Taxation
and International Capital Asset Pricing Theory

Inauguraldissertation

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Abstract

Adler and Dumas (1983) laid the foundation for pricing international assets under deviation from Relative Purchasing Power Parity (PPP). Only Lally (1996) regards the spectrum of international taxation but in his model - he disregards the tremendous impact of exchange gains taxation in International Capital Asset Pricing Theory (IntCAPT). Furthermore, the consensus in economic literature that exchange rates show evidence of a non-linear behavior as elaborated by Dumas (1992), Grauwe (1993) and Serçu and Uppal (1995) and that monetary policy ultimately determines inflation as determined by McCallum (1990) is ignored. In addition to this, a realistic version of the Tax International Capital Asset Pricing Model (Tax – IntCAPM) should incorporate the fact that dividends are stochastic, as developed in the Tax Capital Asset Pricing Model (Tax – CAPM) of Lally (1998), Wiese (2006b) and Mai (2006a).

This dissertation develops a theory of taxation in pricing international assets. To understand this theory, in the first part we introduce and discuss the research question and the conceptual procedure of the dissertation. The review of the status of research provides an extensive overview on research pertaining to taxation in IntCAPT. In the second part, the framework of international taxation is introduced, and by introducing the features of exchange gains taxation a new income type in IntCAPT is presented. The analysis of the international tax system with the features of exchange gains taxation leads to the new result that under the hypothesis of Relative PPP certain constellations of international taxation lead to a Tax-IntCAPM that would be equal to the Tax-CAPM. With the features of exchange gains taxation and the modeling of deviation from Relative PPP by non-linear behavior of exchange rate and inflation determined by monetary policy, an extended model of taxation in IntCAPT – the Tax-IntCAPM – is developed and interpreted. The new result is that the integration of exchange gains taxation into the Tax-IntCAPM leads to an international pricing relationship composed of the risky asset’s excess return and its world risk premium, which is adapted by exchange gains tax factors. The non-linear deterministic behavior of exchange rates and the determination of inflation by monetary policy lead to the integration of the market equilibrium exchange and inflation rate into the Tax-IntCAPM. International tax arbitrage opportunities lead to the derivation of the Tax-IntCAPM under short sale and borrowing restrictions. To implement this new international capital market model, the Tax-IntCAPM with homogeneous
expectations is derived and interpreted. The third part concludes the dissertation with an extensive critique elaborating the boundaries of the models and a conclusion summarizing the main results and analyzing the implications of the findings.

Style manual used was the *MLA Handbook for Writers of Research Papers*, 7th Edition.

Computer software used was LaTeX2e and Microsoft® Excel 2007.
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XIX
1 Introduction

"I am certain there is too much certainty in the world."


International commerce is not a new phenomenon, it dates back into history beyond the Phoenicians around 1200 B.C. (Kotabe (1999)). International capital market integration commenced with the flow of capital from Britain to the United States, Latin America and the British colonies after 1700 (Neal (1990) and Zevin (1992)). In the last and in future decades the dynamic process of globalization initialized by global externalizations of economic, political and social developments enhanced and will enhance the impact of international investments on the world economy.

The process of globalization has led to the integration of national capital markets, yet nationhoods in our world economy are delineated along barriers in the form of taxation of international investment and different inflation and exchange rates generating a form of heterogeneity in international investors and consequently leading to the partial segmentation of markets.

The imposition of taxes extends back into history beyond the Egyptian Pharaohs; the tax collectors – known as scribes – imposed a tax on cooking oil, whereas the Athenians introduced international taxation by imposing a monthly poll tax on foreigners, people who
were not born to an Athenian mother and father (Adams (1999)). The tax systems that emerged in the 19th and 20th centuries were constructed for (relatively) closed economies. Globalization created a new situation leading to the abolition of nation state borders for the movement of capital. Multinational institutions like the United Nations (UN) and Organization for Economic Co-operation and Development (OECD) exert worldwide pressure to remove fiscal barriers and construct model agreements of international taxation (Wahl (2005)). Human beings of different countries have totally different tastes and consume different baskets of goods. The baskets of goods on which they consume the income from their investments face different processes of price. In such a setting, further dimensions of internationality resulting from deviation from PPP arise. In the international conference of Bretton Woods in New Hampshire in July 1944 a fixed exchange rate regime was established pursuing the concept that the expansion of international trade should be framed by the international fixation of exchange rates. In the beginning of the 70s the imbalance of currency deposits led to the suspension of the fixed exchange rate system of Bretton Woods and to free floating exchange rates (James (1996) and Samuelson and Nordhaus (2005)). The passing to floating exchange rates symbolized the approach to a more liberal policy of economy and distance from the theories of Keynes pursuing the idea of a more intervening role of the state (James (1996) and Samuelson and Nordhaus (2005)).

The delineation of national groups of investors by international taxation and deviation

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1 The theoretical framework for income taxation was established by Schanz (1896), Haig (1921) and Simmons (1962) (Schanz – Haigh – Simmons (SHS)). The income tax has a comprehensive character in order to secure the universality of taxation and a synthetic nature, since different incomes are summed up to one tax unit. The income tax pursues the notion that income from all sources is combined to determine the taxable income. A further income conception is the schedular system or the dual income tax; under this hybrid conception capital income is taxed separately from other income. Capital income is taxed at relatively low rates. The reason for the low tax rates is the increasing mobility of capital income; the rate is aligned to the bottom rate of the progressive tax on other income (Head (1997)). Further theoretical conceptions are the consumption or expenditure income while the tax basis is the difference between income and savings (Kaldor (1955) and Andrews (1974)).

2 The exact beginning of the suspension of the fixed exchange rate system of Bretton Woods is controversial in literature. James (1996) is of the opinion that the suspension of the system of Bretton Woods began in August 1971 with Nixon’s announcement of the New Economic Program proclaiming flexible exchange rates and by ending the convertibility of the U.S. dollar to gold whereas Emminger (1986) pursues the opinion that the suspension of the system of Bretton Woods was in March 1973 since the Smithsonian Agreement in December 1971 maintained the system of flexible exchange rates until March 1973.
from Relative PPP cause them to evaluate differently the returns from the same asset and consequently lead to partial segmentation of markets (Adler and Dumas (1982)). The segmentation of markets prevents investors from holding international assets and results in their imperfection, leading to an equilibrium deviating from the perfect market outcome (Dumas (1994a)).

The compelling theoretical and empirical arguments show evidence of the existence of home bias (French and Poterba (1991), Cooper and Kaplanis (1994) and Tesar and Werner (1995)). The delineation of national groups of investors by deviation from Relative PPP and international taxation are explanations for the international market puzzle why domestic assets can offer superior risk and return relationship to foreign assets.

Theoretical models were developed to describe the features of international asset pricing. However, the heterogeneity in investment opportunities for investors from different countries in the form of international taxation changes the classic IntCAPM and desires a model of international asset pricing integrating the features of international taxation.

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3 In contrast to segmentation, incompleteness leads to the impossibility of trading assets (Dumas (1994a)).

4 The terms Intertemporal Capital Asset Pricing Model (ICAPM) and International Arbitrage Pricing Model (IAPM) are used for the International Capital Asset Pricing Model, however models of Intertemporal Capital Asset Pricing Theory and of International Arbitrage Pricing Theory are summarized by these terms (Dumas (1994a) and Zimmermann (2003)).
2 Research Question and Conceptual Procedure

“If we knew what it was we were doing, it would not be called research, would it?”

Albert Einstein, US (German – born) physicist (1879 – 1955)

Various universal models of valuation of international assets were developed, but we are ignorant of the influence of international taxation on the pricing of international assets. Referred to the tension between knowledge and ignorance, the problem of pricing international assets under barriers in the form of taxation arises. We discover a contradiction in our supposed knowledge, since the IntCAPMs are not applicable for international affairs. IntCAPT draws the picture of homogenous investors pursuing the conception of international investment motivated by the benefit of diversification (Grubel (1968) and Levy and Sarnat (1970)). This picture is contradicted by barriers in the form of taxation and the delineation of the world across countries along deviation from Relative PPP leading to heterogeneous market participants. The Capital Asset Pricing Model (CAPM) is considered to be the fundamental model of integration of taxes into asset pricing theory.5 Although the model is often criticized, the Capital Asset Pricing Model remains the best illustration of long-term tradeoffs between risk and return in the financial markets (Cousins (2010)). Although very few investors actually use the CAPM without modification, its

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5The literature on the pricing of risky assets under an integrated tax system is based, either explicitly or implicitly, on the seminal paper of Brennan (1970) on equilibrium asset pricing (Handley and Makeswamy (2005)).
principles are very valuable (Cousins (2010)). It is fully accepted that the influence of international taxation in international investments is not denied, but international finance theory does not extensively focus on the problem of integrating taxation in international market equilibrium models. It is a severe limit of international finance theory that international taxation is not sufficiently considered in IntCAPT, since the relevance of taxes leads to special kinds of market equilibrium.

The dissertation analyzes as research object the influence of taxation on IntCAPT. The dissertation aims to describe taxation in IntCAPT through the construction and interpretation of models. The central research question is: “What Are the Impacts of Taxation on the International Capital Asset Pricing Model?” The aim of the dissertation is to derive IntCAPMs under integration of international taxation. In order to find answers on the research question, a research plan was developed which is illustrated by figure 2.1 on page 6.

The Tax-IntCAPM extends the IntCAPM through the integration of investor’s international income taxation. The aim of this extension is the derivation and interpretation of a pricing relationship under consideration of an investor’s international income taxation in equilibrium.

Based on the research plan the structure and methodology of the dissertation is derived and is illustrated by figure 2.2 on page 7. After the introduction in the first chapter aiming to put the dissertation in a broader context, the research problem leads to the elaboration of the research question and the conceptual procedure in chapter two. An extensive literature review on research pertaining to taxation in IntCAPT is given in chapter three. The discussion of the IntCAPM leads to the hypothesis that the integration of taxation in IntCAPT leads to a new model – the Tax-IntCAPM –.

\footnote{The theme of the doctorate is rooted in diverse contexts and has multifarious aspects. Furthermore many issues are interconnected; the issues cannot be addressed within the confines of a particular discipline, but require an interdisciplinary approach. The author examines the topic through the eyes of specialists in several academic fields and makes use of their research knowledge and tools. He intends to produce transcendent insights and to commit to contextualism with a reaching for the general. He aims to extend the boundaries of academic knowledge by eclecticism (Salter and Hearn (1996), Starkey and Madan (2001) and Lamb (2005)).}
<table>
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Figure 2.1: Structure of Research Plan

In chapter four the conception of international taxation is introduced.\(^7\) Hereby the conception of taxation of the relevant income types in IntCAPT, the dividends, the capital gains, interest and exchange gains are presented.\(^8\) The features of international taxation – the assignment of right of taxation and the methods to avoid double taxation – are extensively discussed in this chapter.

The research object – the derivation and interpretation of the Tax-IntCAPM – starts in chapter five.\(^9\) In order to guarantee a deeper understanding of the models they are illustrated by figures and examples.\(^10\) In chapter five the Tax-IntCAPM is developed

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\(^7\) The conception of integration international taxation in IntCAPM makes use of the three-phase scheme (\textit{Wagner and Dirrigl} (1980), \textit{Schult} (1983) and \textit{Schult} (2002)); in order to assess the tax burden the effective tax calculation according to \textit{Horst} (1971) and \textit{Rose} (1973) is applied. S. also \textit{Montag} (1977).

\(^8\) For the sake of readability we use solely the term gains instead of gains and losses.

\(^9\) The models are constructed according to the constructive and the contemplative system approach (\textit{Graumann} (2004)). In the process of constructing models, the method of decreasing abstraction is used; the complexity of interdependencies is reduced by assumptions and in the course of a more profound analysis the assumptions are gradually dropped.

\(^10\) The examples are illustrated by the German and Russian economies.
under integration of international taxation. The Tax-IntCAPM and the implication of exclusion of international arbitrage opportunities due to differential taxation of assets is discussed in chapter six, leading to a Tax-IntCAPM under short sale and borrowing restrictions.

In chapter seven, the Tax-IntCAPM with homogeneous expectations is derived. We derive hypotheses and the model is illustrated by an example.

The critique in chapter eight intends to give a deeper insight into the interpretation and the limits of taxation in IntCAPT. The main results are summarized in chapter nine, which finishes by an outlook into the future. The proofs and additional information are
given in the appendix.
3 Review of the Status of Research

“Reviewing has one advantage over suicide: in suicide you take it out on yourself; in reviewing you take it out on other people.”

George Bernard Shaw, Irish dramatist and socialist (1856 – 1950)

This chapter provides a literature review on research pertaining to taxation in IntCAPT. The literature review forms the basis for the dissertation. The review of the status of research can be described as an alternate form of presentation of the contributions and of critique. The critique is presented by elaborating the extensions of the successor and by a final conclusion. This chapter is divided into seven sections. In the first section the presentation of literature of Tax-CAPM is elaborated, followed by an extensive review of IntCAPMs in the second section. In the third section taxation in IntCAPM is reviewed. In section four we discuss exchange rate theories and in section five the quantity theory of money. The chapter finishes with an integrative survey of empirical evidence and a conclusion.

3.1 Tax Capital Asset Pricing Model

The CAPM marks the birth of asset pricing theory (Fama and French (2004)). It forms the basis for the integration of taxes in asset pricing theory. There exist further models

\footnote{For an overview, cf. Dimson and Mussavian (1999).}
\footnote{S. footnote 5.}
such as the Time State Preference Model (TSPM) developed by Myers (1968), Kraus and Litzenberger (1975) and Rubinstein (1976) and the Arbitrage Pricing Theory (APT) - developed by Ross (1976). The TSPM is criticized since the derived model cannot be represented as a handy empirical tool (Myers (1968)). The CAPM can be restated in terms of the APT (Ross and Walsh (1983)). Due to its very general concept APT does not develop clear statements about patterns of expected returns in an international setting and is confronted with severe theoretic criticism (Gokey (1991) and Kruschwitz and Löffler (1997)).

The CAPM constitutes a theory for the structure of asset prices in national capital markets and was introduced independently by Sharpe (1963), Lintner (1965b), Mossin (1966) and Treynor (1999). The model is based on the portfolio theory of Markowitz (1959) which analyzes an investor’s composition of a portfolio of risky and risk-free assets. In Markowitz’s model, an investor selects a portfolio at the beginning of the period that produces an insecure return at the end of the period. The model assumes that investors are risk-averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return. As a result, investors choose portfolios which minimize the variance of portfolio return (maximize expected return), given expected return (variance); thus, the Markowitz approach is often called a “mean-variance model” (Fama and French (2004)). The portfolio model provides a condition on asset weights in mean-variance efficient portfolios. The CAPM turns this statement into a testable prediction about the relationship between risk and expected return by identifying an investor’s optimal portfolio composition under market equilibrium (Fama and French (2004)).

Taxes are irrelevant only if the entire income types of CAPT are taxed at the same rate or if there is a special linear relationship between dividend, interest and total return (Mai (2008)). However, both constellations transpire to be implausible: On the one hand under a stylized tax systems the equivalence of all tax rates for all income types is unrealistic (Richter (2004)) and on the other hand – the linear relationship between

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13 The framework of the TSPM was laid by Arrow (1964) and has been expanded and expounded by Debreu (1959) and Hirschleifer (1964) and Hirschleifer (1966) (Myers (1968)).


dividend and total return was empirically rejected. Hence, taxation must be considered in CAPT, otherwise, the pricing relationship would lead to suboptimal decisions (Long (1977) and Wiese (2006b)).

Brennan (1970) relaxed the strong assumption of irrelevance of taxation that underpins the original CAPM and laid the foundation for integration taxation in CAPT. There are different versions of the Tax-CAPM which vary in respect of the tax system, the stochastic of return and restrictions to avoid tax arbitrage. The following review focuses on these aspects of Tax-CAPM variants.

### 3.1.1 Tax Capital Asset Pricing Model under Different Tax Regimes

Brennan (1970) developed the basic version of the Tax-CAPM assuming linear, investor-specific tax rates for interest and deterministic dividends which are unequal to the linear and investor-specific tax rates for capital gains. Two strands of Tax-CAPM under different tax regimes can be differentiated between: The first strand includes models that have been developed to allow for dividends being taxed at a different rate to that of interest, and some of these also allow for capital gains being taxed at a third rate. The second strand ignores the linearity assumption of tax rates. All of these models contain a dividend yield term that is certain.

Ashton (1989), Cliffe and Marsden (1992) and Lally and van Zijl (2003) among many others integrate the imputation system into the Tax-CAPM. Lally and van Zijl (2003) integrate the imputation system into the Tax-CAPM under the assumption of deterministic dividends and equal tax rates for dividends and capital gains which are unequal to the tax rates of interest income.

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16 The linear relationship between dividend and total return is empirically rejected (Friend and Packett (1964), Litzenberger and Ramaswamy (1979) and Kalay (1982)). Only does Blume (1980) find empirical evidence for the linear relationship but solely for the small time horizon between 1947 and 1956 in the United States.

17 S. footnote 5.

To admit the greatest generality, the model should allow for each of the income types in CAPT - dividends, capital gains and interest - to be differently taxed for any investor. Jonas (2004), Wiese (2004), Wiese (2006b), and Dempsey and Partington (2008) extend the models by differentiating tax rates for dividends, capital gains and interest. Lally (1992) and Handley and Maheswaran (2005) develop a Tax-CAPM which is free of all of these restrictions by abstracting from particular tax systems and present an asset pricing model with integrated tax systems. Litzenberger and Ramaswamy (1979) and König (1990) develop versions of the Tax-CAPM under the assumption of progressive equal tax rates for deterministic dividends and interest; capital gains is assumed to be tax-free.

3.1.2 Tax Capital Asset Pricing Model with Stochastic Dividends

A more realistic version of Tax-CAPM should incorporate the fact that dividends are stochastic, since the future is insecure. Schwetzler and Piehler (2004) construct a simple model incorporating stochastic dividends but ignore the stochastic relationship of dividends and capital gains. Lally (1998), Wiese (2006b), Mai (2006a) and Mai (2008) extend the Tax-CAPM with stochastic dividends and linear investor-specific varying tax rates for the entire income types in CAPT.

Mai (2006a) and Mai (2008) analyze the Tax-CAPM with stochastic dividends and consider constellations where the model incorporates a similar structure to the Tax-CAPM.

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19 Wiese (2006b) integrates tax-free allowance into the model by equalizing the tax rate to zero. Wiese (2006b) derives a version of the Tax-CAPM with stochastic capital gains but ignores the aspect that the tax-free allowance must also be stochastic (Mai (2008)).

20 Wiese (2006b) extends this version of Tax-CAPM by differentiating between progressive tax rates for deterministic dividends and interest. The tax rate for capital gains is assumed to be linear. Since tax rates are not stochastic, there is no fundamental difference to the Tax-CAPM with linear tax rates. The linear tax rates are to be replaced by investor-specific marginal tax rates in the equilibrium relationship (Litzenberger and Ramaswamy (1979), König (1990) and Mai (2008)).

21 In an empirical review Schulz (2006) reviews critically the assumption that dividends are deterministic. Wiese (2006b) comes to the conclusion that the extension of the Tax-CAPM to a multiperiod framework requires stochastic dividends, otherwise stocks would be risk-free.
with stochastic dividends and homogeneous expectations, a model where the tax rates are equal for all investors. However, Mai (2006a) and Mai (2008) admit that simplifying transformations of the $\beta$-factor are required which are formally not proved.

### 3.1.3 Tax Capital Asset Pricing Model under Restrictions

The versions of Tax-CAPM were derived in the framework of equilibrium, however it is uncertain if this equilibrium exists due to the existence of tax arbitrage. The exclusion of arbitrage is the central theorem in CAPT. Tax rate differentials across income classes can generate tax arbitrage (McDonald (2001)); since investors are differently taxed, they might use opportunities for tax arbitrage (Schäfer (1982), Dammon and Green (1987) and Ross (1987)). Arbitrage is based on the concept of replication. The market is free of arbitrage, if an asset can be replicated by a portfolio and the asset’s and portfolio’s price are equal (value additivity) (Kruschwitz (2007)). In the case of integration of taxes, different replication portfolios can exist for an asset with deviating prices.\(^{22}\)

Schäfer (1982), Dybvig and Ross (1986) and Ross (1987) among many others\(^{23}\) discuss the relationship of exclusion of arbitrage with and without taxes.\(^{24}\) Market equilibrium can be implemented by the introduction of short sale and borrowing restrictions. These restrictions do not exclude arbitrage, but limit the use of arbitrage opportunities (Schäfer (1982), Auerbach (1983), Dammon and Green (1987), Dammon (1988), Ashton (1989) and Jones and Milne (1992)). The Tax-CAPM under restrictions can be differentiated by short sale and borrowing restrictions.


\(^{22}\)Tax arbitrage can be avoided by special construction of tax systems and reduced by progressive tax rates which is not further analyzed since tax systems and rates are assumed to be given. As far as the analysis of tax arbitrage under the assumption of progressive tax rates is concerned, cf. Schäfer (1982), Dybvig and Ross (1986), Dammon and Green (1987), Dammon (1988), Jones and Milne (1992) and Basak and Croitoru (2001).

\(^{23}\)These include McDonald (2001), Galmeyer and Srivastava (2003), Galmeyer (2006) and Huang (2008).

(1980) and König (1990) assume that capital gains is untaxed, whereas Ashton (1989) develops an equilibrium pricing equation in the case of the dividend imputation system.

If a certain asset is held by one group of investors, then the Tax-CAPM under short sale restrictions can be interpreted as a tax clientele CAPM and assets are distributed among investors depending on their tax rates. If the tax rate on capital gains is less than on dividends, the tax clienteles are formed based on the amount of tax rates and on dividend return: Investors with the highest dividend tax rate will include assets with the lowest dividend return into their portfolio (Litzenberger and Ramaswamy (1980), König (1990), Wiese (2006b) and Mai (2008)).

The integration of short sale restrictions into the Tax-CAPM in the form that the investor is only allowed to hold a positive amount of shares leads to an equilibrium pricing relationship that is applicable in special cases. It assumes that dividends are equally taxed to capital gains and not only that investor’s covariance term of total return and tangential portfolio is proportional to the beta factor but also that the risk premiums of all investors are equal (Mai (2008)). Hence, conditions were introduced in order to derive a market equilibrium which lacks economic interpretation (Litzenberger and Ramaswamy (1980)).

Black (1972a) introduced borrowing restrictions in CAPT. Litzenberger and Ramaswamy (1979), König (1990) and Wiese (2006a) assume the existence of two kinds of borrowing restrictions, the first one pursuing the conception that the borrowed amount is less than an investor-specific investment in risky assets and the second one pursuing the idea that interest shall not exceed dividends, which is assumed to be deterministic.

On account of the assumption of stochastic dividends, Mai (2008) incorporates short sale restrictions on a limited amount and borrowing restrictions into the Tax-CAPM. The entire amount of money invested in risky assets is restricted from acquiring a negative value, implying that the short sale restrictions are not binding on every single risky asset but on an investor’s portfolio of risky assets. The borrowing restrictions incorporate that the borrowed amount shall not exceed an investor-specific amount. He derives an equilibrium pricing relationship which in comparison to the Tax-CAPM is adapted by a term incorporating the market value of the short sale and borrowing restrictions.
3.2 International Capital Asset Pricing Model

In this section a brief discussion on the fundamental problems of asset pricing in the international framework is provided. The issues shall give a first insight of differences between national and international asset pricing theory.

An investor’s motivation for international acquisition lies in the benefit of diversification which was expounded in the models of Tobin (1958) and Markowitz (1959). These insights were extended to an international framework by Grubel (1968) and solidified for international portfolio diversification by Levy and Sarnat (1970) and Solnik (1974b) among many others. Through international diversification investors could attain a higher return (lower variances) for given variances (returns) on international portfolios in comparison to domestic portfolios, which is due to the low correlations between national capital markets.

In which manner is it necessary to extend the CAPM to render feasible an application on an international setting?

1. Inflation rates differ across states,
2. investors residing in different states consume different baskets of goods,
3. investors face different investment opportunity sets.

IntCAPMs can be categorized according to the following criteria (Stulz (1995) and Zimmermann (2003)):

1. Models with identical consumption and investment opportunity sets,
2. models with different consumption and investment opportunity sets,
3. models with barriers to international investment.

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25 These include Lessard (1976), Levy and Sarnat (1993), Santis and Gerard (1997) and Gehrig and Zimmermann (1999).
26 The low correlation is partly related to the states’ specialization in particular industries (Roll (1992)).
The investors’ consumption opportunity set comprises the consumable commodities and their prices, his investment opportunity set is described by the future distributions of wealth (Stulz (1981a)) and (Stulz (1995)). Different consumption and investment opportunity sets arise from the deviation from Relative PPP, which results from varying inflation and exchange rates and implies that asset returns are heterogeneously perceived. Consequently, these differences affect an investor’s portfolio choice and expected returns (Stulz (1995)). Barriers to international investment occur through the integration of taxation in IntCAPM.28

Solnik (1974a), Grauer (1976), Serçu (1980) and Adler and Dumas (1983) followed by others29 extend the CAPM of Sharpe, Lintner and Mossin and the ICAPM of Merton (1973) to an international context and develop International Capital Asset Pricing Models (IntCAPMs).

