse propagation dynamics resulting from the influence of the high nonlinearity. In section 5.3 we aim

to construct a microstructured cladding which modifies strongly the linear properties of the PCF — the GVD exhibits three zero-dispersion wavelengths (ZDW). We show that the existence of a third ZDW influences the propagation of pulses and introduces a new flexibility for the conversion of light into new spectral regions.

5.2 Propagation in highly nonlinear PCFs

This section covers pulse propagation in highly-nonlinear photonic-crystal fibers. Typically, those fibers have a small core, leading to a high nonlinear coefficient and therefore to a very short nonlinear length L_{nl} . As a consequence, the pulse shaping processes happen on a length scale shorter than the dispersion length L_{GVD} , and dispersive effects play a minor role in the propagation dynamics. In earlier studies, small-core waveguides were used to investigate the breakup of ultrashort pulses, soliton collisions and the formation of bound soliton pairs [79, 108]. In section 5.2.1 we demonstrate that the high nonlinear coefficient in PCFs is responsible for an anomalous pulse breakup. This can result in the generation of equally-spaced pulse trains. Then we add a unconventional dispersion profile by fabricating a submicron-core PCF. The broadening mechanism in this fiber and the consequences for wavelength conversion is discussed in section 5.2.2. Emerging from this novel PCF design are strong higher-order linear and nonlinear contributions. Their influence on the propagation of solitons is discussed in section 5.2.3.

5.2.1 Generation of pulse trains in small-core PCF

In the following we consider pulse propagation in a short piece of a small-core fiber. We demonstrate that a pulse breakup in a small-core fiber results in the formation of two highly chirped pulses. This process allows the generation of a train of ultrashort pulses. To begin with, we summarize the formation of spectro-temporal modulations emerging during copropagation of two optical pulses.

5.2.1.1 Linear effects of copropagating chirped pulses

Two pulses with the same carrier frequency ω_0 and a time delay $\Delta T = T_1 - T_2$ gives rise to spectral fringes with a period proportional to $1/\Delta T$. As an example, we show in Fig. 5.1a the spectral and temporal field envelope of two gaussian-shaped pulses, located at $\omega_0 = 2.35 \text{ rad/fs}$ (800 nm)

Figure 5.1: *SH FROG spectrograms of two (a) unchirped and (b) chirped ultrashort pulses at 800 nm, having a temporal delay of $\Delta T = 60$ fs. In (b), the chirp is $\phi_2 = 1000 \text{ fs}^2$, introduced in the frequency domain.*

and separated by $\Delta T = 60$ fs. These plots are placed around the SH FROG spectrogram (see section 4.1.3), a powerful tool to visualize the spectro-temporal behavior. The interference of the two pulses leads to spectral fringes at zero delay. The symmetrically distributed cross-correlation lobes at around 800 nm display the inter-pulse delay of $\Delta T = 60$ fs.

Interesting consequences arise when a frequency chirp is applied across the interfering pulses. Fig. 5.1b shows to pulses with a time delay $\Delta T = 60$ fs and a linear temporal chirp with magnitude $\phi_2 = 1000 \text{ fs}^2$ (see appendix A). This leads to strong modulations in the spectral and temporal domain. The temporal envelope of the light features a train of equally spaced, ultra-short pulses (see the graph on top of Fig. 5.1b). The chirp of the pulses is reflected in the spectrogram, showing a regular arrangement of peaks. The magnitude of the chirp can easily be estimated from the slope of the peaks. This mechanism can be exploited to generate equidistant ultra-short pulses in a small-core PCF.

5.2.1.2 Generation of pulse trains in PCFs

A train of intense ultrashort laser pulses is a powerful tool to probe a sample, permitting larger control over the specimen when compared to a single femtosecond pulse [109, 110]. As a result, the modulated laser light influences directly the dynamics of chemical and physical systems [111, 112]. For example, a train of ultrashort pulses can be used to control the molecular alignment and thus assist in chemical reaction dynamics, surface processing, and ultrafast optics [113]. Moreover, it was shown that pulse trains enhance molecular vibrations due to resonant excitation [114]. Con-

Figure 5.2: *GVD curve of the PCF shown in the SEM image. The dots are measured dispersion values. The line is a polynomial fit to the measured GVD points.*

ventional methods commonly used for shaping pulses employ spatial light modulators or acousto-optical modulators, but the complexity and relatively high cost of these systems are an obstacle to many applications. Other schemes for the generation of pulse trains were devised on the base of optical fibers. The formation of equally spaced trains of picosecond pulses was observed. Typically, GHz repetition rates were obtained in passive fiber cavities [115] and in dispersion-managed optical fibers [116].

Our approach to the generation of pulse train generation is based on the overlap of two chirped pulses. Two time-delayed, phase-shifted and frequency-chirped pulses can be a natural product of the pulse breakup occurring in the anomalous GVD region of small-core fibers [117]. As an advantage, the number of pulses and the inter-pulse spacing can be adjusted by the precise properties of the interfering pulses.

For the experiment, we used a silica-glass PCF with a core diameter of $1.2\ \mu\text{m}$. An SEM image of the fiber is shown in Fig. 5.2, in addition to the GVD profile, measured by white-light interferometry (see section 4.3). The highly nonlinear character of the PCF leads to a ZDW of around $689\ \text{nm}$ so that the pump wavelength lies within the region of anomalous dispersion. The nonlinear coefficient of the fiber is $120\ \text{W}^{-1}\text{km}^{-1}$. In the experiment we used a mode-locked Ti:sapphire laser, operating at $800\ \text{nm}$ — for further details refer to section 4.1. The laser beam was coupled into the PCF using a $60\times$ focusing objective, with launch efficiencies of around 50 %. We used short pieces with lengths ranging from 3 cm to 5 cm, and the output from the fibers was analyzed

Figure 5.3: Experimentally measured spectrogram of the output of a 4.3 cm long small-core PCF, for an input power of 8 kW. The feature in the IR shows the generation of an ultrashort pulse train. Measured by Dr. A. Podlipensky.

with a SH-FROG from APE GmbH (section 4.1). The light after the PCF was filtered with a long-pass filter. This is necessary because radiation in the visible spectral range adds an unwanted background noise to the FROG measurement. We controlled the polarization after the PCF by a Glan-Thomson polarizer and a $\lambda/2$ -waveplate. The dispersion length of the input pulse in the PCF is approximately $L_{\text{GVD}} = 20$ cm. In comparison, the nonlinear length L_{nl} for the peak powers used in the experiments was just a few mm. Since the dispersion length is much longer and the

Figure 5.4: (a) Retrieved temporal profile and temporal phase of the output pulse in Fig. 5.3. (b) Retrieved spectrum and spectral phase of the output pulse.

nonlinear length much shorter than the fiber length, amplitude and phase-modulation of the laser pulse is mainly affected by nonlinear propagation. In Fig. 5.3 we present the FROG spectrogram

of the light after propagating through a 4.3 cm long fiber, displaying strong spectral and temporal modulations. The input power was 8 kW. The FROG spectrogram shows that the output pulse has two main spectral components, located at 750 nm and 880 nm; they are a result of self-phase modulation (SPM) (see section 3.2.1.2). The cross-correlation lobes at around 800 nm indicate an inter-pulse temporal delay of ~ 400 fs. At the same time, the red-shifted FROG band exhibits a quite regular spectral and temporal structure. This feature coincides with the structure of Fig. 5.1, and shows the existence of several equidistant ultra-short pulses. The retrieved spectrum (Fig. 5.4a) and temporal profile (Fig. 5.4b) of the pulse indicates a spectral spacing (between peaks) of ~ 15 nm and a temporal delay of 70 fs between the pulses (~ 14 THz repetition rate). The FWHM of the single pulse is ~ 50 fs, which is at least two times shorter than the pulse duration at the input. Additionally we plot the spectral and temporal phases, and the parabolic shape indicates the linear frequency chirp of the pulse induced by SPM.

5.2.1.3 Physical mechanism of the train generation

We investigated the mechanism of the pulse train formation by solving the generalized nonlinear Schrödinger equation. We have introduced this equation in chapter 3, and we discussed the implementation into a numerical solver in appendix A. The dispersion operator was constructed from the measured dispersion curve in Fig. 5.2. The peak power was 6.4 kW. The deviation from the experimental value stems from the gaussian approximation of the nonlinear coefficient. The numerical temporal and spectral profile after 4 cm propagation is presented in Fig. 5.5. The simu-

Figure 5.5: *Simulated temporal (a) and spectral (b) envelope after 4 cm propagation through the small-core PCF. The launched peak power was 6.4 kW.*

lation shows the same strong modulation in the temporal and spectral domain as the experimental data, confirming the formation of ultrashort pulse trains in the near-IR. However, some differences are visible.

- Firstly, the time scale of the experimental (Fig. 5.4a) and the simulated (Fig. 5.5a) results are different. This is a consequence of the experimental output traversing several optical elements (mainly the lens and a Glan-Thomson polarizer). This resulted in an increase of the temporal delay when the spectrogram was recorded.
- Secondly, the spectrum of the simulated data extends further into the IR. This could be explained by the fact that the achromaticity of the outcoupling lens prevented the IR light from coupling into the FROG.

Nevertheless, the theoretical and the experimental findings show qualitative agreement.

The changes of the temporal pulse profile during propagation is plotted in Fig. 5.6a. The break-up of a pulse in the small-core PCF occurs on propagation distances much shorter than the dispersion length of the fiber. One finds that the input pulse undergoes a breakup at around 1.25 cm. For

Figure 5.6: (a) Simulated temporal evolution of the input pulse for an input power of 6.4 kW. (b) XFROG spectrogram of the pulse after propagation of 1.5 cm. The color scale is normalized and refers to (a) and (b).

longer propagation distances the pulse envelope is strongly modulated. The impact of the breakup is presented in Fig. 5.6b, showing the XFROG spectrogram at $z=1.5$ cm. At this stage, the pulse breakup transformed the initial pulse into two chirped pulses, labeled **A** and **B**. During further propagation the two pulses walk-off and the temporal delay results in a modulation, according to the discussion in section 5.2.1.1. The combination of the frequency chirp and the walk-off effect

are then responsible for the formation of ultra-short pulse trains.

5.2.1.4 Tuning range of the pulse trains

The frequency chirp and the time delay between the sub-pulses determine the spectral and temporal modulation period of the pulse train. In particular, the amount of chirp and the temporal walk-off can be controlled by the fiber and laser parameters. For example, the power of the input pulse controls the nonlinear phase shift the pulse acquires during SPM broadening. Fig. 5.7 shows the spectro-temporal dynamics when the power was lowered to 2 kW. The length of the fiber was 4.5 cm. A reduction of the peak power leads to a smaller spectral broadening and lowers the strength of the frequency chirp. Two consequences arise:

Figure 5.7: *Properties of an ultrashort pulse measured after propagating through a 4.5 cm long PCF. The launched peak power was 2 kW. (a) Temporal profile and temporal phase. (b) Pulse spectrum and spectral phase.*

- Firstly, the pulse train shifts to shorter wavelengths (as compared to Fig. 5.3) and is now around 840 nm. This can be seen in the pulse spectrum Fig. 5.7b.
- Secondly, the reduced walk-off between the pulses leads to a lower number of pulses in the train.

In Fig. 5.7a we show the associated temporal field envelope. It can be seen that two pulses are present, separated by a time delay of approx. 70 fs. The repetition rate of the train and the duration of the pulses are not influenced by the pulse power.

In contrast, a change of the fiber parameters results in a change of the pulse train characteristics. Here, instead of using a different PCF, we simply increased the wavelength of the light to 1200 nm.

The input pulse is thus launched far in the anomalous GVD region of the fiber. The dispersion

Figure 5.8: *Simulated (a) temporal and (b) spectral field envelope of a pulse propagating in a 6 mm long fiber. The launched peak power was 32 kW, launched at 1200 nm.*

length is reduced to 9 cm. We simulated the propagation of a 130 fs pulse in the fiber with a peak power of 32 kW, corresponding to a nonlinear length of just 0.3 mm. Therefore a fiber length of 6 mm was sufficient. The high power accelerates the pulse breakup, occurring after around 3 mm propagation in the PCF. The temporal and spectral profile is plotted in Fig. 5.8, showing a train of four ultrashort pulses. The inter-pulse delay is ~ 30 fs. The FWHM duration of the single pulses is around 8 fs. The number of pulses building the train can be easily changed by the input power. For example, for an input power of around 20 kW the train consists of two distinct pulses. This could be very useful for pump-probe experiments. The designability of the PCF allows then to create the pulse train in a desired spectral range.

5.2.2 Enhanced UV generation in submicron-core PCF

PCFs have the virtue of great flexibility and their properties can be widely tailored. It is an excellent device to confine light to a very small area, and to fabricate unconventional dispersion landscapes. For the study in this section, we combine both advantages and engineered a submicron waveguide, embedded in a high air-filling cladding. This results in an anomalous dispersion region spanning the visible spectral range. The fiber dispersion plays a role in governing the pulse propagation mechanism. In case of anomalous dispersion, solitons can exist and control the supercontinuum generation process (see section 3.3). In this section we discuss the impact of a high nonlinear coefficient on the dynamics of pulses sent into regions of narrow anomalous dispersion. We show the usage of submicron-core PCFs for wavelength conversion.

5.2.2.1 Generation of a submicron-core PCF

The submicron-core waveguide was fabricated from a fused silica PCF with a core diameter $1.23\ \mu\text{m}$ and an air-filling fraction of 85 %. We tapered the fiber down to a core diameter of 590 nm using the flame-brush technique presented in section 4.2. Fig. 5.9 shows scanning electron micrographs of the original and tapered fibers. The tapering process introduced very little change in the fiber geometry, so that the high air-filling fraction was preserved. The GVD was measured experimentally using white-light interferometry, showing that the anomalous range lies in the visible between two closely-spaced zero dispersion wavelengths at 509 nm and 640 nm (Fig. 5.9). Ultrashort pulses in the visible spectral range were launched into the PCF. The wavelength of the light was tuned across the anomalous dispersion region. The laser source was a non-collinear optical parametric amplifier (NOPA) with a tuning range from 490-700 nm (see section 4.1.2). A schematic of the setup is shown in Fig. 4.2. The power of the laser beam was adjusted with a neutral-density filter. The autocorrelator and the FROG system were adjusted for measurements in the visible spectral range. Since our small core PCF was slightly birefringent (modal birefringence of 7.5×10^{-4}) we used a half-waveplate to align the laser polarization parallel to one of the main axes. The laser beam was coupled into the PCF using a $40\times$ aspherical focusing lens with coupling efficiencies up to 25 %. The length of the tapered fiber was 5 cm. To increase the coupling efficiency we launched the light into an untapered part of the fiber with a length of 2.5 cm. This led to a modification of the input pulse as demonstrated in [118]. We do not consider the input pigtail in our simulations since we focus on the effects of the two ZDWs only. For low powers

Figure 5.9: Experimentally measured dispersion of the original and tapered fibers. The inset shows an SEM of the tapered PCF structure, superimposed at the same scale on top of and SEM of the original structure. The core diameter in the tapered fiber is $\sim 0.59 \mu\text{m}$.

(less than 1 kW) the influence of the input pigtailed is negligible. Deviations of the experimental from the simulated data (especially for high power) can be traced back to the untapered input fiber. Previous studies of SCG in fibers with submicron core showed that the second ZDW has a strong influence on the spectrum. For example in a work by Gattass et al. [119], pulse propagation in fibers with different taper waist diameter was investigated. It was found that the maximal spectral broadening were obtained for core diameters between 500 nm and 700 nm, which also correspond to the maximum of the nonlinearity.

5.2.2.2 Pumping at peak of anomalous dispersion

In the following, we study the existence of a second ZDW very close to the first one in a fiber with submicron core. This is done by launching light at different wavelengths into the fiber and investigate the broadening mechanism as well as the conversion efficiency into the UV and IR. We measure the power dependence of the pulse broadening and plot numerical XFROG spectrograms (see section 3.3) as well as the length evolution in the device to clarify the physics of spectral broadening.

We begin by tuning the NOPA output to 575 nm (FWHM pulse duration ~ 32 fs before the in-coupling lens), corresponding to the peak of the dispersion profile (Fig. 5.9). The dispersion at this wavelength ($\beta_2 = -8.8 \text{ ps}^2/\text{km}$) results in a comparatively short dispersion length of $L_{\text{GVD}} = 3.8 \text{ cm}$, and the absence of third-order dispersion means that the third-order dispersion

Figure 5.10: *Experimental (a) and numerical (b) power dependent evolution of the spectrum of a pulse launched at 575 nm. The horizontal dashed black lines show the ZDWs at 509 and 640 nm. The corresponding soliton order of the input pulse is given along the top of the figure. The color scale is in dB and applies to all subsequent plots.*

length is infinite. The experimental and simulated power dependence of the SC spectrum are presented in the Fig. 5.10, showing quite good agreement. It is striking that the spectral evolution is highly symmetric around the zero third-order dispersion wavelength. Initially the pump pulse experiences symmetric broadening as a result of self-phase modulation (SPM). At a peak power of 200 W (soliton order $N \sim 2$), two additional bands simultaneously appear at 460 nm and 740 nm; these can be assigned to dispersive waves. Further increase in the power leads to a continuous blue shift to 400 nm. In contrast, the spectral evolution of the red band is quite complex. First, with increasing power several additional peaks appear close to 800 nm. Then, for powers higher than 600 W, a peak at 750 nm starts to dominate and grows with further increase in power. At this stage we observe almost complete conversion of the pump energy into dispersive waves, in both the experiment and the simulations (Fig. 5.11). Pumping at the maximum of the GVD curve was conducted recently by Benabid et al. [120]. Mediated by the fourth-order dispersion only, the spectrum was mainly generated by soliton dynamics like fission and dispersive wave generation. A huge part of the energy remained around the pump wavelength, although dispersive radiation was detected at 470 nm and 1570 nm. However, in our experiment much stronger dispersive radiation is observed, located much closer to the pump wavelength. The smaller frequency separation (around $\Delta\nu = 160$ THz) of the dispersive wave resonances to the pump frequency leads to a stronger energy transfer into the dispersive waves. We find that an almost full transfer of the energy into dispersive waves is possible. We can identify at least three distinct regimes of

Figure 5.11: *Experimental (a) and numerical (b) spectra after the fiber for a launched pulse with $\lambda_0=575$ nm (horizontal solid line), $P_0 = 3.5$ kW and $T_0 = 32$ fs. The vertical dashed black lines show the ZDWs at 509 and 640 nm.*

spectral broadening in Fig. 5.10: (1) low power (below 100 W), where the spectral broadening is caused by SPM and the whole pump energy remains within the anomalous spectral range; (2) intermediate power (from 100 W to 2 kW), where dispersive radiation appears on both sides of the pump wavelength; and (3) high power, where complete energy transfer to dispersive waves is observed. To clarify the mechanisms behind these effects, we have studied the spectral evolution with distance for different power levels. Figs. 5.12a&b show simulated spectra as a function of propagation distance for an input power of 270 W (intermediate) and 10 kW (high power). The numerical XFROG spectrograms in Fig. 5.12c&d show the spectro-temporal profile at the output of a 5 cm long tapered fiber. As can be seen in Fig. 5.12a, the spectral evolution at lower power is dominated by soliton-related dynamics. Shortly after injection, the pulse spectrum spreads periodically, indicating the breathing behavior characteristic of higher-order (bound) solitons. At each cycle, radiation is shed into dispersive waves on both sides of the ZDWs. This leads to stepwise conversion from the pump to pulsed dispersive waves [68], as can be seen in the XFROG spectrogram of the pulse after propagation along a 5 cm taper (Fig. 5.12c). In this case a pair of bound solitons can be seen at 579 nm, which is responsible for the breathing behavior of the spectrum and for radiation into dispersive waves in the blue and red spectral ranges. The dispersive waves at 419 nm and 745 nm have pulse durations of 200 fs and are temporally shifted from the solitons by 300 fs, confirming that they are generated when the spectral width of the soliton is maximum. The result is trains of dispersive wave fs pulses in the red and blue spectral ranges. As shown in [68], the soliton self-frequency shift (SSFS) shifts the wavelength of the dispersive waves, leading to the

Figure 5.12: Numerical spectral evolution of a pulse launched at 575 nm with $P_0 = 270$ W (left column) and $P_0 = 10$ kW (right column). The XFROG spectrograms after 5 cm of propagation are shown in (c) and (d). The horizontal dashed black lines indicate the ZDWs at 509 and 640 nm. Color scale is the same as in Fig. 5.10.

generation of several spectral bands in the red part of the spectrum (Figs. 5.10a&b) corresponding to each breathing cycle. At high peak power, the mechanism of spectral broadening and frequency conversion to the blue and red is quite different. Fig. 5.12b shows the spectral evolution of a 10 kW laser pulse propagating through 10 cm of tapered fiber. The nonlinear length is now very short in comparison to the dispersion length. The spectral width of the pulse spreads very fast, reaching its maximum after several mm of propagation before any soliton formation can set in. The length at which excessive broadening occurs is now much shorter than the dispersion length. Therefore, in contrast to the previous case, SPM now plays crucial role in the broadening mechanism. SPM produces a spectrum with two main side-lobes, which spread out on both sides into the normal GVD spectral ranges, leaving only a small amount of energy in the anomalous GVD range — not enough to form solitons. In the XFROG spectrogram after 5 cm of propagation (Fig. 5.12d) the whole pulse energy is now concentrated in blue and red spectral ranges with normal GVD, where the pulses disperse and eventually form the observed spectra, without any contribution from soliton dynamics. To summarize, the power of the pulse dictates the strength of the SPM shift and therefore, if the formation of solitons is possible or not.