Two strands of IntCAPMs can be identified: Grauer (1976) among others30 develop IntCAPMs under the postulate of PPP. The other strand of valuation concept in the international setting comprises more general equilibrium models which were introduced by Solnik (1974a), Serçu (1980) and Adler and Dumas (1983) and which develop IntCAPMs without considering PPP. Like Solnik (1974a), Serçu (1980) and Adler and Dumas (1983) assume that exchange rates are stochastic, but in contrast to him they do not assume that exchange rates are independent from asset returns.

Further models of International Asset Pricing Theory comprise the International Consumption Capital Asset Pricing Model (IntCCAPM) developed by Stulz (1981a) and the International Arbitrage Pricing Theory (IAPT) originated by Solnik (1983) and elaborated by Levine (1989) and Ikeda (1991).31 Stulz extends the Consumption Capital Asset Pricing Model (CCAPM) of Breeden (1979) to an international setting,32 whereas

28 Barriers in the form of taxation are explicitly discussed in chapter 3.3.
30 These include Kouri (1977), Frankel (1979), Fama and Farber (1979), Hodrick (1981) and Lucas (1982).
32 A severe limitation of Stulz’s model is the empirical implementation since it requires the observation of an investor’s risk tolerance and time-varying local price indices; cf. Gokey (1991). Furthermore, the consumption rates are endogenous variables that are solved by investors when they optimize their expected lifetime expected utility (Stulz (1984)). For a critical review of consumption-based asset pricing, cf. Cornell (1981), Hansen (1982) and Wheatley (1983).
3 Review of the Status of Research

Solnik (1983) and the other authors transfer the Arbitrage Pricing Theory (APT) of Ross (1976) to an international context and develop the IAPT. 33

3.2.1 Purchasing Power Parity

In the first class of international valuation theories, apart from the general assumptions of CAPT, it is assumed that markets are integrated and PPP is satisfied (Stulz (1995)). The origins of the PPP concept can be traced back to Spanish Salamanca School in the 16th century (Oxelheim and Wihlborg (2008)); the concept of PPP was introduced in economic theory by Cassel (1916). 34 Since then, the concept of PPP has become embedded in the theories of many international economists (Frankel (1981), Taylor (1995), Frankel and Rose (1995), Rogoff (1996)). 35

According to the PPP hypothesis the determination of exchange rates depends on prices of commodities. The PPP is based on the concept of Commodity Price Parity (CPP), which implies in an arbitrage-free setting the same price of a commodity in every country after translation into a common currency (Law Of One Price (LOOP)). The Absolute PPP is defined in terms of the equality of the price of a representative domestic basket of goods to the price of a foreign representative basket of goods (Serçu and Uppal (1995)). The basket of goods contains the goods used for consumption in the domestic and foreign countries.

\[ APPP = P^D S = P^F \]

with \( APPP \) Absolute Purchasing Power Parity

\( P^D \) price of domestic consumption bundle

\( S \) nominal exchange rate

\( P^F \) price of foreign consumption bundle

33 As far as the critique of APT is concerned, c. chapter 3.1.

34 Cf. Cassel (1918) and Cassel (1925). After leaving the gold standard in 1914 countries experienced significantly different rates of inflation; Cassel proposed PPP as a means of adjusting exchange rates.

The ratio of the Absolute PPP at two different instants leads to the dynamic version of the Absolute PPP relationship, the Relative PPP (Serçu and Uppal (1995)).

\[ RPPP = \frac{S}{S_0} = \frac{P_F}{P_F^0} \frac{P_D}{P_D^0} \]  

(3.1)

with \( RPPP \) Relative Purchasing Power Parity

- \( S \) nominal exchange rate at the end of period
- \( S_0 \) nominal exchange rate at the beginning of period
- \( P_F \) price of foreign consumption bundle at the end of period
- \( P_D \) price of domestic consumption bundle at the end of period
- \( P_F^0 \) price of foreign consumption bundle at the beginning of period
- \( P_D^0 \) price of domestic consumption bundle at the beginning of period

The rate of inflation is defined in terms of the appreciation of consumer goods. We can insert the inflation rate in the above equation.

\[ \frac{P_D}{P_D^0} \equiv 1 + \pi_D \quad \frac{P_F}{P_F^0} \equiv 1 + \pi_F \]  

(3.2)

with \( \pi_D \) inflation rate of the domestic state

- \( \pi_F \) inflation rate of the foreign state

We define the appreciation of nominal exchange rates as the quotient of end of period nominal exchange rate and exchange rate in the beginning of period.

\[ \frac{S}{S_0} \equiv 1 + e \]  

(3.3)

with \( e \) appreciation of nominal exchange rate
So, we have

\[ RPPP \equiv 1 + e = \frac{1 + \pi^F}{1 + \pi^D} \Rightarrow RPPP \equiv (1 + e)\frac{1 + \pi^D}{1 + \pi^F} = 1. \quad (3.4) \]

**Example 1** Relative Purchasing Power Parity

Prices in Germany rose by 0.4% and prices in Russia by 8.8% in 2009; the nominal exchange rate Russian Ruble (RUB)/Euro (EUR) should appreciate by 8.36%.\(^{36}\)

According to the hypothesis of Relative PPP, domestic and foreign inflation rates are related to exchange rate changes to the extent that differences in the development of price levels are set off by the appreciation of the nominal exchange rate and the appreciation of the nominal exchange rate is determined by the relation of foreign to domestic inflation. The use of the Relative PPP allows for a different basket of goods and varying weights to be applied towards the goods within the consumption bundle. This relation is necessary but not sufficient for the validity of Absolute PPP, but not vice versa, so the Relative PPP is a weaker version of the Absolute PPP.

### 3.2.2 International Capital Asset Pricing Model under Purchasing Power Parity?

Under the assumption of the prevalence of Relative PPP, the IntCAPM can be derived (\textit{Grauer} (1976), \textit{Stulz} (1995), \textit{Zimmermann} (2003) and \textit{Armitage} (2005)).\(^{37}\) The assumption of prevalence of Relative PPP leads to the conclusion of full integration of national capital and goods markets; the real returns on international assets are the same across countries.

\[ E[\tilde{r}_i] = r_f + \frac{\text{Cov}[\tilde{r}_i, \tilde{r}_w]}{\text{Var}[\tilde{r}_w]} (E[\tilde{r}_w] - r_f) \quad (3.5) \]

\(^{36}\)As far as the data for the German and Russian inflation rate is concerned, s. \url{www.destatis.de} and \url{www.cbr.ru/eng/analytics/macro/}.

\(^{37}\)S. chapter 5.3.
with \( E \) expected value
\[ \tilde{r}_i \] real risky asset \( i \)'s rate of return
\( r_f \) international real risk-free rate of return
\( Cov \) covariance
\( Var \) variance
\( \beta_{r_i,r_w} \) beta factor, sensitivity factor of risky asset \( i \)'s and world market return
\( \tilde{r}_w \) real world market rate of return

Figure 3.1: Security Market Line of IntCAPM under the Assumption of Relative PPP

Figure 3.1 represents eq. (3.5). Any investor determines the expected return of an asset by the international real risk-free rate and a risk premium, composed of the product of the beta-factor and the real expected return on the world market portfolio in excess of the international real risk-free rate.\(^{38}\) As a result the expected return of a risky asset is invariant to investors residing in different states.

\(^{38}\)The world market portfolio comprises the entire securities of the world in proportion to their capitalization relative to the world wealth by use of consumption goods as the numeraire to measure world wealth and each security's capitalization (Stulz (1995)). Under the assumptions of the existence of an asset with a risk-free return as well as a zero world market beta in real terms and a zero covariance between inflation rate and nominal asset returns, an adequate asset pricing equation in nominal terms can be derived (Zimmermann (2003)).
Heckscher (1916) proposed the idea of deviation from (Relative) PPP. Siegel (1972) first raised the question of asset pricing when deviations from PPP are the source among market participants. The main reasons for (Relative) PPP deviation lie in

1. the presence of non-traded goods and services,
2. the existence of significant transaction costs,
3. differences in relative prices of goods, the composition of national consumption baskets (heterogeneous preferences),


Since Relative PPP does not hold to the extent that the exchange rate does not set off differences in the development of price levels, investors’ portfolio composition differs and
real returns are differently measured. The fact that Relative PPP may fail introduces a new dimension of internationality in international valuation which does not exist in the framework of domestic pricing models.

3.3 Taxation and International Capital Asset Pricing Theory

The general form of IntCAPM ignores the existence of taxation; however, rational investors judge the return and risk of assets after taxes. Two strands of Tax-IntCAPM can be differentiated. The first models ignored the existence of exchange rates and the question of deviation from PPP, the other strand implemented deviation of PPP in their models.

Stapleton and Subrahmanyam (1977) raised the question of the effect of taxation in IntCAPT under irrelevance of exchange rates. In Stapleton and Subrahmanyam (1977) numerical examples of the effects of taxes on foreign assets are analyzed, but no pricing relationship is presented. The first approach of modeling the segmentation of markets in the form of taxation based on the concept of capital market equilibrium was initiated by Black (1974). In a two country one period CAPM, a proportional tax is imposed on an investor’s long position of foreign risky assets, whereas the investor with a short position of foreign risky assets receives a subsidy. Thus, in the Black model taxes are paid in proportion to the net holdings of risky foreign assets. The tax may differ from asset to asset or from country to country. It may be assessed by the investor’s own country or by the country in which the asset is located. The tax is intended to represent a barrier to international investment on the value of an individual’s holdings of assets in foreign countries. He states that the direction of deviations from CAPM is consistent with the results of empirical studies. In comparison with the CAPM Black’s model implicates for the equilibrium pricing relationships that assets with a low (high) systematic risk have a higher (lower) expected return; the capital market line adopts a flatter line. The optimal portfolio choice implies a mixture of the international market portfolio, the minimum

\[ \text{Empirical studies show that high } \beta \text{ securities with expected returns are lower than predicted by the CAPM and vice versa (Jensen (1972) and Black (1972b)).} \]
variance zero-beta portfolio of risky assets and a minimum variance country-portfolio subject to a given level of tax being held. It is possible for the risk-free asset to be non-traded (Stulz (1981)). The model can only be applied in a frame when taxes are assumed to be modest in amount,\textsuperscript{46} since an increase in the level of taxation does not lead to abstention from holding foreign assets; in contrast, it would lead to an enlargement of holding foreign assets short since they are subsidized. Thus the model does not sufficiently explain the effects of barriers in the form of taxation on IntCAPT.

Stulz (1981) extended Black’s framework by constructing a model of asset pricing in which different taxes on long and on short positions of foreign risky securities are imposed.\textsuperscript{47} Thus, in contrast to Black’s model, short positions of foreign assets are not subsidized and the wealth tax is imposed on an investor’s absolute holdings of foreign assets. In contrast to Black’s model the world market portfolio is inefficient on account of the existence of non-traded foreign assets, which fall below a critical beta and consequently do not provide expected returns large enough to equalize the costs of holding them. The Capital Market Line is replaced by a Security Market Band. Taxed long (short) positions of foreign assets plot on a security market line lying above (below) parallel to the CAPM security market line for risky domestic assets, nontraded foreign risky assets plot between these ones. Byun and Chen (1997) explicitly regard Stulz’s model in the form of differential taxation for the domestic and foreign investor. Tax positions from short and long holdings are asymmetrically taxed. Wood (1997) develops Stulz’s model under incorporation of the imputation system. In the model’s framework the exchange rate is regarded as irrelevant, since the introduction of the exchange rate would not change the results; the effect of barriers to international investment would be the same so long as those barriers to international investment were of a type which in the limit can produce complete segmentation (Stulz (1981) and Byun and Chen (1997)).

Wheatley (1983) and Stulz (1984) integrate the taxation of end of period price of a risky asset in the CCAPM and derive identical solutions as in the above mentioned model. In Sellin (1989) and Sellin (1990), a two country asset pricing model is developed where the domestic investor’s holding is subject to a variable tax. The asset pricing equation consists of two components, the asset’s covariance with the world market portfolio and

\textsuperscript{46}The question of how to quantify the term modest in amount is raised.

\textsuperscript{47} Cf. Stulz (1995).
the covariance of the asset with the tax rate divided by the variance of the world market portfolio. In equilibrium the market participant’s holding consists of the risk-free rate, the world market portfolio and a portfolio designed to hedge against changes in the stochastic tax rate.

Lally (1996) was the first to raise the question of taxation in IntCAPT under consideration of exchange rates which can deviate from Relative PPP. In Lally (1996) and Lally (1998) an IntCAPM is developed incorporating the dividend imputation system. Based on the model of Solnik (1974a) he assumes that exchange rate movements are stochastic but independent from assets returns and ignores inflation.48 Lally (1996) assumes unrestricted international capital flows which in combination with the assumption of homogeneous expectations implies that home bias is solely dictated by personal taxation and exchange rate uncertainty. Personal taxes vary across investors and for stochastic capital gains and deterministic interest and cash yield (dividends etc.). Given these assumptions then, the pre-tax expected rate of return on asset \( i \) is

\[
E[\tilde{r}_{i,n}] = r_{f,n} (1 - T) + r_{C,n} T_i + \frac{Cov[\tilde{r}_{i,n}, \tilde{r}_{w,n}]}{Var[\tilde{r}_{w,n}]} (E[\tilde{r}_{w,n}] - r_{C,n} T_W - r_{f,w,n} (1 - T)). \tag{3.6}
\]

with

- \( \tilde{r}_{i,n} \) nominal risky rate of return of asset \( i \)
- \( r_{f,n} \) nominal risk-free rate of return in asset \( i \)'s economy
- \( T \) weighted average of \( \frac{t_{I,j} - t_{CG,j}}{1-t_{CG,j}} \) over all investors world wide
- \( t_{I,j} \) tax rate of investor \( j \) on interest
- \( t_{CG,j} \) effective tax rate of investor \( j \) on capital gains (allowing for deferral opportunities)
- \( r_{C,n} \) nominal deterministic cash yield on asset \( i \)
- \( T_i \) weighted average of \( \frac{t_{I,j} - t_{CG,j}}{1-t_{CG,j}} \) for asset \( i \) over all investors \( j \) world wide
- \( t_{C,j} \) tax rate of investor \( j \) on risky asset \( i \)'s cash yield
- \( r_{w,n} \) nominal return on the world portfolio of risky assets exclusive of exchange rates
- \( r_{C,w,n} \) nominal cash yield (dividends etc.) of world market portfolio
- \( T_W \) weighted average over all \( T_i \), where the weights are the relative

48 Since exchange rate movements are stochastic and unequal to unity, the hypothesis of Relative PPP is rejected, s. chapter 5.2.2 and eq. 5.11.
3 Review of the Status of Research

cash payouts (dividends etc.) of the risky assets

$r_{f_w,n}$ world risk-free rate, weighted average of risk-free rate of return,

where the weights are the relative market values of risky assets

in each economy

Equivalent to the IntCAPM under PPP in chapter 3.2.2, the market risk premium involves terms such as $E[r_{w,n}]$ and $r_{f_w,n}$ which reflect the extension to the world market. Since deterministic cash yield is considered, the equilibrium pricing relationship is adapted by the terms $r_{C,n}$ and $r_{C_w,n}$. The tax parameters $T$, $T_i$ and $T_W$ involve averaging over all investors in the world and $T_W$ relates to the world market portfolio. Imputation in any country reduces $T_i$ and hence the expected return. The extent of this effect will depend upon what proportion of investors can benefit from the credits and the value weight of these investors.\(^{49}\) Furthermore, imputation in any state reduces $T_i$ and hence $T_W$ with a flow-on effect to the expected return.

According to Lally (1996) the only manifestation of stochastic exchange rates is in the world risk-free rate.\(^{50}\) Under the assumption of fixed exchange rates and no restrictions on capital flow, only one risk-free rate can prevail throughout the world.

The model of Lally (1996) is the most elaborated Tax-IntCAPM since it assumes varying tax rates for capital gains, deterministic interest and cash yield under the assumption of deviation from Relative PPP.

3.4 Exchange Rate Theories

The currency market is the largest financial market whose transactions are made up of short term, intra-daily transactions and results from the interaction of traders with

\(^{49}\) In most cases, only domestic investors benefit from imputation credits; unless these investors have a significant value weight in the world, the impact of imputation on $T_i$ and hence on the expected return is trivial (Lally (1996)).

\(^{50}\) This statement cannot be concluded from the derivation of the model in the appendix, since the world risk-free rate is derived by weighting the risk-free rate of return which is derived in the absence of exchange rates. In the implementation of the model, Lally (1996) ignores the consideration of exchange rates for the world risk-free rate as well.
different time-horizons, risk-profiles or regulatory constraints (Guillaume (1997)). The price of a foreign currency is determined by the supply and demand of the currency which in turn is related to the trade between countries (Vélez-Pareja (2003)). However, currency markets are characterized by abrupt changes traceable to central bank interventions and attempts to control the value of currencies contrary to the natural market forces (Vaga (1994) and Peters (1994)). Exchange rate markets are different from capital markets since their intention is not to raise capital but to create the ability to trade in stocks and bonds; as pure trading markets they are a zero sum game (Peters (1994)). In contrast to the stock market, whose asset values rise and fall with the economy, currencies have no stable relationship with the economy (Peters (1994)).

The first approach of exchange rate theories can be categorized into macroeconomic rational expectations news and structural models. The macroeconomic rational expectations news and the structural model are confronted with severe theoretic criticism, since these models contain an infinity of possible information which reveal themselves to be unstable and imply the selection of one particular solution. Furthermore, they face severe empirical criticism. A first empirical puzzle is that the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. The asset market approach together with the rational expectation assumption imply that there is a long-run equilibrium relationship between the exchange rate and its fundamentals, which is rejected by the results of tests performed by Baillie and Selover (1987), Abel and Mishkin (1983),

51 For an overview of exchange rate regime classification, cf. Baillie and McMahon (1989), McDonald (1989), Mussa (1991), Gruwe (1993), Gruwe and Grimaldi (2002) and Bhatti and Moosa (2010). According to the rational expectations news model the exchange rate is determined by the present value of the expected future path of the fundamental variable driving the exchange rate while structural models specify the economic structure of the fundamental variable (Gruwe (1993)). The structural models can be categorized into monetary models with flexible prices developed by Mussa (1976), Frankel (1976), Bilson (1978a), Bilson (1978b) and Mussa (1979), monetary models with sticky prices developed by Dornbusch (1976b), Dornbusch (1976a) and extended by Frankel (1979) and the portfolio balance model which was developed by McKinnon and Oates (1966), Branson (1969), Branson (1975), Branson (1977), Allen and Kenen (1978), Isard (1980) and Dornbusch and Fischer (1980). For an explicit overview, cf. Bhatti and Moosa (2010).

52 This selection is based on information not contained in the model; ad hoc assumptions – information outside the model – must be introduced (Gruwe (1993) and Hofstadter (1999)).

53 This is a contradiction to the rational news expectations models, which predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases (Baxter and Stockman (1989), Flood and Rose (1995) and Gruwe and Grimaldi (2002)). Goodhart (1989) and Goodhart and Figliuoli (1991) found that exchange rate movements appear to occur in the absence of observable news.
and **Booth and Glassman** (1987) among others. A third puzzle relates to PPP and is closely related to the previous one. **Dumas** (1992) has stressed the long time needed to adjust to PPP. The transaction cost hypothesis implies a non-linearity in the adjustment process. This hypothesis has been confirmed by the empirical evidence based on **Michael** (1997).

The theoretical and empirical criticism of the macroeconomic rational expectations news and structural exchange rate models has led to new attempts to model the exchange rate (**Bhatti and Moosa** (2010)). These attempts have led to three different modeling approaches.

Microeconomic models characterize explicitly the process of trading in the foreign exchange market (**Grauwe and Grimaldi** (2002) and **Bhatti and Moosa** (2010)). A number of authors, e.g. **Grauwe and Grimaldi** (2002), **Evans and Lyons** (2002), **Jeanne and Rose** (2002) and **Kilian and Taylor** (2003) among many others, endogenize the market structure and explicitly model the behavior of different types of traders such as chartists and fundamentalists. Central issues like the sources and persistence of heterogeneous beliefs, excess volatility and exchange rate determination have to be further researched (**Frankel and Rose** (1995)).

In **Harvey** (1999) and **Harvey** (2006) a Post-Keynesian approach is used to explain the determination of exchange rates. The argument is that exchange rates are determined by international investors’ demand for currency as they act to adjust their portfolios with an emphasis on psychological and institutional factors. In the Post-Keynesian model, exchange rates are determined by the international supply and demand for each currency. Demand comes from imports, foreign direct investment, portfolio investment, and official reserve management.

Finally, a third approach recognizes that heterogeneous agents have different beliefs about the behavior of the exchange rate. These different beliefs introduce non-linear features in the dynamics of the exchange rate. The appeal of non-linear dynamics lies in the

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54. **Cutler** (1989) report significant autocorrelation at short horizons of a month but negative significant autocorrelation at lower frequencies.

55. For an overview of the PPP, s. chapter 3.2.1.


plethora of behaviors they allow, in the sense that the simplest non-linear deterministic system is capable of generating complex dynamics (Ellis (1992)). A variable can converge to a fixed point, to a cycle, where it oscillates between points, explode, or lead to chaotic systems (Ellis (1992) and Grawe (1993)). This is motivated by the notion that the real exchange rate follows a deterministic nonlinear process which generates output that mimics the output of stochastic systems. In other words, it is possible for the real exchange rate to appear random but not to be really random (Assaf (2006)).

Empirical research has convincingly demonstrated the importance of non-linearities in exchange rates. Dumas (1992) and Serçu and Uppal (1995) among many others58 suggest that exchange rates adjust towards PPP equilibrium in a nonlinear fashion. Obstfeld and Taylor (1997) among many others59 provide evidence using Threshold Autoregression (TAR) models in favor of nonlinearity in real exchange rates.60 Obstfeld and Taylor (1997) consider the Consumer Price Index (CPI) of 24 countries at monthly frequencies from 1980 to 1995. Meanwhile, Michael (1997) employs Smooth Transition Autoregression (STAR) models. Their interwar data set comprises monthly observations on wholesale price indices for the United Kingdom, United States, France, and Germany, and spot exchange rates for sterling against the U.S. dollar, French franc, and German mark. The sample period covers January 1921 to May 1925. The low-frequency data consist of annual observations on exchange rates together with the wholesale price indices from 1791 to 1992 Michael (1997) also found evidence in support of nonlinearity in PPP.61

3.5 Quantity Theory of Money

There is a wide consensus in economic literature that, in the long run, inflation is a monetary phenomenon; monetary policy ultimately determines inflation, by influencing directly or indirectly the rate at which the monetary side of the economy expands (McCallum

59 These include O’Connell (1998) and Enders and Falk (1998).
According to the quantity theory of money, traded goods valued in money are equivalent to transferred money. The idea of constant velocity in money and number in transactions leads to the conception of quantity of money as the only variable of consumption goods prices and inflation rate (Friedman (1987)). Central banks adopt the apparatus of monetary regime to inflation targeting (Masson (2000), Mishkin (2000) and Ball and Reyes (2008)).

Hillinger and Süssmuth (2008) test the empirical distribution against a normal distribution with unity mean and find strong evidence for the quantity theory of money. Their study comprised 164 countries for a period of 44 years from 1960 to 2003. By use of the augmented Dickey–Fuller (ADF) test Ajuzie (2008) also finds strong evidence of the quantity theory of money for the period from January 1993 through June 2007 for the US.

Mumtaz and Surico (2006) among many others investigate the impact of global factors on inflation. The panel of the study of Mumtaz and Surico (2006) includes 164 quarterly series of prices for 13 countries: United Kingdom, United States, Sweden, Spain, Netherlands, New Zealand, Japan, Italy, Germany, France, Finland, Canada and Australia. The full sample covers the period from 1961 to 2004 and they use the Bayesian methods described by Kim and Nelson (2000).

They find evidence that inflation appears to have become driven by global factors such as labor costs and tradable and non-tradable goods and services. Borio and Filardo (2007) argue that the robustness of the findings should be assessed further and argue that

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62 Even fiscal policies imply different ongoing inflation rates only if they result in different money stock growth rates (McCallum (1990)).


64 Empirical evidence thus far supports the presence of a unit root in inflation rates, the results indicate that most shocks to inflation rates are temporary and that the targeting of inflation rates is the result of profound policies of central banks, cf. MacDonald and Murphy (1989), Moazzami (1991), King and Watson (1992), Lai (1997) and Loayza and Soto (2002).

65 These include Morimoto (2003), Ciccarelli and Mojon (2005) and Borio and Filardo (2007).

studies about the influence of global factors need better information, better measures of the contestability of markets and more micro-data.

Furthermore, the empirical study of Adler and Dumas (1983) shows that inflation risk represents a negligible factor in IntCAPT.\footnote{Other studies reveal that the empirical evidence on the pricing of inflation risk is inconclusive. Moerman and van Dijk (2010) finds empirical support for the pricing of inflation risk whereas Javid and Ahmad (2009) produces mixed results.}

## 3.6 Empirical Evidence

Most empirical investigations test the segmentation or integration of asset markets intending to find or reject barriers to international investments. In general, neither evidence for segmentation nor integration can be found at significant levels. Empirical studies assuming that markets are integrated against the hypothesis that markets are segmented by barriers do not appear to present a very powerful framework; the (abnormal) returns implied by barriers to international investment are not supported in empirical studies (Stulz (1995)). The lack of power of asset pricing tests with unspecified null hypotheses has lead to a new approach of empirical tests. Models that explicitly quantify the impact of barriers to international investment have been developed and tested (Stulz (1995)).

In their empirical test of the conditional version of the IntCAPM Dumas and Solnik (1995) use the Generalized Methods of Moments (GMM) of Hansen (1982). Four countries are taken into account: Germany, the United Kingdom, Japan, and the United States and the data covers the period from March 1970 to December 1991. Santis (1998) tests the conditional version of an IntCAPM using a multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process. They include ten countries (Austria, Belgium, France, Germany, Italy, Japan, The Netherlands, Spain, UK and US) and the sampling period covers 288 observations from January 1974 through December 1997. Ramchand and Susmel (1998) use a specification of the Switching Autoregressive Conditional Heteroskedasticity (SWARCH) model. The data cover the period from January 1980 through April 1996 and includes the countries Australia, Hong Kong, Japan, France, Germany, Sweden, Switzerland, UK, US and Canada. Wu (2008) employs the

\textit{Cappiello and Fearnley} (2000) test the conditional version of the IntCAPM by application of a multivariate GARCH process. In \textit{Cappiello and Fearnley} (2000), the observations cover the period from February 1986 to December 1998; the USA, Europe and Japan are included in the sample. \textit{Fearnley} (2002) tests the conditional version of the IntCAPM by application of a multivariate GARCH process. The sample data used for estimation cover the period January 1993 to October 2001. US, Japanese and European stocks and government bonds, two Euro currency deposits (Yen and a European currency basket), and the world market portfolio of stocks and government bonds are estimated. \textit{Dahlquist and Sallstrom} (2002) use the GMM of \textit{Hansen} (1982). The sample period is July 1973 to June 1999 and the set of assets consists of total market returns in 20 developed equity markets.\textsuperscript{68}


\textit{Cooper and Kaplanis} (1994) use the GMM, the study comprising the time period from January 1978 to December 1987 includes France, Italy, Japan, Spain, Sweden, UK, USA and Germany. The study indicates that deviations from Relative PPP alone are insufficient to account for the degree of home bias observed. \textit{Fearnley} (2002) comes to the same conclusion by rejecting the IntCAPM of \textit{Adler and Dumas} (1983) and suggests that additional, unidentified pricing factors contribute to return expectations.

### 3.7 Conclusions

The failure of the state of art is based on the discrepancy of theoretical and empirical results and models of taxation in IntCAPT.

\textsuperscript{68}The countries included are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, the U.K., and the U.S.
The introduction of taxation was intended to represent a general kind of barrier to international investment, but taxation adopts a much more complex form than considered in the theoretical models. Only *Lally* (1996) regards the international taxation spectrum by differential taxation of capital gains, dividends and interest. Nevertheless, he assumes exchange gains to be equivalently taxed to the underlying nature of income. In his model, capital gains and exchange gains on capital, dividends and exchange gains on dividends as well as interest and exchange gains on interest are regarded as only three income types. Exchange gains are taxed by characterization of the recognition, character, nature, source and hedging features, so, capital gains, exchange gains on capital, dividends, exchange gains on dividends, interest and exchange gains on interest must be regarded as six different income types which are differently taxed.69 The characteristic features of the exchange gains tax system – namely, the rise of exchange gains taxes depending on the characterization of the its recognition, character, nature, source and hedging features are ignored in his model.

Furthermore, *Lally* (1996) admits that the return and cash-yield return on world portfolio and weighted average of risk-free rate of return may vary substantially due to inflation rates. Hence, the consensus in economic literature that in the long run, monetary policy ultimately determines inflation, by influencing directly or indirectly the rate at which the monetary side of the economy expands is neglected.

As *Solnik* (1974a) he assumes exchange rates to be stochastic but independent from assets returns and ignores the extensions by *Serçu* (1980) and *Adler and Dumas* (1983) who do not assume that exchange rates are independent from assets returns. Furthermore, he ignores the exchange rate effects on the $\beta$-factor. In contrast to the derived models of *Solnik* (1974a), *Serçu* (1980) and *Adler and Dumas* (1983) exchange rates reveal not to be stochastic.60 Empirical studies show evidence of a non-linear behavior of exchange rates, suggesting that they are not random.71

In contrast to *Lally* (1998), *Wiese* (2006b), *Mai* (2006a) and *Mai* (2008) he does not assume stochastic dividends. However, a realistic version of Tax-IntCAPM should

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69 S. chapter 4.4.
70 S. chapter 3.4.
71 S. chapter 3.2.
incorporate the fact that dividends are stochastic, because the future is insecure.\textsuperscript{72}

These assumptions reveal to be unrealistic.\textsuperscript{73} The theoretical signs indicate that further components have to be integrated and the divergent empirical results may be attributed to a severe theoretical lack of the IntCAPM - the ignorance of the framework of exchange gains taxation and non-linear behavior of exchange rates in the IntCAPM -. The theoretical and empirical studies give the ball back to theorists, we need to know more about the framework of taxation of exchange gains and the integration of non-linear behavior of exchange rates in IntCAPT.

\textsuperscript{72}S. chapter 3.1.2.
\textsuperscript{73}S. chapter 3.1.2, 3.4 and 3.5.
4 International Taxation

“The hardest thing in the world to understand is the income tax.”

Albert Einstein, US (German-born) physicist (1879 – 1955)

The aim of integration taxation in IntCAPM is the derivation of a pricing relationship. The assumption that taxes are identical in their treatment of the entire income types in IntCAPT - dividends, capital and exchange gains and interest - reveals itself to be invalid under the prevailing tax systems. In most countries these parts of income are differently taxed.

How to quantify the features of international taxation

The tax systems generally differ from country to country and are subject to constant changes. The valuation approaches typically used do not account for such differences or changes - a general approach for deriving the equilibrium pricing relationship is needed which can be adjusted to any situation depending on the characteristics of taxation in the given country. In this chapter, we abstract from the mechanics of particular international tax systems and present a model for a reasonably general integrated international tax framework. The characteristic feature of international taxation is foreign investors’ tax treatment. The right of taxation of each income type in IntCAPT - dividends, capital and exchange gains and interest - can be separately assigned to the source state or to the state of residence.\textsuperscript{74} The right of taxation of dividends, capital gains and interest can be assigned to both states. If the right of taxation is assigned to both states, two fiscal

\textsuperscript{74}For the sake of readability we use solely the term capital and exchange gains.
jurisdictions clash and the foreign investor can be subject to double taxation. In order to avoid this (negative) characteristic, international tax law incorporates methods to avoid double taxation.

By deriving a model integrating the features of international taxation, we have the possibility of quantifying and analyzing the features of international taxation. After a discussion of the assumptions we introduce the national tax system that the domestic and the foreign investor is subject to. We present the corporate and dividend, the capital gains and interest tax system. After introducing the national tax systems we focus on the taxation of exchange gains. We conclude this chapter by presenting the methods to avoid and reduce double taxation.

4.1 Assumptions

Our world is too complex to be understood, so models are constructed based on hypotheses assuming away the complexities which are thought to have a minor impact on our world. Hence, the assumptions of international taxation are presented which correspond to the necessities of taxation in IntCAPT.

Assumption 1 World
The world consists of a domestic $D$ and foreign $F$ state, the states are delineated along different tax systems.\footnote{Cf. Black (1974), Stulz (1981) and Byun and Chen (1997). The model is applied in a binational framework, since international taxation is based on national and binational tax agreements. Consequently, the two country IntCAPM forms the basis in gaining new scientific insight into taxation in IntCAPT. Factors of heterogeneity such as transaction costs, legal restrictions of foreign access such as tariffs, the control of capital and exchange rates, limitation of foreigners’ ownership, political risk, control of import and export of capital, reserve requirements on bank deposits and other assets and limited information are excluded. For a discussion of these barriers, cf. Ross (1978), Garman (1981), Sellin (1990), Dermody and Prisman (1993), Jouini and Kallal (1995), Ardalan (1999), Zhang (2002) and Bodnar (2003). The tax rate can be regarded as a surrogate for the certainty-equivalent cost of the above barriers (Stulz (1984)).}

Assumption 2 Investors
Each state is populated by one representative investor $d$ and $f$ which is an artificial agent
who behaves the same as the aggregate ones (local representative).76

**Assumption 3 Time**

There is a period of one year, in the beginning of the period \( t=0 \) the decisions are made and the outcome occurs at the end of period \( t=1 \), the insecure future.77

**Assumption 4 Investment opportunities**

As far as the investment opportunities are concerned we consider only equity financed corporations.78 The investor has the opportunity to invest in risky and risk-free assets. We assume that the return of the risky securities is composed of insecure dividends \( \bar{D}_i \) and capital gains \( \bar{C}G_i \) - the difference between the value of the risky asset at the end of period \( \bar{V}_i \) and at the beginning of period \( V_{i,0} \) - and the principal \( V_{i,0} \).79 The value of the risky security at the end of period and the dividends are assumed to be normally distributed.80 The return of the risk-free security consists of nominal interest \( I \) - the difference between the value of the risk-free asset at the end of period \( V_0 \) and at the beginning of period \( V_{0,0} \) - and the principal \( V_{0,0} \).81 In contrast to the domestic investor the foreign investor realizes exchange gains. Exchange gains on the risky asset can be differentiated by the exchange gains on the principal, on capital gains and on dividends \( (S - S_0) V_{i,0} + (S - S_0) (\bar{V}_i - V_{i,0}) + \)

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76 Cf. Lengwiler (2006). Investors' behavior is different within each state. However, in order to gain new scientific insight in the form of international delineation the concept of local representative reveals itself to be sufficient.

77 For a discussion of a multiperiod model, s. chapter 8. No assumptions are made as far as the number and the structure of states are concerned; the states can be finite or infinite (Jonas (2004)).


79 This assumption is restrictive but customary. cf. Mai (2006a), Wiese (2006b) and Mai (2008). In a one period model the entire return should consist of dividends, cf. Blanfus (2002). If the end of period value decreased by the entire dividends, arbitrage opportunities would arise, cf. Elton and Gruber (1970) and Elton (2005).

80 Cf. Lally (1998), Mai (2006a), Wiese (2006b) and Mai (2008). The normal distribution seems not to be descriptive for capital asset pricing because of limited liability and because empirical distributions of returns have tails that are leptokurtic (Balvers (2001)). As far as the critique of the normal distribution assumption is concerned, see chapter 8.

81 This is an assumption generally used in CAPT, cf. Sharpe (1963), Lintner (1965b), Mossin (1966) and Treynor (1999). In reality, assets cannot be risk-free. Black (1972a) developed the CAPM without the assumption of risk-free assets.
\[(S - S_0) \tilde{D}_i \text{ which can be summarized as follows } (S - S_0) \tilde{V}_i + (S - S_0) \tilde{D}_i \text{ and in the case of the risk-free asset by } (S - S_0) V_0.\]

The market approach requires the typification of tax. We ignore specializations and instead regard stylized income tax systems of a representative domestic and foreign investor which incorporate the pricing relevant aspects of tax without considering every detail of tax law (Ollmann and Richter (1999) and Mai (2008)). In determining the tax burden, three elements are critical: the definition of income, the applicable tax rate and the rules for netting the income with other sources (Desai and Gentry (2003)). Commission Of The European Communities (1992), Kluge (2000), Rohatgi (2002), Lorié (2006) and Jacobs (2007) among many others provide a good overview of the features of international taxation.

**Assumption 5 Principle of taxation**

It is assumed that the domestic and foreign state raises income taxes. According to the sovereignty principle and general international rules, states can raise taxes in the case of a concrete point of contact on their territory. The point of contact of persons is the principle of residence and the principle of nationality. In case of residence or nationality, the person is unlimited tax liable with his entire income (world income principle, universality principle), otherwise, the person is limited tax liable with his national income (territoriality principle). Possibilities for tax evasion are excluded.

**Assumption 6 Tax base**

Income is regarded as a net size, the expenses to yield gross income - the investments -

85 Residence means the existence of a person’s domicile and that the person is living for a certain period in the country. The principle of nationality takes effect in the case of citizenship according to national law (Rohatgi (2002) and Jacobs (2007)).
87 Foreign jurisdictions - tax havens - offer potential for tax evasion as regards tax rates, legal concepts, standards of administration, reporting and enforcement and governmental attitudes towards the liberty and privacy of taxpayers and the confidentiality of financial and business transactions (Wang (1995) and Niemann (2007)). Tax evasion shall solely have a tax rate effect and can be implemented into the Tax-Int CAPM by a supplementary factor.
are deducted from gross income (objective net presentation principle). We ignore non-deductible expenses and tax-free income (Barcis (2000)). It will be assumed that the tax is raised at the end of period - the flow of income to the investors at the end of period results in immediate taxation and that portfolio switching at the beginning of the period is not taxable. Expenses resulting from tax compliance are regarded as irrelevant. Redistribution effects of taxes to investors are ignored. Furthermore, we assume that gains and losses are symmetrically taxed (full loss offset). Equivalently, symmetry in the tax treatment of long and short positions is assumed (Lally (1996)).

Assumption 7 Tax scale

The tax scale is assumed to be linear and deterministic. Since taxes are proportional, the average and the marginal tax-rate are identical. Stochastic returns lead to a stochastic tax basis, which implies that average tax rates and marginal tax rates are stochastic in the case of progressive tax; we assume linear tax rates to exclude stochastic tax rates (Mai (2008)).

89 This is an assumption generally used in Tax-CAPT, cf. e.g. Brennan (1970), Wiese (2006a) and Mai (2008). We assumed a time horizon of one year, but in reality investors can determine by themselves the point of time of acquisition and selling of assets. In a multiperiod option pricing model Constantinides (1983) develops a model to determine the optimal point of time of realization.
91 This assumption is restrictive but customary. For the integration of redistribution effects of taxes to investors, cf. Eikseth and Lindset (2009) and Kruschwitz and Löfler (2009).
92 Cf. Long (1977), Mai (2006b) and Dierkes (2007). This assumption implies that gains and losses can be netted for tax purposes (Dierkes (2007)). This assumption also implies that the differential tax treatment of debt and credit interest is neglected. If the tax on interest income differs in respect of debt and credit interest, then the tax payment depends on whether the investor borrows or lends. For the integration of differential taxation of debt and credit interest, cf. Elton (2007) and Hüper (2009).
93 In the CAPM of Wood (1997) the symmetry assumption of short and long positions is abandoned. He assumes that if an investor shorts an asset offering imputation credits, then the long holder is compensated. Wood (1997) shows that the expected return on an asset offering imputation credits depends upon whether that economy's domestic investor has net long, net short or net zero positions in that asset. However, this requires that the portfolio holdings of all investors must be observed (Lally (1996)).
94 We assumed the existence of one representative investor in each state. Tax rates can vary according to the personal situation, so the after tax returns could be different for every national investor. The concept of a representative investor in each state reveals itself to be fully sufficient to gains new scientific insight into taxation in IntCAPT, so varying tax rates for national investors are neglected. For a discussion of the Tax-CAPM with heterogeneous national investors, cf. chapter 3.1.
95 For a discussion of progressive and stochastic tax rates, s. chapter 8.
Assumption 8 Tax effects

The return is unaffected by tax.\textsuperscript{96}

The principles of taxation lay the foundation for an integrated tax framework. It is assumed that the tax rates are different in respect of the income classes, so all relevant income types in the IntCAPM - dividends, capital and exchange gains and interest - are differently taxed.\textsuperscript{97}

In the following the dividends, capital gains, interest, exchange gains tax system and the methods of double taxation reduction are presented.

4.2 Corporate and Dividend Tax System

Corporations are taxed according to the deferral principle, corporations are taxed on the corporate level and investors on the personal level (Rohatgi (2002), Jacobs (2007), Scheffler (2007) and Schreiber (2008)).\textsuperscript{98} In an integrated tax system, the corporate and personal tax systems interact. We present the corporate tax system and derive the dividend tax system. The corporate tax system consists of the classical, shareholder relief, (full) imputation and dividend exemption system.\textsuperscript{99} The dividend tax system can be categorized into a full tax, tax-reducing and tax-free system. Measures to reduce and to avoid double taxation can be considered on the corporate and on the personal level. The fundamental feature common to all double taxation reducing or avoiding systems is the provision of a valuable tax benefit to shareholders on dividend income (Handley and Maheswaran (2005)). Figure 4.1 on page 40 illustrates the corporate and dividend tax systems.\textsuperscript{100} Profits (gross dividends) are the tax base for the corporate tax, whereas


\textsuperscript{98} For a discussion of taxing capital income on the corporate or personal level, cf. OECD (2007).


\textsuperscript{100} Cf. Jacobs (2007) for a comparable illustration.
dividends are defined as the profit after corporate tax.

\[ \tilde{D}_i \equiv \tilde{P}_i (1 - t_C) \]  

(4.1)

with \( \tilde{D}_i \) nominal dividends
\( \tilde{P}_i \) nominal profit
\( t_C \) corporate tax rate with \( 0 < t_C < 1 \)

In the case of the classical system corporate tax is levied on profits and dividend tax is levied on dividends.\(^\text{101}\)

\(^{101}\) The term classical system was introduced by Tempel (1970).
\[ \tilde{P}_i^{CS} = \tilde{P}_i - (t_C + t_D - t_C t_D) \tilde{P}_i \]
\[ \tilde{D}_i^{CS} = \tilde{D}_i - t_D \tilde{D}_i \] (4.2)

with \( \tilde{P}_i^{CS} \) nominal profit after tax in the classical system
\( t_D \) dividend tax rate with \( 0 < t_D < 1 \)
\( \tilde{D}_i^{CS} \) nominal dividends after tax in the classical system

The classical system can be categorized as a full dividend tax system since neither tax reducing nor avoiding measures are installed.

In the case of the shareholder relief system (classical system with tax scale) a reduced dividend tax rate is applied on dividends.

\[ \tilde{P}_i^{SR} = \tilde{P}_i - t_C \tilde{P}_i - (1 - \phi) t_D \left( \tilde{P}_i - t_C \tilde{P}_i \right) \]
\[ \tilde{D}_i^{SR} = \tilde{D}_i - (1 - \phi) t_D \tilde{D}_i \] (4.3)

with \( \tilde{P}_i^{SR} \) nominal profit after tax in the shareholder relief system
\( \tilde{D}_i^{SR} \) nominal dividends after tax in the shareholder relief system
\( \phi \) relief factor of dividend tax with \( 0 < \phi < 1 \)

A further root of the shareholder relief system is the possibility of partial exemption from dividend tax.

\[ \tilde{P}_i^{SR} = \tilde{P}_i - t_C \tilde{P}_i - t_D \left( \tilde{P}_i - t_C \tilde{P}_i - \phi \left( \tilde{P}_i - t_C \tilde{P}_i \right) \right) \]
\[ \tilde{D}_i^{SR} = \tilde{D}_i - t_D \left( \tilde{D}_i - \phi \tilde{D}_i \right) \] (4.4)

The equations (4.3) and (4.4) are equal. The shareholder relief system can be classified as a dividend tax-reducing system.
The (full) imputation system is designed to reduce the total tax burden on corporate income by compensating shareholders for tax paid by corporations. The compensation takes the form of a tax imputation attached to dividend payments (Rohatgi (2002)). In the case of the imputation method the tax base of the dividend tax is increased by part of the corporate tax imposed on dividends and this part encumbering on dividends is credited. Depending on the full imputation system (conduit system, dividend imputation system or credit system), the tax base of the dividend tax is increased by the total corporate tax imposed on the dividends (grossing-up) and the corporate tax encumbering on the dividends is credited.¹⁰²

The classification of the full imputation system $\psi = 1$ depends on the relationship of corporate and dividend tax rate. If the dividend tax rate is higher than the corporate tax rate the full imputation system reveals itself to be a dividend tax-reducing system. In the case of equality of the tax rates the full imputation system is a dividend tax-free system. If the dividend tax rate is lower than the corporate tax rate, the voluntary announcement of dividend income is to be expected, since the shareholder would stand to gains to the amount of the difference between corporate and dividend tax (alignment method) (Head

¹⁰² The imputation system is subject to various restrictions. Smith (1993) and Rohatgi (2002) provide a comprehensive overview of full imputation systems to be found internationally. A comparable structure has the shareholder tax (Engels and Stützel (1968)).
(1997)). The measures of reducing the total tax burden are executed on the level of the dividend tax. In that way only the dividend tax is levied on the profit. The corporate tax is an advanced payment by the company on the shareholders’ dividend tax and it takes a withholding role (Mintz (1995) and Lally (2000)). The full imputation system transfers the comprehensive income conception, that taxes are to be imposed on individuals. In comparison with the imputation method the imputation factor of corporate tax of the full imputation method is one, since the corporate tax imposed on dividends is fully credited. The main rationale for a dividend tax imputation lies in the impossibility to tax unrealized capital gains. Corporations and shareholders have the choice between the distribution of dividends and the sale of shares. By eliminating the taxation of unrealized capital gains or a corporate level tax, tax-payments could be deferred indefinitely if the corporation were to retain all earnings. A corporate tax eliminates the possibility of deferring the tax; the integration is achieved by rebating the corporate tax to shareholders when they pay tax on corporate distributions (McDonald (2001)). Another explanation for levying a corporate tax is that it captures part of the benefits of public expenditures on goods and services, of public expenditures on education and training and of the legal provisions that are offered to the corporation (Mintz (1995) and OECD (2007)). The corporate tax also acts as a withholding tax on equity income earned by non-resident shareholders, which would otherwise escape taxation in the source country in the absence of a withholding tax on dividends distributed to foreign shareholders (Mintz (1995) and OECD (2007)). If the right of taxation is assigned to the source state, the foreign investor may not be able to (fully) utilize imputation credits. The (full) imputation system leaves the foreign investor liable to full taxation on dividends and hence the classical system (Treisch (2004) and Jacobs (2007)). Assuming that the foreign investor cannot capture the value associated with the imputation credit, the foreign investor is disadvantaged in relation to the domestic investor.

In the case of the dividend exemption method, dividends are exempted from the tax base of the dividend tax. The principal advantage of the dividend exclusion prototype is its simplicity and relative ease of implementation.

103 For a discussion, s. footnote 91.
104 For further reasons for levying the corporate tax, cf. OECD (2007).
105 Imputation is internationally not installed on account of evading imputation systems, administrative and fiscal reasons (Jacobs (2007)).
\[ \tilde{P}^{DE}_i = \tilde{P}_i - t_c \tilde{P}_i - t_D (0) \]  
\[ \tilde{D}^{DE}_i = \tilde{D}_i - t_D (0) = \tilde{D}_i \]  

with \( \tilde{P}^{DE}_i \) nominal profit after tax in the dividend exemption system

\( \tilde{D}^{DE}_i \) nominal dividends after tax in the dividend exemption system

The dividend exemption method can be classified as a dividend tax-free system.

The different corporate and dividend tax systems are modeled by assigning the relief factor of dividend tax and the imputation factor of corporate tax numbers from the interval from \([0; 1]\) (Bareis (2000) and Sureth and Niemann (2005)). Table 4.1 on page 44 illustrates the incorporation of the factors.

\[ \tilde{P}^{DTS}_i = \tilde{P}_i (1 - t_c) - (1 - \phi) t_D \tilde{P}_i (1 - t_c) + \psi (1 - t_D) t_c \tilde{P}_i \]  
\[ \tilde{D}^{DTS}_i = \tilde{D}_i - (1 - \phi) t_D \tilde{D}_i + \psi (1 - t_D) t_c \tilde{D}_i \]  

with \( \tilde{P}^{DTS}_i \) nominal profit after tax in the corporate tax systems

\( \tilde{D}^{DTS}_i \) nominal dividends after tax in the dividend tax systems

<table>
<thead>
<tr>
<th>Corporate and dividend tax systems</th>
<th>Relief factor of dividends tax ( \phi )</th>
<th>Imputation factor of corporate tax ( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical system</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Shareholder relief method</td>
<td>((0; 1))</td>
<td>0</td>
</tr>
<tr>
<td>Imputation system</td>
<td>0</td>
<td>((0; 1))</td>
</tr>
<tr>
<td>Full imputation system</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Dividend exemption system</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1: Relief and Imputation Factors Depending on the Corporate and Dividend Tax Systems
The classical system is modeled by assigning the factors the number zero. In the case of the shareholder relief method the imputation factor of the corporate tax is zero and the relief factor of the dividend tax is assigned a number from the interval \((0; 1)\). In the event of the imputation system it is reversed. The full imputation system is characterized by an imputation factor of the corporate tax of one and an imputation factor of dividend tax of zero, whereas in the case of the dividend exemption method it is reversed.