5.2.2.3 Pumping close to short wavelength ZDW

In the next experiment we tuned the NOPA to 523 nm, close to the short-wavelength ZDW (509 nm). The duration of the pulse before the in-coupling lens was measured to be 39 fs (FWHM). At this frequency $\beta_2 = -3.6 \times 10^{-27} \text{ s}^2/\text{m}$, resulting in a dispersion length $L_{\text{GVD}} = 0.15 \text{ m}$. Fig. 5.13 shows good agreement between the experimentally measured and numerically modeled spectral evolution of the SC with increasing peak power. Just as in the previous case, increasing the input power leads first to SPM broadening of the pump pulse. At $\sim 230 \text{ W}$ the spectrum shows a band of dispersive radiation at 770 nm. At 800 W a second band of red radiation appears at 710 nm. At the same time the blue edge of the spectrum shifts to 420 nm. With further increase in power the intensity of the blue band grows and the edge of the spectrum shifts to shorter wavelength. In contrast, the red band shows a more complicated behavior. The two dominant spectral features, which can be traced back to resonant radiation and the compensating radiation due to the SSFS cancellation (see section 3.2.3), merge into a single spectral band at $\sim 750 \text{ nm}$ for powers higher than 2 kW ($N \sim 15$). Tapered fibers with a ZDW in the blue and one in the red were used in a study by Leon-Saval et al. [121]. The restriction to nanosecond pulses missed the opportunity to

Figure 5.13: *Experimental (a) and numerical (b) power dependent evolution of pulses launched at 523 nm. The horizontal dashed black lines show the ZDWs at 509 and 640 nm. The corresponding soliton order of the input pulse is given along the top of the figure. Color scale is the same as in Fig. 5.10.*

observe soliton dynamics and the associated dependence on power. The spectral broadening was governed by four-wave mixing. The generated spectrum was flat and extended over 500 nm, with a significant part of the energy located around the pump wavelength. The spectra measured in our experiment are generated by different mechanisms and are asymmetric, with most of the energy in the UV dispersive wave. The remarkable dispersion of the fiber and the small core can even lead to the complete depletion of the pump. The proximity of the ZDW at 640 nm to the blue ZDW prevents an excessive broadening into the IR. In the following we show the complex dynamics of pulse broadening during increase of the power. We once again study the spectral evolution with distance for different power levels, considering both intermediate (300 W) and high (10 kW) power cases (Figs. 5.14a&b). For low power, SPM dominates the dynamics of pulse broadening in first few cm. The spectrum develops two side-lobes, one traveling in a regime of anomalous GVD and the other entering a regime of normal GVD in the blue, where it disperses. The combination of weak dispersion and huge nonlinearity is responsible that one SPM lobe of the pulse is transformed rapidly into a soliton, even for low input peak power (since the soliton order N goes as $N^2 \propto P\gamma/|\beta_2|$, see section 3.2.2). At the moment when the spectral width of the pulse is maximum, dispersive radiation appears at 776 nm. This marks the intermediate power regime, where the pulse forms a soliton in the anomalous GVD spectral range. This soliton undergoes SSFS and overlaps with the dispersive wave resonances. The XFROG spectrogram of the pulse (Fig. 5.14c) shows the generation of a dispersive wave at 776 nm. At the same time, a band of radiation at 445 nm (within

Figure 5.14: Numerical spectral evolution of a pulse launched at 523 nm with $P_0 = 300$ W (left column) and $P_0 = 10$ kW (right column). The XFROG spectrograms after 5 cm of propagation are shown in (c) and (d). The horizontal dashed black lines show the ZDWs at 509 and 640 nm. Color scale is the same as in Fig. 5.10.

the regime of normal GVD), created from the pump pulse by SPM, broadens with time. After 20 cm of propagation the soliton approaches the second ZDW at 640 nm (Fig. 5.14a), resulting in the appearance of a "fin-shaped" feature at 700 nm, caused by SSFS cancellation. However, in contrast to the dispersive wave at 776 nm, here the dispersive wave is emitted continuously. Both dispersive waves at 700 nm and 776 nm are present in the experimental and calculated spectra in the Fig. 5.13. For high power the SPM acts on the outermost lobes of the pulse and shifts the majority of the pulse energy out of the narrow anomalous dispersion regime. This leaves to little energy left in the anomalous dispersion regime to form a soliton. In this respect, the mechanism of spectral broadening for the high peak power regime (Fig. 5.14b) is similar to that seen when pumping at the GVD maximum (Fig. 5.12b). The nonlinear length is very short compared to the dispersion length. The spectrum of the pulse is therefore able to broaden extensively before any soliton formation sets in. Strong broadening sets in at distances much shorter than the dispersion length. Under these circumstances, SPM dominates over soliton formation. The frequency separation between the outermost peaks is proportional to the peak power. Thus the spectrum reaches its maximum width after a few mm of propagation, showing two bands at 400 nm and at 750 nm in the range of normal GVD. Meanwhile, the spectro-temporal profile of the pulse suffers strongly from third-order dispersion and self-steepening, resulting in asymmetric spectral broadening with an intensity maximum on the blue side. The corresponding XFOG spectrogram (Fig. 5.14d) shows temporal spreading on the blue and red sides of the pulse after 5 cm propagation. This results in a decrease in the pulse intensity with propagation distance, after which the spectrum does not change.

5.2.2.4 Pumping close to the long-wavelength ZDW

Finally, we tuned the NOPA to the second ZDW, located at 640 nm. The FWHM pulse duration in this case was 38 fs. The dispersion length in the fiber is infinite, since $\beta_2 \sim 0$. The experimental and modeled power dependence of the spectrum is shown in the Fig. 5.15a and Fig. 5.15b. In general the simulation reproduces the spectral width of the SC. However, some differences are apparent, especially in the anomalous GVD range. This can be explained by the initial untapered input part, having the ZDW at 636 nm. For this case, we implemented a short input pigtail and transition (overall length 2 cm) in our simulations and show the result in Fig. 5.16. From the simulations we found that for powers up to 1 kW the pulse undergoes temporal compression. This prevents

Figure 5.15: *Experimental (a) and numerical (b) power dependent evolution of pulses launched at 640 nm. The horizontal dashed black lines show the ZDWs at 509 and 640 nm. The corresponding soliton order of the input pulse is given along the top of the figure. Color scale is the same as in Fig 5.10.*

the experimental observation of the SPM splitting for increasing power, visible in the simulation in Fig. 5.15b for $P < 1$ kW. For higher power, the pulse starts to broaden considerably. This leads to the development of a strong red shoulder for $P > 2$ kW. The simulated results, including the input pigtail, recovers the experimental measurement very accurately. However, in the following we neglect the effect of the pigtail and focus on the taper waist.

Launching pulses at the ZDW of a fiber typically results in spectral splitting due to SPM. Harbold et al. [122] conducted experiments by launching pulses about the second ZDW of a fiber. They observed a broad SC which is centered at the pump wavelength. In this work, a large depletion of the central region was observed and the spectrum spans 700 nm. In contrast, we observe a clear difference since the blue ZDW is very close to the ZDW we pump at. One major consequence is the development of dispersive radiation and therefore a pronounced asymmetry in the spectrum, likewise in the previous case. Similar to pumping at 523 nm, at low peak power the broadening leads to a transfer of pulse energy into the normal GVD spectral range. This results in an extension of the red spectral edge. The other part of the pulse travels in the anomalous GVD range, showing a shift to shorter wavelengths with increasing input power. The phenomenon of the soliton blue shift has been observed recently in small core PCF [123] and is caused, as we will show later, by SPM broadening during the initial stages of propagation. At 350 W, we observe the occurrence of two bands at 430 nm and 750 nm in the normal GVD spectral range. These bands can be assigned to dispersive waves, and grow in strength with increasing input power. However, the intensity of

Figure 5.16: Numerical power dependent evolution of pulses launched at 640 nm. In addition to the 5 cm long taper waist, a 2 cm long input pigtail and associated transition was included. The horizontal dashed black lines show the ZDWs of the taper waist and are at 509 and 640 nm. The corresponding soliton order of the input pulse is given along the top of the figure. Color scale is the same as in Fig. 5.10.

the band at 750 nm rises more steeply than the dispersive wave in the blue part of the spectrum. This eventually leads again to a complete conversion of the pump power into the dispersive waves. The Fig. 5.17a demonstrates evolution of the pulse spectrum with the propagation length at a peak power of 655 W. Just as in the two previous cases, SPM causes side-lobes to appear in the spectrum over the first few cm. One side-lobe travels again in the anomalous GVD spectral range and the other enters the range of normal GVD, which in this case is in the red part of the spectrum. After propagating 2.5 cm in the fiber one SPM side-lobe (in the anomalous dispersion range) shows spectral broadening and we find the onset of accompanying dispersive radiation at 408 nm. The side-lobe exhibits solitonic features but is still blue-shifting during further propagation. This corresponding to the above mentioned case of the soliton blue shift. At 5 cm the pulse achieves its maximum spectral width, with lobes at 550 nm and 720 nm. At this moment two additional bands appear in the spectrum at 425 nm and 780 nm, associated with dispersive waves corresponding to the soliton-like pulse at 550 nm. The XFROG spectrogram in the Fig. 5.17c shows that the dispersive wave is generated as a pulse at the same moment when the spectrum of the input pulse has maximum width. The lobe in the anomalous GVD range splits into two solitons (Fig. 5.17a). One of these experiences SSFS, resulting in the appearance of an additional dispersive wave at ~ 660 nm. This demonstrates that the unique combination of weak dispersion and high nonlinearity

Figure 5.17: Numerical spectral evolution of a pulse launched at 640 nm with $P_0 = 655$ W (left column) and $P_0 = 10$ kW (right column). (c) and (d) show the spectrogram after 5 cm of propagation. The horizontal dashed black lines show the ZDWs at 509 and 640 nm. Color scale is the same as in Fig. 5.10.

in our fiber rapidly forms a soliton out of one of the SPM sidelobes, responsible for the observed soliton features in the spectra. For high power (Fig. 5.17c) the pulse reaches its maximum spectral width after a very short distance. This is also in contrast to earlier observed spectra [122, 124]. In our experiment, we show that it is possible to transport energy over the anomalous dispersion regime into the normal dispersion regime in the blue due to the strong SPM shift. The spectral lobes caused by SPM are transferred into the normal GVD range within just a few mm. As a result, the associated pulses disperse in time, as shown in the XFROG spectrogram after 5 cm propagation (Fig. 5.17d). This decreases the peak power of the pulses, after which the spectra do not change further. At the same time, a small amount of pulse energy appears in the anomalous GVD range, where we observe formation of very weak soliton pulses, which do not affect the spectral evolution.

5.2.2.5 Efficiency of conversion

Figure 5.18: Power dependent conversion efficiencies for pulses with $\lambda_0 = 523$ nm (blue line), $\lambda_0 = 575$ nm (green line) and $\lambda_0 = 640$ nm (red line). (a) shows conversion into the UV (350 to 470 nm), (b) shows conversion into the IR (700 to 900 nm).

The above discussion shows that the broadening mechanism depends more on the input power than on the excitation wavelength. However, the conversion efficiency into the blue or red spectral ranges can be controlled by tuning the wavelength of the laser pulse. In Figs. 5.18a&b we present experimental data for the conversion efficiency into the blue (350 to 470 nm) and red (700 to 900 nm) spectral bands. A conversion efficiency of 40 % the blue dispersive waves is achieved at peak powers greater than 3 kW for excitation at 523 nm (close to the short-wavelength ZDW). On the other hand, for the same powers a maximum conversion efficiency of 80 % to the red band is

observed for excitation at 640 nm, close to the second ZDW.

5.2.3 Soliton collisions and enhanced dispersive wave generation in PCFs

In the previous section we showed that ultrashort pulses, launched in the region of anomalous dispersion between the two ZDWs, give rise to two dispersive waves, blue- and red-detuned from the pump wavelength. The spectral broadening depends on both the pump power and the width of the anomalous dispersion region, showing either predominantly soliton-like dynamics (at low power) or pure self-phase-modulation (SPM) broadening (at high power). When solitons exist in the anomalous dispersion region, the spectral extent of the SC is determined by the dispersive waves. The narrowness of the anomalous GVD region between the two ZDWs confines solitons spectrally, enhancing the likelihood of soliton interactions. The collision of two red-shifting solitons, in the presence of higher-order dispersion terms can result in the generation of a dispersive wave with magnitude much larger than that emitted by a single soliton [125]. Breakup of a launched ultrashort pulse results in the generation of two solitons followed by pulsating soliton dynamics, accompanied by bursts of dispersive wave radiation. SSFS cancellation (see section 3.2.3) at the red ZDW induces a series of soliton collisions, triggering the growth of strong dispersive waves in the IR. The proximity of the blue ZDW at 509 nm guarantees enhancement of the dispersive wave band in the UV.

5.2.3.1 Linear and nonlinear properties of the PCF

The numerical study is based on the GVD curve of the tapered submicron-core PCF used in section 5.2.2. The highly nonlinear PCF had a core diameter of 590 nm. An SEM image of the taper waist can be found in Figs. 5.19a. The GVD spectrum as a function of frequency is presented in Fig. 5.19a, showing ZDWs at 468.4 THz (640 nm) and 589.0 THz (509 nm). Fig. 5.19b plots the frequency dependence of the nonlinear coefficient γ of the PCF, calculated using the plane-wave method (see section 2.1.2.2). The magnitude of γ is highest in UV, falling off in the visible and infrared due to an increasing effective mode area (see section 2.1.2.1). The propagation of pulses in the PCF was simulated by the GNLSE (chapter 3). The nonlinear coefficient within the anomalous dispersion band is well approximated by a linear frequency dependence

$$\gamma(\omega) = \gamma_0 + \omega\gamma_1, \quad (5.1)$$

with $\gamma_0 = 0.285 \text{ W}^{-1}\text{m}^{-1}$ and $\gamma_1 = 1.5 \times 10^{-16} \text{ sW}^{-1}\text{m}^{-1}$, see Fig. 5.19b.

Figure 5.19: (a) Group velocity dispersion of the PCF on which our numerical study is based. The tapered PCF has ZDWs at 468.4 THz (640 nm) and 589.0 THz (509 nm). The inset shows SEM images of the tapered fiber. The wavelength of the pump was chosen to be at the maximum of the dispersion curve, i.e. 521.4 THz (575 nm). (b) Frequency dependence of the nonlinear coefficient. The slanted dotted line visualizes the linear approximation of the nonlinear coefficient. The vertical lines are the ZDWs of the PCF.

5.2.3.2 Anomalous pulse break-up

We treat the case when the pump pulses are tuned to the peak of the dispersion curve, thus avoiding the effects of third-order dispersion (TOD). The pulse duration $T_{\text{FWHM}} = 35$ fs, yielding a second-order dispersion length $L_{\text{GVD}} = 3.8$ cm and a fourth-order dispersion (FOD) length $L_{\text{FOD}} = 1.3$ m. The pump pulse energy used corresponds to soliton order $N = 4$. In Fig. 5.20 the evolution of the temporal pulse shape with propagation length is shown. Since the TOD is negligible, higher-order soliton effects dominate the pulse dynamics, showing first pulse compression, followed by splitting into two and then three distinct peaks. The periodic recurrence of a $N = 4$ soliton in the PCF without the frequency dependence of the dispersion and nonlinearity is shown in Fig. 5.21. The soliton reaches its initial profile at the soliton length $L_{\text{sol}} = 6$ cm. During the soliton evolution, SPM broadens the spectrum significantly, creating a pair of symmetrically-detuned spectral side-lobes. The strong fourth-order dispersion and nonlinearity slope (β_4 and γ_1) exert different dispersive and nonlinear forces on each side-lobe, pulling them apart temporally. This can be seen in Fig. 5.20, where after breakup two branches appear, each moving at the same velocity but in opposite directions.

After pulse breakup, the peaks separate from each other and evolve into fundamental solitons, as seen in Fig. 5.20a, where the temporal profile at a distance of 5.5 cm is plotted. The presence of

Figure 5.20: Simulated temporal evolution of a pulse with carrier frequency at the maximum of the dispersion curve. (a) Pulse breakup into three solitons after 5.5 cm of propagation. The red curve shows the solitons when the dispersive radiation is filtered out. The soliton characteristics are given in Table 5.1. (b) Soliton collision after 27 cm.

Figure 5.21: (a) Temporal and (b) spectral evolution of a fourth-order soliton in a fiber with constant $\beta_2 = -1.05 \times 10^{-26} \text{ s}^2/\text{m}$. In (a) the color scale refers to the power, normalized to the input power. In (b) the color scale is in dB.

Soliton	delay	duration	wavelength	soliton order
A	-70.6 fs	9.72 fs	557 nm	0.95
B	0 fs	7.03 fs	589 nm	1.19
C	30.4 fs	10.78 fs	584 nm	1.17

Table 5.1: *Parameters of the solitons shown in Fig. 5.20. The delay is relative to soliton B.*

three solitons can best be seen when the strong resonant radiation component is extracted from the field by spectrally gating the light in the anomalous dispersion region. This allows determination of the precise characteristics of each soliton, given in Table 5.1. It is interesting to note that reducing the input power leads to remarkably different dynamics. Fig. 5.22 shows the evolution when the soliton order is reduced to $N = 2.7$. The spectra of lower-order solitons broaden less and the higher-order contributions do not break the distinct peaks apart. The soliton is thus able to finish the conventional soliton breathing, leading to pulsating emission of resonant radiation [68]. This can be seen in the spectral evolution in Fig. 5.22b.