**Example 2 Corporate and Dividend Taxation**

*The dividend tax rate amounts to 35%, the corporate tax rate is 15%.*

![Figure 4.2: Tax Rate Depending on the Relief and Imputation Factors in the Corporate Tax System](image)

The shareholder relief system can be classified between the classical system and the dividend exemption system. With the decrease of the relief factor the shareholder relief system approaches the classical system, while with the increase of the relief factor the shareholder relief system approaches the dividend exemption method.\(^{106}\) With the increase of the imputation factor of corporate tax the imputation system approaches the full imputation system, whereas a decrease of the imputation factor means the approach to the classical system.

\(^{106}\)Motivated by administrative complexities the trend of a shift from imputation tax based systems to dividend exclusion based systems can be observed (Hoek (2003)).
The different dividend tax systems are due to a tax rate effect, hence, the term \((1 - \phi) t_D - \psi (1 - t_D) \frac{t_C}{1-t_C}\) can be replaced by \(t_D^{D/F}\), so dividends after tax are expressed as follows:

\[
\tilde{D}_i^{DTS} = \tilde{D}_i - t_D^{D/F} \tilde{D}_i. \tag{4.10}
\]

with

- \(D_i^{DTS}\) nominal dividends after tax in the dividend tax system
- \(t_D^{D/F}\) integrated tax rate on dividends with \(0 \leq t_D^{D/F} < 1\)

### 4.3 Capital Gains and Interest Tax System

After describing the dividend tax systems, the capital gains and interest tax system are presented.\(^{107}\)

There are a number of considerations in the design of capital gains tax rules. The design dimensions can be categorized according to the following criteria (OECD (2006)).

---

1. Taxation on a realization basis: Most countries apply taxation on a realization basis rather than accrual basis.\textsuperscript{108}

2. Setting of the statutory tax rate: Individuals can face lower tax rates on capital gains income than on personal income or capital gains can be exempted from taxation (Rohatgi (2002), Jacobs (2007) and Keen (2008)).

3. Treatment of losses: Ring-fencing rules restricting capital loss claims can be implemented to protect the tax base. Certain assets can be restricted from capital loss deductions.\textsuperscript{109}

4. Rollover provision: Taxpayers are allowed to defer payment of capital gains tax that might be triggered otherwise.\textsuperscript{110}

5. Treatment of gains on a taxpayer’s personal residence: Some countries provide a full exemption for capital gains on homes.

6. Treatment of the inflation component of (nominal) capital gains: Only real capital gains can be included in the tax base. Nominal capital gains is adjusted to net out the inflation component.\textsuperscript{111}

7. Treatment of non-residents. Non-residents can face lower tax rates on capital gains income or they can be exempted from taxation.\textsuperscript{112}

8. Transitional considerations: Capital gains tax can be introduced on an entirely prospective basis. The tax is applied only to gains on assets acquired after the

\textsuperscript{108} The realization-based system recognizes that accrual taxation poses significant difficulties by requiring periodic valuation of assets. Realization-based taxation can create liquidity problems for taxpayers who do not have sufficient cash to cover tax on accrued but unrealized gains, requiring them to borrow or sell assets to pay their tax liability, cf. OECD (2006). In our model a realization-based approach is adopted, s. assumption 6.

\textsuperscript{109} In our model gains and losses are symmetrically taxed, s. assumption 6.

\textsuperscript{110} If rollover relief is provided valuation and cash-flow problems can occur. For a description of the different types of rollovers, cf. OECD (2006). In our model a realization-based approach is adopted, s. assumption 6.

\textsuperscript{111} Most countries do not attempt to adjust nominal capital gains to net out the inflation component, since inflationary gains are not prevalent and because of the complexity of calculation, cf. OECD (2006).

\textsuperscript{112} Certain countries have special realization rules that apply to taxable accrued gains where a resident becomes a non-resident, while others apply tax at the time of realization, cf. OECD (2006).
Capital gains after tax is modeled by the following formula:

\[
\tilde{CG}_{i,t} = (\tilde{V}_i - V_{i,0}) - t^{D/F}_{CG} (\tilde{V}_i - V_{i,0}) = \tilde{CG}_i - t^{D/F}_{CG} \tilde{CG}_i. \tag{4.11}
\]

with \( \tilde{CG}_{i,t} \) nominal capital gains after tax
\( t^{D/F}_{CG} \) capital gains tax rate with \( 0 \leq t^{D/F}_{CG} < 1 \)

Interest is subject to interest income taxation. In schedular systems individuals face flat tax rates for interest income (Hess-Ingrassa (1995), Gordon (2003) and Benshalom (2008)).\(^{114}\) On account of the linearity of tax rates the comprehensive income as well as the schedular system are comprised by the formula. Interest after tax is modeled as follows:

\[
I_t = (\tilde{V}_0 - V_{0,0}) - t^{D/F}_I (\tilde{V}_0 - V_{0,0}) = I - t^{D/F}_I I. \tag{4.12}
\]

with \( I_t \) nominal interest after tax
\( I \) nominal interest
\( t^{D/F}_I \) interest tax rate with \( 0 \leq t^{D/F}_I < 1 \)

### 4.4 Exchange Gains Tax System

Foreign exchange gains and losses arise on the foreign investor’s cross-border transactions. Foreign exchange gains taxation can be effective in calming exchange rate volatility and avoiding currency crises (Bird and Rajan (1999)). However, the taxation of exchange gains and losses varies widely; the tax treatment of exchange gains and losses are, in most countries, neither simple nor rational (Chown (1990)). These rules differ on how

\(^{113}\) On account of the assumption of a one period model, s. assumption 3, we regard this aspect as irrelevant.

exchange gains are calculated, the timing when they are taxable, and their characterization for tax purposes (Rohatgi (2002)). The key exchange gains conversion issues for the foreign investor can be characterized as follows (L’Association Fiscale Internationale (1986), International Bureau Of Fiscal Documentation (1988), OECD Committee On Fiscal Affairs (1988) and Rohatgi (2002) among many others):

1. Recognition: Exchange gains are recognized at the realization of exchange gains or losses. The exchange rate is based on the transaction date.

2. Character: Exchange gains can be taxed as ordinary income, as capital gains, as a separate type of income or as outside the scope of taxation. If they are taxed as ordinary income, they can be taxed at different rates.

3. Nature: Exchange gains can be affected by the nature of the underlying transaction.

4. Source: Exchange gains can be regarded as of either foreign or domestic source. If exchange gains are regarded as income from the source state it can be non-taxable.

5. Hedging: Hedged transactions can have both exchange gains and losses at different times with differing tax treatment.

Exchange gains are only taxable in the state of residence. The overview 4.4 on page 50 illustrates the exchange gains tax system in various countries. In many countries such as Russia, Austria and Canada exchange gains are taxed according to the underlying

---


116 In a one-period model, the aspect that year-end losses can be recognized in the current year while gains can be carried forward is regarded as irrelevant.

117 We neglect the fact that certain states apply for taxation exchange rates which deviate from the market rates.

118 The distinction might depend on the nature and source of the underlying transaction. The tax treatment may differ between trading and non-trading items, capital and revenue items, and monetary and non-monetary items, s. International Bureau Of Fiscal Documentation (1988) and Rohatgi (2002).

119 Exchange gains and losses and assets and liabilities can be asymmetrically taxed. Exchange gains are regarded as taxable whereas exchange loss is regarded as out of the scope of taxation. The asymmetric tax treatment of gains and loss is neglected on account of assumption 6.

120 The exchange gains conversion issue of hedging with the asymmetric treatment of exchange gains and losses is neglected, on account of the assumption of equal treatment of gains and losses, s. assumption 6.

121 S. I would like to thank ECOVIS AG, and especially Prof. Dr. Peter Lüdemann who provided me with his survey of exchange gains taxation. S. also www.kpmg.com/Global/en/IssuesAndInsights/ArticlesPublications/TIES/Pages/Taxation-of-international-executives-2010.aspx.
transaction, so it is equivalently taxed to dividend, capital gains and interest. In countries where taxes seem not to be the primary source of revenue, exchange gains as well as the underlying transaction are not subject to taxation. Other states as e.g. the USA, Australia, Italy, the Netherlands and Belgium regard exchange gains as a separate income type. In contrast to the USA, Australia and Italy, in the Netherlands, Belgium and other countries exchange gains are not taxable. The exchange gains on capital gains and on the principal after tax can be expressed by the following formula:

\[ \tilde{E}_{CG,P,t} = (S - S_0) \tilde{V}_i - t_{CG,P,e}^F (S - S_0) \tilde{V}_i. \]  

(4.13)

with \( \tilde{E}_{CG,P,t} \) exchange gains after tax on capital gains and on principal with \( t_{CG,P,e}^F \) exchange gains tax rate on capital gains and on principal with \( 0 \leq t_{CG,P,e}^F < 1 \)
Exchange gains on dividends after tax are expressed by the following formula:

$$\tilde{E}_{D,t} = (S - S_0) \tilde{D}_t - t_{D,e}^F (S - S_0) \tilde{D}_t.$$  (4.14)

with $\tilde{E}_{D,t}$ exchange gains on dividends after tax

$$t_{D,e}^F$$ tax rate on dividend exchange gains with $0 \leq t_{D,e}^F < 1$

For the exchange gains after tax on interest and on the principal we can derive the following expression:

$$E_{I,P,t} = (S - S_0) V_0 - t_{I,P,e}^F (S - S_0) V_0.$$  (4.15)

with $E_{I,P,t}$ exchange gains after tax on interest and principal

$$t_{I,P,e}^F$$ exchange gains tax rate on interest and on the principal

with $0 \leq t_{I,P,e}^F < 1$

### 4.5 Methods of Double Taxation Reduction

The foreign investor’s subjection to the domestic and foreign state can lead to international double taxation, since the world income principle and the territoriality principle clash together. The domestic state (source state) and the foreign state (residence state) impose taxes on the income of the foreign investor. Economic double (multiple) taxation is defined as the taxation of an economic size by two (multiple) identical types of taxes by two fiscal sovereignties in the same period (Siegel and Bareis (2004)).\footnote{Economic undertaxation can be defined as the lower international in comparison to national taxation (Jacobs (2007)).} Double taxation can be avoided by the conception of incorporating identical criteria of allocation and assigning the right of taxation either to the domestic state or to the foreign state.\footnote{These identical definitions of tax principles can be incarnated if the point of contact is solely the tax object or the tax subject (Henselmann (1996)).} Alternatively, the domestic state can impose a withholding tax on the income of the foreign...
investor. Bilateral methods are incarnated in order to avoid or reduce double taxation. Bilateral and national tax laws incorporate different methods in order to avoid or reduce double taxation (Rohatgi (2002), Schult (2002) and Jacobs (2007)). The methods of avoiding or reducing double taxation are illustrated in figure 4.5 on page 52.\footnote{Cf. Henselmann (1996) and Bareis and Siegel (2004) for a comparable illustration.} The ex-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{methods_of_avoiding_and_reducing_double_taxation.png}
\caption{Methods of Avoiding and Reducing Double Taxation}
\end{figure}

emption and credit method can lead to the elimination of double taxation, whereas the deduction, remission and flat tax lead to a reduction of double taxation. The exemption method can be differentiated according to with progression (relative exemption) and without progression.\footnote{The differentiation by with and without progression is irrelevant in our model on account of the assumption of linear tax rates, s. assumption 7.} The credit method can be differentiated by unlimited, fictive, indirect
and limited credit.\textsuperscript{126} The limited credit method can be differentiated by per country limitation, per basket limitation or a combination of both methods.\textsuperscript{127}

**Assumption 9** Per basket limitation

*The per basket limitation is incorporated, capital gains, exchange gains on capital gains, dividends, exchange gains on dividends, interest and exchange gains on interest are regarded as separate income types.*

In the following we analyze the effects of the methods of avoidance and reducing double taxation.

According to the exemption method the residence state does not take into consideration foreign investors’ income yielded in the source state.\textsuperscript{128} The exemption method is equivalent to the assignment of the taxation right to the domestic state.

\[
\begin{align*}
    t_{t^F, Em}^F &= t^F(0) = 0 \\
    &= t^F - t^F = 0 
\end{align*}
\]  

\textbf{(4.16)}

with $t_{t^F, Em}^F$ tax rate of the foreign state after application of the exemption method

$t^F$ foreign state’s tax rate

As far as the exemption method is concerned the location of the investment is the determining factor, so the concept of capital import neutrality (territoriality principle) is pursued (\textit{Scheffler} (2009)).

In the case of the credit method, the source tax of the domestic state is credited to the income tax of the foreign state. A credit is allowed for domestic income taxes paid on domestic source income where such income is also subject to foreign tax. The base point of the credit method is the tax amount, the foreign state deduces the source tax from the income tax.

\textsuperscript{126} The indirect credit can be regarded as irrelevant, since we do not consider affiliates in our model, s. assumption 4.

\textsuperscript{127} The per country limitation can be regarded as irrelevant, since only two countries are regarded, s. assumption 1.

In the case of unlimited credit, the entire tax of the source (domestic) state is deducted from the tax of the foreign state.\footnote{Cf. \textit{Gordon and Hines} (2002) and \textit{Scheffler} (2009).}

\[ t_{t_{F},Cm}^{F} = t_{F} - t_{St}^{D} \]  

(4.17)

with \( t_{t_{F},Cm}^{F} \) tax rate of the foreign state after application of the unlimited credit method

\( t_{St}^{D} \) domestic state’s source tax

The conception of unlimited credit leads constantly to the tax burden of the state of residence, the foreign state. The concept of capital export neutrality is pursued (\textit{Siegel and Bareis} (2004) and \textit{Scheffler} (2009)). If the tax rate of the source state is higher than the tax rate of the residence state, the residence state refunds the excess tax payment and yields lower tax income than in comparison with a national investment.

In the case of limited credit the deduction shall not exceed the part of the income tax, as computed before the deduction is given.\footnote{Cf. \textit{Gordon and Hines} (2002) and \textit{Scheffler} (2009).} The maximum credit may be computed either by apportioning the total tax on total income according to the ratio between the income for which the credit is to be given and the total income, or by applying the tax rate for total income to the income for which credit is to be given. On account of the assumption of linear tax rates, the computation of the maximum credit leads to the same result.

\[ t_{t_{F},Cml}^{F} = t_{F} - Min(t_{St}^{D}; t_{F}) \]  

(4.18)

with \( t_{t_{F},Cml}^{F} \) tax rate of the foreign state after application of the limited credit method

\( Min \) minimum

\footnote{Cf. \textit{Gordon and Hines} (2002) and \textit{Scheffler} (2009).}
If the income tax of the residence state exceeds the source tax, the principle of capital export neutrality is pursued and in the opposite case the principle of capital import neutrality is pursued. The total tax burden of the foreign income measured by the highest tax demands of the involving states corresponds at least to the level of tax of the residence state. As a result, source tax reductions of the domestic state are not for the benefit of the shareholder. Creditable foreign taxes, exceeding the income tax of the residence state, are not taken into account (credit overhang). The conception of the (unlimited) credit consists in raising the (lower) tax level of the domestic state to the tax level of the foreign state.

In order to make effective tax allowances, a fictive credit can be given. In the case of fictive credit, it is not the effective tax of the source state, but a fictive (higher) tax amount that is credited to tax in the foreign state.\footnote{A credit overhang is found, if the source tax of the domestic state exceeds the income tax of the foreign state. The credit overhang is calculated by deducting the source tax from the income tax.}

\[
t_{t_{F,FCm}}^F = t^F - t_{FCm}^F
\]  \hspace{1cm} (4.19)

with \( t_{t_{F,FCm}}^F \) tax rate of the foreign state after application of the fictive credit method

\( t_{FCm}^F \) tax rate of fictive credit method

In the case of the remission method a reduced tax rate on foreign income is applied.\footnote{Cf. Scheffler (2009).}

The remission method leads to a reduction of double taxation.

\[
t_{t_{F,Rm}}^F = t^F - \tau t^F
\]  \hspace{1cm} (4.20)

with \( t_{t_{F,Rm}}^F \) tax rate of the foreign state after application of the remission method

\( \tau \) tax remission factor

\footnote{Cf. Scheffler (2009).}
The concept of the flat tax is that the foreign state raises a (lower) flat tax. The flat tax method leads to the reduction of double taxation.

\[
\begin{align*}
t_{t,Ft}^F & = t_{Ft}^F \\
& = t^F - \varrho t^F
\end{align*}
\]  

(4.21)

with \( t_{t,Ft}^F \) tax rate of the foreign state after application of the flat tax method

\( t_{Ft}^F \) flat tax rate

\( \varrho \) flat tax factor

In the case of the deduction method the tax of the source state is reduced from the tax base of the residence state. A deduction is allowed for domestic income taxes paid on domestic source income where such income is also subject to foreign tax.

\[
\begin{align*}
t_{t,F,Dr}^F & = t^F - t_{St}^D
\end{align*}
\]  

(4.22)

with \( t_{t,F,Dr}^F \) tax rate of the foreign state after application of the deduction method

The deduction method leads to a reduction of double taxation and it limits the double tax relief to the foreign tax paid as multiplied by the tax rate.

The different methods of double taxation reduction are due to a tax rate effect, so we define them by the following equation:

\[
\begin{align*}
t_{t,F,Int}^F & = t^F - t_{Int}^F
\end{align*}
\]  

(4.23)

with \( t_{t,F,Int}^F \) tax rate of the foreign state after application of the relief methods

\( t_{Int}^F \) relief tax rate

---

Exchange gains affect only one jurisdiction and hence does not lead to double taxation (Rohatgi (2002)). Since exchange gains are not taxed in the source state, the exemption, the credit and the deduction method are not applicable for the taxation of exchange gains. The different methods of double taxation reduction for exchange gains taxation are due to a tax rate effect and can be summarized by the following equation:

\[ t_{tF,Int,e}^F = t_e^F - t_{Int,e}^F. \] (4.24)

with  
- \( t_{tF,Int,e}^F \) tax rate on exchange gains of the foreign state after application of the relief methods
- \( t_e^F \) tax rate on exchange gains of the foreign state
- \( t_{Int,e}^F \) relief tax rate

In our model, the exemption and the credit method in the case of equivalence of source and residence tax rate lead to an equivalent tax burden of zero in the foreign state; foreign investors’ income is solely subject to tax in the source state.
5 Tax International Capital Asset Pricing Model

“The government’s view of the economy could be summed up in few short phrases: If it moves, tax it. If it keeps moving, regulate it. And if it stops moving, subsidize it.”


Most IntCAPMs consider either different consumption and investment opportunity sets - deviations from PPP - or barriers to international investment - taxes on foreign investment - when PPP is assumed to hold. Rational investors judge the real return after taxes, which implies that a more realistic model should incorporate both, deviations from PPP and barriers in the form of taxes on international investment. The incorporation of international tax systems has the important benefit of not only being descriptive on the macro level but also of better describing the micro behavior of the international investor. Tax effects are regarded as a very important factor in determining the investor’s international asset return and it is evident that deviation from PPP exists in the short run.

How is international asset pricing affected by international tax systems?

We analyze the general market equilibrium with stochastic dividend and tax rates that are investor-specific for all income types in IntCAPM, dividend, interest, capital and exchange gains. By integrating international taxation into the IntCAPM, the effect of international
tax systems on individual investor behavior and on equilibrium returns can be studied by derivation of the Tax-IntCAPM.\textsuperscript{136}

The construction of the Tax-IntCAPM will allow the determination of the relevant measure of risk for any asset and the relationship between expected return and risk for assets under integration of different taxes and deviation from Relative PPP when the capital market is in equilibrium. In order to analyze the effects of taxes in pricing international assets, a theoretically sound model of international market valuation under international taxation is required to develop a normative theory of taxation in IntCAPT.

Based on the concepts of Adler and Dumas (1983) and Mai (2008) a binational version of the Tax-IntCAPM is presented. In contrast to the model of Adler and Dumas (1983) the Tax-IntCAPM is formulated in real terms in order to incorporate deviation from Relative PPP. The framework of a discrete time model is applied, since the framework of the nonlinear exchange rate behavior with the option to integrate chaotic models is incorporated, and which is founded on the basis of discrete time.

After discussion of the assumptions and set-up, we analyze different versions of the Tax-IntCAPM. The solution of the model derives the equilibrium pricing equation and an intensive interpretation elaborates the economic concepts and implications.

### 5.1 Assumptions

Here, the assumptions of the Tax-IntCAPM are introduced. On the one hand, they build on the assumptions of international taxation and on the other hand, they correspond to the necessities of IntCAPT.

The economies in the domestic and foreign state can be comprised by making assumptions for the capital, goods and currency market, describing the investor's investment and consumption opportunities and the barriers in the form of tax on international investment. The assumptions are rounded off with the description of investors' behavior and a conclusion.

\textsuperscript{136}Furthermore, the derived Tax-IntCAPM should lay the basis for the derivation of a Tax-IntCAPM in a multiperiod context where dividends obviously are stochastic (Lally and van Zijl (2003), Jonas (2004) and Wiese (2006b)).
Assumption 10 Exchange gains and inflation effects
The nominal return is unaffected by exchange gains and inflation.\textsuperscript{137}

Assumption 11 Capital market
In the domestic state, there is a perfectly competitive capital market. A perfect capital market implies that there are no arbitrage opportunities.\textsuperscript{138}

Assumption 12 Efficient market
The Efficient Market Hypothesis (EMH) is pursued and the prices on assets reflect immediately and completely the entire relevant information.\textsuperscript{139}

Assumption 13 Heterogeneous expectation
The concept of the Heterogeneous Expectation Model (HEM) is pursued.\textsuperscript{140} Market participants differ in degrees of international taxation and impact of deviation from Relative PPP and hence in their risk aversion.

Assumption 14 Capital market equilibrium
The capital market is characterized by equilibrium in a world with taxes and deviation from Relative PPP. Market participants are in a state of optimum and prices for all risky

\textsuperscript{137}C. Mai (2008). This is an assumption generally accepted in Tax-CAPT. For a discussion, c. Jones and Lamont (2001) and Wiese (2004).

\textsuperscript{138}No-arbitrage is a generally accepted condition in finance. In general, if there is any arbitrage opportunity, the market force would act as an invisible hand to drive the prices change and bring the market back to an arbitrage-free setting. In reality, however, financial markets are never short of friction (Garman (1981)). The exclusion of tax arbitrage in the Tax-IntCAPM is discussed in chapter 6.

\textsuperscript{139}C. Fama (1970a). The EMH is associated with the idea of a random walk, which is a term to characterize a price series where all subsequent price changes represent random departures from previous prices. The logic of the random walk idea is that if the flow of information is unimpeded and information is immediately reflected in stock prices, then tomorrow’s price change will reflect only tomorrow’s news and will be independent of the price changes today. But news is by definition unpredictable and, thus, resulting price changes must be unpredictable and random. As a result, prices fully reflect all known information, and even uninformed investors buying a diversified portfolio at the tableau of prices given by the market will obtain a rate of return as generous as that achieved by the experts (Malkiel (2003)). Empirical evidence has been mixed, but has generally not supported strong forms of the EMH, c. Ross (1978), Garman (1981) and Sellin (1990). As far as the theoretical critique of the EMH is concerned, c. chapter 8.

\textsuperscript{140}The extensive body of empirical evidence shows that investors do not behave the way homogeneous expectation model predicts they should. The HEM assumes that individual investors hold differing beliefs about an asset’s future prospects (Miller (1977)).
assets are assumed to move in such a way that an exogenous quantity of each risky asset \( n_{i(k)}^0 \) equals the aggregate demand for the risky asset \( n_i^D + n_i^F \) and the capital market is cleared.\(^{141}\) World wealth is taken to be exogenous (endowment economy) (Lucas (1978) and Pennacchi (2007)).