Figure 5.22: (a) *Temporal and (b) spectral evolution of a soliton with $N = 2.7$.*

5.2.3.3 Soliton collision

The soliton dynamics in the two-ZDW system (Fig. 5.20) differs markedly from the conventional case. The ZDW at 640 nm is responsible for cancellation of the SSFS, which limits the red shift of the solitons. This leads to frequent interactions between the solitons **B** and **C**, accompanied by the shedding of dispersive radiation. To understand this phenomenon, the propagation of two pulses

Figure 5.23: Propagation of two pulses simulated with (a) identical (590 nm) and (b) differing (580 and 590 nm) wavelengths. The initial temporal delay was 30 fs. For (a), the individual peak powers were 250 W each, for (b) the peak powers were 250 W and 300 W. We refer to the soliton interaction of (a) as complete collision and of (b) as incomplete collision.

is simulated, separated by a time-shift corresponding to the pulse delay in Table 5.1. In Fig. 5.23a the dynamics of two solitons with the same carrier frequency are shown, separated by 30 fs. The well-known attraction between in-phase solitons is seen, causing them to collide periodically as they propagate [126]. If the carrier frequencies of the two solitons differ, however the interaction is weaker, since their relative phase changes with time (Fig. 5.23b) [126]. In Fig. 5.20 we observe that solitons **B** and **C** undergo incomplete interactions at 8, 14 and 18 cm, the mechanism of which can be understood as follows. After being generated at the ~ 5 cm point (see Fig. 5.20)), the solitons have different carrier frequencies and propagate with different group velocities. The red ZDW cancels the SSFS and prevents the temporal and spectral walk-off of solitons **B** and **C**. The differing wavelengths of the two solitons cause several incomplete collisions. At longer propagation distances both solitons have the same carrier frequency due to SSFS cancellation and a complete collision according to Fig. 5.23a takes place. This is demonstrated in Fig. 5.20b. During the collision a strong burst of dispersive wave radiation appears in two bands, with frequencies determined by the phase-matching condition Eq. (3.31). In our case, the energy loss is strong enough to prevent both solitons from reappearing, with the result that only one soliton survives. The multiple soliton collisions are accompanied by multiple bursts of dispersive radiation, which transfer energy into the UV and IR. This is demonstrated in Fig. 5.24, where the length dependence of the conversion efficiencies into the UV (400 nm to 440 nm) and IR (680 nm to 800 nm) are shown. Each collision causes a step increase in the conversion efficiency to both bands, with

values as high as 22 % for the UV and 21 % for the IR.

Figure 5.24: Conversion efficiencies into the UV (400 to 440 nm) and IR (680 to 800 nm). Both curves show a step-wise efficiency increase, occurring when soliton interaction takes place. The step at 27 cm is due to equal-frequency soliton collision. The slanted arrows indicate the collision-induced step-enhancements in the dispersive wave signals.

5.3 Unconventional wavelength conversion in PCF with three zero-dispersion points

So far we made heavy use of the dispersion tunability of PCFs and generated fibers with a small core, surrounded by a cladding with a high-air filling fraction. We saw that the tailored GVD curves proved very useful for the efficient and systematic manipulation of ultrashort pulses, yielding ultrashort pulse trains, a large energy transfer into the UV or allowed a controlled collision of solitons. The dispersion was mainly determined by the core size and the core material, very similar to a silica strand. However, the cladding of the submicron waveguide prevented an interaction of the guided mode with the environment.

The versatile stack-and-draw technique of the PCF fabrication has enabled the production of fibers with previously unattainable core or cladding geometries. For example, it was demonstrated that a cladding structure with concentric rings of differently sized air holes can lead to a third ZDW in the transmission window of silica [127]. In another work, it was found that a small bore in the center of the core leads to a very unusual waveguide contribution to the overall curve, resulting in as many as four ZDWs [128].

In this section, we are going to show that even a simple ESM PCF can be designed to yield, not just two, but three ZDWs. The presence of a third dispersion zero creates a rich phase-matching topology, enabling enhanced control over the spectral locations of the four-wave-mixing (FWM) and resonant-radiation bands emitted by solitons and short pulses. The greatly enhanced flexibility in the positioning of these bands has applications in wavelength conversion, supercontinuum generation and pair-photon sources for quantum optics.

5.3.1 Dispersion characteristics

Some years ago it was shown that the structure of small-hole endlessly single-mode PCF can be tailored to provide an ultraflat GVD profile [26]. The cladding of the PCF had a hexagonal lattice with holes of diameter $d \sim 0.6 \mu\text{m}$, requiring many rings to keep the confinement loss low, see section 2.1.2.3. By choosing appropriate values of the hole diameter d and inter-hole spacing Λ , the shape of the GVD can be flattened in the 1550 nm wavelength range. A broad-band single-mode operation has advantages in applications where well-controlled interactions between signals at widely different wavelengths are required, such as optical parametric amplifiers [129] and in

supercontinuum-based infrared communications [130]. In this section we are going to show that this type of PCF can also exhibit three ZDWs, with the result that the PM curves not only exhibit additional phase-matched frequencies, but have a complicated algebraic topology.

In Fig. 5.25 we show numerical calculations of GVD profiles, obtained using the fixed-frequency plane wave (FFPW) method (see section 2.1.2.2). The fiber geometry follows closely the structure of the fiber reported in [26], where it was experimentally shown that the dispersion curve is very flat and small in magnitude over a broad wavelength region. By carefully choosing the inter-hole spacing at a hole diameter of 600 nm we have found that the GVD curves have three ZDWs (see Fig. 5.25). We will show that the PM wavelengths depend on the relative degree of asymmetry between the widths $\Delta\nu_a$ and $\Delta\nu_n$ of the anomalous and normal dispersion regions, parameterized by $\alpha = (\Delta\nu_n - \Delta\nu_a)/\Delta\nu_n$. The value of α is very sensitive to the pitch, and can be both positive and negative (see Fig. 5.25). The curves labeled A–C in Fig. 5.25 are the GVD curves used in the following PM study, showing that an increase of the pitch entails to a increase of α .

Figure 5.25: Dispersion profiles for different inter-hole spacings at hole-to-pitch ratio $d/\Lambda = 0.12$. The red arrow marks the general trend for increasing pitch (from $2.5\ \mu\text{m}$ to $2.7\ \mu\text{m}$). A–C labels the curves used for the PM calculations, with $2.6\ \mu\text{m}$ (A), $2.612\ \mu\text{m}$ (B) and $2.62\ \mu\text{m}$ (C). The shading indicates the anomalous dispersion regions and curve B is used to define the widths $\Delta\nu_a$ and $\Delta\nu_n$.

5.3.2 Four-wave mixing

Phase-matching topology

In degenerate FWM, energy conservation determines the frequencies of the signal (anti-Stokes, frequency ν_{AS}) and idler (Stokes, frequency ν_S) waves generated from the pump light (frequency ν_0). The process is most intense when wave momentum is conserved. This leads to the governing phase-matching equation, which has been discussed in section 3.2.4. We now present PM curves for FWM processes in fibers with 3 ZDWs and show that their topology has several unique features. Fig. 5.26 shows the PM curves for different values of α and input pump power. We find

Figure 5.26: *Topology of PM curves for fibers with three ZDWs. (a) corresponds to fiber A, (b) to fiber B and (c) to fiber C shown in Fig. 5.25. The black curves are for $\gamma P_0 = 0$. The other curves are labeled by their corresponding value γP_0 . We used $\gamma = 10 \text{ W}^{-1} \text{ km}^{-1}$, obtained with the FFPW method. The shaded regions have anomalous dispersion.*

that the PM curves can provide parametric gain at up to six frequencies, for certain pump frequencies within $\Delta\nu_a$. In addition, the FWM PM topology depends on the parameter α . In Fig. 5.26a we show the PM for $\alpha = -1.43$ (i.e., $\Delta\nu_n > \Delta\nu_a$) which is associated with the GVD curve A in Fig. 5.25. The topology features two PM curves, labeled **I** and **II**. The PM curve **I** is confined between z_1 and z_2 and can provide gain to up to four PM sidebands, depending on the frequency and power. In contrast to this behavior, the newly arising third (long-wavelength) ZDW z_3 causes the formation of an additional PM branch (**II**), which is phase-matched over a broad frequency range. It extends beyond the high frequency anomalous GVD region, bringing the total number of PM sidebands between z_1 and z_2 up to a total of six. When the pump power P_0 exceeds the critical value $3\beta_2/(2\beta_4\gamma)$, only branch **II** is phase-matched and branch **I** vanishes. When the spacing between z_1 and z_2 falls, the two PM curves approach each other until they finally touch, as can be seen in Fig. 5.26b ($\alpha = -0.23$). A further increase of α leads then to *avoided crossings* of the

Figure 5.27: Simulated propagation of 50 ps pulses in the fiber C (see Fig. 5.25). The lower plots show the spectral evolution along the fiber length. The input spectrum (zero detuning) is too narrow to be clearly visible. The upper plots show the spectrum after 2 m (red) and 4 m (black). In (a) $\nu_0 = 220.4$ THz, resulting in 6 PM sidebands as expected from Fig. 5.26c. (b) Decrease of the input frequency to $\nu_0 = 206.8$ THz, showing only two sidebands. The color scale is in decibels. The dashed lines indicate the ZDWs, separating the anomalous (A) and the normal (N) dispersion region.

PM curves, creating a new topology. We demonstrate this in Fig. 5.26c, where the fiber dispersion profile is such that $\Delta\nu_n < \Delta\nu_a$ ($\alpha = +0.36$), and the avoided crossings result in the appearance of two new PM curves, labeled **III** and **IV**. The frequency regions for which the pump photons can generate six sidebands are now strongly limited, appearing near z_3 . It is very interesting to see that in the limit of vanishingly small power (black curve) the PM curve **IV** supplies gain for all frequencies in $\Delta\nu_n$.

Near-CW pulse propagation

In the following we support our findings by studying the propagation of near-CW pulses. The governing equation is the generalized nonlinear Schrödinger equation, introduced in chapter 3. In our simulations we study the dynamics of 50 ps pulses, long enough to limit dispersive effects in the pump pulses and to prevent temporal walk-off from the generated sidebands. The fiber we used has zero dispersion points at 165.2 THz, 186.5 THz and 233.3 THz (see Fig. 5.25, fiber C). In Fig. 5.27 we show the spectral evolution along the fiber length for light launched at different

frequencies. In Fig. 5.27a the input frequency is $\nu_0 = 220.4$ THz and, from Fig. 5.26c, six PM sidebands are expected. This is confirmed in the simulations, six sidebands appearing after 2 m of propagation. When the pump frequency is decreased to $\nu_0 = 206.8$ THz, only two PM sidebands are expected, a prediction that is confirmed in Fig. 5.27b, the frequencies of the two sidebands exactly corresponding with the PM calculations.

The presented richness of the FWM topology is valid for all fiber with 3 ZDW. In contrast to fibers with one or two ZDWs, we gained a new degree of freedom to tailor the distribution of the six sidebands over a very wide frequency range. The existence of an additional PM branch, able to undergo an avoided crossing with the other PM branch, allows to adjust the gain-to-gain spacing and can be placed into the anomalous or the normal dispersion region. The parametric gain can be found over a broad pump frequency range, even within the normal dispersion regions and can be found with considerable detuning from the pump frequency.

5.3.3 Resonant radiation

Phase-matching

We investigate the ability of 3 ZDW PCFs to modify the propagation of ultrashort pulses and to generate light at new frequencies. In general, solitons with peak amplitude $\sqrt{P}$ adapt to external perturbations, such as higher-order dispersion, by acquiring an oscillating tail, the resonant radiation (RR). This radiation builds up constructively when the resonance PM condition $\beta(\nu) - \beta(\nu_0) - \beta_1(\nu_0) \cdot (\nu - \nu_0) = \gamma P/2$ is satisfied (section 3.3.1). This PM condition differs from the one for parametric FWM in that the sidebands are no longer restricted to symmetric positions on both sides of the pump frequency. Although bright solitons only exist when the GVD is anomalous, we find it very helpful to present the RR topology over the full range between 150 and 240 THz, including both normal and anomalous regions. Fig. 5.28 shows the RR topology for increasing values of α [from (a) to (c)], for the same PCFs as in Fig. 5.26. In general, regions can be identified where solitons are able to phase-match to three resonant frequencies. A change in the parameter α or the power dramatically modifies the RR topology.

Asymmetry $\alpha = -1.43$

For this value of asymmetry the topology consists of two curves, labeled **I** and **II**, which are clearly separated in the limit $P \rightarrow 0$ (the black curves in Fig. 5.28a). In this limit the PM curve **I**

Figure 5.28: *Resonant radiation topology for fibers with three ZDWs. The dashed arrows indicate how the curves change for increasing power. Curves for $\gamma P_0 = 0$ are marked with a black circle. Calculations based on: (a) dispersion profile A in Fig. 5.25 for $\gamma P_0 = 0, 10, 50$ and 70 m^{-1} ; (b) dispersion profile B in Fig. 5.25 for $\gamma P_0 = 0, 1, 10$ and 25 m^{-1} ; (c) dispersion profile C in Fig. 5.25 for $\gamma P_0 = 0, 1, 5$ and 10 m^{-1} . The shaded regions have anomalous dispersion.*

is localized between z_1 and z_2 , allowing PM to two resonances. The existence of the third zero-dispersion point adds the additional PM branch **II** to the topology, which is not limited to certain frequencies but extends into the anomalous GVD region between z_1 and z_2 . This means that solitons which are located in $\Delta\nu_a$ can shed energy into three dispersive waves. In contrast, solitons in the low-frequency anomalous dispersion region can phase-match to only one resonance. When the soliton peak power is increased, the topology changes strongly and is indicated by the arrows in Fig. 5.28a. At first, PM curve **I** expands since the rising power causes a larger detuning of the RR from the pump frequency. Thus the low frequency sections of curves **I** and **II** approach each other. For sufficiently high power, the two curves **I** and **II** create an *avoided crossing*. Because of this, further increase in the power reduces the frequency range where solitons can phase-match to three RR frequencies (see the curve for highest power).

Asymmetry $\alpha = -0.23$ and 0.36

In Figs. 5.28b&c we show how increasing α changes the RR topology. The excitation of three RR bands is now feasible for solitons in both anomalous GVD regions, in contrast to the case in Fig. 5.28a. For example, for $\alpha = -0.23$, a narrow frequency range can be identified in the high frequency anomalous GVD region where solitons can phase-match to three RR bands, all blue-detuned from the pump frequency. Around z_1 , however, solitons can phase-match to only one RR band. For even higher pump power in the high frequency anomalous GVD region, the topology resembles the one shown in Fig 5.28a and solitons can radiate into only one RR band. Moving z_2

closer to z_3 leads to further modifications in the PM diagram. For $\alpha = 0.36$ (Fig. 5.28c) two new PM curves appear, labeled **III** and **IV**. In this case broad regions exist in both anomalous GVD regions where solitons can shed energy into three RR bands. Increasing the soliton peak power has a strong effect on curve **IV**; it splits, frequency separation between soliton and RR steadily decreasing.

Ultrashort pulse dynamics in PCFs with three ZDWs

RR can cause pronounced reshaping of the spectrum during soliton propagation. The introduction of a third ZDW opens up new opportunities for modulating ultrashort pulses in both temporal and spectral domains. Here we study the behavior of sech^2 -pulses with carrier frequency set to 200 THz, within the high frequency anomalous GVD region of fiber C. We have chosen this particular set of input conditions to demonstrate the complex ultrafast dynamics resulting from the RR topology in Fig. 5.28c. For this carrier frequency only one RR band is phase-matched. The

Figure 5.29: Simulated propagation of 25 fs pump pulses in the fiber C (see Fig. 5.25). (a) Spectral evolution along the fiber length; the dashed lines mark the three ZDWs and the pump frequency was 200 THz. (b) Inverse group velocity of the fiber; the inset zooms in on the region around 158 THz (the vertical scale is fs/m). (c) The IR pulse after 1 m (upper) and 10 m (lower) of propagation. The dashed lines indicate the ZDWs (z_i), separating the anomalous (A) and the normal (N) dispersion region.

numerically modeled spectral evolution with propagation distance (Fig. 5.29) shows a dramatic redistribution of the pulse energy to new frequencies. Initially, after the launch of the pulses,

a radiation band appears at 284 THz as expected by the PM calculations. However, due to the Raman self-frequency red-shift and energy dissipation the soliton moves into a parameter space where it is able to emit radiation into the IR. From Fig. 5.28c we see, that the PM curve **IV** can suddenly be phase-matched when the soliton frequency lowers. This effect is visible in our pulse propagation simulations, resulting in strong energy transfer into the IR. Within the central region of anomalous dispersion, the light oscillates up and down both spectrally and spatially, creating a pattern of slanting fringes. The origin of this can be understood by means of a simplified model in which two pulses with constant carrier frequencies (corresponding to phase-matched soliton and RR bands at $\omega_1/2\pi = 197$ THz and $\omega_2/2\pi = 158$ THz in Fig. 5.29) walk-off from each other temporally at a constant rate. We give a demonstration of this model in Fig. 5.30, where we analyze the temporal and spectral profile at different propagation distances. Longer propagation increases the temporal delay ΔT (Fig. 5.30a) of the two pulses. The copropagation of the pulses gives rise to beat notes in the Fourier domain. We can calculate the spectrum analytically and plot the evolution along distance in Fig. 5.30b; the spectrum shows the same slanted pattern as in Fig. 5.29. The periodicity of the pattern in Fig. 5.30b is marked by the white lines and can be analytically derived.