**Assumption 15** Goods market

In each state there is a goods market which is characterized by bilateral trade. The price process of the consumption bundle \( P \) is based on the quantity theory of money. The inflation rates adopt different price processes in the states which are due to the state’s different policies of money quantity providence. The states announce their policy of money quantity providence at the beginning of the period.\(^{142}\)

**Assumption 16** Deviation from Relative PPP

The economic concept of nationhood is characterized by deviations from Relative PPP (Adler and Dumas (1983)). At the beginning of the period PPP is assumed to hold but the heterogeneous process of the inflation and exchange rate leads to the irrelevance of PPP at the end of period and hence the irrelevance of Relative PPP. Consequently, this leads to the separation of the goods markets.\(^{143}\)

**Assumption 17** Currency market

The exchange rate process is characterized by a market approach in which demand for foreign currency equals the supply. The non-linear behavior of exchange rates is assumed to be determined by the interaction of speculators and traders.\(^{144}\)

In the vein of the new paradigm we treat the capital, goods and currency market as complex, interdependent systems which are characterized by the coexistence of randomness and determination (Peters (1996)).

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\(^{141}\) The assumption of capital market equilibrium is a central theorem in CAPT. For a discussion of disequilibrium in capital markets, c. chapter 8.

\(^{142}\) For a discussion of pricing inflation risk and integration of global factors, c. chapters 3.5 and 8.

\(^{143}\) The good markets are considered to be separated if goods are differently evaluated.

\(^{144}\) S. chapter 3.4. The assumptions of the currency market are given in chapter 5.2.3 in detail.
Assumption 18 Investment opportunity set
The representative investors face an equal investment opportunity set in the sense that the available assets are the same for both. The capital market contains I+1 assets, i=0 is a riskless asset, whereas i=1...I are risky assets. The capital market comprises all assets except currencies. The investors have the opportunity of borrowing and lending at the risk-free rate and short-selling is admitted.

Assumption 19 Wealth
The investors possess initial real wealth \( W_0 \) which equals unity in terms of the consumption basket of the domestic state. Initial wealth is used in order to invest in exogenously given, perfectly divisible and non-redundant risky stocks and risk-free bonds.

Assumption 20 Consumption opportunity set
The investors consume the consumption bundle of their home state.

Assumption 21 Investors
Capital market investors behave as homo economicus, they optimize the well-being for

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145 It is assumed that all assets are marketable and so, the world capital market portfolio shall consist of all types of capital (except currencies), including real estate, privately held capital, publicly held capital (roads, parks, etc.), human capital which transpires not to be marketable. Balvers (2001) derives a CAPM integrating non-marketable assets.
146 Short sales are necessary for efficiently functioning markets. Effective short sale constraints likely decrease informational efficiency and fundamental value efficiency, and may lead to increased volatility and market bubbles. Short sellers also increase liquidity, facilitate market making, and help markets to identify corporate fraud (Jones and Lamont (2001), Francis (2005)). On the other hand short sales can lead to arbitrage opportunities where investors and securities are subject to differential taxation and deviation from Relative PPP. In reality, investors may be charged a higher interest rate for borrowing money than the interest rate for lending money (Zhang (2004)). Brennan (1971) examined the case where the borrowing rate differed from the lending rate and these rates were different for each investor. The Tax-IntCAPM under short sale and borrowing restrictions is discussed in chapter 6.
147 Real wealth is nominal wealth divided by the price of the consumed commodity (Stulz (1984)).
149 The term homo economicus is a model of the human being. The model refers to the concept of Economic Man introduced by Mill (1909).
themselves in the sense that they maximize their expected utility.\textsuperscript{150} The different utility concepts of the individuals include future consumption as their sole argument. The argument is assumed to be normally distributed. Investors’ preferences display homotheticity, non-satiation and variance-aversion.\textsuperscript{151} Investors are considered to be price takers.\textsuperscript{152}

Investors’ decisions are based on homogeneous availability and interpretation of information, leading to homogenous estimation of state probabilities and hence homogeneous expectations as far as the expectation value, variances and covariances are concerned for pre-tax returns. However on account of investor-specific taxes, different inflation and exchange rates asset returns are heterogeneous for the domestic and foreign investor (\textit{Balvers (2001)}).

The structure of the Tax-IntCAPM is illustrated in figure 5.1 on page 64. The capital, goods and currency market are separated from each other, implying that the prices of assets, goods and currencies are separately determined. We ignore the question of uniqueness of equilibrium (\textit{Wiese (2004) and Mai (2008)}).\textsuperscript{153} Since the nominal before tax return is unaffected by tax, inflation and exchange rates, it is possible to determine the expected return on any risky asset endogenously, whereas the risk-free rate, the inflation and exchange rate is exogenously determined. The investor’s consideration of one period implies, that end of period consumption is equal to end of period real wealth, since at the end of period the entire real wealth is used for consumption (\textit{Balvers (2001)}). So, the expected utility depends solely on the end of period wealth.

\textsuperscript{150}The concept of expected utility (Neumann-Morgenstern utility) is the best instrument to make rational decisions in the state of insecurity (\textit{Kruschwitz (2007)}). The concept of expected utility was introduced by \textit{Bernoulli (1738)} and justified axiomatically by \textit{Neumann and Morgenstern (1944)}. The concept of expected utility says that the utility of the investment corresponds to the expected value of utility. Under the assumption of normal distribution of wealth the expected utility depends solely on expected return and variance irrespective of the type of utility function (\textit{Balvers (2001) and Kruschwitz (2007)}), for proof s. appendix A.

\textsuperscript{151}This implies that the utility function is continuous, twice differentiable, increasing and concave (\textit{Huang and Litzenberger (1988)}). \textit{Epstein (1985)} shows that mean-variance utility functions are implied by a set of decreasing absolute risk aversion postulates. The assumption of homotheticity implies that a proportional increase in consumption of all goods yields a proportional increase in utility. For given prices the same consumption mix is optimal regardless of income. A homothetic utility function can be represented by monotonic transformation. As far as asset pricing models under the assumption of non-homotheticity are concerned, c. \textit{Breen (1979) and Balvers (2001)}.

\textsuperscript{152}For a discussion of non-price-taking behavior, c. \textit{Basak (1994)}.

\textsuperscript{153}The existence of the uniqueness of equilibrium is regarded in \textit{Hens (2002)}. \textit{Botazzi (1998)} shows that the CAPM can have several equilibria.
Figure 5.1: Structure of the Tax-IntCAPM
5.2 Model Set-up

The set-up of the model allows us to construct the foundations of the Tax-IntCAPM. After the formulation of domestic investor’s real rate of return, the concept of deviation from Relative PPP, the integration in the real exchange rate, the non-linear behavior of exchange rate is derived. The set-up of the model is concluded with the derivation and implementation of foreign investor’s rate of return.

5.2.1 Domestic Investor’s Rate of Return

The stock return consists of capital gains and dividend. The nominal capital gains and dividend rate of return are defined as:

\[
\tilde{r}_{CG,i} = \frac{\overline{CG}_i}{V_{i,0}}, \quad \tilde{r}_{D,i} = \frac{\overline{D}_i}{V_{i,0}}. \tag{5.1}
\]

with
\[
\tilde{r}_{CG,i} \quad \text{nominal capital gains rate of return}
\]
\[
\tilde{r}_{D,i} \quad \text{nominal dividend rate of return}
\]

As far as the domestic investor is concerned, we formulate the stock rate of return after integration of the national tax system comprising the dividend and the capital gains tax system.\(^{154}\)

\[
1 + \tilde{r}^D_{CG,i} + \tilde{r}^D_{D,i} \equiv \frac{V_{i,0} + \overline{CG}_i (1 - t^D_{CG}) + \overline{D}_i (1 - t^D_{D})}{V_{i,0} (1 + \pi^D)} \equiv \tilde{r}_{CG,i} + \tilde{r}_{D,i} \frac{1 - t^D_{CG}}{1 + \pi^D} + \frac{1}{1 + \pi^D} \tag{5.2}
\]

with
\[
\tilde{r}^D_{CG,i} \quad \text{domestic investor’s real capital gains rate of return after tax}
\]
\[
\tilde{r}^D_{D,i} \quad \text{domestic investor’s real dividend rate of return after tax}
\]
\[
t^D_{CG} \quad \text{domestic investor’s tax rate on capital gains}
\]

\(^{154}\)S. chapter 4.2 and chapter 4.3. The rate of returns are derived according to the concept of internal rate of return (Walker and Bos (2009)).
The nominal interest rate of return is defined as:

$$r_{I,n} \equiv \frac{I}{V_{0,0}}.$$  \hfill (5.3)

with $r_{I,n}$ nominal interest rate of return

The domestic investor’s risk-free borrowing and lending is subject to the interest tax system of the domestic state.\(^{155}\)

$$1 + r_{I}^{D} \equiv \frac{V_{0,0} + I (1 - t_{I}^{D})}{V_{0,0} (1 + \pi^{D})} \equiv r_{I,n} \frac{1 - t_{I}^{D}}{1 + \pi^{D}} + \frac{1}{1 + \pi^{D}}$$  \hfill (5.4)

with $r_{I}^{D}$ domestic investor’s real interest rate of return

$t_{I}^{D}$ domestic investor’s tax rate on interest

The inflation rate is determined according to the quantity theory of money.\(^{156}\) We assume a constant velocity in money and constant number in transactions.\(^{157}\) The price level of consumption goods and hence the domestic inflation rate is determined by a proportionate increase of money supply of the domestic state. We can calculate the domestic inflation rate, which adopts numbers from the interval (-1;\(\infty\)). Negative rates imply deflation.

$$\frac{MS^{D}}{MS_{0}^{D}} \equiv \frac{P^{D}}{P_{0}^{D}} = 1 + \pi^{D}$$  \hfill (5.5)

\(^{155}\) C. chapter 4.3.

\(^{156}\) C. chapter 3.5.

\(^{157}\) The velocity in money depends on payment habits such as salaries and taxes. The number in transactions depends on income. These factors change solely in the long run, so it is reasonable to assume constant velocity in money and constant number in transactions, c. Gordon (2009).
5.2.2 Deviation from Purchasing Power Parity and Real Exchange Rate

The real exchange rate is defined in terms of the consumption of goods and is the relative price of the foreign consumption basket to the domestic one. It is the nominal spot exchange rate multiplied by the ratio of domestic to foreign prices (Serçu and Uppal (1995) and Chinn (2008)).

\[ R \equiv S \frac{P_D}{P_F} \]  (5.6)

with \( R \) real exchange rate (at the end of period)

The real exchange rate measures the value of the domestic state’s goods against those ones of the foreign state at the prevailing nominal exchange rate and is a measure to compare living standards across countries. If the real exchange rate is equal to unity the goods in the domestic country have the same value as the goods in the foreign country and PPP is assumed to hold. According to PPP if the real exchange rate is unequal to unity, there will be pressure on the nominal exchange rate to adjust, because the same good can be purchased more cheaply in one country than in the other one.

Example 3 The Big Mac

The nominal Euro-Russian Ruble exchange rate is \( S = 42.1691 \frac{RUB}{EUR} \) as of January 1, 2009.\(^{158}\) In Germany the Big Mac costs 2.99 EUR and in Russia the Big Mac costs 100 RUB.\(^{159}\) The real exchange rate amounts to \( R \equiv S \frac{P_D}{P_F} = 42.1691 \frac{RUB}{EUR} \frac{2.99 EUR}{100 RUB} = 1.2609 \),

\(^{158}\) S. www.oanda.com/convert/fxhistory.
\(^{159}\) McDonalds is a franchise enterprise, so the prices differ from town to town. We chose as representative Big Mac prices the ones in Berlin and Kazan.
so the Big Mac in Germany is 26.09% more expensive than in Russia. If the Big Mac in Germany costs 2.37 EUR, the real exchange rate would be equal to unity and the Big Mac in Germany would cost the same as in Russia.

The appreciation of the real exchange rate is the quotient of the real exchange rate at the end of period and at the beginning of the period (Balvers (2001)).

\[
\frac{R}{R_0} = \frac{S_{PD}}{S_{PD}^0} \frac{P_D}{P_D^0} \quad (5.7)
\]

with \( R_0 \) real exchange rate at the beginning of period

The appreciation of the real exchange rate measures the value of the foreign state’s goods against those of the domestic state at the prevailing nominal exchange rate at two different points of time and expresses its appreciation. We can insert the definition of the inflation rate (eq. (3.2)) in the above equation.

\[
\frac{R}{R_0} = S (1 + \pi_D) S_0 (1 + \pi_F) \quad (5.8)
\]

We insert the rate of nominal exchange appreciation in the above equation. The appreciation of the real exchange rate is the appreciation of the nominal exchange rate and the relation of domestic and foreign inflation.

\[
\frac{R}{R_0} = (1 + e) \frac{1 + \pi_D}{1 + \pi_F} \quad (5.9)
\]

Under the assumption of prevalence of Relative PPP the rate of appreciation of the real exchange rate would equal unity as can be derived from the definition of Relative PPP (eq. (3.4)).

\[
R_{PPP} = (1 + e) \frac{1 + \pi_D}{1 + \pi_F} = 1 \quad (5.10)
\]
The hypothesis of Relative PPP says that domestic and foreign inflation rates are related to exchange rate changes to the extent that differences in the development of price levels are balanced by the appreciation of the exchange rate.

For theoretical and empirical reasons we reject the hypothesis of Relative PPP. \[ RPPP \equiv (1 + e) \frac{1 + \pi^D}{1 + \pi^F} \neq 1 \] (5.11)

Deviation from Relative PPP indicates a change of consumers’ living standards and in the relative competitiveness of countries (Serçu and Uppal (1995)). Figure 5.2 on page 70 illustrates the concept of deviation from Relative PPP.

### 5.2.3 Non-linear Behavior of Exchange Rates

The price process of the exchange rate is described by non-linearity and is not determined by Relative PPP. We follow the model of Ellis (1992), since it retains the salient features of the market. \(161\) The exchange rate is determined by the interaction of speculators and traders and is characterized by a market approach in which the change of demand for foreign currency equals the change of supply.

**Assumption 22** Demand for foreign currency

*The speculator’s demand for foreign currency is determined by the percentage deviation of the expected future exchange rate from the current exchange rate.* \(162\)

\[ D_{S_d} = \alpha \left( \frac{E[S_{d+1}]}{S_d} - 1 \right) \] (5.12)

---

\(160\) C. chapter 3.2.1.


\(162\) Ellis (1992) does not specify how the expected exchange rate is determined. The domestic (foreign) speculators’ demand for foreign (domestic) currency generates chaotic dynamics for some parameter values (Ellis (1992), Karaguler (2000) and Vlad (2010)).
Figure 5.2: Deviation from Relative PPP

with  
\[ D_{S_d} \]  
the demand for foreign currency on the present day 
\[ \alpha \]  
sensitivity factor with \( \alpha \geq 0 \) 
\[ E\left[ \tilde{S}_{d+1} \right] \]  
expected exchange rate of the next day 
\[ S_d \]  
exchange rate of the present day
The demand for foreign (domestic) currency generates non-linearity. If the expected exchange rate is higher than the spot exchange rate and the sensitivity factor $\alpha$ is unequal to zero, this will result in a speculators' demand. If the sensitivity factor $\alpha$ is equal to zero or the expected future exchange rate is equal to the spot exchange rate, there is an absence of speculative demand for the currency. $\alpha$ is a term generating non-linear demand. If $\alpha \neq 1$ the demand is non-linear in expected change of exchange rate. For $\alpha > 1$ the demands are proportionally larger. For $0 < \alpha < 1$ the demands are proportionally smaller if the expected exchange rate is higher than the spot exchange rate.

Assumption 23 Trade balance

It is assumed that the trade balance is determined by spot and previous day exchange rates, the corresponding expected values and sensitivity factors.

According to Ellis (1992) the presence of contract arrangements could justify the use of this specification.

$$T_d = \delta \left( S_d - E \left[ \tilde{S}_d \right] \right) + \gamma \left( S_{d-1} - E \left[ \tilde{S}_d \right] \right)$$

with $T_d$ trade balance of the present day
$\delta$ sensitivity factor with $\delta > 0$
$\gamma$ sensitivity factor with $\gamma > 0$
$S_{d-1}$ previous day's exchange rate

Ellis (1992) leaves the question open if a supply of foreign currency will be generated in the case of a higher value of the spot exchange rate than of the expected future exchange rate. According to Ellis (1992) if $\alpha = \infty$, the infinite demand will be eliminated instantaneously.

In Ellis (1992) the presence of transaction costs is given as an explanation for this type of behavior.

The trade balance must be interpreted as the trade surplus, which is the difference between the monetary value of imports and exports in the domestic or foreign economy over the period of one day. The term balance is misleading since the values are flow figures. The domestic (foreign) nation's trade surplus is equal to the supply or deficit of domestic (foreign) currency which is provided by traders. For a discussion of the assumption, s. chapter 8.
The trade balance is not only influenced by the present exchange rate but also by the previous day’s exchange rate. The previous day’s exchange rate may have a greater effect on the trade balance than the current exchange rate (Ellis (1992)). If the expected exchange rate is equal to the spot or to the previous day’s exchange rate,

\[ S_{d-1} = S_d = E \left[ \tilde{S}_d \right] \]  

the exchange rate is said to be at its steady state value. In this state, speculators do not intend to borrow or lend, and the expected exchange rate is determined by exogenous fundamentals and the trade balance becomes zero.\(^{166}\)

The time frame of the model is short run, weekly at most, because the fundamentals are not expected to change in the planning horizon.

**Assumption 24** Currency market equilibrium  
*The foreign exchange market is cleared at each period, so that the deviation of demand is equal to the trade surplus.*\(^{167}\)

\[ D_{S_d} - D_{S_{d-1}} = T_d \]  

with \( D_{S_{d-1}} \) demand for foreign currency on the previous day.

Figure 5.3 on page 73 illustrates the non-linear behavior of exchange rates. We solve this

\(^{166}\) Ellis (1992) mentions that fundamentals like the interest rate differential or relative money supplies can determine the expected exchange rate. For \( \alpha > 0 \), it follows that \( S_{d-1} = S_d = E \left[ \tilde{S}_d \right] = E \left[ \tilde{S}_{d+1} \right] \) and for \( \alpha = 0 \), \( E \left[ \tilde{S}_{d+1} \right] \) can adopt every value.

\(^{167}\) The assumption that the expected exchange rate is determined by Relative PPP leads to constant deviation of demand is found to be implausible.
equation for the present exchange rates.

\[
\alpha \left( \frac{E[\tilde{S}_{d+1}]}{S_d} - 1 \right) - \alpha \left( \frac{E[\tilde{S}_d]}{S_{d-1}} - 1 \right) = \delta \left( S_d - E[\tilde{S}_d] \right) + \gamma \left( S_{d-1} - E[\tilde{S}_d] \right) \tag{5.16}
\]

We place all terms on the left side and multiply by the current and previous exchange rates to prepare the resolution of the quotients.
\[
\alpha \left( \frac{E\left[\tilde{S}_{d+1}\right]}{S_d} - 1 \right) - \alpha \left( \frac{E\left[\tilde{S}_d\right]}{S_{d-1}} - 1 \right) - \delta \left( S_d - E\left[\tilde{S}_d\right] \right) - \gamma \left( S_{d-1} - E\left[\tilde{S}_d\right] \right) = 0 \quad (5.17)
\]

\[
\alpha \left( \frac{E\left[\tilde{S}_{d+1}\right]}{S_d} - 1 \right) S_d S_{d-1} - \alpha \left( \frac{E\left[\tilde{S}_d\right]}{S_{d-1}} - 1 \right) S_d S_{d-1} - \delta \left( S_d - E\left[\tilde{S}_d\right] \right) S_d S_{d-1} - \gamma \left( S_{d-1} - E\left[\tilde{S}_d\right] \right) S_d S_{d-1} = 0 \quad (5.18)
\]

The multiplication of the terms in parenthesis with the spot and previous day exchange rates leads to the resolution of the quotients.

\[
\alpha \left( E\left[\tilde{S}_{d+1}\right] S_d - S_d S_{d-1} \right) - \alpha \left( E\left[\tilde{S}_d\right] S_d - S_d S_{d-1} \right) - \delta \left( S_d - E\left[\tilde{S}_d\right] \right) S_d S_{d-1} - \gamma \left( S_{d-1} - E\left[\tilde{S}_d\right] \right) S_d S_{d-1} = 0 \quad (5.19)
\]

We expand the terms in parenthesis and rearrange in respect of the future exchange rate.

\[
\alpha E\left[\tilde{S}_{d+1}\right] S_{d-1} - \alpha E\left[\tilde{S}_d\right] S_d - \delta S_d^2 S_{d-1} + \delta E\left[\tilde{S}_d\right] S_d S_{d-1} - \gamma S_{d-1}^2 S_d + \gamma E\left[\tilde{S}_d\right] S_d S_{d-1} = 0 \quad (5.20)
\]

\[
\delta S_d^2 S_{d-1} - \delta E\left[\tilde{S}_d\right] S_d S_{d-1} - \gamma E\left[\tilde{S}_d\right] S_d S_{d-1} - \gamma S_{d-1}^2 S_d + \alpha E\left[\tilde{S}_{d+1}\right] S_{d-1} = 0 \quad (5.21)
\]

\[
\delta S_d^2 S_{d-1} - \left( \delta + \gamma \right) E\left[\tilde{S}_d\right] S_d S_{d-1} - \gamma S_{d-1}^2 - \alpha E\left[\tilde{S}_d\right] S_d \quad (5.22)
\]

There are two roots for \(S_d\); since the exchange rate cannot be negative the positive root is chosen (Ellis (1992)).\(^{168}\)

\(^{168}\)For the correct derivation, cf. Vlad (2010).
\[ S_d = \frac{1}{2\delta S_{d-1}} \left( (\delta + \gamma) E \left[ \tilde{S}_d \right] S_{d-1} - \gamma S_{d-1}^2 - \alpha E \left[ \tilde{S}_d \right] \right) + \sqrt{\frac{\left( (\delta + \gamma) E \left[ \tilde{S}_d \right] S_{d-1} - \gamma S_{d-1}^2 - \alpha E \left[ \tilde{S}_d \right] \right)^2 + 4\alpha \delta E \left[ \tilde{S}_{d+1} \right] S_{d-1}^2}{2\delta S_{d-1}}} \]

(5.23)

The non-linear process is formulated by a recursive function. In the model, if the exchange rate is known, the next day’s exchange rate can be determined. By iterating the present exchange rate we receive the process of the exchange rate and can calculate the appreciation of the exchange rate.\(^{169}\)

**Example 4** Exchange rate process

As in Ellis (1992) we assume that the parameters adopt values of \( \alpha = 5 \), \( \delta = 5 \), and \( \gamma = 25 \) and for reasons of tractability that the expected exchange rates \( E \left[ \tilde{S}_{d+1} \right] \) and \( E \left[ \tilde{S}_d \right] \) are equal to unity.\(^{170}\) The starting point for the exchange rate is 0.8. We insert the values into the formula leading to:

\[ S_d = \frac{30S_{d-1} - 25S_{d-1}^2 - 5}{10S_{d-1}} + \sqrt{\frac{(30S_{d-1} - 25S_{d-1}^2 - 5)^2 + 100S_{d-1}^2}{10S_{d-1}}} \]

(5.24)

The time variation of the exchange rate for 25 days is shown in figure 5.4. It displays non-linear behavior. The maximum value is 2.0220, whereas the minimum value is 0.2078 for the first 25 days. The value table of exchange rate time series is given in the appendix C.1 on page 154. The return of the nominal exchange rate is -0.7370%.\(^{171}\) The non-linear process assumes a totally different process in the case of a change in starting parameters.

---

\(^{169}\) In the case of much larger values of the sensitivity factor \( \gamma \) than of the sensitivity factor \( \delta \) the model can exhibit chaotic behavior (Ellis (1992)).

\(^{170}\) To apply this model the parameters must be estimated.

\(^{171}\) S. value table of exchange rate time series in the appendix C.1 on page 154. The time frame of the model is short run, weekly at most. Since we did not estimate the fundamentals, we refrain from changing them in the planning horizon.
If $\alpha$ adopts the value of 4, the exchange rate adopts a totally different path, as shown in appendix C.1 on page 155.

5.2.4 Foreign Investor’s Rate of Return

The features of international taxation, comprising the dividend, interest, capital and exchange gains tax system and the methods of double taxation reduction, have to be considered in order to formulate a foreign investor’s rate of return.\textsuperscript{172}

By integrating the taxing right factors the different constellations of assignment of the taxing right of all income types are considered. Foreign investor’s taxation is modeled by assigning the right of taxation factors the numbers 0 and 1. Figure 5.1 illustrates the taxation right.