This accumulating temporal delay $\Delta T(z)$ leads to a z -dependent change in relative phase between

Figure 5.30: *Analytical evolution of two pulses with different frequencies. (a) The walk-off increases the temporal delay between the pulses during propagation. (b) Evolution of the spectrum during the walk-off shown in (a).*

the pulses, given by

$$\Delta\phi = (\kappa_2 - \kappa_1)z + (\omega_2 + \omega_1)\Delta T(z)/2 \quad (5.2)$$

where κ_1 and κ_2 are the average propagation constants of the two pulses. During propagation, new spectral lines progressively accumulate at a typical length scale L_B , forming a modulated spectral comb in the anomalous band between z_1 and z_2 in Fig 5.29 and leading to a recurrence at the pump frequency when $\Delta\phi$ increments by integer multiples of 2π (see also Fig. 5.30c). Given that phase-matching requires $\kappa_1 = \kappa_2$, and that ΔT is related to the inverse group velocity difference between the pulses, i.e.,

$$\Delta T = z \cdot (\beta_1(\omega_2) - \beta_1(\omega_1)) = z \cdot (v_{g2}^{-1} - v_{g1}^{-1}), \quad (5.3)$$

the spatial beat length of the pattern is:

$$L_B = \frac{4\pi}{(\omega_2 + \omega_1)(\beta_1(\omega_2) - \beta_1(\omega_1))} \quad (5.4)$$

For the parameters in Fig 5.29 we have $\beta_1(\omega_2) - \beta_1(\omega_1) \simeq 15 \text{ fs/m}$ and Eq. (5.4) predicts $L_B = 37.5 \text{ cm}$, which is close to the $\sim 35 \text{ cm}$ period seen in the numerical simulations (Fig 5.29a); note that this value is very sensitive to the precise values of frequencies ω_1 and ω_2 . The analysis requires constant velocities of the two pulses, guaranteed by the novel characteristics of the fiber dispersion: (a) The soliton velocity is constant because the red-shift is canceled by z_2 and (b) the frequency of the RR does not change because it is a linear wave. Since the RR at $\omega_2/2\pi = 158 \text{ THz}$ travels in the anomalous GVD region, the dispersive broadening is limited due to the slow transformation into a soliton. This pulse in the IR carries more than 88% of the input energy and shows no pronounced change in width or duration over several meters of propagation. After its appearance, the RR pulse in the IR is Gaussian in shape with duration 10.75 fs and energy 13.2 pJ (Fig 5.29c). During propagation, the pulse disperses, finally forming a fundamental soliton. This is shown in Fig 5.29d, where the pulse fits well to a sech^2 envelope with duration 48 fs and energy 12.8 pJ. Since the central pulse frequency has not shifted, the pulse corresponds to a soliton of order $N = 1.5$. For longer propagation the pulse undergoes the usual soliton self-frequency shift (not shown). This effect has been termed spectral soliton tunnelling [74, 128, 131, 132], and the

mechanism observed in our simulations agrees well with the findings in [74].

Temporal modulations in fibers with three ZDWs

The existence of solitons can lead to the emergence of resonant radiation. When the GVD has two zero-dispersion points, one radiation band can be found blue-detuned to the pump frequency, and the other band is red-detuned, see for example section 5.2.2. In this case, the resonant fields have strongly different group velocities (GVs) and walk off rapidly. In the case of fibers with three ZDW, two RR bands are always blue- or red-detuned in respect to the pump frequency. The flat GV profile leads then to a temporal overlap which is sustained over long propagation distances. This results in the generation of a train of ultrashort pulses. In the following we studied the propagation of a 25 fs pulse in the fiber A. The pump frequency was 222.4 THz and $\gamma P_0 = 4.5 \text{ m}^{-1}$. The temporal and spectral dynamics is shown in Fig. 5.31. In the spectral evolution (Fig. 5.31a) one finds the existence of three RRs, labelled 1–3 in the top part of the figure. Here, the copropagation of the fields **1** (with angular frequency ω_1) and **2** (with angular frequency ω_2) leads to a modulation of their temporal envelope, with a period proportional to $1/\Delta\omega$ (see section 5.2.1.1). The temporal length evolution is shown in Fig. 5.31b. One find the generation of two bright radiation fields, quickly dispersing. However, the beatings of the RR **1** and **2** are responsible for the oscillations close to the zero delay. In Fig. 5.31c the temporal field after 4 m propagation is plotted. One finds a train of pulses with intrapulse spacing of 27 fs is generated, each having a duration of 16 fs. The frequency difference of the radiation bands **B** and **C** is $\Delta\omega = 179 \text{ THz} - 134 \text{ THz} = 45 \text{ THz}$ and the resulting temporal modulation is

$$\Delta T = 2\pi/\Delta\omega = 22 \text{ fs.} \quad (5.5)$$

This matches quite well with the intrapulse spacing shown in Fig. 5.31c. Variation of the power or the pump frequency changes strongly of the RR (see Fig. 5.28a) and allows thus a tuning of the repetition rate. Fibers with 3 ZDW turned out to be very interesting for pulse propagation dynamics. The three ZDWs result in a considerably reduced frequency-dependence of the group velocity, when compared to a PCF with one ZDW. This reduces strongly the walk-off between copropagating spectral components, leading to very long interaction lengths. In combination with the wide designability of the resonant frequencies, this results in novel SC processes.

Figure 5.31: *Simulated (a) spectral and (b) temporal length dependence of a pulse propagating in the fiber A. The pulse was launched at 222.4 THz. The color scale in (a) is in dB and is normalized in (b). (c) The temporal field at $z = 4$ m, showing the generation of ultrashort pulse trains due to the interference of two resonant radiations*

5.4 Conclusions

We studied the capacity of PCFs with tailored linear and nonlinear properties to modify an optical field. We demonstrated the occurrence of novel propagation phenomena by means of different types of PCFs. The designability of the PCF allowed to construct a device with a high nonlinear coefficient. The initial propagation dynamics of the input pulse are then governed by the nonlinearity. Due to action of SPM the pulse accumulates temporal phase which is not balanced out by dispersive effects. The resulting breakup results in a pair of temporally shifted pulses with an uncompensated chirp. The emerging interference pattern leads to ultrafast fluctuations on the temporal profile. This corresponds to a train of ultrashort pulses, the duration and repetition rate of which can be tuned by the pulse and fiber parameters.

Interesting consequences arise when a second ZDW is introduced. For example, we created a tapered PCF with two closely-spaced ZDWs in the visible spectral range. The device was fabricated by tapering. The input pigtail allowed a high launch efficiency into the tapered core as small as 500 nm in diameter. When excited with fs pulses from a tunable visible laser, the tapered fiber provides a fruitful playground to investigate pulse evolution for different pump powers and wavelengths. The large nonlinearity can lead to a significant redistribution of spectral energy. We find that the spectral broadening is influenced by the pump power rather than by the pump wavelength. For high input powers, the broadening is determined by SPM. In contrast, for low power, the spectral width is governed by soliton dynamics. The pump wavelength itself does not influence the spectral width and determines the conversion efficiencies into UV and IR DWs. Conversion

efficiencies as high as 40 % and 80 % can be achieved into blue (350 to 470 nm) and red (700 to 900 nm) spectral bands.

PCFs with two closely lying ZDWs can support unusual soliton dynamics, which undergo a series of strong collisions. The process develops from initial higher-order soliton dynamics in the pump pulse and requires vanishingly small third-order dispersion. The close proximity of the two ZDWs influences the collision, causing strong generation of dispersive waves. The multiple bursts of resonant radiation produced during the multiple soliton collisions results in a step-wise increase of the conversion efficiency into the UV and near-IR spectral ranges.

Last, we discussed novel methods to tailor the spectro-temporal properties of light. We exploited the subtle interplay between waveguide and material GVD and engineered three ZDWs in an endlessly single-mode PCF. This broadens the scope of highly unusual supercontinuum spectra, soliton dynamics and resonant radiation properties. Multiple FWM peaks appear, the location of which can be effectively manipulated by slightly changing the input wavelength or power. These new peaks are likely to be of great interest in quantum optics, since the correct design of the fiber allows to optimally place the sidebands in frequency regions (far from the pump) where the Raman scattering is less important, thus reducing the impact of Raman decorrelations and leading to an increase of the rate of entangled photons in PCFs. The fabrication of PCFs with three ZDWs will require control of the PCF microstructure to better than 1%, which is within the tolerances of the drawing process.

Chapter 6

Pulse dynamics in axially-varying PCFs

6.1 Introduction

The last chapter indicated that photonic-crystal fibers (PCFs) constitute a large playground to manipulate the spectral and temporal characteristics of ultrashort pulses. The dispersion properties of PCFs are highly tunable and the anomalous dispersion region can span different wavelength regions. The formation of solitons was observed and found to be one of the key objects in nonlinear optics and telecommunications. Solitons are nondispersing wavepackets which behave rather unusually during propagation in PCFs. For example, solitons can be described as particle-like objects, referring to the absence of interaction during collisions [42]. They were identified as important ingredients in the generation of octave-spanning supercontinua. In the case of modulation-instability seeded supercontinua, the formation of solitons can show rare events where the soliton intensity is unexpectedly large [105, 106]. Their distribution follows a L-shaped statistics and has its foundation in the stochastic nature of the modulation instability. The occurrence of such freak solitons is called rogue wave formation. In addition, the characteristics of PCFs can account for an irregular breakup of higher-order solitons [79] and the formation of bound solitons [108, 133]. This chapter studies the potential to modify soliton dynamics in axially varying PCFs. Fibers with slowly decreasing dispersion have initially been used for adiabatic compression of solitons [134–136]. The length of the varying region can range from several millimeters [137] to several meters [138] and offer unexpected and intriguing physics. For example, the spectral extend of the supercontinuum (SC) can be increased due to trapping of linear radiation by solitons or by cascaded processes, resulting in spectra extending as low as 395 nm when picosecond-

pumped [86, 138]. Further, very short pieces of transitions in the range of 6 mm have been used to generate coherent, low-noise fs-pulses in the visible [137]. We study the impact of a continuously changing dispersion profile on the spectral broadening of ultrashort pulses and the conversion into the UV spectral range.

In section 6.2 the parameters of the PCFs used to fabricate taper transitions are presented. Two different types of PCFs were used. The small-core PCF has a zero-dispersion wavelength (ZDW) at 735 nm, and section 6.3 covers the propagation dynamics of a 800 nm pump pulse in the tapered PCF. The ZDW of the large-core fiber is 835 nm. Launching pulses at 800 nm corresponds to the normal dispersion region and the broadening mechanism is shown in section 6.4. The propagation in the untapered PCFs is then shown in section 6.5.

6.2 Fabrication and numerical simulation of taper transitions

6.2.1 Fiber characteristics

Figure 6.1: *GVD curves of the untapered fibers. The dots are the experimentally measured points, and the solid curves are simulated dispersion profiles obtained from a silica strand model. The inset shows the SEM images of the fiber geometry.*

For our study we fabricated two kinds of fused-silica PCFs, with a core diameter of $1.76\ \mu\text{m}$ (labeled PCF_A) and $2.74\ \mu\text{m}$ (labeled PCF_B), respectively. Scanning-electron micrographs (SEM) images of the PCFs can be found in the inset of Fig. 6.1. Additionally the group-velocity dispersion (GVD) of the two fibers are presented in Fig. 6.1. The GVD was measured by white-light interferometry (section 4.3). The ZDWs are located at 735 nm (fiber PCF_A) and 835 nm

(fiber PCF_B). In the next step, we used a home-made taper rig to fabricate taper transitions of different lengths (see section 4.2). The target taper waist diameter r_{end} was adjusted to support a vanishingly small anomalous dispersion region. The maximum of the GVD touches the zero-dispersion line at p_0 and is blue-detuned from the initial ZDW (see Fig. 6.2a). Further reduction of the core is not considered, because a small core can lead to a strong leakage of the light (section 2.1.2.3). The two PCFs have different air-filling fractions, allowing us to place the point p_0 at different wavelengths, the position of which dictates the dispersion landscape towards the end of the transition. For example, a high air-filling fraction (PCF_B) shifts p_0 far into the blue (see Fig. 6.2a). In contrast the lower air-filling fraction of PCF_A places p_0 into the green spectral range. From fiber PCF_A we fabricated a 10 cm long transition, labeled $Tr_{A,10}$. The shape of the fiber transition was measured with a light microscope and is shown in Fig. 6.2b. The diameter of the taper waist diameter was 45 μm , giving a core radius of around $r_{\text{end}} = 0.32 \mu\text{m}$. The target waist diameter was found by measuring the GVD of a set of fiber tapers, each having different taper diameters. In Fig. 6.2a we show the simulated dispersion profile of the target waist diameter. Further we plot a measured GVD curve of a 48 μm taper waist made from PCF_A, which is close to the dispersion profile we aimed for. Fiber PCF_B was used to fabricate a 14 cm long transition,

Figure 6.2: (a) Outer-diameter variation of the transition region fabricated from PCF_A and PCF_B. (b) Dispersion of the taper waist. Here p_0 indicates the single ZDW point and is different for both fibers. The dotted curve is the measured GVD curve of a 48 μm taper waist made from PCF_A.

$Tr_{B,14}$, and the outer diameter is given in Fig. 6.2b. The taper waist diameter was 34 μm , giving a core diameter of around $r_{\text{end}} = 0.3 \mu\text{m}$. In both cases, the core radius of the taper waist is in the range where the maximum nonlinear coefficient reaches its maximum value (see section 2.1.2.1). We study the propagation of 130 fs pulses obtained from a commercially available mode-locked

Ti:Sapphire laser system (see section 4.1.1). A $\lambda/2$ -waveplate was used to launch the light parallel to one of the fiber axis the fibers exhibit a small amount of birefringence. For the experiments with the PCF_A (PCF_B) we used a $40 \times (30 \times)$ aspherical incoupling lens, achieving launching efficiencies of 60 % (70 %).

6.2.2 Numerical methods

The varying core size was resolved in 0.5 nm steps. The propagation constant for each step was calculated by the silica-strand model, and approximated the effective index n_{eff} of the cladding region by a simple geometric average

$$n_{eff}^2(\lambda) = f + (1 - f) \cdot n_{silica}^2(\lambda), \quad (6.1)$$

where f is the air-filling fraction of the cladding region and n_{silica} is the frequency-dependent index of the glass. The accuracy of this model is limited, since it does not consider the frequency dependence of the microstructuring contribution to the dispersion (see chapter 1 and Ref. [139]). However, the advantage is that a vast amount of dispersion curves can be obtained quickly, for a broad range of core radii and air-filling fractions. In addition, this model turns out to be adequate and it matches nicely to the experimentally measured GVD curves. We fitted GVD profiles obtained by Eq. (6.1) to the experimental data in a least-squares sense. In the limit of Eq. (6.1) we found that the fiber PCF_A can best be matched to a core radius of $r = 0.902 \mu\text{m}$ and an air-filling fraction value of $f = 75 \%$. The fiber PCF_B fits nicely to a silica strand in air with a radius of $1.521 \mu\text{m}$. Both results are plotted in addition to the experimental GVD curves in Fig. 6.1. We can additionally determine r_{end} , by assuming conservation of the air-filling fraction during tapering. For the fiber PCF_A we found $r_{\text{end}} = 0.376 \mu\text{m}$ and $p_0 = 638 \text{ nm}$. In contrast, the higher air-filling fraction of fiber PCF_B leads to $r_{\text{end}} = 0.236 \mu\text{m}$ and $p_0 = 492 \text{ nm}$.

6.3 Pumping in the anomalous dispersion region

The reduction of the core size within a taper transition leads to a steady change of the dispersion landscape. The initial ZDW (labeled p_{blue}) experiences a continuous blue-shift and, at a specific taper position, a second ZDW appears (p_{red}) which is also blue-shifting. In combination, this yields an anomalous GVD region which reduces its extend along the transition and shifts its centre

towards lower wavelength (see Fig. 6.3, where the dotted lines trace the position of the ZDW). Therefore, pulses launched at the red or blue side of the ZDW are expected to adapt differently to the changing dispersion. In the following, we fix the wavelength and change the position of the initial ZDW. We investigate the spectral broadening when the pulse enters the taper transition within the normal or anomalous dispersion region. We vary the strength of the taper gradient and study the impact of the transition length on the energy conversion into the blue spectral range.

6.3.1 Experimental power dependence of spectral broadening

We begin the study by launching light into the anomalous dispersion region of the input facet. This scenario is experimentally available by the taper transition $Tr_{A,10}$. In Fig. 6.3 we show ex-

Figure 6.3: *Length dependence of an ultrashort pulse launched into the transition $Tr_{A,10}$ (see Fig. 6.2). The peak power was (a) 660 W, (b) 1.65 kW and (c) 6.5 kW. White curves show the positions of the two zero-dispersion points, obtained from FFPW simulations of the cleaved fiber tips. The confinement loss leads to a strong increase of the attenuation in the IR for propagation distances over 12 cm.*

perimental cutback measurements of the spectral evolution along the taper transition $Tr_{A,10}$ for different input powers. In all cases we see that the initial broadening mechanism is dictated by the anomalous dispersion, leading to dispersive wave (DW) generation and soliton fission. The input power determines the position of the soliton fission. For low powers the soliton fission occurs far within the taper transition. The small core at this stage should lead to DW generation close to the UV. However, the large detuning of the DW frequency from the pump reduces strongly the energy conversion [71]. An increase of the input power shifts the soliton fission closer to the input facet, and fission occurs at a larger core radius. Accordingly, the DW is shed in the green spectral range

and the energy conversion increases strongly. The magnitude of the input power determines additionally the number of solitons and the rate of the SSFS (see sections 3.2.2 and 3.3.1). However, several unconventional features are visible in the experimental data, for example the increase of the blue-spectral edge of the supercontinuum. In addition, we see that the solitons are strongly influenced by the blue shifting zero-dispersion wavelengths, and undergo a reversal of the SSFS. In the following, we study the unconventional features by means of $P_0 = 1.65$ kW. The cutback measurement revealed that the device had an untapered input pigtail of more than 4 cm length, which leads to some modifications of the spectrum when compared to the following numerical result. However, we neglected the input pigtail in our simulation and we focused on the transition part only. The effect of the short untapered piece can be found in our publication [140].

6.3.2 Initial dynamics

We simulated the evolution of a $\lambda_0 = 800$ nm, $T_0 = 130$ fs pulse with power $P_0 = 2.5$ kW, and show the spectral and temporal characteristics in Fig. 6.4. The deviation from the experimental value (1.65 kW) originates from the gaussian model used to calculate the nonlinear coefficient, which gave $\gamma = 0.12 \text{ W}^{-1}\text{m}^{-1}$.

The initial dynamics in the temporal and spectral domain correspond to the initial stage of su-

Figure 6.4: Simulated (a) spectral and (b) temporal evolution of a 2.5 kW, 130 fs pump pulse launched in a 10 cm long transition, comparable to the transition $Tr_{A,10}$. The dashed lines show the zero-dispersion points, changing due to the progressive core reduction. Soliton A refers to the most intense soliton. The L_i mark the onset of characteristic spectral changes. The dotted lines trace the phase-matched points for dispersive wave generation (see text).

percontinuum generation and spectral broadening due to self-phase modulation occurs [27, 76].

At the fission length $L_{\text{DW}} = 2.0$ cm strong temporal compression sets in (see Fig. 6.4b, showing a pulse as short as $T_0 = 3.5$ fs) and is accompanied by soliton fission and DW generation (see section 3.3.1). The wavelength of the dispersive wave (λ_{DW}) is determined by the core size at L_{DW} , which is around 665 nm and has a ZDW at 689 nm. The wavelength of the DW can be calculated by the phase-matching equation (section 3.3.1),

$$\beta(\lambda) - \beta(\lambda_0) - \beta_1(\lambda_0) \cdot (\lambda - \lambda_0) = \gamma \cdot P/2, \quad (6.2)$$

which is satisfied for $\lambda = \lambda_{\text{DW}}$. In our case, Eq. (6.2) gives $\lambda_{\text{DW}} = 524$ nm and does not entirely coincide with the numerical result, giving 505 nm. By using an advanced phase-matching equation [72]

$$\beta_{nl}(z, \lambda) = \beta(z, \lambda) + \gamma \left(1 - \frac{1}{\omega_0} \cdot (\omega - \omega_0) \right) \text{Re} \left\{ \frac{FT [A(z, \tau) \otimes |A(z, \tau)|^2]}{\tilde{A}(z, \omega)} \right\}, \quad (6.3)$$

we obtain 509 nm, which agrees well with the simulations. Here, Re refers to the real part and FT is the Fourier transform of the convolution of $A(z, \tau)$ and $|A(z, \tau)|^2$.

The wavelength of the dispersive wave in Fig. 6.4 is shorter than the experimental value of Fig. 6.3b, $\lambda_{\text{DW}} = 550$ nm, and can be traced back to the input pigtail. When the power is increased, the fission process advances and occurs at a larger core diameter. As a consequence, the wavelength of the DW increases.

After the fission, the propagation dynamics are governed by the evolution of the emerging solitons (see temporal evolution in Fig. 6.4b). Each soliton experiences a Raman red-shift at a rate determined by the z -dependent $\beta_2(z, \lambda_{\text{sol}})$ value at the soliton wavelength λ_{sol} . Note that $\beta_2(z, \lambda_{\text{sol}})$ changes its sign for increasing z , going from positive to negative. The negative value results in an accelerated red-shift, performed by the soliton A in Fig. 6.4a. The red-shifting soliton is responsible for the increase of the short-wavelength spectral edge and is described in the following section.

6.3.3 Intrapulse four-wave mixing

In Fig. 6.4a one finds an increase of the blue spectral edge at the propagation distance $L_I = 4.4$ cm. The phenomenon occurs also in untapered fibers and was initially described by cross-phase modulation between the soliton and the dispersive wave [141, 142]. In a later work by Gorbach et al. [143], it was found that an intrapulse four-wave mixing (FWM) process between the soliton

and the dispersive wave is responsible for the energy transfer into the blue. The frequency shift can be evaluated by applying wavenumber matching conditions and can be found in [144],

$$\beta(\omega - \omega_{ref}) = \beta(\omega_{DW} - \omega_{ref}) + \beta_1(\omega_{sol} - \omega_{ref}) \cdot (\omega - \omega_{DW}), \quad (6.4)$$

Here, β is the propagation constant, $\beta_1 = \partial_\omega \beta$ and ω_{ref} is a reference frequency which we chose as the frequency of the dispersive wave ω_{DW} ; ω_{sol} is the frequency of the soliton. In Fig. 6.5 we visualize the scattering process, occurring from the dispersive wave into the blue. We show numerical XFROG spectrograms at different positions along the fiber and focus on soliton A (see Fig. 6.4). The XFROG spectrograms used a 20 fs gate pulse.

When the DW is created at L_I , the DW travels slower than the remaining light and starts to fall

Figure 6.5: *Simulated XFROG spectrograms at different propagation distances for the results shown in Fig. 6.4. (a) At 4.4 cm, the soliton A catches up with the leading part of the chirped dispersive radiation, starting at $\lambda_{DW,lead} = 515$ nm. The trailing edge is at $\lambda_{DW,tr} = 472$ nm. (b) Scattering of the leading part of the DW to $\lambda_I = 458.8$ nm. The right-hand side figure represents the phase-matching Eq. (6.4). The circle denotes the pump wavelength (given by the DW) and the square is the resulting FWM resonance. The horizontal axis corresponds to the wavenumber. (c) At 6.6 cm, the blue shift stops, because the soliton accelerates relative to the dispersive radiation. The dotted lines show the position of the ZDW.*

back. However, the change of the GVD profile and the SSFS decelerate continuously the soliton which can catch up with the DW (Fig. 6.5a). At this stage, the DW has transformed into a strongly up-chirped wave packet. The soliton A interacts with the leading edge of the dispersive radiation, inducing an energy transfer into the lower wavelength region at $\lambda_I = 458.8$ nm (see Fig. 6.5b). We have applied the phase-matching condition of Eq. (6.4) and show it at the right-hand side of Fig. 6.5b, agreeing with the simulation. During further propagation, the soliton A sweeps across the dispersive wave, shifting more energy into the blue. The wavelengths of the generated

light reduces continuously because the wavelength of the soliton increases (due to SSFS) and the instantaneous wavelength [$\lambda_{DW} = \lambda_{DW}(\tau)$] of the dispersive wave shrinks. The magnitude of the blue-shift depends on the soliton wavelength and on the wavelength of the linear radiation [see Eq. (6.4)]. The FWM scattering process ends at a distance L_{canc} , because the overlap of the two fields is arrested. This feature is reproduced in the experimental data of Fig. 6.3b. During the propagation, the blue edge of the spectrum shifts from λ_{DW} into the UV to $\lambda_I = 478$ nm. This blue-shift progresses until the soliton approaches the red zero-dispersion point. In agreement with the numerical results, the blue-shift stops, and the spectral edge now extends to 420 nm.

6.3.4 Soliton blue-shift in tapered PCF

For propagation distances longer than L_{canc} the soliton **A** experiences the presence of the long-wavelength ZDW. It has been shown that a ZDW on the red side of a soliton arrests the SSFS (see for example section 5.2.2). The soliton stabilizes its frequency by emitting continuously radiation into the IR. However, in our case the ZDW $p_{\text{red}}(z)$ is falling along the fiber length. The soliton adjusts to the changing GVD landscape and transforms into a blue-shifting, soliton-like pulse. In Fig. 6.6 we show the evolution of the centre wavelengths λ_{sol} of the solitons **A** and **B** with

Figure 6.6: *Evolution of the center wavelength of solitons **A** and **B** during propagation. After the maximum red-shift due to SSFS, a blue-shift occurs in response to the blue-shifting ZDW p_{red} .*

propagation distance, obtained by gating the solitons in the temporal domain. The SSFS shifts the center wavelength of soliton **A** to 960 nm at around 6.5 cm (see downward arrow in Fig. 6.6). This is accompanied by an increase of the peak power, as it is expected from a negative value of

the third-order dispersion β_3 (see section 3.2.3). Then, as the long-wavelength ZDW reduces its wavelength, the soliton follows the change of the ZDW and shifts as low as 660 nm. This blue-shift is gained at the expense of the peak power, because the proximity of the second zero-dispersion point causes the wave packet to shed energy continuously into the IR, starting at $\lambda_{\text{DW,IR}} = 1.5 \mu\text{m}$. The soliton and the ZDW $p_{\text{red}}(z)$ lower their wavelengths, filling the spectral gap as the resonant wavelength falls. The position of the dispersive wave in the IR can be calculated with Eq. (6.2) and are superimposed on the spectrum in Fig. 6.4a. The formation of the fin-shaped band follows precisely the course of the position-dependent resonant wavelength. Discrepancies result from the other solitons, which add to the IR band at a later stage. For example, we see in Fig. 6.6 that the soliton **B** moves to 870 nm at 7.5 cm (see upward arrow in Fig. 6.6), when the SSFS reversal sets in. At this point, soliton **B** sheds dispersive radiation into the IR. This leads then to the formation of spectral fringes in the IR in Fig. 6.4a, which spread towards shorter wavelengths as the core size falls. These fringes are visible in the experimental data in Fig. 6.3a&b. The blue-shifting soliton is not only responsible for the generation of light in the IR, but leads to an energy transfer into the UV. If λ_{sol} is close to $p_{\text{blue}}(z)$, a phase-matched generation of dispersive waves on the blue side of the soliton is possible. We calculated the phase-matched wavelengths according to Eq. (6.2), and show the results additionally in Fig. 6.4a. The generation of dispersive radiation starts at 405 nm and increases with further propagation. This demonstrates that the blue shifting soliton can be used to increase the conversion efficiency into the blue spectral range.

6.3.5 Physical model of the soliton blue-shift

In the following we study the physics of the SSFS reversal. We are going to show that the blue-shift of soliton-like pulses results from a reshaping process, controlled by the blue-shifting anomalous dispersion region. To start with, we focus on soliton **A** shown in Figs. 6.4&6.5. After 5 cm, this soliton has a FWHM duration $T_{\text{sol}} = 12.3$ fs and the center wavelength is at $\lambda_{\text{sol}} = 904$ nm. The core radius of the PCF, at that stage, is $r(z = 5\text{cm}) = 0.639 \mu\text{m}$, leading to $\beta_2(\lambda = \lambda_{\text{sol}}) = -58.9 \text{ ps}^2/\text{km}$. We simulated the propagation of this soliton in the taper transition, starting at 5 cm, in absence of other solitons or linear waves. The spectral evolution is plotted in Fig. 6.7a and shows clearly the blue-shift of the pulse center, accompanied by DW generation. In the temporal domain, we see that the pulse duration shortens and the peak power increases. However, due to DW generation the soliton-like pulse loses energy and the peak power starts to shrink.

Next, we studied the impact of the varying GVD on the soliton propagation. We fixed the nonlinear coefficient for all propagation steps. Then we remove the higher-order dispersion terms by setting $\beta_2(z, \omega) = \beta_2^0(z)$. This corresponds to the absence of ZDWs and the soliton is not influenced by DW generation or SSFS cancellation. The transition effect is introduced by a linear variation of the second order term β_2 from $\beta_2 = -58.9 \text{ ps}^2/\text{km}$ at 5 cm to $\beta_2 = 0$ at 10 cm. We show the spectral and temporal evolution in Figs. 6.7c&d. The continuous reduction of the absolute value $|\beta_2(z)|$ leads to a continuous decrease of the soliton duration, and results in an increase of the spectral bandwidth. Due to energy conservation, the soliton power in Fig. 6.7d increases.

By comparison of Figs. 6.7a&b and 6.7c&d we see the consequence of the existence of the short-wavelength and long-wavelength ZDWs. The spectral broadening of the soliton is constricted. Spectral components on the red side of the soliton are cut and shed far into the IR. In contrast, the blue spectral components are continuously generated in the anomalous dispersion region. They provide the seed to shift the soliton envelope into the blue.

The formation of blue-shifting soliton-like pulses is also visible in the experimental data. For all

Figure 6.7: (a) Spectral and (b) temporal evolution of soliton **A** in the second half of the taper transition. In (c) and (d) the GVD profile has been approximated by a frequency independent $\beta_2(z)$, varying linearly from $\beta_2 = -58.9 \text{ ps}^2/\text{km}$ to $\beta_2(z = 10\text{cm}) = 0$. The nonlinear coefficient γ was kept constant at $0.15 \text{ m}^{-1} \text{ W}^{-1}$ for (c) and (d).

power levels we find that the spectrum adjusts to $p_{\text{red}}(z)$, as expected by the simulations (Fig. 6.4). For example, for $P_0 = 6.5$ kW the outermost soliton approaches p_{red} at around 10 cm. The pulse center shifts into the blue and sheds radiation into the IR. In addition, at around 12 cm propagation, the soliton-like pulse shifted to 700 nm, accompanied by an increase of radiation in the UV. Similar behavior can be found for $P_0 = 1.65$ kW. Further reduction of the peak power to 660 W leads to an interaction of the soliton with p_{red} at around 11 cm. The spectral fringes result from the interference of the blue-shifting pulse with the remaining pump wave. Further propagation leads to the formation of a nose towards the blue.

In Figs. 6.3b&c a substantial part of light in the IR is cut. This results from the submicron core size, leading to a strong growth in the confinement loss (section 2.1.2.3). This can be prevented by increasing the number of holes in the fiber cladding. In the numerical simulation we neglected a wavelength-dependent loss. At this stage, we were not able to observe the DW generation at the end of the taper. This is due to the poor length resolution of our experiment, as we measured the spectrum every 5 mm. We are going to investigate longer transitions, where this effect occurs over longer propagation distances.

6.3.6 Dependence of the spectral broadening on the transition length

So far we demonstrated that a given length of a taper transition increases the blue edge of the generated spectrum by intrapulse FWM. Additionally, soliton-like pulses can move towards the blue and raise the energy transfer into the blue. The transition-induced effects are mainly determined by the rate of the soliton red- or blue-shift. This is governed by the magnitude of the ZDW change. Therefore we expect different spectral transformations for different gradients of the ZDWs. Here, we investigate the impact of the taper gradient by simulating different lengths of the taper. However, the radius of the core at the start and at the end of the transition will be kept the same for all cases. In Fig. 6.8 we show the simulated spectral evolution for two transitions and the length in Fig. 6.8a was 2 cm and in Fig. 6.8b the length was chosen to be 35 cm. The input peak power was $P_0 = 2.5$ kW, as in Fig. 6.4.

In the case of a short transition, the propagation dynamics lacks features which are induced by the red- or blue shift of solitons (Fig. 6.8a). The fission process occurs close to the end of the transition. The proximity of two ZDWs ($p_{\text{blue}} = 612$ nm and at $p_{\text{red}} = 1079$ nm) leads to the generation of two dispersive waves, at $\lambda_{\text{DW,UV}} = 397$ nm and $\lambda_{\text{DW,IR}} = 1575$ nm. Shortly after, a second burst

Figure 6.8: *Simulated spectral evolution for transition lengths of (a) 2 cm and (b) 35 cm. In both cases, the input power was 2.5 kW. The black lines trace the evolution of the zero-dispersion points.*

of radiation takes place and leads to an additional energy transfer into the IR and VIS [68]. The steep gradient does not modify the further propagation of the solitons.

In contrast, the 35 cm long transition leads to a more gentle gradient of the ZDWs. This places the soliton fission process close to the beginning of the device, at $L_{\text{DW}} = 2.5$ cm, generating DW at 533 nm. The emerging solitons can propagate over a longer distance until the interaction with the approaching second ZDW takes place. This leads to a larger SSFS red-shift, causing a twofold blue-shift of the dispersive wave. The first blue shift is the same as discussed in section 6.3.3. However, in a long taper transition the soliton **A** is able to pass over the entire dispersive radiation. At the propagation distance $L_{\text{II}} = 14$ cm, the soliton overlaps with the blue-shifted light at λ_I , and a second energy transfer sets in. This mechanism causes the continuous extension of the blue edge to as low as 407 nm. However, at the propagation distance L_{canc} this soliton, now located at $\lambda_{\text{sol}} \sim 1100$ nm, adapts to the presence of the red ZDW. Firstly, the blue-shift of the spectral edge is arrested. No temporal overlap can be established anymore. Secondly, a strong energy transfer into the IR sets in and the long-wavelength edge of the SC extends to 1600 nm. Thirdly, as discussed before, the solitons undergo a strong blue-shift. Close to the end of the transition, for $L_{\text{UV}} = 30$ cm, a transfer of energy into dispersive waves in the UV occurs and is sustained until the end of the fiber.

It was shown that two competitive competitive effects determine the energy transfer into the UV – the DW generation and the scattering of radiation into the blue. In this respect, a short transition length places the DW generation close to the end of the transition, and the resulting small core

allows multiple transfers of energy into the UV spectral range around 400 nm (see Fig. 6.8a). The intrapulse FWM processes are absent. On the other side, a gentle transition leads to the appearance of DWs in the green (see Fig. 6.8b). However, the long propagation length of the solitons results in pronounced spectral reshaping. On one hand, multiple scattering events shift part of the dispersive wave deep into the blue. On the other hand, at the end of the transition, the blue-shifting soliton leads to a second DW generation process. The ideal length for UV radiation generation can be taken from Fig. 6.9, where we show the numerical energy conversion into three different spectral regions as a function of the transition length. It is a strongly peaking function, having peak efficiency for transitions lengths of around 3 cm.

Figure 6.9: *Simulated conversion efficiencies into three spectral bands as a function of the transition lengths.*

6.4 Pumping in the normal dispersion region

In the previous sections we investigated the effects of pulses launched into the anomalous GVD region of a taper transition. By using fibers with a larger core size we can set the pump wavelength initially in the normal dispersion region. Therefore, the initial dynamics of the input pulse is governed by normal GVD. As the core size falls, the light can enter the anomalous dispersion region. The following numerical and experimental results are based on the taper transition made from PCF_B (see Fig. 6.1). The spectra were measured by direct coupling of the tapered PCF to an OH-doped multi-mode fiber with core diameter 500 μm .

In Fig. 6.10 we discuss the numerical and the experimental spectral evolution of an ultrashort

Figure 6.10: (a) The figure on top shows the simulated spectral evolution in a 14 cm long taper transition. The input power was 1.4 kW. The bottom plot shows the experimentally measured spectral evolution in the 14 cm long transition $Tr_{B,14}$ for a power of 1.2 kW. (b) The same as in (a) only for input powers of 3.5 kW (theo.) and 3.0 kW (exp.). The dashed lines are the ZDWs.

pulse in a 14 cm long transition, launched in the normal dispersion regime at 800 nm. We show the spectral evolution for two different input powers. Although the two PCFs we used for our study are different, the spectral evolution exhibits the same features. The low power case of Fig. 6.10a is absent of strong dispersive wave generation. The initial normal dispersion region delays the solitonic pulse compression to around 7 cm. The soliton in the anomalous dispersion region propagates until it collides with the red ZDW p_{red} , which is rapidly blue-shifting. The soliton follows the blue-shift of the red ZDW and moves close to p_0 , lying at around 500 nm. The experimental result exhibits the same features as the simulated data. The first soliton breather occurs at around 7 cm. Then, shortly before the strong IR spectral broadening, a soliton is visible,

red-shifting at an accelerated rate. After the energy transfer into the IR, some part of the spectrum extends into the blue, and is a sign of the soliton blue-shift. This is accompanied by interference fringes in the near-IR.

Maximization of the spectral extend into the UV

Fig. 6.10b shows the spectrum for a power around 3 kW. The DW generation is occurring at an earlier stage, at around $L_{DW} = 5.5$ cm. Now a noticeable amount of radiation is shed into the green at $\lambda_{DW} = 530$ nm. During subsequent propagation, a very strong shift of this radiation sets in. This can be understood by multiple FWM scattering events, as described for Fig. 6.8b. In detail, we find a four-fold shift of the blue edge of the SC in Fig. 6.10b. This is demonstrated in the XFROG shown in Figs. 6.11a&b. At 7.5 cm, the soliton overlaps with the leading part of the DW. The DW constitutes a short burst of radiation, and the soliton is able to sweep across the complete wave. Fig. 6.11b shows the spectrogram at propagation distance 11.5 cm. We see the action of the fourth scattering event, shifting the blue-edge of the spectrum to as low as $\lambda = 330$ nm. We numerate the linear waves pertaining to the different scattering events. As discussed before, the FWM interaction stops with the arrest of the temporal overlap, which occurs when the soliton **A** collides with the red ZDW p_{red} , which is close to the end of the transition. Additionally, the spectrum starts to extend far in the IR because the soliton sheds phase-matched radiation at around 1350 nm. We demonstrate the large amount of blue-shift of the soliton in Fig. 6.11c and show the

Figure 6.11: (a) Simulated XFROG spectrogram at different distances of the spectral evolution shown in Fig. 6.10. (a) At 7.5 cm, the soliton overlaps with the leading edge of the dispersive wave, starting the energy transfer into the UV. (b) At 11.5 cm the soliton **A** has induced a four-fold shift of the blue spectral edge of the SC. (c) At 13.5 cm, the solitons **A** and **B** have lowered their center wavelengths.

spectrogram at 13.5 cm in a linear scale. Whereas at 11.5 cm the soliton was at $\lambda_{sol} = 910$ nm, it

has now moved to $\lambda_{\text{sol}} = 575$ nm. The intense radiation tail accompanying the soliton in the IR is the DW. The energy loss of the soliton into this radiation reduces the soliton peak power. In addition, from Fig. 6.11c we find that soliton **B** reduced its center wavelength as well; however, it has entered the normal dispersion region and is dispersing. The experimental result of Fig. 6.10b shows the same blue-shift of the short-wavelength spectral edge. From around 7 cm onwards, several intrapulse FWM scattering events set in. This is accompanied by the red-shift of the most intense soliton. We observe a progressive increase of the blue edge of the supercontinuum, until it reaches the bandwidth of our spectrum analyzer.