\textsuperscript{172} S. chapter 4.
<table>
<thead>
<tr>
<th>Right of taxation factors</th>
<th>$\epsilon$</th>
<th>$\zeta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic state</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Foreign state</td>
<td>0</td>
<td>1</td>
<td>(0/1)</td>
</tr>
<tr>
<td>Domestic and foreign state</td>
<td>1</td>
<td>1</td>
<td>(0/1)</td>
</tr>
</tbody>
</table>

Table 5.1: Taxing Right Factors

By assigning $\epsilon$ the value one and the other factors the value zero, the rate of return under the assumption of assignment of the taxing right to the domestic state is modeled. By assigning $\zeta$ the value one and $\epsilon$ the value zero, the assignment of the taxing right to the foreign state is modeled. By assignment of unity to the factors $\epsilon$ and $\zeta$ the taxation in the domestic and foreign state is modeled. If $\eta$ is equal to one the methods of avoiding and reducing double taxation are considered. Capital gains after tax is modeled by the following formula:

$$\text{CG}_{i,t}^{F} = \text{CG}_{i} - (\epsilon_{St,CG} t_{St,CG}^{D} + \zeta_{CG} t_{CG}^{F} - \eta_{CG} t_{Int,CG}^{F}) \text{CG}_{i}. \quad (5.25)$$

with

- $\text{CG}_{i,t}^{F}$ foreign investor’s capital gains after international taxation
- $\epsilon_{St,CG}$ domestic state’s taxing right factor on capital gains
- $t_{St,CG}^{D}$ foreign investor’s (source) capital gains tax rate
- $\zeta_{CG}$ foreign state’s assignment taxing right factor on capital gains
- $t_{CG}^{F}$ foreign investor’s capital gains tax rate
- $\eta_{CG}$ foreign state’s taxing right factor on capital gains relief
- $t_{Int,CG}^{F}$ foreign investor’s capital gains relief tax rate

The taxation in the domestic and foreign state is due to a tax rate effect, so we can summarize the above equation as follows:

$$\text{CG}_{i,t}^{F} = \text{CG}_{i} - t_{eff,CG}^{F} \text{CG}_{i}. \quad (5.26)$$

with

- $t_{eff,CG}$ foreign investor’s effective capital gains tax rate
5 Tax International Capital Asset Pricing Model

The taxation of the exchange gains on capital gains and on the principal can be modeled by the following equation:

$$\tilde{E}_{CG,P,t}^F = (S - S_0) \tilde{V}_i - (\zeta_{CG,P,e} t_{CG,P,e} - \eta_{CG,P,e} t_{Int,CG,P,e}) (S - S_0) \tilde{V}_i$$

$$\quad = (S - S_0) \tilde{V}_i - t_{eff,CG,P,e} (S - S_0) \tilde{V}_i. \quad (5.27)$$

with $$\tilde{E}_{CG,P,t}^F$$ foreign investor’s exchange gains on capital gains and principal after national tax

$$\zeta_{CG,P,e}$$ foreign state’s exchange gains taxing right factor on capital gains and principal

$$t_{CG,P,e}$$ exchange gains tax rate on capital gains and principal

$$\eta_{CG,P,e}$$ foreign state’s relief exchange gains taxing right factor on capital gains and principal

$$t_{Int,CG,P,e}$$ foreign investor’s relief exchange gains tax rate on capital gains and principal

$$t_{eff,CG,P,e}$$ foreign investor’s effective exchange gains tax rate on capital gains and principal

The taxation of foreign investor’s dividend can be comprised by the following formula:

$$\tilde{D}_{i,t}^F = \tilde{D}_i - (\epsilon_{St,D} t_{St,D}^D + \zeta_D t_{D}^F - \eta_D t_{Int,D}^F) \tilde{D}_i. \quad (5.28)$$

with $$\tilde{D}_{i,t}^F$$ foreign investor’s dividend after international taxation

$$\epsilon_{St,D}$$ domestic state’s taxing right factor on dividend

$$t_{St,D}^D$$ foreign investor’s (source) dividend tax rate

$$\zeta_D$$ foreign state’s taxing right factor on dividend

$$t_{D}^F$$ foreign investor’s dividend tax rate

$$\eta_D$$ foreign state’s taxing right factor on dividend relief

$$t_{Int,D}^F$$ foreign investor’s relief tax rate on dividend
The taxation in the domestic and foreign state is due to a tax rate effect, so we can summarize the above equation as follows:

\[ \tilde{D}_{i,t} = \tilde{D}_i - t^F_{eff,D} \tilde{D}_i. \] 

(5.29)

with \( t^F_{eff,D} \) foreign investor's dividend effective tax rate

The taxation of the exchange gains on dividend can be modeled by the following equation:

\[ \tilde{E}^F_{D,t} = \left( S - S_0 \right) \tilde{D}_i - \left( \zeta_{D,e} t^F_{D,e} - \eta_{D,e} t^F_{int,D,e} \right) \left( S - S_0 \right) \tilde{D}_i 
= \left( S - S_0 \right) \tilde{D}_i - t^F_{eff,D,e} \left( S - S_0 \right) \tilde{D}_i. \] 

(5.30)

with \( \tilde{E}^F_{D,t} \) foreign investor's exchange gains on dividend after tax

\( \zeta_{D,e} \) foreign state's taxing right factor on exchange gains on dividend

\( t^F_{D,e} \) foreign investor's exchange gains tax rate on dividend

\( \eta_{D,e} \) foreign state's taxing right factor on exchange gains on dividend relief

\( t^F_{int,D,e} \) foreign investor's exchange gains relief tax rate on dividend

\( t^F_{eff,D,e} \) foreign investor's dividend effective tax rate

Foreign investor's stock return is summarized by the following equation:

\[ 1 + \tilde{r}^F_{CG_i} + \tilde{r}^F_{D_i} \equiv \tilde{r}^F_{CG_i,n} \frac{1 - t^F_{eff,CG}}{1 + \pi^F} + e \tilde{r}^F_{CG_i,n} \frac{1 - t^F_{eff,CG,P,e}}{1 + \pi^F} + 1 + e \tilde{r}^F_{D_i,n} \frac{1 - t^F_{eff,D,e}}{1 + \pi^F} - e t^F_{eff,CG,P,e} \frac{1 + \pi^F}{1 + \pi^F} + e \tilde{r}^F_{D_i,n} \frac{1 - t^F_{eff,D,e}}{1 + \pi^F}. \] 

(5.31)

with \( \tilde{r}^F_{CG_i} \) foreign investor's capital gains rate of return after tax

\( \tilde{r}^F_{D_i} \) foreign investor's dividend rate of return after tax

Interest after tax is modeled by the following formula:

\[ I_t = I - \left( \epsilon_{st,t} t^D_{st,t} + \zeta_{I} t^F_{I} - \eta_{I} t^F_{int,I} \right) I. \] 

(5.32)
The taxation in the domestic and foreign state is due to a tax rate effect, so we can summarize the above equation as follows:

\[ I_t = I - t_{eff,I} I. \] (5.33)

with \( t_{eff,I} \) foreign investor’s interest effective tax rate

The taxation of the exchange gains on capital gains and on the principal can be modeled by the following equation:

\[
\begin{align*}
E_{I,P,t} &= (S - S_0) V_0 - (\zeta_{I,P,e} t_{I,P,e}^F - \eta_{I,P,e} t_{Int,I,P,e}^F) (S - S_0) V_0 \\
&= (S - S_0) V_0 - t_{eff,I,P,e} (S - S_0) V_0.
\end{align*}
\] (5.34)

with \( E_{I,P,t} \) foreign investor’s exchange gains after tax on interest and principal
\( \zeta_{I,P,e} \) foreign state’s taxing right factor on exchange gains on capital gains and principal
\( t_{I,P,e}^F \) foreign investor’s exchange gains tax rate on interest and on the principal
\( \eta_{I,P,e} \) foreign state’s taxing right factor on exchange gains on capital gains and principal relief
\( t_{Int,I,P,e}^F \) foreign investor’s relief exchange gains tax rate on interest and on the principal
\( t_{eff,I,P,e} \) foreign investor’s effective exchange gains tax rate on capital gains and on the principal
The integration of the taxing right factors into the risk-free asset is equivalent.

\[ 1 + r_I^F \equiv r_{I,n} \frac{1 - t_{D,I}}{1 + \pi^F} + r_{I,n}^e \frac{1 - t_{D,I,P,e}}{1 + \pi^F} + \frac{1 + \pi^F}{1 + \pi^F} - \epsilon^F \]

(5.35)

with \( r_I^F \) foreign investor’s interest rate of return after tax

The first term of foreign investor’s rate of return incorporates the taxation of capital gains (interest). In the second term the taxation of exchange gains on capital gains (interest) is incarnated. In contrast to the dividend rate of return the third and fourth part of the capital gains and interest rate of return incorporate the fact that the principal is solely subject to exchange gains taxation.

The inflation rate is determined according the quantity theory of money.\(^\text{173}\) The foreign investor’s inflation rate is determined by the money supply of the foreign state. Equivalently to the domestic inflation rate the foreign inflation rate is assumed to adopt numbers from the interval \((-1; \infty)\).

\[ \frac{MS^F}{MS_0^F} \equiv \frac{P^F}{P_0^F} = 1 + \pi^F \]

(5.36)

with \( MS^F \) money supply of the foreign state at the end of period

\( MS_0^F \) money supply of the foreign state at the beginning of the period

Since the tax, inflation and exchange rates are different in the economies, asset rate of returns are heterogeneous for the investors.

### 5.2.5 Implementation of Foreign Investor’s Rate of Return

We implement foreign investor’s rate of return. As the nominal risk-free asset we consider the eonia.\(^\text{174}\) As the risky asset we consider the K+S AG stock (WKN 716200) from

\(^\text{173}\) S. chapter 3.5.

Xetra. On September 14, 2007, 6.75 % of the K+S AG stock were acquired by MCC Holding Ltd. (Linea Ltd) whose holding manages assets of the Russian investor Andrey Melnichenko. The share of K+S AG was increased to 15.001% until November 28, 2008. The data comprises the period from November 1, 2007 to December 31, 2009. In order to determine the risk-free rate we consider the average of the first month eonia. In order to estimate the capital gains, we follow the methodology of Schmid and Trede (2006) and apply the logarithm to use the additivity characteristic of continuous returns. We correct the capital gains rate of return by the influences of dividend and stock-split and annualize the return.

\[
    r_{CGK+S,n} = \frac{12}{M} \sum_{m=1}^{M} \ln \left( 1 + \frac{V_{K+S,m} - V_{K+S,m-1}}{V_{K+S,m-1}} \right)
\]

(5.37)

with \( r_{CGK+S,n} \) nominal capital gains rate of return of the K+S AG stock

\( V_{K+S,m} \) value of the K+S AG stock at the end of month \( m \)

\( V_{K+S,m-1} \) value of the K+S AG stock at the beginning of month \( m \)

Dividends are distributed yearly. Equivalent to the capital gains rate of return of the K+S AG stock we apply the logarithm.

\[
    r_{DK+S,n} = \frac{\sum_{t=1}^{T} \ln \left( 1 + \frac{D_{K+S}}{V_{K+S,t-1}} \right)}{T}
\]

(5.38)

with \( r_{DK+S,n} \) nominal dividend rate of return of the K+S AG stock

\( D_{K+S} \) dividend of the K+S AG stock in the month


176 Restricting the period of investigation to these years results from the consideration that the Tax-Int CAPM shall reflect the implications of the international tax system (Dausend and Schmitt (2007)). In November 2007 the K+S AG stock became part of the MSCI ACWI. s. financial report 2007 of the K+S AG. In order to build on the implementation of of foreign investor’s rate of return the data starts from November 1, 2007.


The appreciation of nominal exchange rate is determined on a monthly basis. Equivalent to the determination of the capital gains rate of return of the K+S AG stock we apply the logarithm and annualize the return.

\[
\epsilon_{\text{RUB-EUR}} = 12 \sum_{m}^{M} \ln \left( 1 + \frac{S_m - S_{m-1}}{S_{m-1}} \right)
\]

with \( \epsilon_{\text{RUB-EUR}} \) appreciation of the Ruble-Euro exchange rate
\( S_m \) exchange rate at the end of month \( m \)
\( S_{m-1} \) exchange rate at the beginning of month \( m \)

According to §2 German Fiscal Code respectively Art.7 Tax Code of the Russian Federation the Double Taxation Convention (DTC) Federal Republic of Germany - Russian Federation takes precedence over German and Russian tax law. According to Art. 1 DTC the convention is valid for the German limited liability corporation and the Russian investor. The German Russian DTC assigns the right of capital gains and interest taxation to the Russian Federation (Art. 13 No. 4 and Art. 11 No.1 DTC). Dividends are subject to a source tax \( t_{St} \) of 15% (Art. 10 I No. 2). According to Art. 208 Item 3 I in connection with Art. 214 Item 1 and Art. 224 Item 4 RITL the tax rate on foreign dividends \( t_D^F \) amounts to 6%. According to Art. 224 Item 1 and 2 in connection with 214.2 RITL the tax rate on interest amounts to 13%, since the nominal interest rate of return falls below the critical margin of 9%. The tax rate on capital gains amounts to 13% according to Art. 224 Item I. According to Art. 214 Item 1 RITL foreign taxes are credited, if this is allowed in the DTC. Taxes imposed in Germany are to be credited on Russian taxes (Art. 23 I DTC). According to Art. 23 I DTC the limited credit method is applied. Incomes and expenses accepted for deduction of the taxpayer expressed (nominated) in foreign currency are converted into rubles at the rate of the Central Bank of

\(^{180}\) We refrain from determining the exchange rate according to the model of Ellis (1992), since its application requires the determination of several sensitivity parameters for the EUR-RUB exchange rate.

\(^{181}\) In contrast to the OECD model convention Art. 10 I DTC does not explicitly incorporate the confirmation of the right of taxation to the state of residence. However, there is no material deviation from the model convention, since Art. 10 I Model Convention does not contain an independent material regulatory content, cf. Tischbirek (2003).
the Russian Federation established on the date of actual receipt of the incomes (Art. 210 Item 5 RITL).\(^{182}\) The tax base income (loss) under a securities purchase/sale deal shall be determined as a difference between the sums received from the sale of the securities and the expenses towards acquiring the securities (Art. 214.1 Item 3 RITL).

**Example 5** Foreign Investor’s Rate of Interest Return after Tax

The eonia and hence the nominal risk-free rate of return \( r_{i,n} \) amounts to 2.52%. The **expected nominal capital gains rate of return of the K+S AG stock** \( r_{CGK+S,n} \) is 47.31\% whereas the **expected nominal dividend rate of return of the K+S AG stock** amounts to \( r_{DK+S,n} \) 1.77\%. The appreciation of the Rouble-Euro exchange rate \( e_{RUB-EUR} \) amounts to -8.99\% whereas the inflation rate for the Russian economy is 8.8\%.\(^{183}\) Since the taxing right on capital gains and interest is assigned to the Russian Federation and no double taxation relief or avoiding methods are implemented Germany’s taxing right factor on capital gains \( \epsilon_{St,CG} \) and on interest \( \epsilon_{St,I} \) as well as Russia’s taxing right factor on capital gains \( \eta_{CG} \) and interest relief \( \eta_{I} \) are zero whereas the Russian state’s taxing right factor on capital gains \( \zeta_{CG} \) and on interest \( \zeta_{I} \) are equal to unity. The effective tax rate on capital gains \( t^{F}_{eff,CG} \) and effective tax rate on interest \( t^{F}_{eff,I} \) amount to 13\%. The taxing right on dividend is assigned to Germany and Russia, hence Germany’s taxing right factor on dividend \( \epsilon_{St,D} \), Russia’s taxing right factor on dividend \( \zeta_{D} \) as well as its taxing right factor on dividend relief \( \eta_{CG} \) are equal to unity. The Russian state applies the limited credit method, hence the effective tax rate on dividend \( t^{F}_{eff,D} \) is 15\%. Incomes and expenses in foreign currency are converted into rubles at the exchange rate. The effective exchange gains tax rate on capital gains and on the principal \( t^{F}_{eff,CG,P,e} \) and the effective exchange gains tax rate on interest and on the principal \( t^{F}_{eff,I,P,e} \) amount to 13\%, whereas the effective exchange gains tax rate on dividend \( t^{F}_{eff,D,e} \) is 15\%.

\(^{182}\) The official exchange rates of foreign currencies against the rouble are set by the Central Bank of the Russian Federation without assuming any liability to buy or sell foreign currency at the above rate. [www.cbr.ru/eng/currency_base/monthly.aspx](http://www.cbr.ru/eng/currency_base/monthly.aspx). Hence we can assume that the exchange rate for tax purposes is the market rate.

\(^{183}\) S. example 1.
We can calculate foreign investor’s rate of risky and risk-free return.

\[ r_{CGK+s}^F + r_{DK+s}^F = 0.4731 \frac{1 - 0.13}{1 + 0.088} + 0.4731(-0.0899) \frac{1 - 0.13}{1 + 0.088} + \frac{1 + (-0.0899)}{1 + 0.088} - (-0.0899) \frac{0.13}{1 + 0.088} + 0.0177 \frac{1 - 0.15}{1 + 0.088} + 0.0177(-0.0899) \frac{1 - 0.15}{1 + 0.088} - 1 = 0.2041 \] (5.40)

\[ r_I^F = 0.0252 \frac{1 - 0.13}{1 + 0.088} + 0.0252(-0.0899) \frac{1 - 0.13}{1 + 0.088} + \frac{1 + (-0.0899)}{1 + 0.088} - (-0.0899) \frac{0.13}{1 + 0.088} - 1 = -0.1344 \] (5.41)

with

- \( r_{CGK+s}^F \) real capital gains rate of return of the K+S AG stock after tax
- \( r_{DK+s}^F \) real dividend rate of return of the K+S AG stock after tax

Foreign investor’s real K+S stock rate of return amounts to 20.41% whereas the real risk-free rate of return after tax is -13.44%.

5.3 Versions of Tax International Capital Asset Pricing Model

The Tax-IntCAPM incorporates special constellations leading to different versions of the model.\(^{184}\) We equalize domestic and foreign investor’s real rate of return after tax in order to find the implications for the taxation of the different income types - capital and exchange gains, dividend and interest - in IntCAPT which can lead to different versions of the Tax-IntCAPM.

\[ 1 + \tilde{r}_{DG}^D + \tilde{r}_{D_i}^D = 1 + \tilde{r}_{CG}^F + \tilde{r}_{D_i}^F \] (5.42)

\(^{184}\)Hereby we ignore constellations implying the irrelevance of tax, exchange and inflation rates which lead to the CAPM, IntCAPM and (international) Tax-CAPM. The international version of the Tax-CAPM is characterized by domestic and foreign investors’ residence in the same monetary zone and subjection to different income classes.
We apply this concept in order to compare domestic and foreign investor’s rate of return and to find out the implications for the tax system.\textsuperscript{185}

\[
\tilde{r}_{CG,i,n} \left( 1 - t_D^{CG} \right) + 1 + \tilde{r}_{D,i,n} \left( 1 - t_D^{D} \right) = \tilde{r}_{CG,i,n} \frac{1 - t_F^{CG}}{1 + \pi^D} + \tilde{r}_{D,i,n} \frac{1 - t_D^{D}}{1 + \pi^D} \\
+ \frac{1 + e}{1 + \pi^F} + \frac{\tilde{r}_{CG,i,n} \left( 1 - t_F^{CG} \right) \left( 1 + \pi^D \right) \left( 1 - t_F^{CG} \right)}{1 + \pi^F} \\
+ \frac{\tilde{r}_{D,i,n} \left( 1 - t_F^{CG} \right) \left( 1 + \pi^D \right) \left( 1 - t_F^{CG} \right)}{1 + \pi^F} \quad (5.43)
\]

We regard the central theorem in IntCAPT - deviation from Relative PPP - as irrelevant and assume that the hypothesis of Relative PPP holds. This approach enables us to concentrate solely on tax effects.

In a first step, we multiply domestic and foreign investor’s capital gains rate of return with the domestic investor’s inflation rate $1 + \pi^D$ in order to incorporate the appreciation of the real exchange rate and the concept of Relative PPP.

\[
\tilde{r}_{CG,i,n} \left( 1 - t_F^{CG} \right) \left( 1 + \pi^D \right) = \tilde{r}_{CG,i,n} \frac{1 - t_F^{CG}}{1 + \pi^F} \left( 1 + \pi^D \right) \\
+ \left( 1 + e \right) \frac{1 + \pi^D}{1 + \pi^F} - e \tilde{r}_{CG,i,n} \frac{1 - t_F^{CG} \left( 1 + \pi^D \right)}{1 + \pi^F} \\
+ \tilde{r}_{D,i,n} \left( 1 - t_F^{CG} \right) \left( 1 + \pi^D \right) \left( 1 - t_F^{CG} \right) + \tilde{r}_{D,i,n} \left( 1 - t_F^{CG} \right) \left( 1 + \pi^D \right) \left( 1 - t_F^{CG} \right) \quad (5.44)
\]

Since we assume the existence of Relative PPP, which implies that the appreciation of

\textsuperscript{185} Since the tax structure of dividends and interest rate of return is comprised by the tax structure of the stock rate of return, it is sufficient to regard the stock rate of return.

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The equivalence of domestic and foreign investors’ rate of return depends decisively on the features of international taxation. Under the assumption of Relative PPP and a certain constellation of the international tax system the decision problem would be equivalent to the one of the Tax-CAPM with investor-specific tax rates. If capital gains and the principal are not taxable in either state and the tax rate on dividends is equal to the exchange gains tax rate on dividends in the foreign state, then the decision problem would be equivalent to the one of the Tax-CAPM with investor-specific tax rates.

\[
\tilde{r}_{CG_{i,n}} (1 - t_{CG}^D) + \tilde{r}_{D_{i,n}} (1 - t_{D}^D) = \tilde{r}_{CG_{i,n}} \frac{(1 - t_{eff,CG}^F) (1 + \pi^D)}{1 + \pi^F} + e\tilde{r}_{CG_{i,n}} \frac{(1 - t_{eff,CG,P,e}^F) (1 + \pi^D)}{1 + \pi^F} - \epsilon t_{eff,CG,P,e} (1 + \pi^D) + \tilde{r}_{D_{i,n}} \frac{(1 - t_{eff,D}^F) (1 + \pi^D)}{1 + \pi^F} + e\tilde{r}_{D_{i,n}} \frac{(1 - t_{eff,D,e}^F) (1 + \pi^D)}{1 + \pi^F}
\]  

(5.45)

The solution of the Tax-IntCAPM will allow us to determine the relationship for international assets between expected return and risk. The CAPM can alternatively be derived by aggregating investors’ individual optimum or by theoretical concepts of portfolio theory. In general, the Tax-IntCAPM cannot be derived based on theoretical concepts of portfolio theory, since the tangential portfolio is investor-specific on account of the assumption

\[
\tilde{r}_{CG_{i,n}} + \tilde{r}_{D_{i,n}} (1 - t_{D}^D) = \tilde{r}_{CG_{i,n}} + \tilde{r}_{D_{i,n}} (1 - t_{eff,D}^F)
\]  

(5.47)

Under the assumption of Relative PPP certain tax constellations lead to the equivalence of Tax-IntCAPM and the Tax-CAPM with investor-specific tax rates.

### 5.4 Model Solution

The solution of the Tax-IntCAPM will allow us to determine the relationship for international assets between expected return and risk. The CAPM can alternatively be derived by aggregating investors’ individual optimum or by theoretical concepts of portfolio theory. In general, the Tax-IntCAPM cannot be derived based on theoretical concepts of portfolio theory, since the tangential portfolio is investor-specific on account of the assumption
Derivation of Tax-IntCAPM

Market Equilibrium

Individual Optimum

Maximization of expected utility

Constraints
- Value shares sum to unity
- Terminal wealth is set up of the portfolio return

Differentiation in respect of the risky asset value

Integration of the constraints into the maximization problem

Transformation

- Application of the definition of the covariance
- Application of Stein's Lemma

Individual optimum

Definition of the world market return

The world market portfolio return is equal to the value weighted return of risky assets

Market equilibrium

- Aggregation

Supply is equal to aggregated demand

Integration of international factors

Application of the world market portfolio

Tax-IntCAPM

Figure 5.5: Model Solution of Tax-IntCAPM
of investor-specific tax, inflation and appreciation of exchange rate.\textsuperscript{186} The process of the derivation of the Tax-IntCAPM is illustrated in figure 5.5 on page 88. The decision problem of maximization expected utility of end of period real wealth after international tax with the constraints that the value shares sum to unity and end of period wealth is set up of the portfolio return is differentiated in respect of the value weighted shares of the risky assets. After transformation of the derivation by application of the definition of the covariance and Stein’s Lemma we derive the individual optimum of the domestic and foreign investor. We define the world market return and derive the market equilibrium by aggregation and equalization of supply and demand. By integration of international factors and application of the world market portfolio we derive the equilibrium pricing relationship of the Tax-IntCAPM.

The solution of the model will allow us to determine the relationship for international assets between expected return and risk under the framework of international tax system. We hypothesize that the integration of exchange gains taxation, deterministic inflation and non-linear exchange rates in IntCAPT leads to the integration of new factors whereas the integration of the dividend, capital gains and interest tax system and the methods of avoiding and reducing double taxation have solely a tax rate effect.

The derivation of investor’s individual optimum forms the basis for the Tax-IntCAPM. The domestic and foreign consumer maximize their expected utility of end of period real wealth after national and international tax and hence, they achieve individual optimum.

### 5.4.1 Individual Optimum

The representative consumers intend to maximize their expected utility of end of period real wealth after tax and hence, they achieve individual optimum. The formulas are constructed in such a way that they are valid for the domestic as well as for the foreign investor.

\[
\text{Max } \left\{ \omega \right\}_{i=0}^{T} \mathbb{E} \left[ U \left( \tilde{W}_t \right) \right]
\]

\textsuperscript{186} Cf. (Mai (2008)).
with \( \omega_i \) investor’s value portion of assets in the portfolio

Max Maximum

\( U \) investor’s utility function

\( \tilde{W}_t \) investor’s end of period real wealth after tax

The decision problem is subject to constraints. At the beginning of the period each investor makes the decision about the composition of the portfolio under the budget constraint that the real value of the portfolio equals initial real wealth which is assumed to be unity. PPP is assumed to hold at the beginning of period, so the real price of asset \( i \) is the same as for the domestic investor and the foreign investor’s portfolio at the beginning of the period differs from the one of the domestic investor solely in respect of the demanded number of each asset \( i \).

\[
\sum_{i=0}^{I} n_i^D \frac{V_{i,0}}{P_{0}^D} = 1 \quad \sum_{i=0}^{I} n_i^F \frac{V_{i,0}}{S_{0}P_{0}^F} = 1 \quad (5.49)
\]

with \( n_i^D \) demanded number of asset \( i \) by the domestic investor

\( V_{i,0} \) price of asset \( i \) at the beginning of period

\( P_{0}^D \) price of domestic consumption bundle at the beginning of period

\( n_i^F \) demanded number of asset \( i \) by the foreign investor

The value weighted shares of the portfolio are made of the product of demanded number and real price of the risky respectively the risk-free asset.

\[
\omega_i = \frac{n_i^D \frac{V_{i,0}}{P_{0}^D}}{\sum_{i=0}^{I} n_i^D \frac{V_{i,0}}{P_{0}^D}} = n_i^D \frac{V_{i,0}}{P_{0}^D} \quad \omega_0 = \frac{n_0^F \frac{V_{0,0}}{S_{0}P_{0}^F}}{\sum_{i=0}^{I} n_i^F \frac{V_{i,0}}{S_{0}P_{0}^F}} = n_0^F \frac{V_{0,0}}{S_{0}P_{0}^F} \quad (5.50)
\]

with \( \omega_0 \) investor’s value weighted share of the risk-free asset in the portfolio

\( n_0 \) investor’s number of the risk-free asset in the portfolio

\( V_{0,0} \) price of the risk-free asset at the beginning of period
Initial real wealth equals one, so the value shares sum to unity. Since the entire wealth is invested in risky and risk-free assets, the price of the portfolio equals initial wealth. Short-selling is permitted, so the shares of the portfolio can also be negative.

\[ \text{s.t. } \sum_{i=0}^{I} \omega_i = 1 \quad (5.51) \]

Terminal real wealth after taxes is set up of the value weighted rate of return after tax of the portfolio, consisting of risky and risk-free assets.\(^{187}\)

\[ \text{s.t. } \tilde{W}_t = \sum_{i=1}^{I} \omega_i (1 + \tilde{r}_{CG_i} + \tilde{r}_{D_i}) + \omega_0 (1 + r_I) \quad (5.52) \]

One part of the initial real wealth is invested in risky assets, the other part is invested into risk-free assets (Huang and Litzenberger (1988), Balvers (2001) and Pennacchi (2007)). The constraints, that the value shares sum to unity (eq. (5.51)) and that terminal real wealth after taxes is set up of the return after tax of the portfolio, consisting of risky and risk-free assets (eq. (5.52)) are integrated into the maximization problem.

\[ \text{Max} \ \{ \omega_i \} \quad \text{s.t. } \quad E \left[ U \left( (1 - \sum_{i=1}^{I} \omega_i) (1 + r_I) + \sum_{i=1}^{I} \omega_i (1 + \tilde{r}_{CG_i} + \tilde{r}_{D_i}) \right) \right] \quad (5.53) \]

The expansion and rearrangement of the above equation to express the excess rate of return lead to the following result:

\[ \text{Max} \ \{ \omega_i \} \quad \text{s.t. } \quad E \left[ U \left( (1 + r_I) + \sum_{i=1}^{I} \omega_i (\tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I) \right) \right]. \quad (5.54) \]

Investor’s aim consists in determining the optimal share of each asset for the portfolio in order to maximize the expected value of terminal real wealth after tax. We differentiate partially in respect of the risky assets’ shares by application of the chain rule and set the

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\(^{187}\)According to the theorem of linear transformation of normally distributed variables, the end-of-period wealth is normally distributed, since the returns are assumed to be normally distributed (Balvers (2001)). Consider in constraint (eq. (5.52)) that initial wealth equals unity and can be omitted.
derivative equal to zero.\(^{188}\)

\[
\frac{\partial E}{\partial \omega_i} \left[ U \left( (1 + r_I) + \sum_{i=1}^{I} \omega_i \left( \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right) \right) \right] = 0
\]

\[
E \left[ \frac{\partial U}{\partial \omega_i} \left( (1 + r_I) + \sum_{i=1}^{I} \omega_i \left( \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right) \right) \right. \\
\left. \left( \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right) \right] = 0
\]

(5.55)
for \( i \in 1..I. \)

According to constraint (5.52) we can replace the variable of the utility function by terminal real wealth after tax.

\[
E \left[ \frac{\partial U}{\partial \omega_i} \left( \tilde{W}_t \right) \right. \\
\left. \left( \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right) \right] = 0
\]

(5.56)

The expected marginal utility of real wealth after tax has to be zero, what implies that the change of the value utility function does not lead to further expected utility and that the investor achieved individual optimum. Since the utility function is concave, the first order necessary condition leads to the maximization of investor’s expected utility. The concavity of the utility function implies that further increase of wealth leads to less enhancement of investor’s utility.\(^{189}\) The investors undertake risky investments only if the rate of return on at least one risky asset exceeds the risk-free rate.\(^{190}\)

By using the definition of covariance and the result of the first order condition that the expected marginal utility of real wealth after taxes has to be zero (eq. (5.56)) we obtain

\(^{188}\) We do not differentiate in respect of the risk-free asset share. The optimal share of the risk-free asset can be determined by deduction of the sum of shares of the risky assets from unity.

\(^{189}\) In mathematical terms this is expressed by the negativity of the second derivation which is necessary for a maximum, cf. Huang and Litzenberger (1988) and Pennacchi (2007).

\(^{190}\) For proof and for derivation of the minimum risk premium for entire investment in the risky asset, cf. Huang and Litzenberger (1988).
\[
\text{Cov} \left[ \frac{\partial U \left( \tilde{W}_t \right)}{\partial \omega_i}, \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right] = E \left[ \frac{\partial U \left( \tilde{W}_t \right)}{\partial \omega_i} \left( \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right) \right] \\
- E \left[ \frac{\partial U \left( \tilde{W}_t \right)}{\partial \omega_i} \right] E \left[ \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right] 
\]

\[
- \text{Cov} \left[ \frac{\partial U \left( \tilde{W}_t \right)}{\partial \omega_i}, \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right] = E \left[ \frac{\partial^2 U \left( \tilde{W}_t \right)}{\partial \omega_i \partial W_t} \right] \text{Cov} \left[ \tilde{W}_t, \tilde{r}_{CG_i} + \tilde{r}_{D_i} \right] 
\]

By virtue of Stein’s Lemma we can reformulate the negative of the covariance. Stein’s Lemma is a linearization result for covariances under the assumption of normal distribution when one argument is a (possible) non-linear function of a normally distributed variable. Stein’s Lemma equates the covariance of a function of normal random variable to the underlying covariance times the expectation number of the derivative function. The gains from using Stein’s Lemma lies in the separation of the utility risk components from the covariances and hence, to derive tractable, interpretable results (Gron (2004), Söderlind (2006a) and Söderlind (2006b)).

\[
- \text{Cov} \left[ \frac{\partial U \left( \tilde{W}_t \right)}{\partial \omega_i}, \tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I \right] = E \left[ \frac{\partial^2 U \left( \tilde{W}_t \right)}{\partial \omega_i \partial W_t} \right] \text{Cov} \left[ \tilde{W}_t, \tilde{r}_{CG_i} + \tilde{r}_{D_i} \right] 
\]

We replace the left hand side of equation (5.58) by the right hand side of equation (5.59) where we applied Stein’s Lemma.

\[
E[\tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I] E \left[ \frac{\partial U \left( \tilde{W}_t \right)}{\partial \omega_i} \right] = - E \left[ \frac{\partial^2 U \left( \tilde{W}_t \right)}{\partial \omega_i \partial W_t} \right] \text{Cov} \left[ \tilde{W}_t, \tilde{r}_{CG_i} + \tilde{r}_{D_i} \right] 
\]

191 For proof of Stein’s Lemma, s. appendix B.
and define the risk factor.$^{192}$

$$A = \frac{E \left[ \frac{\partial^2 U(\tilde{W}_t)}{\partial \omega_i \partial W_t} \right]}{E \left[ \frac{\partial U(\tilde{W}_t)}{\partial \omega_i} \right]}$$

(5.60)

with $A$ expected coefficient of world risk aversion

We can derive the pricing relationship for the investors in individual optimum.

$$E[\tilde{r}_{CG_i} + \tilde{r}_{D_i}] = r_I + ACov[\tilde{r}_{CG_i} + \tilde{r}_{D_i}, \tilde{W}_t]$$

(5.61)

The above equation describes uniquely investor’s expected rate of return for each asset. It says that the investor’s expected rate of return of an asset depends on two components in individual optimum: The risk-free rate of return and the risk premium.

The individual optimum of the Tax-IntCAPM differs from the IntCAPM only in respect of use of real returns after tax.

The real risk-free rate of return after tax represents investor’s price of time or the rate of return that is required for delaying consumption ($Lengwiler$ (2006) and $Elton$ (2007)). The risk premium expresses investor’s marginal rate of substitution between expected excess return and return variance. The investor expects a risk premium since he is risk averse. The risk premium depends on the covariance of wealth and the real capital gains and dividend rate of return after tax of the asset. Asset $i$’s contribution to the risk of the portfolio is expressed in the form of the covariance. The variance and hence the risk of

$^{192}$According to $Balvers$ (2001) the term $A$ is similar to the coefficient of world absolute risk aversion which expresses the subjective degree of risk aversion. The term is not equal due to the expectations that are taken. The coefficient expresses investor’s risk tolerance and serves as a measure to plausible utility functions. For a description of the concept of absolute risk aversion, see $Pratt$ (1964), $Zeckhauser$ and $Keeler$ (1970), $Arrow$ (1970), $Huang$ and $Litzenberger$ (1988) and $Kruschwitz$ (2007).
asset $i$ is irrelevant, asset $i$’s effect on the variance of the portfolio is decisive and influences if the variance of the portfolio decreases or increases (Elton (2007)). If the real capital gains and dividend rate of return after tax of the asset is positively correlated with end of period wealth, the expected rate of return has to be higher than the risk-free rate of return to compensate for the risk. Adversely, the expected rate of return of the asset has to be lower than the risk-free rate, if the rate of return is negatively correlated with end of period real wealth after tax. This can be interpreted, that the investor accepts a lower rate of return for assets whose rate of return is negatively correlated with end of period wealth, since these assets have high pay-offs when the economy is in a relatively poor state (Elton (2007)).

For the domestic investor we can express the following individual optimum relationship.

$$E \left[ \tilde{r}_D^{CG} + \tilde{r}_D^{D_i} \right] - r_D^0 = A^D \text{Cov} \left[ \tilde{W}_t^D, \tilde{r}_D^{CG} + \tilde{r}_D^{D_i} \right] \quad (5.62)$$

We consider solely the covariance term of the domestic investor. We decompose the covariance term by application of the linearity property.

$$\text{Cov} \left[ \tilde{W}_t^D, \tilde{r}_D^{CG} \right] + \text{Cov} \left[ \tilde{W}_t^D, \tilde{r}_D^{D_i} \right] \quad (5.63)$$

We consider the first covariance term. By replacing investor’s terminal real wealth after taxes by the definition of the portfolio return (eq. (5.52)), the following equation can be derived.

$$\text{Cov} \left[ \sum_{i=1}^{I} \omega_i^D \left( 1 + \tilde{r}_D^{CG} + \tilde{r}_D^{D_i} \right) + \omega_0^D \left( 1 + r_D^0 \right), \tilde{r}_D^{CG} \right] \quad (5.64)$$

Since the covariance with the risk-free asset is zero, we can derive the following term.

$$\text{Cov} \left[ \sum_{i=1}^{I} \omega_i^D \left( 1 + \tilde{r}_D^{CG} + \tilde{r}_D^{D_i} \right), \tilde{r}_D^{CG} \right] \quad (5.65)$$

We decompose the covariance of the domestic investor by application of the linearity prop-
\[ \text{Cov} \left[ \sum_{i=1}^{I} \omega_i^{P} \tilde{r}_{CG,i}^{P}, \tilde{r}_{CG,i}^{D} \right] + \text{Cov} \left[ \sum_{i=1}^{I} \omega_i^{D} \tilde{r}_{D,i}^{D}, \tilde{r}_{CG,i}^{D} \right]. \] (5.66)

The first covariance describes the value weighted real capital gains rate of return with the real capital gains rate of return after tax, the other one the value weighted real dividend rate of return with the real capital gains rate of return after tax. By considering domestic investor’s nominal capital gains and dividend rate of return, we can separate the tax system and inflation rate from the risky real rate of return from the covariance.

\[ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n}^{D} \omega_i^{P} \frac{1 - t_{CG}^{D}}{1 + \pi^{D}}, \tilde{r}_{CG,i,n}^{D} \frac{1 - t_{CG}^{D}}{1 + \pi^{D}} \right] + \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D,i,n}^{D} \omega_i^{D} \frac{1 - t_{D}^{D}}{1 + \pi^{D}}, \tilde{r}_{CG,i,n}^{D} \frac{1 - t_{CG}^{D}}{1 + \pi^{D}} \right]. \] (5.67)

Equivalently, we derive the covariance term with the dividend rate of return (See appendix C.3.1).

\[ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n}^{D} \omega_i^{P} \frac{1 - t_{CG}^{D}}{1 + \pi^{D}}, \tilde{r}_{D,i,n}^{D} \frac{1 - t_{D}^{D}}{1 + \pi^{D}} \right] + \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D,i,n}^{D} \omega_i^{D} \frac{1 - t_{D}^{D}}{1 + \pi^{D}}, \tilde{r}_{D,i,n}^{D} \frac{1 - t_{D}^{D}}{1 + \pi^{D}} \right]. \] (5.68)

We aggregate the tax system and separate inflation rates in order to derive an expression depending solely on the nominal capital gains and dividend return. The left hand side of
equation 5.62 and the above terms lead to domestic investor’s individual optimum.

\[
\frac{(1 + \pi_D)^2}{A^D} (E[\tilde{r}_{CG_i} + \tilde{r}_{D_i}] - r_I^D)
= \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG_i,n} \omega_i^D (1 - t_{CG}^D)^2, \tilde{r}_{CG_i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D_i,n} \omega_i^D (1 - t_{D}^D) (1 - t_{CG}^D), \tilde{r}_{CG_i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG_i,n} \omega_i^D (1 - t_{CG}^D) (1 - t_{D}^D), \tilde{r}_{D_i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D_i,n} \omega_i^D (1 - t_{D}^D)^2, \tilde{r}_{D_i,n} \right]
\]

After deriving the pricing relationship term of the domestic investor, we derive analogously the equilibrium pricing relationship for the foreign investor (See appendix C.3.2).

\[
\frac{(1 + \pi_F)^2}{A^F} (E[\tilde{r}_{CG_i} + \tilde{r}_{D_i}] - r_I^F)
= \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG_i,n} \omega_i^F \left(1 - t_{eff,CG} + e \left(1 - t_{eff,CG,P,e}^F\right)\right)^2, \tilde{r}_{CG_i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D_i,n} \omega_i^F \left(1 - t_{eff,D} + e \left(1 - t_{eff,D,e}^F\right)\right), \tilde{r}_{CG_i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG_i,n} \omega_i^F \left(1 - t_{eff,CG} + e \left(1 - t_{eff,CG,P,e}^F\right)\right) \\
\left(1 - t_{eff,D} + e \left(1 - t_{eff,D,e}^F\right)\right), \tilde{r}_{D_i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D_i,n} \omega_i^F \left(1 - t_{eff,D} + e \left(1 - t_{eff,D,e}^F\right)\right)^2, \tilde{r}_{D_i,n} \right]
\]
5.4.2 Market Equilibrium

The capital market is in a state of equilibrium which implies that market participants maximize expected utility - to achieve individual optimum - and that the endogenous price process for all risky assets is assumed to develop such a way that an exogenous quantity of each asset equals the aggregate demand for the asset (Nielsen (1990)). By aggregation we develop an equilibrium relationship which is effective for the domestic as well as for the foreign investor. According to the first welfare theorem the equilibrium allocation leads principally to Pareto efficiency and solves a specific Social Welfare Function, where the individual’s weights are determined by their shadow prices of wealth (competitive Social Welfare Function) (Lengwiler (2006)). The concept of Walras’ law is behind the idea of equilibrium, Walras’ law is associated with the concept of Walrasian auction. The Walrasian auction is a form of auction where each participant calculates his demand for the good at every possible price and submits it to an auctioneer. Then a price is determined through tatonnement process equalizing the entire demand and supply across all agents of the good. According to the concept of Walras it is sufficient to regard the equilibrium of risky assets. An equilibrium of risky assets will lead to an equilibrium of the riskless asset (Kruschwitz and Löffler (2009)).

According to equation (5.49) the demanded numbers for asset $i$ can be expressed by the following equations.

\[ n_{iD} = \omega_{iD} \frac{P_D^0}{V_{i,0}} \]
\[ n_{iF} = \omega_{iF} \frac{P_D^0}{V_{i,0}} \]  

(5.71)

The aggregate demanded number of each risky asset has to correspond to the exogenously given number (Brennan (1970), Kruschwitz (2007) and Albrecht and Maurer (2008)).

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193 According to the first welfare theorem competitive equilibrium (Walrasian equilibrium) leads to Pareto efficiency what implies that no other allocation of (risky) assets would be preferred by the domestic and foreign investor. The shadow price is the change in expected utility of investor’s individual optimum by relaxing the constraints by one unit, cf. Stiglitz (1991), Sen (1993), Barr (2004) and Kanbur (2008).
\[ n^D_i + n^F_i = \left( \omega^D_i + \omega^F_i \right) \frac{P^D_0}{V_{i,0}} = n^0_i \]  \hspace{1cm} (5.72)

\[ (n^D_i + n^F_i) \frac{V_{i,0}}{P^D_0} = \omega^D_i + \omega^F_i = n^0_i \frac{V_{i,0}}{P^D_0} \]  \hspace{1cm} (5.73)

with \( n^0_i \) exogenous number of asset \( i \)

As long as investors can raise their expected utility by acquiring or selling assets, the market is characterized by excess demand or supply. The price changes in such a way that demand of both investors is equal to the supply of each asset and none of the investors is able to enhance expected utility. We reformulate equation (5.61) to express the excess rate of return and sum the individual optimum of domestic and foreign investors.\(^{194}\)

\[ E \left[ \tilde{r}^D_{CG, i} + \tilde{r}^D_{Di} \right] - \tilde{r}^D_i + E \left[ \tilde{r}^F_{CG, i} + \tilde{r}^F_{Di} \right] - \tilde{r}^F_i = A^D \text{Cov} \left[ \tilde{r}^D_{CG, i} + \tilde{W}^D_i, \tilde{W}^D_t \right] + A^F \text{Cov} \left[ \tilde{r}^F_{CG, i} + \tilde{W}^F_i, \tilde{W}^F_t \right] \]  \hspace{1cm} (5.74)

Equations (5.72) and (5.74) express the market equilibrium in a general manner. The left hand side of equation (5.74) contains the real excess rate of return of the domestic and foreign investor. The right hand side is determined by the covariances of the investors which are multiplied by the risk aversion parameter \( A \). Based on the aggregation of individual equilibrium we derive a pricing relationship which contains complex factors. The market equilibrium depends on the expectations and risk preferences of investors.

We derive separately the international excess return and the world equity risk premium. We define the domestic and foreign risk aversion factor under inflation.

\[ A^D_{\pi_D} = \frac{1 + \pi^D}{A^D} \quad A^F_{\pi_F} = \frac{1 + \pi^F}{A^F} \]  \hspace{1cm} (5.75)

with \( A^D_{\pi_D} \) domestic investor’s risk aversion factor under inflation

\( A^F_{\pi_F} \) foreign investor’s risk aversion factor under inflation

and the world aggregate risk aversion factor under inflation $A^W_\pi$.

$$A^W_\pi = A^D_\pi + A^F_\pi$$ (5.76)

with $A^W_\pi$ world aggregate risk aversion factor under inflation

After aggregation of the left hand side of the individual optimum of the domestic and foreign investor, we separate the national and international tax, inflation and exchange rates of the domestic and foreign investor. By defining the weighted tax factors, the weighted foreign risk aversion factor and the foreign exchange gains tax factors we derive the excess return of the Tax-IntCAPM. We define the weighted tax factor on capital gains, dividend and interest.

$$T_{CG} = \frac{t^D_{CG} A^D_\pi + t^F_{eff,CG} A^F_\pi}{A^W_\pi} \quad T_D = \frac{t^D_{D} A^D_\pi + t^F_{eff,D} A^F_\pi}{A^W_\pi} \quad T_I = \frac{t^D_{I} A^D_\pi + t^F_{eff,I} A^F_\pi}{A^W_\pi}$$ (5.77)

Under internationalization, the tax parameters reflect the tax rates of all investors worldwide. The value of the factors depends decisively on the features of international taxation and the taxing right factors.\textsuperscript{195} If the domestic investor’s tax rate is equal to foreign investor’s effective tax rate then the tax factors become tax rates.\textsuperscript{196} We define the weighted foreign risk aversion factor as

$$A^F_W = \frac{A^F_\pi}{A^W_\pi}. \quad (5.78)$$

We define the foreign tax factor on capital gains, dividend and interest exchange gains.

$$ET^F_{CG} = \frac{t^F_{eff,CG,P,e} A^F_\pi}{A^W_\pi} \quad ET^F_D = \frac{t^F_{eff,D,P,e} A^F_\pi}{A^W_\pi} \quad ET^F_I = \frac{t^F_{eff,I,P,e} A^F_\pi}{A^W_\pi}$$ (5.79)

\textsuperscript{195} S. figure 5.1.
\textsuperscript{196} Cf. Mai (2008).
After some mathematical transformation the following formulation for the excess return can be derived (See appendix C.2.1).

\[
E[\tilde{r}_{CG,i,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e (A^F_W - ET^F_{CG}) - eET^F_{CG} \\
+ E[\tilde{r}_{D,i,n}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e (A^F_W - ET^F_D) \\
-r_{I,n} (1 - T_I) - r_{I,n} e (A^F_W - ET^F_I) + eET^F_I
\] (5.80)

The excess return consists of an expression of capital gains, exchange gains on capital gains and principal, dividend, exchange gains on dividend, interest and exchange gains on interest and principal return.

The capital market is in a state of equilibrium. Both market participants maximize expected utility to achieve individual optimum and an exogenous quantity of each asset equals the aggregate demand for the asset. In order to derive the equilibrium pricing equation further capital market relationships are to be explained. We define the world market equity portfolio in the absence of tax, inflation and appreciation of exchange rate (Ibbotson and Sinquefield (1976), Lally (1996)). The nominal value of the world market portfolio at the beginning of the period is given by

\[
M_0 \equiv \sum_{i=1}^{I} n^0_i V_{i,0}.
\] (5.81)

with \( M_0 \) nominal value of the world market portfolio at the beginning of period

The world market portfolio comprises the entire risky securities. The nominal value of the world market equity portfolio at the end of period is the sum of capital gains and dividend return of all risky assets.
\[ \tilde{M} \equiv \sum_{i=1}^{I} n_i^0 \left( \tilde{C}G_i + \tilde{D}_i \right) \]  

(5.82)

with \( \tilde{M} \) nominal value of the world market equity portfolio at the end of period

Based on equation (5.81), our definition of the world market equity portfolio at the beginning of the period and equation (5.82) and our definition of the world market equity portfolio at the end of period, we define the nominal capital gains and dividend rate of return of the world market equity portfolio.

\[ \tilde{r}_{CG,n} + \tilde{r}_{D,n} = \frac{\tilde{M} - 1}{M_0} = \frac{\sum_{i=1}^{I} n_i^0 \left( \tilde{C}G_i + \tilde{D}_i \right)}{\sum_{i=1}^{I} n_i^0 V_{i,0}} - 1 \]  

(5.83)

with \( \tilde{r}_{CG,n} \) nominal world market capital gains rate of return

\( \tilde{r}_{D,n} \) nominal world market dividend rate of return

The idea consists in expressing the nominal rate of return of the world market equity portfolio by the sum of dividend and capital gains rate of return. Hence, we reformulate equation (5.83) by considering the definition of the nominal world market equity portfolio (eq. (5.81)) and the nominal rate of return of risky assets (eq. (5.1)).

\[ \tilde{r}_{CG,n} + \tilde{r}_{D,n} = \frac{\sum_{i=1}^{I} n_i^0 \left( \tilde{C}G_i + \tilde{D}_i \right)}{\sum_{i=1}^{I} n_i^0 V_{i,0}} - 1 \]

\[ = \sum_{i=1}^{I} \left( \frac{n_i^0 \left( \tilde{C}G_i + \tilde{D}_i \right)}{n_i^0 V_{i,0}} - 1 \right) \frac{n_i^0 V_{i,0}}{M_0} \]

\[ = \sum_{i=1}^{I} \left( \tilde{r}_{CG,i,n} + \tilde{r}_{D,i,n} \right) \frac{n_i^0 V_{i,0}}{M_0} \]

(5.84)

The term \( \frac{n_i^0 V_{i,0}}{M_0} \) expresses the share of each asset in the nominal world market equity
portfolio. The world market equity portfolio rate of return is the sum of value weighted capital gains and dividend rate of return of each risky asset.