In the experimental data of Fig. 6.10b we did not find a spectral extension to 330 nm. This was caused by the tapering process, which led to a change of the air-filling fraction towards the end of the transition. For example, this can be seen by help of soliton **A**. In the experimental data, L_{canc} is around 11 cm. This is shorter than in the simulation and results from the increasing air-filling fraction close to the taper transition. Nevertheless, by applying pressure to the PCF cladding during the tapering process the shrinkage of the air holes can be prevented [98]. To conclude this section, we discuss the effect of different transition lengths on the energy conversion. Fig. 6.12 shows the conversion efficiency into three spectral bands as a function of the transition length. Comparable to the result shown in Fig. 6.9 we find that the maximum conversion occurs for a transition of around 3 cm. However, pumping in the normal dispersion region leads to a larger energy transfer into the 300-400 nm range, whereas the energy transfer into wider spectral bands is reduced when compared to the case of anomalous pumping.

Figure 6.12: Simulated conversion efficiencies into three spectral bands as a function of the transition lengths.

6.5 Propagation in the untapered PCFs

In Fig. 6.13a we show the spectral evolution in PCF_A for an input power of 1.65 kW. This power value was used to study the spectral broadening in the transition $Tr_{A,10}$. The pump wavelength corresponds to anomalous dispersion in the fiber and we observe the emergence of dispersive radiation at around 600 nm for propagation distances longer than 6 cm. A soliton forms, which undergoes a slight red-shift.

The propagation of ultrashort pulses in PCF_B is displayed in Fig. 6.13b. The pump wavelength was 800 nm, lying in the normal dispersion region. Due to the large core, the nonlinear coefficient of the fiber is as low as $30 \text{ W}^{-1}\text{m}^{-1}$. As a result, we chose a pump power as high as 12 kW, much larger than the powers used for the investigation in $Tr_{B,14}$ (section 6.4). The spectral broadening is determined by SPM. For propagation distances longer than 10 cm, the spectrum starts to extend into the anomalous dispersion region. For longer propagation distances, the red-detuned SPM sidelobe can transform into a soliton, analogous to Fig. 5.13.

Figure 6.13: *Spectral broadening in untapered pieces of (a) PCF_A (b) PCF_B. The pump power was (a) 1.65 kW and (b) 12 kW. The dashed lines are the ZDWs, and are at (a) 735 nm and (b) 835 nm.*

6.6 Summary

We investigated the influence of axially-varying PCF on the spectral properties of ultrashort pulses. The reducing core size results in a continuously changing GVD landscape; two ZDWs appear and the spanned anomalous dispersion region shifts continuously to lower wavelengths. We have found that the spectral broadening is governed by soliton dynamics — as long as the light enters the anomalous dispersion region. For example, we demonstrated by means of a smoothly varying red ZDW that the solitons can undergo a pronounced blue-shift. This is accompanied by strong emission of dispersive waves in the UV and IR. The mechanism was identified as a reshaping process, where continuously generated spectral components at the blue edge of the soliton move the pulse envelope towards the blue. The tapering of a large-core fiber with a high air-filling fraction lead to strongly varying ZDWs. Such a device has the potential to create a supercontinuum extending to close to 300 nm. The underlying broadening process was identified as multiple intrapulse FWM scattering events, transferring part of the DW energy due to the interaction of the solitons with the DWs. The optimum transition length for blue-light generation can be found from a trade-off. For short transition lengths the soliton fission occurs close to the end of the transition and the dispersive wave is generated in the UV. When the transition length is in the tens of cm, soliton fission occurs in the initial part of the transition, and the DW is generated in the green spectral range. We found that efficient wavelength conversion can be achieved for transitions with length ~ 3 cm.

Chapter 7

Summary

In the course of this thesis we showed that the high designability of the PCFs paves the way to new light sources. We studied the existence of novel pulse propagation phenomena in dispersion-tailored PCFs. The possibility to tune the fiber parameters over a broad range leads to unconventional behavior of optical pulses in PCFs, constituting new sources for supercontinuum generation, wavelength conversion and pulse shaping. In this work, we introduced the concept of optical waveguides in chapter 2. We found that optical modes exist, confined to the core region and transmitted with very low loss numbers. The constriction of light to an area spanning only a few μm^2 gives rise to interactions of the light with the molecules of the glass. As a result linear and nonlinear effects emerge, the impact of which on the propagating light can be tuned over a broad range, as discussed in chapter 3. In short, linear effects lead to a reshaping of the temporal envelope of the electric field, whereas nonlinear effects result in the generation of new frequency components. When both act together, fibers support the propagation of solitons which are non-dispersive wavepackets. Nonlinear effects in fibers, and PCFs in particular, can be used to modify dramatically the properties of an electric field, resulting ultimately in the formation of octave spanning spectra. We discussed the ramifications of the fiber nonlinearity using the generalized nonlinear Schrödinger equation. An implementation of the solver can be found in appendix A.

In chapter 4 we summarized the experimental techniques we used to study the pulse propagation in novel types of PCFs. Firstly, the laser sources and their characterization tools were described. Our light sources included a 800 nm mode-locked Ti:Sapphire laser and an parametric amplifier, tunable between 500 nm and 650 nm. Secondly, we established the postprocessing technique as an important method to modify the properties of stock PCFs. The typical length scale of tapered

devices can be up to several tens of centimeters. This corresponds to the length scales typically connected to femtosecond pulse propagation. Lastly, we outline the principle of the measurement of the group-velocity dispersion, one of the most important quantities of an optical fiber — besides the fiber attenuation.

In chapter 5 we discussed the impact of fibers with tailored dispersion properties on the propagation of optical pulses. We started with small core fibers, featuring a high nonlinear coefficient. This allows us to study the propagation of ultrashort pulses in devices where the nonlinearity dominates over the dispersive; even for moderate pump power levels. To begin with, we studied the consequences of a pulse breakup in the presence of a dominating nonlinear influence. The input pulse acquires a pronounced spectral phase, which cannot be compensated by dispersive effects. At the stage of the pulse breakup, this causes the formation of two strongly chirped pulses. Additional propagation leads to a time shift between the pulses. The temporal and the spectral envelope then shows strong modulations, corresponding to a train of ultrashort pulses. The pulse breakup is strongly influenced by the input wavelength or the pump pulse power. This allows a certain amount of control over the pulse duration or the inter-pulse spacing of the pulse train. The short fiber length involved makes this device a promising, compact source for pump-probe experiments. The repetition rate can be matched to the molecular resonances and drive the molecular system. Further applications could lie in the preparation of matter in its excited states (for example Raman processes). A further advantage of small-core fibers lies in their ability to shape supercontinua, governed by their unusual dispersion properties. We fabricated PCFs with an anomalous dispersion region spanning part of the visible spectrum. We observed that for pulse powers exceeding 3 kW the spectral energy splits efficiently into two spectral bands, one in the UV and one in the near-IR spectral range. This process is driven by self-phase modulation and constitutes a supercontinuum source of high spectral brightness. In the end, this could lead to a multistage all-optical device, where the spectral distribution of the light can be tailored to suit various experimental demands. The first stage involves the conversion of the laser light into a pump light. The pump wavelength falls then within a narrow anomalous dispersion region of the second stage, generating bright spectral bands. The anomalous GVD region can be formed at almost any spectral range, owing to the wide tunability of the PCF structure or by using fibers of different materials. This could be very useful for medical or biological uses or for very sensitive frequency combs. Strongly different behavior can be observed when the pump power of the input pulse is lowered. In this case, soliton dynamics govern the spectral shaping process by dispersive wave generation.

However, the confinement of the solitons to a narrow wavelength region can then lead to multiple soliton collisions, enhancing the energy conversion into linear radiation. The energy loss can be large enough to destroy the existence of a soliton completely. The second part of chapter 5 was focusing on PCFs with more than the conventional one or two ZDWs. The presence of a third ZDW leads to unusual phase-matching behavior for four-wave mixing (FWM) and dispersive waves. In general, an additional phase-matching branch appears, influencing the propagation of near-CW or ultrashort pulses in these fibers. We discussed the potential for nonlinear wavelength conversion. For example, anti-crossings between the phase-matching branches gives rise to an increased tuning range of the FWM sidebands. When considering ultrashort pulse propagation, the unusual GVD profile modifies the spectral evolution and raises novel concepts of spectral shaping. A challenge will be to widen the range where we can find three ZDWs. However, promising candidates involves fibers made from different glass material. In addition the filling of the cladding holes with various materials changes the waveguide dispersion and could lead to the emergence of several ZDWs.

In chapter 6 we studied the dynamics of solitons in axially-varying PCFs. The continuously changing dispersion landscape and the progressive increase of the nonlinearity adds new possibilities to shape the characteristics of the propagating electric field. An impressive result involved the observation of the reversal of the soliton-self frequency shift (SSFS). The center of a soliton-like pulse can shift towards the blue; the SSFS always results in a red-shift. During this process, strong emission of dispersive waves can be observed, which are generated in the IR and the UV spectral ranges. This is favored by a gentle gradient of the long-wavelength ZDW. Another spectral change due to the continuous change of the dispersion is a very strong extension of the SC into the blue. This process is mediated by action of intrapulse FWM between the solitons and dispersive waves. It is favored by a strong gradient of the short-wavelength ZDW, for example accessible in a large-core PCF. So far we found that the spectrum can extent to the UV as low as 330 nm. However, in the future we will consider even larger gradients by starting at larger core radii. In principle the extent is then only limited by the strong absorption resonances of silica in the UV. This obstacle can be overcome by the use of a core material suitable for UV guidance.

Appendix A

Appendix A - Numerical implementation of the nonlinear Schrödinger equation

A.1 The split-step Fourier method

Direct numerical solutions of the generalized nonlinear Schrödinger equation (GNLSE) can be obtained by the split-step Fourier algorithm (SSFA) [145]. In this section we outline the implementation of a numerical solver, written in MATLAB. The basics of the SSFA lies in the separation of the linear and nonlinear terms. The continuous propagation is then reduced to an advancement in discrete steps, see Fig. A.1a. Therefore, Eq. (3.1) is written as

$$\frac{\partial A(z, \tau)}{\partial z} = (\hat{D} + \hat{N}(A(z, \tau)))A(z, \tau). \quad (\text{A.1})$$

The operator $\hat{D}$ accounts for the linear properties of the medium, and $\hat{N}(A)$ performs the nonlinear interaction of the medium on the pulse. Eq. (A.1) can be used to advance the field $A(z, t)$ to $A(z + h, t)$, where h denotes the discrete propagation step. The formal solution is given by

$$A(z + h, \tau) = e^{h(\hat{D} + \hat{N})} A(z, \tau) \simeq e^{\frac{h}{2}\hat{D}} \cdot e^{\int_z^{z+h} \hat{N}(z') dz'} \cdot e^{\frac{h}{2}\hat{D}} A(z, \tau). \quad (\text{A.2})$$

In the second step of Eq. (A.2) we used the Baker-Hausdorff approximation. The operators $\hat{D}$ and $\hat{N}(A)$ are now applied in series and act independently on the field envelope A . The single steps are as follows:

Figure A.1: (a) Schematic of the split-step Fourier method. The linear and nonlinear parts are treated separately and the field is advanced from $z=0$ to $z=h$ by (I) linear propagation to $h/2$, (II) execution of the nonlinear operator and (III) linear propagation from $h/2$ to h . (b) Calculation of the local error for the adaptive step size determination. The propagation of the field envelope from $z=0$ to $z=h$ is carried out twice, each with a different step size. Both results are compared and the step size accordingly adapted, see text.

- First, the nonlinear operator $\hat{N}$ is neglected, and the input field $A(z, t)$ is advanced to $A(z + h/2, t)$. This is done in the frequency domain, and the linear properties like the dispersion and loss are incorporated. This leads to

$$\tilde{A}(z + h/2, \omega) = e^{\frac{h}{2}\hat{D}} \tilde{A}(z, \omega) \quad (\text{A.3})$$

- The action of the nonlinearity over the full propagation step h is included in the middle of the segment. The operation is done in the time domain,

$$A(z + h/2, \tau) = e^{\frac{h}{2}\hat{N}} A(z + h/2, \tau). \quad (\text{A.4})$$

For $h \rightarrow 0$ the integral can be approximated by $\hat{N}(z)$.

- Last, the pulse amplitude at the end of the step h is calculated by removing the nonlinear operator from the GNLSE,

$$\tilde{A}(z + h, \omega) = e^{\frac{h}{2}\hat{D}} \tilde{A}(z + h/2, \omega) \quad (\text{A.5})$$

The step size h can be chosen to be fixed for all positions in the fiber, i.e. $h(n) = h$, where n is the number of the step. Alternatively, it may vary during propagation (adaptive scheme), as it is done by the simulations presented in this thesis. The distribution $h(n)$ of the discrete step size is determined by the amount of change in $A(z + h, t)$, and will be discussed later.

A.2 Setting up the amplitude and the operators

Analytic signal

The complex-valued envelope $A(z, t)$ represents the optical field launched into the fiber and is called the analytic signal. It is related to the (real) electric field $E(t)$ of the pulse [92, 93].

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} d\omega. \quad (\text{A.6})$$

Since $E(t)$ is real-valued, $\tilde{E}(\omega)$ is Hermitian and satisfies $\tilde{E}(\omega) = \tilde{E}^*(-\omega)$. The pulse is therefore completely characterized by the knowledge of the spectrum for positive frequencies. We define

$$\tilde{E}^+(\omega) = \begin{cases} \tilde{E}(\omega) & \text{for } \omega \geq 0 \text{ and} \\ 0 & \text{for } \omega < 0 \end{cases} \quad (\text{A.7})$$

$$\tilde{E}^-(\omega) = \begin{cases} \tilde{E}(\omega) & \text{for } \omega < 0 \text{ and} \\ 0 & \text{for } \omega \geq 0 \end{cases} \quad (\text{A.8})$$

and with

$$E^+(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}^+(\omega) e^{i\omega t} d\omega \quad (\text{A.9})$$

$$\tilde{E}^+(\omega) = \int_{-\infty}^{\infty} E^+(t) e^{-i\omega t} dt \quad (\text{A.10})$$

we end up with the analytic signal $E^+(t)$, related to the real electric field by

$$E(t) = E^+(t) + E^-(t). \quad (\text{A.11})$$

The analytic signal is complex-valued and can be represented in terms of amplitude and phase,

$$E^+(t) = \frac{1}{2} \bar{A}(t) e^{i\Phi_0} e^{i\omega_0 t} e^{i\Phi(t)} = \frac{1}{2} A(t) e^{i\omega_0 t}. \quad (\text{A.12})$$

Here $\bar{A}(t)$ is the real field envelope and $A(t)$ is the complex field envelope function. The constant phase term $e^{i\Phi_0}$ is the carrier-envelope phase, $e^{i\omega_0 t}$ describes the carrier oscillations and $\Phi(t)$ is the time-dependent phase function. It is very instructive to decompose the Fourier transform of the analytic signal into an amplitude and phase term

$$\tilde{E}^+(\omega) = |\tilde{E}^+(\omega)| e^{-i\Phi(\omega)}, \quad (\text{A.13})$$

where $|\tilde{E}^+(\omega)|$ is the spectral amplitude and $\phi(\omega)$ is the spectral phase. The spectral phase can be expanded into a Taylor-series,

$$\phi(\omega) = \sum_{i=0}^{\infty} \frac{\partial_{\omega}^i \Phi(\omega)|_{\omega_0}}{i!} \cdot (\omega - \omega_0)^i \quad (\text{A.14})$$

$$= \Phi(\omega_0) + \Phi'(\omega_0)(\omega - \omega_0) + \frac{1}{2} \Phi''(\omega_0)(\omega - \omega_0)^2 + \frac{1}{6} \Phi'''(\omega_0)(\omega - \omega_0)^3 + \dots \quad (\text{A.15})$$

Here, $\Phi(\omega_0)$ corresponds to the carrier envelope phase and can be neglected. The first-order term $\Phi'(\omega_0)$ leads to a temporal translation of the envelope. More important are the higher-order coefficients, which result in changes of the temporal envelope. For example, the Φ'' term adds a linear frequency variation across the input pulse and the pulse is said to be chirped. In chapter 3 we have seen that changes in the spectral phase result in changes in the electric field strength $E(t)$, leaving the spectral amplitude unchanged. A typical example is dispersive propagation in media, which induces modulations in the spectral phase.

The complex field envelope function is used to define the input pulse prior to the simulations (for example pulse shape, the pulse duration and the peak power). Additionally, a temporal delay or linear or higher-order chirp functions can be applied to the pulse.

Discretization of the time domain

Rather than using a continuous function to simulate the pulse propagation, the temporal (and therefore the spectral) domain is discretized. Each temporal bin is then numerically evaluated

during propagation from z to $z + h$. The size of the temporal and spectral bins depends on the size of the temporal window and the number of discretization points. The temporal window needs to be large enough to comprise all temporal features¹. One way of building the temporal domain in MATLAB is

$$T = \text{linspace}(-x \cdot T_0, x \cdot T_0, N), \quad (\text{A.16})$$

where T_0 is the input pulse duration and parameter N is the number of discretization points. Typically N is in the order of 2^{13} to 2^{16} . The value x is a free parameter, used to adjust the size of the temporal and spectral domain and will be determined later. Note that in Eq. (A.16) the temporal domain is symmetric around $T_0=0$. From Eq. (A.16) we can determine the grid size dT of the temporal domain

$$dT = 2 \cdot x \cdot T_0 / N. \quad (\text{A.17})$$

The next step is to build the spectral domain. The elimination of the carrier oscillations (contained in the factor $e^{-i\omega_0 t}$ in Eq. A.12) places the center of the spectral amplitude at $\omega = 0$ (instead of ω_0). The spectral grid size $d\nu$ is determined by

$$d\nu = 1/\Delta T = 1/(dT \cdot N) = 1/(2 \cdot x \cdot T_0), \quad (\text{A.18})$$

where ΔT labels the size of the temporal window. An implementation in MATLAB necessitates that the frequency domain is build in a wrap-around manner,

$$\begin{aligned} \nu &= [0 : N/2 - 1; -N/2 : -1] \cdot d\nu \\ &= [0 : N/2 - 1; -N/2 : -1]/(dT \cdot N); \\ \lambda &= 2 \cdot \pi \cdot \nu / (2 \cdot \pi \cdot \nu + \omega_0); \end{aligned} \quad (\text{A.19})$$

where c is the speed of light. The connection of the temporal and spectral domains becomes clear. A very large temporal window ΔT is connected with a high resolution in the spectral domain, but gives poor resolution in the temporal domain. The temporal domain should be discretized in a way that ultrafast features can be resolved, usually requiring resolutions of a few femtoseconds or even smaller. The size of the spectral domain, $\Delta\nu = 1/dT = N \cdot \pi (x \cdot T_0)$, is also influencing

¹A quick remark here: When periodic boundary conditions are used (rather than absorbing), rapid dispersing components cycle through the window. This is acceptable as long as no artificial collisions are generated.

the size of the temporal domain by the free parameter x . Generally, $\Delta\nu$ is set according to the prospective spectral broadening. The higher frequency boundary ν_{high} of the spectrum is then used to define the free parameter x . Reasonable choices for ν_{high} depend on the high-frequency edge of the simulated spectrum. Since the spectral window is centered about the pulse carrier (see Eq. A.19), we write $\Delta\nu = 2 \cdot (\nu_{high} - \nu_0)$, leading to a suitable choice for x ,

$$x = N \cdot \pi (2 \cdot T_0 \cdot (\nu_{high} - \nu_0)). \quad (\text{A.20})$$

Since $(\nu_{high} - \nu_0)$ can be very large, the low-frequency boundary of the spectral domain can extend into negative frequencies. This must be avoided, because the analytic signal was defined for positive frequencies. A fixed positive value for the low-frequency boundary is introduced. Potentially surfacing artifacts are avoided by multiplication of the protruding part of the spectral envelope with zero. However, the spectral boundaries should be chosen to contain the complete pulse spectrum. If not, a sharp edge develops at the boundaries, corresponding to a convolution of the spectrum with a Heaviside function. Since the Fourier transformation of a step function will yield a considerable amount of high-frequencies features, pronounced artifacts can appear in the simulated field.

A.3 The propagator — implementation of the length propagation

We outline the implementation of the numerical simulator. After the analytical signal has been created, the propagator is used to advance the solution from $z = 0$ to $z = L$, where L is the fiber length. The outline of the SSFA has been presented in section A.1. First, the dispersive step is applied to the complex analytic signal. The dispersion operator comprises the fiber dispersion. The dispersion operator $\hat{D}$ is easily derived

$$\hat{D} = i \cdot (\beta' - \beta_1 \cdot \omega' - \beta_0); \quad (\text{A.21})$$

Note that β' is evaluated around the carrier frequency ω_0 and $\omega' = \omega - \omega_0$. The parameter β_1 is the frequency derivative of β' . Note that ω is constructed in the wrap-around order. The field

$A(z = 0, t) = U_0$ is then propagated to $h/2$ by the operation

$$U_d = \text{ifft}(\text{fft}(U_0) \cdot \exp(0.5 \cdot h \cdot \hat{D})), \quad (\text{A.22})$$

Here, U_d denotes the field at $z = h/2$, and is used for the next step.

The nonlinear operator $\hat{N}$ is executed in the temporal domain and comprises the Kerr and the Raman effect, written as

$$R(t) = (1 - fR)\delta(t - t_e) + fR \cdot h_R(t), \quad (\text{A.23})$$

see section 2.2.3. The Raman function can be implemented directly in the frequency domain, or one uses the simple gaussian approximation,

$$h_R = \left((\tau_1^2 + \tau_2^2) / (\tau_1 \cdot \tau_2) \right) \cdot \exp(-T/\tau_2) \cdot \sin(T/\tau_1) \cdot H(T). \quad (\text{A.24})$$

Here, $\tau_1 = 12.2 \times 10^{-15}$ fs and $\tau_2 = 32 \times 10^{-15}$ fs. H refers to the Heaviside function. The Raman response function is normalized by setting the temporal integral of h_R to unity. The nonlinear response function A.23 can then be evaluated with help of the field A.22, and the nonlinear operator is constructed

$$\begin{aligned} R_{nl} &= (1 - fR) \cdot \text{abs}(U_d)^2 + fR \cdot (h_R \star U_d) \\ \hat{N} &= i \cdot \gamma_0 \cdot R_{nl} - \gamma_1 \cdot \text{gradient}(R_{nl} \cdot U_d) / U_d \\ U_{nl} &= U_d \cdot \exp(h \cdot \hat{N}) \end{aligned} \quad (\text{A.25})$$

Here, $\star$ refers to the convolution of h_R and U_d . This corresponds to multiplication of the Fourier transforms of h_R and U_d . The function R_{nl} carries out the convolution of the Raman response function A.24 and the optical field A.22. The field U_{nl} is then advanced to position h

$$A(z = h, t) = U_d = \text{ifft}(\text{fft}(U_{nl}) \cdot \exp(0.5 \cdot h \cdot \hat{D})). \quad (\text{A.26})$$

The field $A(z = h, t)$ has been numerically evaluated, starting from the field at $A(z = 0, t)$.

At this point, the performance of the step size h needs to be analyzed. This is done by propagating

a second time from $z = 0$ to $z = h$. This time, the step size h is halved. After twofold application of the propagator a fine solution A' is achieved:

$$A(z = 0, t) \rightarrow A'(z = h/2, t) \rightarrow A'(z = h, t).$$

After that, both fields at the position $z = h$ are compared. A schematic of this process is shown in Fig. A.1b. Then the relative local error is calculated, as presented in [146]

$$\Delta = \frac{\sqrt{\text{trapz}(\text{abs}(A' - A)^2)}}{\sqrt{\text{trapz}(\text{abs}(A')^2)}}. \quad (\text{A.27})$$

The step size is adapted such that the relative local error within a specified range, typically 10^{-4} to 10^{-6} . When the error is too large, the step size is reduced, and the local error is evaluated once more. Otherwise the step size is increased for the next step. In Fig. A.2 we show the step-size distribution of the higher-order soliton propagation shown in Fig. 3.3. The step size is minimized around the points of strongest temporal compression. In general, the step size can be increased by using a fourth-order Runge-Kutta algorithm to advance the fields by the step size h .

Figure A.2: Step-size distribution of the higher-order soliton propagation presented in Fig. 3.3.

Bibliography

- [1] J.C. Knight, T.A. Birks, P. St.J. Russell, and D.M. Atkin, "All-silica single-mode optical fiber with photonic crystal cladding," *Optics Letters* **21**, 19, 1547 (1996).
- [2] P. St.J. Russell, "Photonic crystal fibers," *Science* **299**, 5605, 358 (2003).
- [3] E. Yablonovitch, "Inhibited spontaneous emission in solid-state physics and electronics," *Physical Review Letters* **58**, 20, 2059 (1987).
- [4] S. John, "Strong localization of photons in certain disordered dielectric superlattices," *Physical Review Letters* **58**, 23, 2486 (1987).
- [5] P. St.J. Russell, "Photonic-crystal fibers," *Journal of Lightwave Technology* **24**, 12, 4729 (2006).
- [6] P. St.J. Russell, T.A. Birks, and F.D. Lloyd-Lucas, "Photonic Bloch waves and photonic band gaps," E. Burstein and C. Weisbuch eds. (Birkhäuser, 1995).
- [7] M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, (Cambridge University Press, 2000).
- [8] T.A. Birks, J.C. Knight, and P. St.J. Russell, "Endlessly single-mode photonic crystal fiber," *Optics Letters* **22**, 13, 961 (1997).
- [9] A.W. Snyder and J.D. Love, *Optical waveguide theory*, (Springer, 1983).
- [10] B. Povazay, K. Bizheva, A. Unterhuber, B. Hermann, H. Sattmann, A.F. Fercher, W. Drexler, A. Apolonski, W.J. Wadsworth, J.C. Knight, P. St.J. Russell, M. Vetterlein, and E. Scherzer, "Submicrometer axial resolution optical coherence tomography," *Optics Letters* **27**, 20, 1800 (2002).

- [11] A. Aguirre, N. Nishizawa, J. Fujimoto, W. Seitz, M. Lederer, and D. Kopf, "Continuum generation in a novel photonic crystal fiber for ultrahigh resolution optical coherence tomography at 800 nm and 1300 nm," *Optics Express* **14**, 3, 1145 (2006).
- [12] H. Yokoyama, H. Tsubokawa, H. Guo, J. Shikata, K. Sato, K. Takashima, K. Kashiwagi, N. Saito, H. Taniguchi, and H. Ito, "Two-photon bioimaging utilizing supercontinuum light generated by a high-peak-power picosecond semiconductor laser source," *Journal of Biomedical Optics* **12**, 5, 054019 (2007).
- [13] D. Wildanger, E. Rittweger, L. Kastrup, and S.W. Hell, "STED microscopy with a supercontinuum laser source," *Optics Express* **16**, 13, 9614 (2008).
- [14] K. Lindfors, T. Kalkbrenner, P. Stoller, and V. Sandoghdar, "Detection and spectroscopy of gold nanoparticles using supercontinuum white light confocal microscopy," *Physical Review Letters* **93**, 3, 037401 (2004).
- [15] C.F. Kaminski, R.S. Watt, A.D. Elder, J.H. Frank, and J. Hult, "Supercontinuum radiation for applications in chemical sensing and microscopy," *Applied Physics B: Lasers and Optics* **92**, 3, 367 (2008).
- [16] D. Meshulach and Y. Silberberg, "Coherent quantum control of multiphoton transitions by shaped ultrashort optical pulses," *Physical Review A* **60**, 2, 1287 (1999).
- [17] R. Bartels, S. Backus, E. Zeek, L. Misoguti, G. Vdovin, I.P. Christov, M.M. Murnane, and H.C. Kapteyn, "Shaped-pulse optimization of coherent emission of high-harmonic soft X-rays," *Nature* **406**, 6792, 164 (2000).
- [18] R.W. Boyd, *Nonlinear optics*, (Academic Press, 2008).
- [19] J.D. Jackson, *Classical electrodynamics*, (Wiley, 1999).
- [20] G. He and S.H. Liu, *Physics of nonlinear optics*, (World Scientific, 1999).
- [21] A. Yariv and P. Yeh, *Photonics: optical electronics in modern communications*, (Oxford University Press, 2007).
- [22] M. Artiglia, G. Coppa, P. Di Vita, M. Potenza, and A. Sharma, "Mode field diameter measurements in single-mode optical fibers," *Journal of Lightwave Technology* **7**, 8, 1139 (1989).

- [23] M. Foster, K. Moll, and A. Gaeta, "Optimal waveguide dimensions for nonlinear interactions," *Optics Express* **12**, 13, 2880 (2004).
- [24] S. Afshar and T.M. Monro, "A full vectorial model for pulse propagation in emerging waveguides with subwavelength structures part I: Kerr nonlinearity," *Optics Express* **17**, 4, 2298 (2009).
- [25] F. Mitschke, *Fiber Optics: Physics and Technology*, (Springer, 2010).
- [26] W.H. Reeves, D.V. Skryabin, F. Biancalana, J.C. Knight, P. St.J. Russell, F.G. Omenetto, A. Efimov, and A.J. Taylor, "Transformation and control of ultra-short pulses in dispersion-engineered photonic crystal fibres," *Nature* **424**, 6948, 511 (2003).
- [27] J.M. Dudley, G. Genty, and S. Coen, "Supercontinuum generation in photonic crystal fiber," *Reviews of Modern Physics* **78**, 4, 1135 (2006).
- [28] G.J. Pearce, T.D. Hedley, and D.M. Bird, "Adaptive curvilinear coordinates in a plane-wave solution of Maxwell's equations in photonic crystals," *Physical Review B* **71**, 19, 195108 (2005).
- [29] G.J. Pearce, *Plane-wave methods for modelling photonic crystal fibre*, Ph.D. thesis, University of Bath (2006).
- [30] M. Ohashi, K. Shiraki, and K. Tajima, "Optical loss property of silica-based single-mode fibers," *Journal of Lightwave Technology* **10**, 5, 539 (1992).
- [31] W. Heitmann, "Attenuation analysis of silica-based single-mode fibers," *Journal of optical communications* **11**, 4, 122 (1990).
- [32] J. Zhou, K. Tajima, K. Nakajima, K. Kurokawa, C. Fukai, T. Matsui, and I. Sankawa, "Progress on low loss photonic crystal fibers," *Optical Fiber Technology* **11**, 2, 101 (2005).
- [33] T.P. White, R.C. McPhedran, C.M. de Sterke, L.C. Botten, and M.J. Steel, "Confinement losses in microstructured optical fibers," *Optics Letters* **26**, 21, 1660 (2001).
- [34] H.C. Nguyen, B.T. Kuhlmei, E.C. Mägi, M.J. Steel, P. Domachuk, C.L. Smith, and B.J. Eggleton, "Tapered photonic crystal fibres: properties, characterisation and applications," *Applied Physics B: Lasers and Optics* **81**, 2, 377 (2005).

- [35] B.T. Kuhlmei, H.C. Nguyen, M.J. Steel, and B.J. Eggleton, "Confinement loss in adiabatic photonic crystal fiber tapers," *Journal of the Optical Society of America B* **23**, 9, 1965 (2006).
- [36] D. Milam, "Review and assessment of measured values of the nonlinear refractive-index coefficient of fused silica," *Applied Optics* **37**, 3, 546 (1998).
- [37] R.H. Stolen, C. Lee, and R.K. Jain, "Development of the stimulated Raman spectrum in single-mode silica fibers," *Journal of the Optical Society of America B* **1**, 4, 652 (1984).
- [38] R. Shuker and R.W. Gammon, "Raman-scattering selection-rule breaking and the density of states in amorphous materials," *Physical Review Letters* **25**, 4, 222 (1970).
- [39] R.H. Stolen and E.P. Ippen, "Raman gain in glass optical waveguides," *Applied Physics Letters* **22**, 6, 276 (1973).
- [40] V.I. Karpman and E.M. Krushkal, "Modulated waves in nonlinear dispersive media," *Soviet Journal of Experimental and Theoretical Physics* **28**, 277 (1969).
- [41] A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion," *Applied Physics Letters* **23**, 3, 142 (1973).
- [42] L.F. Mollenauer and J.P. Gordon, *Solitons in optical fibers: fundamentals and applications*, (Academic Press, 2006).
- [43] G.P. Agrawal, *Nonlinear fiber optics*, (Academic Press, 2007).
- [44] A.C. Newell and J.V. Moloney, *Nonlinear optics*, (Westview Press, 2003).
- [45] L.F. Mollenauer, R.H. Stolen, and J.P. Gordon, "Experimental observation of picosecond pulse narrowing and solitons in optical fibers," *Physical Review Letters* **45**, 13, 1095 (1980).
- [46] H. Nakatsuka, D. Grischkowsky, and A.C. Balant, "Nonlinear picosecond-pulse propagation through optical fibers with positive group velocity dispersion," *Physical Review Letters* **47**, 13, 910 (1981).
- [47] J.P. Gordon, "Theory of the soliton self-frequency shift," *Optics Letters* **11**, 10, 662 (1986).
- [48] F.M. Mitschke and L.F. Mollenauer, "Discovery of the soliton self-frequency shift," *Optics Letters* **11**, 10, 659 (1986).

- [49] F.M. Mitschke and L.F. Mollenauer, "Experimental observation of interaction forces between solitons in optical fibers," *Optics Letters* **12**, 5, 355 (1987).
- [50] Y. Kodama and K. Nozaki, "Soliton interaction in optical fibers," *Optics Letters* **12**, 12, 1038 (1987).
- [51] J.K. Ranka, R.S. Windeler, and A.J. Stentz, "Visible continuum generation in air-silica microstructure optical fibers with anomalous dispersion at 800 nm," *Optics Letters* **25**, 1, 25 (2000).
- [52] J.M. Dudley and J.R. Taylor, *Supercontinuum Generation in Optical Fibers*, (Cambridge University Press, 2010).
- [53] T. Brabec and F. Krausz, "Nonlinear optical pulse propagation in the single-cycle regime," *Physical Review Letters* **78**, 17, 3282 (1997).
- [54] M. Kolesik, J.V. Moloney, and M. Mlejnek, "Unidirectional optical pulse propagation equation," *Physical Review Letters* **89**, 28, 283902 (2002).
- [55] M. Kolesik and J.V. Moloney, "Nonlinear optical pulse propagation simulation: From Maxwell's to unidirectional equations," *Physical Review E* **70**, 3, 036604 (2004).
- [56] F. Lu, Q. Lin, W.H. Knox, and G.P. Agrawal, "Vector soliton fission," *Physical Review Letters* **93**, 18, 183901 (2004).
- [57] S.N. Vlasov, V.A. Petrishchev, and V.I. Talanov, "Averaged description of wave beams in linear and nonlinear media (the method of moments)," *Radiophysics and Quantum Electronics* **14**, 9, 1062 (1971).
- [58] Y.S. Kivshar and G.P. Agrawal, *Optical solitons: from fibers to photonic crystals*, (Academic Press, 2003).
- [59] D.V. Skryabin, F. Luan, J.C. Knight, and P. St.J. Russell, "Soliton self-frequency shift cancellation in photonic crystal fibers," *Science* **301**, 5640, 1705 (2003).
- [60] S.A. Akhmanov, R.V. Khokhlov, and A.P. Sukhorukov, "Self-focusing, self-defocusing and self-modulation of laser beams," F.T. Arecchi and E.O. Schulz-DuBois (eds.) eds. (North-Holland, Amsterdam, 1972).

- [61] C. Naguram, A. Sudam, H. Kawanom, M. Obaram, and K. Midorikawa, "Generation and characterization of ultrafast white-light continuum in condensed media," *Applied Optics* **41**, 18, 3735 (2002).
- [62] V.P. Kandidov, O.G. Kosareva, I.S. Golubtsov, W. Liu, A. Becker, N. Akozbek, C.M. Bowden, and S.L. Chin, "Self-transformation of a powerful femtosecond laser pulse into a white-light laser pulse in bulk optical media (or supercontinuum generation)," *Applied Physics B: Lasers and Optics* **77**, 2-3, 149 (2003).
- [63] K. Cook, A.K. Kar, and R.A. Lamb, "White-light supercontinuum interference of self-focused filaments in water," *Applied Physics Letters* **83**, 19, 3861 (2003).
- [64] M. Kolesik, E.M. Wright, and J.V. Moloney, "Dynamic nonlinear X waves for femtosecond pulse propagation in water," *Physical Review Letters* **92**, 25, 253901 (2004).
- [65] P.B. Corkum, C. Rolland, and T. Srinivasan-Rao, "Supercontinuum generation in gases," *Physical Review Letters* **57**, 18, 2268 (1986).
- [66] T.R. Gosnell, A.J. Taylor, and D.P. Greene, "Supercontinuum generation at 248 nm using high-pressure gases," *Optics Letters* **15**, 2, 130 (1990).
- [67] N. Aközbek, A. Becker, M. Scalora, S.L. Chin, and C.M. Bowden, "Continuum generation of the third-harmonic pulse generated by an intense femtosecond IR laser pulse in air," *Applied Physics B: Lasers and Optics* **77**, 2-3, 177 (2003).
- [68] T.X. Tran and F. Biancalana, "Dynamics and control of the early stage of supercontinuum generation in submicron-core optical fibers," *Physical Review A* **79**, 6, 065802 (2009).
- [69] V.I. Karpman, "Radiation by solitons due to higher-order dispersion," *Physical Review E* **47**, 3, 2073 (1993).
- [70] N. Akhmediev and M. Karlsson, "Cherenkov radiation emitted by solitons in optical fibers," *Physical Review A* **51**, 3, 2602 (1995).
- [71] F. Biancalana, D.V. Skryabin, and A.V. Yulin, "Theory of the soliton self-frequency shift compensation by the resonant radiation in photonic crystal fibers," *Physical Review E* **70**, 1, 016615 (2004).

- [72] D.R. Austin, C.M. de Sterke, B.J. Eggleton, and T.G. Brown, "Dispersive wave blue-shift in supercontinuum generation," *Optics Express* **14**, 25, 11997 (2006).
- [73] A.V. Husakou and J. Herrmann, "Supercontinuum generation of higher-order solitons by fission in photonic crystal fibers," *Physical Review Letters* **87**, 20, 203901 (2001).
- [74] E.N. Tsoy and C.M. de Sterke, "Theoretical analysis of the self-frequency shift near zero-dispersion points: Soliton spectral tunneling," *Physical Review A* **76**, 4, 043804 (2007).
- [75] K. Tai, A. Hasegawa, and N. Bekki, "Fission of optical solitons induced by stimulated Raman effect," *Optics Letters* **13**, 5, 392 (1988).
- [76] P. Beaud, W. Hodel, B. Zysset, and H. Weber, "Ultrashort pulse propagation, pulse breakup, and fundamental soliton formation in a single-mode optical fiber," *IEEE Journal of Quantum Electronics* **23**, 11, 1938 (1987).
- [77] A.B. Grudinin, E.M. Dianov, D.V. Korobkin, A.M. Prokhorov, V.N. Serkin, and D.V. Khaidarov, "Decay of femtosecond pulses in single-mode optical-fibers," *JETP Lett* **46**, 5 (1987).
- [78] Y. Kodama and A. Hasegawa, "Nonlinear pulse propagation in a monomode dielectric guide," *IEEE Journal of Quantum Electronics* **23**, 5, 510 (1987).
- [79] A. Podlipensky, P. Szarniak, N.Y. Joly, and P. St.J. Russell, "Anomalous pulse breakup in small-core photonic crystal fibers," *Journal of the Optical Society of America B* **25**, 12, 2049 (2008).
- [80] J.M. Dudley, X. Gu, L. Xu, M. Kimmel, E. Zeek, P. O'Shea, R. Trebino, S. Coen, and R.S. Windeler, "Cross-correlation frequency resolved optical gating analysis of broadband continuum generation in photonic crystal fiber: simulations and experiments," *Optics Express* **10**, 21, 1215 (2002).
- [81] R. Trebino, *Frequency-resolved optical gating: the measurement of ultrashort laser pulses*, (Springer, 2000).
- [82] A.V. Gorbach and D.V. Skryabin, "Soliton self-frequency shift, non-soliton radiation and self-induced transparency in air-core fibers," *Optics Express* **16**, 7, 4858 (2008).

- [83] A.V. Gorbach and D.V. Skryabin, "Bouncing of a dispersive pulse on an accelerating soliton and stepwise frequency conversion in optical fibers," *Optics Express* **15**, 22, 14560 (2007).
- [84] N. Nishizawa and T. Goto, "Characteristics of pulse trapping by ultrashort soliton pulse in optical fibers across zerodispersion wavelength," *Optics Express* **10**, 21, 1151 (2002).
- [85] N. Nishizawa and T. Goto, "Pulse trapping by ultrashort soliton pulses in optical fibers across zero-dispersion wavelength," *Optics Letters* **27**, 3, 152 (2002).
- [86] J.C. Travers and J.R. Taylor, "Soliton trapping of dispersive waves in tapered optical fibers," *Optics Letters* **34**, 2, 115 (2009).
- [87] P.F. Moulton, "Spectroscopic and laser characteristics of Ti:Al₂O₃," *Journal of the Optical Society of America B* **3**, 1, 125 (1986).
- [88] A.E. Siegman, *Lasers*, (University Science Books, 1986).
- [89] E. Riedle, M. Beutter, S. Lochbrunner, J. Piel, S. Schenkl, S. Spörlein, and W. Zinth, "Generation of 10 to 50 fs pulses tunable through all of the visible and the NIR," *Applied Physics B: Lasers and Optics* **71**, 457 (2000).
- [90] M.K. Reed, M.K. Steiner-Shepard, M.S. Armas, and D.K. Negus, "Microjoule-energy ultrafast optical parametric amplifiers," *Journal of the Optical Society of America B* **12**, 11, 2229 (1995).
- [91] I.A. Walmsley and C. Dorrer, "Characterization of ultrashort electromagnetic pulses," *Advances in Optics and Photonics* **1**, 2, 308 (2009).
- [92] J.-C. Diels and W. Rudolph, *Ultrashort laser pulse phenomena: fundamentals, techniques, and applications on a femtosecond time scale*, (Academic Press, 2006).
- [93] F. Träger, *Springer handbook of lasers and optics*, (Springer, 2007).
- [94] R. Trebino and D.J. Kane, "Using phase retrieval to measure the intensity and phase of ultrashort pulses: frequency-resolved optical gating," *Journal of the Optical Society of America A* **10**, 5, 1101 (1993).

- [95] K.W. DeLong, R. Trebino, J. Hunter, and W.E. White, "Frequency-resolved optical gating with the use of second-harmonic generation," *Journal of the Optical Society of America B* **11**, 11, 2206 (1994).
- [96] K.W. DeLong and Rick Trebino, "Improved ultrashort pulse-retrieval algorithm for frequency-resolved optical gating," *Journal of the Optical Society of America A* **11**, 9, 2429 (1994).
- [97] T.A. Birks and Y.W. Li, "The shape of fiber tapers," *Journal of Lightwave Technology* **10**, 4, 432 (1992).
- [98] W. Wadsworth, A. Witkowska, S. Leon-Saval, and T. Birks, "Hole inflation and tapering of stock photonic crystal fibres," *Optics Express* **13**, 17, 6541 (2005).
- [99] J.D. Love, W.M. Henry, W.J. Stewart, R.J. Black, S. Lacroix, and F. Gonthier, "Tapered single-mode fibres and devices. I. Adiabaticity criteria," *IEE Proceedings J. Optoelectronics* **138**, 5, 343 (1991).
- [100] W. Drexler, "Ultrahigh-resolution optical coherence tomography," *Journal of Biomedical Optics* **9**, 1, 47 (2004).
- [101] G. Chang, T.B. Norris, and H.G. Winful, "Optimization of supercontinuum generation in photonic crystal fibers for pulse compression," *Optics Letters* **28**, 7, 546 (2003).
- [102] B. von Vacano, W. Wohlleben, and M. Motzkus, "Actively shaped supercontinuum from a photonic crystal fiber for nonlinear coherent microspectroscopy," *Optics Letters* **31**, 3, 413 (2006).
- [103] J. Rarity, J. Fulconis, J. Duligall, W. Wadsworth, and P. St.J. Russell, "Photonic crystal fiber source of correlated photon pairs," *Optics Express* **13**, 2, 534 (2005).
- [104] J.E. Sharping, J. Chen, X. Li, P. Kumar, and R.S. Windeler, "Quantum-correlated twin photons from microstructure fiber," *Optics Express* **12**, 14, 3086 (2004).
- [105] D.R. Solli, C. Ropers, P. Koonath, and B. Jalali, "Optical rogue waves," *Nature* **450**, 7172, 1054 (2007).
- [106] J.M. Dudley, G. Genty, and B.J. Eggleton, "Harnessing and control of optical rogue waves in supercontinuum generation," *Optics Express* **16**, 6, 3644 (2008).

- [107] B. Kibler, J. Fatome, C. Finot, G. Millot, F. Dias, G. Genty, N. Akhmediev, and J.M. Dudley, "The Peregrine soliton in nonlinear fibre optics," *Nature Physics* **6**, 10, 790 (2010).
- [108] T.X. Tran, A. Podlipensky, P. St.J. Russell, and F. Biancalana, "Theory of Raman multipeak states in solid-core photonic crystal fibers," *Journal of the Optical Society of America B* **27**, 9, 1785 (2010).
- [109] A.M. Weiner, D.E. Leaird, G.P. Wiederrecht, and K.A. Nelson, "Femtosecond pulse sequences used for optical manipulation of molecular motion" *Science* **247**, 4948, 1317 (1990).
- [110] N.F. Scherer, R.J. Carlson, A. Matro, M. Du, A.J. Ruggiero, V. Romero-Rochin, J.A. Cina, G.R. Fleming, and S.A. Rice, "Fluorescence-detected wave packet interferometry: Time resolved molecular spectroscopy with sequences of femtosecond phase-locked pulses," *The Journal of Chemical Physics* **95**, 3, 1487 (1991).
- [111] A. Assion, T. Baumert, M. Bergt, T. Brixner, B. Kiefer, V. Seyfried, M. Strehle, and G. Gerber, "Control of chemical reactions by feedback-optimized phase-shaped femtosecond laser pulses," *Science* **282**, 5390, 919 (1998).
- [112] H. Rabitz, R. Vivie-Riedle, M. Motzkus, and K. Kompa, "Whither the Future of Controlling Quantum Phenomena?," *Science* **288**, 5467, 824 (2000).
- [113] M. Leibscher, I.S. Averbukh, and H. Rabitz, "Molecular alignment by trains of short laser pulses," *Physical Review Letters* **90**, 21, 213001 (2003).
- [114] J. Hauer, T. Buckup, and M. Motzkus, "Enhancement of molecular modes by electronically resonant multipulse excitation: Further progress towards mode selective chemistry," *The Journal of Chemical Physics* **125**, 6, 061101 (2006).
- [115] S. Coen and M. Haelterman, "Continuous-wave ultrahigh-repetition-rate pulse-train generation through modulational instability in a passive fiber cavity," *Optics Letters* **26**, 1, 39 (2001).
- [116] S.V. Chernikov, J.R. Taylor, and R. Kashyap, "Comblake dispersion-profiled fiber for soliton pulse train generation," *Optics Letters* **19**, 8, 539 (1994).

- [117] A. Nazarkin, R. Netz, and R. Sauerbrey, "Interference-induced transparency and coherent control of quantum systems by frequency-chirped pulses," *Physical Review A* **67**, 4, 041401 (2003).
- [118] D. Törke, W. Wöhlleben, J. Teipel, M. Motzkus, B. Kibler, J. Dudley, and H. Giessen, "Chirp-controlled soliton fission in tapered optical fibers," *Applied Physics B* **83**, 1, 37 (2006).
- [119] R.R. Gattass, G.T. Svacha, L. Tong, and E. Mazur, "Supercontinuum generation in submicrometer diameter silica fibers," *Optics Express* **14**, 20, 9408 (2006).
- [120] F. Benabid, F. Biancalana, P.S. Light, F. Couny, A. Luiten, P.J. Roberts, J. Peng, and A.V. Sokolov, "Fourth-order dispersion mediated solitonic radiations in HC-PCF cladding," *Optics Letters* **33**, 22, 2680 (2008).
- [121] S. Leon-Saval, T. Birks, W. Wadsworth, P. St.J. Russell, and M. Mason, "Supercontinuum generation in submicron fibre waveguides," *Optics Express* **12**, 13, 2864 (2004).
- [122] J.M. Harbold, F. Ilday, F.W. Wise, T.A. Birks, W.J. Wadsworth, and Z. Chen, "Long-wavelength continuum generation about the second dispersion zero of a tapered fiber," *Optics Letters* **27**, 17, 1558 (2002).
- [123] N.Y. Joly, F.G. Omenetto, A. Efimov, A.J. Taylor, J.C. Knight, and P. St.J. Russell, "Competition between spectral splitting and Raman frequency shift in negative-dispersion slope photonic crystal fiber," *Optics Communications* **248**, 1-3, 281 (2005).
- [124] M. Foster and A. Gaeta, "Ultra-low threshold supercontinuum generation in sub-wavelength waveguides," *Optics Express* **12**, 14, 3137 (2004).
- [125] M. Erkintalo, G. Genty, and J.M. Dudley, "Giant dispersive wave generation through soliton collision," *Optics Letters* **35**, 5, 658 (2010).
- [126] A. Hasegawa and M. Matsumoto, *Optical solitons in fibers*, (Springer, 2003).
- [127] K. Saitoh, M. Koshiba, T. Hasegawa, and E. Sasaoka, "Chromatic dispersion control in photonic crystal fibers: application to ultra-flattened dispersion," *Optics Express* **11**, 8, 843 (2003).

- [128] B. Kibler, P.-A. Lacourt, F. Courvoisier, and J.M. Dudley, "Soliton spectral tunnelling in photonic crystal fibre with sub-wavelength core defect," *Electronics Letters* **43**, 18, 967 (2007).
- [129] J. Hansryd and P.A. Andrekson, "Broad-band continuous-wave-pumped fiber optical parametric amplifier with 49-dB gain and wavelength-conversion efficiency," *IEEE Photonics Technology Letters* **13**, 3, 194 (2001).
- [130] Y. Takushima and K. Kikuchi, "10-GHz, over 20-channel multiwavelength pulse source by slicing super-continuum spectrum generated in normal-dispersion fiber," *Photonics Technology Letters, IEEE* **11**, 3, 322 (1999).
- [131] V.N. Serkin, V.A. Vysloukh, and J.R. Taylor, "Soliton spectral tunnelling effect," *Electronics Letters* **29**, 1, 12 (1993).
- [132] F. Poletti, P. Horak, and D.J. Richardson, "Soliton spectral tunneling in dispersion-controlled holey fibers," *Photonics Technology Letters, IEEE* **20**, 16, 1414 (2008).
- [133] A. Podlipensky, P. Szarniak, N.Y. Joly, C.G. Poulton, and P. St.J. Russell, "Bound soliton pairs in photonic crystal fiber," *Optics Express* **15**, 4, 1653 (2007).
- [134] S.V. Chernikov and P.V. Mamyshev, "Femtosecond soliton propagation in fibers with slowly decreasing dispersion," *Journal of the Optical Society of America B* **8**, 8, 1633 (1991).
- [135] S.V. Chernikov, E.M. Dianov, D.J. Richardson, and D.N. Payne, "Soliton pulse compression in dispersion-decreasing fiber," *Optics Letters* **18**, 7, 476 (1993).
- [136] J.C. Travers, J.M. Stone, A.B. Rulkov, B.A. Cumberland, A.K. George, S.V. Popov, J.C. Knight, and J.R. Taylor, "Optical pulse compression in dispersion decreasing photonic crystal fiber," *Optics Express* **15**, 20, 13203 (2007).
- [137] F. Lu and W.H. Knox, "Generation, characterization, and application of broadband coherent femtosecond visible pulses in dispersion micromanaged holey fibers," *Journal of the Optical Society of America B* **23**, 6, 1221 (2006).
- [138] A. Kudlinski, A.K. George, J.C. Knight, J.C. Travers, A.B. Rulkov, S.V. Popov, and J.R. Taylor, "Zero-dispersion wavelength decreasing photonic crystal fibers for ultraviolet-extended supercontinuum generation," *Optics Express* **14**, 12, 5715 (2006).

- [139] J.C. Knight, T.A. Birks, P. St.J. Russell, and J.P. de Sandro, "Properties of photonic crystal fiber and the effective index model," *Journal of the Optical Society of America A* **15**, 3, 748 (1998).
- [140] "Soliton blue-shift in tapered photonic crystal fiber," *arXiv:1012.3670* .
- [141] G. Genty, M. Lehtonen, and H. Ludvigsen, "Effect of cross-phase modulation on supercontinuum generated in microstructured fibers with sub-30 fs pulses," *Optics Express* **12**, 19, 4614 (2004).
- [142] T. Schreiber, T. Andersen, D. Schimpf, J. Limpert, and A. Tünnermann, "Supercontinuum generation by femtosecond single and dual wavelength pumping in photonic crystal fibers with two zero dispersion wavelengths," *Optics Express* **13**, 23, 9556 (2005).
- [143] A.V. Gorbach, D.V. Skryabin, J.M. Stone, and J.C. Knight, "Four-wave mixing of solitons with radiation and quasi-nondispersive wave packets at the short-wavelength edge of a supercontinuum," *Optics Express* **14**, 21, 9854 (2006).
- [144] D.V. Skryabin and A.V. Yulin, "Theory of generation of new frequencies by mixing of solitons and dispersive waves in optical fibers," *Physical Review E* **72**, 1, 016619 (2005).
- [145] F.D. Tappert and C.N. Judice, "Recurrence of nonlinear ion acoustic waves," *Physical Review Letters* **29**, 19, 1308 (1972).
- [146] O.V. Sinkin, R. Holzlöhner, J. Zweck, and C.R. Menyuk, "Optimization of the split-step Fourier method in modeling optical-fiber communications systems," *Journal of Lightwave Technology* **21**, 1, 61 (2003).

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- S. Stark, A. Podlipensky, and P. St.J. Russell, '*THz pulse train generation in small-core photonic crystal fiber,*' in preparation
- S. Stark, F. Biancalana, A. Podlipensky, P. St.J. Russell, '*Nonlinear wavelength conversion in photonic crystal fibers with three zero dispersion points,*' Phys. Rev. A **83**, 023808 (2011).
- S. Stark, A. Podlipensky, P. St.J. Russell, '*Soliton blue-shift in tapered photonic crystal fiber,*' Phys. Rev. Lett. **106**, 083903 (2011).
- C. Conti, S. Stark, P.St.J. Russell, and F. Biancalana (2010), '*Multiple hydrodynamical shocks induced by Raman effect in photonic crystal fibres,*' Phys. Rev. A **82**, 013838 (2010).
- F. Biancalana, T.X. Tran, S. Stark, M.A. Schmidt, and P.S. Russell, '*Emergence of geometrical optical nonlinearities in photonic crystal fiber nanowires,*' Phys. Rev. Lett. **105**, 093904 (2010).
- S. Stark, A. Podlipensky, N.Y. Joly, and P. St.J. Russell, '*Ultraviolet-enhanced supercontinuum generation in tapered photonic crystal fiber,*' J. Opt. Soc. Am. B **27**, 592-598 (2010).

Conference Papers

- S. Stark, H. Hundertmark, P. St.J. Russell, R. Probst, T. Wilken, T.W. Hnsch, Th. Udem, T. Steinmetz, R. Holzwarth, '*14 GHz Supercontinuum Generation: Calibration Sources for Astronomical Spectrographs,*' CLEO Conference, Munich, Germany, 2011.
- T. Wilken, R. Probst, T.W. Hnsch, Th. Udem, T. Steinmetz, R. Holzwarth, A. Manescau, G. Lo Curto, L. Pasquini, S. Stark, H. Hundertmark, P.St.J Russell, '*Suppressed mode recovery in nonlinear fibers of a Fabry-Perot-filtered frequency comb,*' CLEO/QELS Conference, Baltimore, USA, 2011.
- S. Stark, A. Podlipensky and P. St.J. Russell, '*Novel aspects of pulse propagation in photonic-crystal fibers,*' SPIE Photonics West, San Francisco, USA, 2011.
- N. Joly, P. Hölzer, J. Nold, K.L.G. Wong, S. Stark, W. Chang, A. Podlipensky and P. St.J. Russell, '*Efficient generation of deep UV in air-filled hollow-core photonic crystal fibre,*' Frontiers of Nonlinear Physics, Nizhny Novgorod, Russia, 2010.

- A. Podlipensky, S. Stark and P. St.J. Russell, '*THz Pulse Train Generation in Small Core PCF*,' CLEO/Europe-EQEC Focus Meeting 2010, Torino, Italy, 2010.
- S. Stark, F. Biancalana, A. Podlipensky, and P.St.J. Russell, '*Topology of four-wave mixing and resonant radiation in PCFs with three zero-dispersion points*,' Nonlinear Photonics, Karlsruhe, Germany, 2010.
- P. Hölzer, J. Nold, N.Y. Joly, K.L.G. Wong, M. Scharrer, S. Stark, W. Chang, A. Podlipensky and P.St.J. Russell, '*4% conversion of sub- μ J near-IR pulses to deep UV in fundamental mode of Ar-filled PCF*,' CLEO/QELS Conference , USA, 2010.
- S. Stark, A. Podlipensky, N.Y. Joly and P. St.J. Russell, '*Spectral broadening of visible ultrashort pulses in tapered PCF*,' CLEO/QELS Conference, Munich, Germany, 2009.