After deriving the excess return, we derive the covariance terms in equilibrium. We extend the right hand side of expression 5.69 with the market equity portfolio and consider that the value weighted shares of the portfolio are made of the product of demanded number and real price of the risky asset (eq. 5.50), so we have

\[
\begin{align*}
& \frac{M_0}{P_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG, i, n} n_i P_i V_{i,0} \left( 1 - t_{CG} \right)^2, \tilde{r}_{CG, i, n} \right] \\
& + \frac{M_0}{P_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{D, i, n} n_i D_i V_{i,0} \left( 1 - t_{D} \right) \left( 1 - t_{CG} \right), \tilde{r}_{CG, i, n} \right].
\end{align*}
\]

(5.85)

Domestic investor’s first capital gains covariance term

Domestic investor’s second capital gains covariance term

The above covariances incorporate stochastic dividends and capital gains. Equivalently, we can derive the following expressions for the covariance term with the dividend (eq. 5.69).

\[
\begin{align*}
& \frac{M_0}{P_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG, i, n} n_i P_i V_{i,0} \left( 1 - t_{CG} \right) \left( 1 - t_{D} \right), \tilde{r}_{D, i, n} \right] \\
& + \frac{M_0}{P_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{D, i, n} n_i D_i V_{i,0} \left( 1 - t_{D} \right)^2, \tilde{r}_{D, i, n} \right].
\end{align*}
\]

(5.86)

Domestic investor’s first dividend covariance term

Domestic investor’s second dividend covariance term

For the foreign investor we can equivalently derive the covariance of nominal capital gains
and dividend rate of return (5.70). For the covariance with the capital gains, we have

\[
\frac{M_0}{P_0} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,n} V_i V_i,0 \frac{1 - t e_{eff,CG} + e \left( 1 - t e_{eff,CG,P,e} \right)}{M_0} , \tilde{r}_{CG,n} \right]
\]

\text{Foreign investor's first capital gains covariance term}

\[
+ \frac{M_0}{P_0} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D,n} V_i V_i,0 \frac{1 - t e_{eff,D} + e \left( 1 - t e_{eff,D,e} \right)}{M_0} , \tilde{r}_{D,n} \right]
\]

\text{Foreign investor's second capital gains covariance term}

whereas the covariance with dividend is as follows:

\[
\frac{M_0}{P_0} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,n} V_i V_i,0 \frac{1 - t e_{eff,CG} + e \left( 1 - t e_{eff,CG,P,e} \right)}{M_0} , \tilde{r}_{CG,n} \right]
\]

\text{Foreign investor's first dividend covariance term}

\[
+ \frac{M_0}{P_0} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D,n} V_i V_i,0 \frac{1 - t e_{eff,D} + e \left( 1 - t e_{eff,D,e} \right)}{M_0} , \tilde{r}_{D,n} \right].
\]

\text{Foreign investor's second dividend covariance term}

We aggregate the first covariance term of capital gains of the domestic investor (5.85) and of the foreign investor (5.87), so we have

\[
\frac{M_0}{P_0} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,n} n_i V_i,0 \frac{1 - t e_{eff,CG} + e \left( 1 - t e_{eff,CG,P,e} \right)}{M_0} , \tilde{r}_{CG,n} \right]
\]

\text{Foreign investor's first capital gains covariance term}

\[
+ \frac{M_0}{P_0} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D,n} n_i V_i,0 \frac{1 - t e_{eff,D} + e \left( 1 - t e_{eff,D,e} \right)}{M_0} , \tilde{r}_{D,n} \right].
\]

\text{Foreign investor's second capital gains covariance term}

In contrast to the derivation of Wiese (2006b) it is not possible to aggregate domestic and foreign investors’ value weighted nominal capital gains rate of return in order to derive an expression of capital gains world market equity return (Mai (2006a) and Mai (2008)). By regarding the above equation we can state that it is not possible to derive
an expression of

\[
\frac{M_0}{P_0^D} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} D V_{i,0} \frac{1 - t_{CG}^D}{M_0} \tilde{r}_{CG,i,n} \right] \\
+ \frac{M_0}{P_0^D} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} F V_{i,0} \frac{1 - t_{eff,CG}^F + e (1 - t_{eff,CG,P,e})}{M_0} \tilde{r}_{CG,i,n} \right] \\
\neq \text{MCov} [\tilde{r}_{CGm,n}, \tilde{r}_{CGi,n}] X
\]

since after separation of the tax and appreciation of exchange rate

\[
(1 - t_{CG}^D)^2 \frac{M_0}{P_0^D} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} D V_{i,0} \frac{1 - t_{CG}^D}{M_0} \tilde{r}_{CG,i,n} \right] \\
+ (1 - t_{eff,CG}^F + e (1 - t_{eff,CG,P,e}))^2 \frac{M_0}{P_0^D} \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} F V_{i,0} \frac{1 - t_{eff,CG}^F + e (1 - t_{eff,CG,P,e})}{M_0} \tilde{r}_{CG,i,n} \right] \\
\]

we have an expression of

\[
ACov [\bar{X}, \bar{Z}] + BCov [\bar{Y}, \bar{Z}]
\]

which cannot be aggregated. The tax rate and the appreciation of the exchange rate as well as the demanded numbers of risky asset are investor-specific and cannot be aggregated in order to derive an expression of capital gains world market equity return. We can derive the same result for the other terms. Nevertheless, it is possible to derive an equilibrium pricing equation. We define the aggregate nominal capital gains and dividend rate of return for the four combinations. We define the first aggregate capital gains rate of return as follows:

\[
\tilde{r}_{CG, CG}^{\text{agg}} = \sum_{i=1}^{I} \tilde{r}_{CG,i,n} D V_{i,0} \frac{1 - t_{CG}^D}{M_0} (1 - t_{CG}^D)^2 \\
+ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} F V_{i,0} \frac{1 - t_{eff,CG}^F + e (1 - t_{eff,CG,P,e})}{M_0} (1 - t_{eff,CG}^F + e (1 - t_{eff,CG,P,e}))^2.
\]
We aggregate the first covariance term of capital gains of the domestic investor (5.85) and of the foreign investor (5.87), so we have

\[ \frac{M_0}{P^D_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG, i, n} D V_{i, 0} \frac{n_i^D V_{i, 0}}{M_0} (1 - t_{CG}^D)^2, \tilde{r}_{CG, i, n} \right] \]

\[ + \frac{M_0}{P^D_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG, i, n} F V_{i, 0} \frac{n_i^F V_{i, 0}}{M_0} (1 - t_{eff, CG}^F + e (1 - t_{eff, CG, P, e}^F))^2, \tilde{r}_{CG, i, n} \right] \] (5.94)

and sum them to one covariance

\[ \frac{M_0}{P^D_0} Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG, i, n} D V_{i, 0} \frac{n_i^D V_{i, 0}}{M_0} (1 - t_{CG}^D)^2 \right. \]

\[ \left. + \sum_{i=1}^{I} \tilde{r}_{CG, i, n} F V_{i, 0} \frac{n_i^F V_{i, 0}}{M_0} ((1 - t_{eff, CG}^F) + e (1 - t_{eff, CG, P, e}^F))^2, \tilde{r}_{CG, i, n} \right] \] (5.95)

Having aggregated the first capital gains covariance term (eq. (5.93)), we insert the defined return into the above term.

\[ \frac{M_0}{P^D_0} Cov \left[ \tilde{r}_{CG, CG, t, aggr}, \tilde{r}_{CG, i, n} \right] \] (5.96)

Equivalently, we aggregate the other covariance terms and insert the defined returns into the above terms (See appendix C.3.3). After that, we sum the covariances of nominal capital gains and dividend rate of return.

\[ \frac{M_0}{P^D_0} Cov \left[ \tilde{r}_{CG, CG, t, aggr}, \tilde{r}_{CG, i, n} \right] + \frac{M_0}{P^D_0} Cov \left[ \tilde{r}_{CG, D, t, aggr}, \tilde{r}_{CG, i, n} \right] \] (5.97)

We can sum the covariance terms, so we have

\[ \frac{M_0}{P^D_0} Cov \left[ \tilde{r}_{CG, CG, t, aggr} + \tilde{r}_{D, CG, t, aggr}, \tilde{r}_{CG, i, n} \right] \]

\[ + \frac{M_0}{P^D_0} Cov \left[ \tilde{r}_{D, CG, t, aggr} + \tilde{r}_{D, D, t, aggr}, \tilde{r}_{D, i, n} \right] \] (5.98)
We put the derived formulations of the excess return \((5.80)\) and the covariance terms \((5.98)\) together and consider that we divided the excess return by the world aggregate risk aversion factor under inflation \(A_W^W\) (See appendix C.2.1) and hence, adapt the covariance terms.

\[
E[\tilde{r}_{CG,i,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e \left( A_W^F - ET_{CG}^F \right) - e ET_{CG}^F \\
+ E[\tilde{r}_{D,i,n}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e \left( A_W^F - ET_D^F \right) \\
- r_{I,n} (1 - T_I) - r_{I,n} e \left( A_W^F - ET_I^F \right) + e ET_I^F \\
= \frac{M_0}{P_0^W A_W^W} \left( Cov \left[ \tilde{r}_{CG,CG}^t,aggr + \tilde{r}_{D,CG}^t,aggr, \tilde{r}_{CG,i,n} \right] + Cov \left[ \tilde{r}_{CG,D}^t,aggr + \tilde{r}_{D,D}^t,aggr, \tilde{r}_{D,i,n} \right] \right)
\]

(5.99)

In view of the fact that the covariance is taken with respect to the world market portfolio, the world market portfolio with expected real rate of return \(E[\tilde{r}_{w,n}]\) is taken as the benchmark in order to identify the world market risk (Serçu and Uppal (1995) and Fernandez (2005)). CAPT prices every risky asset, so the world market equity portfolio can be applied. This benchmark is perfectly correlated with the world market equity return. We multiply the above equation with the share of each asset in the market portfolio \(\frac{n_i^0 V_{i,0}}{M_0}\).

We aggregate over all risky assets and recall that the world market equity portfolio rate of return is the aggregated sum of value weighted capital and dividend rate of return of each risky asset (eq. (5.107)) and consider the definition of the nominal value of the world market equity portfolio at the beginning of period (eq. (5.81)) and that the aggregated quotient \(\sum_{i=1}^{I} \frac{n_i^0 V_{i,0}}{M_0}\) equals unity. Finally, we eliminate the world aggregate risk aversion factor under inflation (S. appendix C.3.4). After these transformations we can derive the Tax-IntCAPM.
Under the framework of international taxation, deviation from Relative PPP and stochastic dividends we developed the Tax International Capital Asset Pricing Model.

5.5 Interpretation

We derived the Tax-IntCAPM under the postulate of international tax and violation of Relative PPP leading to heterogeneous equilibrium pricing relationships for international market participants. The equilibrium depends on the preferences of the investors, since investor-specific terminal wealth results from investor-specific tax, inflation and appreciation of exchange rates. Hence, the Tax-IntCAPM incorporates investor-specific expectations which lead to an equilibrium pricing equation depending on the preferences of investors.\textsuperscript{197} The derived Tax-IntCAPM is a totally new model revealing the incorporation of a fundamentally different structure to the Tax-CAPM and the IntCAPM. We derived a Tax-IntCAPM whose equilibrium is expressed by the above equation and the capital market equilibrium pricing relationship that aggregated demand has to correspond to the

\textsuperscript{197}The CAPM was derived for investor-specific preferences, cf. \textit{Lintner} (1969) and \textit{Sharpe} (1999). The investor-specific CAPM is based on the assumption that investors estimate differently the returns, so the expected excess returns are investor-specific.
supply of risky assets. Nevertheless, the securities market line of the Tax-IntCAPM is comparable to the concept of CAPM which says that the expected return depends on two components: the risk-free rate of return and the risk premium.

\[
E[r_{CG,m,n}] (1 - T_{CG}) + E[r_{CG,i,n}] e \left( A^F_W - ET^F_CG \right) - eET^F_CG \\
+ E[r_{D,m,n}] (1 - T_D) + E[r_{D,i,n}] e \left( A^F_W - ET^F_D \right)
\]

\[
= r_{I,n} (1 - T_I) + r_{I,n} e \left( A^F_W - ET^F_I \right) - eET^F_I \\
(E[r_{CG,m,n}] (1 - T_{CG}) + E[r_{CG,m}] e \left( A^F_W - ET^F_CG \right) - ET^F_CG \\
+ E[r_{D,m,n}] (1 - T_D) + E[r_{D,m}] e \left( A^F_W - ET^F_D \right) \\
- r_{I,n} (1 - T_I) - r_{I,n} e \left( A^F_W - ET^F_I \right) + eET^F_I)
\]

\[
\beta - factor
\]

The above equation describes an investor’s expected rate of return for each asset. Due to the assumption of differential taxation of all income types in IntCAPT, the expected return is decomposed into the expected return of capital gains, exchange gains on capital gains, dividend and exchange gains on dividend. The investor’s risk-free rate of return represents the investor’s price of time or the rate of return that is required for delaying consumption. In contrast to Lally (1996) and as in Mai (2006a) and Mai (2008) due to stochastic dividend the systematic risk is determined by a complex term incorporating the stochastic relationship of dividend and capital gains (market) return. Furthermore the

---

198 Due to mathematical reasons, s. eq. (5.90) to eq. (5.92) it was impossible to integrate the market equilibrium theorem of CAPT so that the exogenous quantity of each asset has to equal the aggregate demand for the asset into the asset pricing formula.

199 S. also investors’ individual optimum condition (eq. (5.61)).

200 Since capital gains and dividend are differently taxed, dividend policy proves not to be irrelevant (Hamada and Scholes (1985) and Wiese (2006a)). For empirical evidence, cf. Holmen (2008).
beta factor is dependent on tax and exchange rates. It is impossible to separate the tax and appreciation of exchange rate from the covariances of the beta-factor, leading to the conclusion that stochastic capital gains and dividends in the case of differential taxation of dividend and capital gains lead to a beta factor whose covariances are influenced by tax and exchange rates.\footnote{S. eq. 5.90 to eq. 5.92 (Mai (2008)).}

The concept of asset pricing can be explained by comparing assets with equal mean payoffs. Those assets which pay off most when ex-post wealth is highest are, of course, the assets that co-vary strongly with the market. However high ex-post wealth means low marginal utility. Thus, those assets pay off most when the payoff is least useful (and least when the payoff is most useful). Those assets are considered riskier. The relationship between beta and its expected rate of return is linear. The higher beta is for an asset, the higher is supposed to be its equilibrium rate of return. The investor is rewarded for bearing systematic risk that cannot be diversified away, whereas the unsystematic risk is not priced, since it can be averaged away in a diversified portfolio (Elton (2007)).

The term

\[
E[\tilde{r}_{CG,m}] (1 - T_{CG}) + E[\tilde{r}_{CG,m}] e (A_{W}^{F} - ET_{CG}^{F}) - eET_{CG}^{F} \\
+ E[\tilde{r}_{D,m}] (1 - T_{D}) + E[\tilde{r}_{D,m}] e (A_{W}^{F} - ET_{D}^{F}) \\
- r_{I,n} (1 - T_{I}) - r_{I,n} e (A_{W}^{F} - ET_{I}^{F}) + eET_{I}^{F}
\] (5.101)

is the risk premium of the world equity market, the excess rate of return of the world market equity portfolio. For the risk the investor takes, he is compensated by the risk premium of the world equity market per unit of risk. $E[\tilde{r}_{CG,m}]$ and $E[\tilde{r}_{D,m}]$ are expected to be smaller than their counterparts in the national Tax-CAPM, since the world market implies greater diversification and hence lower portfolio variance (Lally (1996)).

Equivalently to the Tax-IntCAPM of Lally (1996), the excess return and the world market equity risk premium are adapted by factors but they adopt a more complicated structure on account of the features of international taxation. The excess rate of return and the risk premium are adapted by the weighted tax factor on capital gains, dividend and interest, the foreign risk aversion and the foreign tax factor on capital gains, dividend and interest exchange gains. In contrast to Lally (1996) differential inflation rates are considered.
and are integrated into the risk aversion factors. The domestic tax rates are weighted by the domestic investor's risk aversion factor under inflation and the foreign tax rates are weighted by the foreign investor's risk aversion factor under inflation. The factors can be interpreted as the equilibrium market value of a unit of domestic and foreign average tax from security \(i\).

The exchange rate is determined by the interaction of speculators and traders and is characterized by a market approach in which the demand for foreign currency equals the supply. The price process of the exchange rate is described by non-linearity and is not determined by Relative PPP. Equivalently to the tax factors, the exchange gains rate multiplied by the foreign risk aversion factor can be interpreted as the equilibrium value of a unit exchange gains from security \(i\).

In contrast to the Tax-IntCAPM of Lally (1996) it was found to be important to consider the features of exchange gains taxation and the characteristic features of the exchange gains tax system: The increase exchange gains taxes in dependence of the recognition, character, nature, source and hedging.\(^{202}\) Foreign investors consider exchange gains taxation of dividend, capital gains and interest in the pricing of international assets, which leads to the integration of the foreign tax factor on the exchange gains of capital gains, dividend and interest in the excess return and the market risk premium.

\[
\begin{align*}
ET_{CG}^F &= \frac{t_{\text{eff},CG,P,e}^F A_{\pi e}^F}{A_{\pi}^W} \\
ET_D^F &= \frac{t_{\text{eff},D,e}^F A_{\pi e}^F}{A_{\pi}^W} \\
ET_I^F &= \frac{t_{\text{eff},I,P,e}^F A_{\pi e}^F}{A_{\pi}^W}
\end{align*}
\]  

\(5.102\)

The principal is subject to exchange gains taxation, so the capital gains (interest) rate of return is adapted by the exchange gains factor on capital gains and interest.

The effect of exchange gains taxation into the Tax-IntCAPM cannot be limited by the integration of the foreign tax factor on exchange gains. However, exchange gains taxation also has an effect on the weighted tax factors, the foreign risk aversion factor and the beta-factor. Exchange gains taxation is integrated into the weighted tax factors and the foreign risk aversion factor via the utility function in the expected coefficient of world risk aversion. In the beta factor we can find exchange gains taxation in the aggregate return. It was found to be impossible to separate exchange gains tax from these factors to analyze.

\(^{202}\) S. chapter 4.4.
further the effects of exchange gains taxation, so we need to derive more simplified models which enable the separation of exchange gains effects to pursue further analysis.

5.5.1 Deterministic Dividends

We derived the Tax-IntCAPM under the assumption of stochastic dividend. The question of how far the Tax-IntCAPM changes under the assumption of deterministic dividends is raised. If we assume deterministic dividends, we can derive the following equation for the domestic and foreign investors’ individual optimum.\(^{203}\)

\[ E[\tilde{r}_{CG}] + r_D = r_I + ACov[\tilde{W}, \tilde{r}_{CG}] \] (5.103)

We consider domestic and foreign investors’ individual optimum and separate the risk aversion factor \(\alpha\) from the covariance. In contrast to the Tax-IntCAPM under stochastic dividend apart from the inflation rates we can separate the tax and appreciation of exchange rate from the covariance term, so the sum over the domestic and foreign investor leads to the following equation.\(^{204}\)

\[
\begin{align*}
\frac{(1 + \pi_D)^2}{A_D (1 - t_D^D)^2} \left(E[\tilde{r}_{CGD}] + r_D^D - r_I^D\right) + \\
\frac{(1 + \pi_F)^2}{A_F (1 - t_{eff,CG} + e (1 - t_{eff,CG,P,e}))^2} \left(E[\tilde{r}_{CGF}] + r_D^F - r_I^F\right) \\
= \sum_{k=1}^K Cov[\tilde{r}_{CGi,n}, \tilde{r}_{CGk,n}] \omega_k + \sum_{k=1}^K Cov[\tilde{r}_{CGi,n}, \tilde{r}_{CGk,n}] \omega_k
\end{align*}
\] (5.104)

Having transformed the equation and defined the factors we can derive the excess return under deterministic dividend. Compare appendix C.2 for the process of transformation.


\(^{204}\) For mathematical purposes we redefine the index.
and appendix C.2.2 for the definition of the factors.

\[
E[\tilde{r}_{CG,i,n}] (1 - T_{CG,dt}) + E[\tilde{r}_{CG,i,n}] e \left( A^F_{W,dt} - ET^F_{CG,dt} \right) - e ET^F_{CG,dt} \\
+ r_{D,i,n} (1 - T_{D,dt}) + r_{D,i,n} e \left( A^F_{W,dt} - ET^F_{D,dt} \right) \\
- r_{I,n} (1 - T_{I,dt}) - r_{I,n} e \left( A^F_{W,dt} - T_{I,dt} \right) + e ET^F_{I,dt}
\]

(5.105)

The excess return is similar to the one of the Tax-IntCAPM under stochastic dividend. Apart from the deterministic dividends the Tax-IntCAPM differs in respect of the factors which are weighted by different risk aversion terms.

In contrast to the Tax-IntCAPM under stochastic dividend it is possible to derive an asset pricing relationship under integration of the market equilibrium postulate that an exogenous quantity of each asset equals the aggregate demand for the asset. We consider the condition for market equilibrium that apart from individual equilibrium the endogenous price process for all risky assets is assumed to develop in such a way that an exogenous quantity of each asset equals the aggregate demand for the asset.\footnote{\textsuperscript{205} S. eq. 5.72.}

\[
\left( n_i^D + n_i^F \right) \frac{V_{i,0}}{P_0^D} = \omega_i^D + \omega_i^F = n_k^0 \frac{V_{k,0}}{P_0^D}
\]

(5.106)

We define the capital gains world market return (eq. 5.84)

\[
\tilde{r}_{CGw,n} = \sum_{k=1}^{K} \frac{n_k^0 \tilde{r}_{CG,k,n}}{M_0} - 1 = \sum_{k=1}^{K} \left( \left( \frac{n_k^0 \tilde{r}_{CG,k,n}}{n_k^0 V_{k,0}} - 1 \right) \frac{n_k^0 V_{k,0}}{M_0} \right) \\
= \sum_{k=1}^{K} \frac{n_k^0 V_{k,0}}{M_0} \tilde{r}_{CG,k,n}
\]

(5.107)

and aggregate the covariance terms of the left hand side of (eq. 5.104).

\[
= Cov \left[ \tilde{r}_{CG,n}, \sum_{k=1}^{K} \tilde{r}_{CG,k,n} \left( \omega_k^D + \omega_k^F \right) \right].
\]

(5.108)

According to the concept of market equilibrium we equalize demand and supply by considering that the value weighted shares of the portfolio are made of the product of demanded
number and real price of the risky asset $i$.

$$
\text{Cov} \left[ \tilde{r}_{CG,i,n}, \sum_{k=1}^{K} \tilde{r}_{CG,k,n} \left( \omega_k^D + \omega_k^F \right) \right] = \text{Cov} \left[ \tilde{r}_{CG,i,n}, \sum_{k=1}^{K} \tilde{r}_{CG,k,n} \frac{n_0}{P_0} V_{i,0} \right] 
$$

The expansion of the right hand side of the above equation by the definition of the world market portfolio (eq. 5.81) and the consideration of the definition of the capital gains world market return (eq. 5.107) leads to a covariance term incorporating solely the capital gains (world market) rate of return.

$$
\text{Cov} \left[ \tilde{r}_{CG,i,n}, \tilde{r}_{CG,w,n} \frac{M_0}{P_0^D} \right] 
$$

The application of the world market capital gains portfolio leads to the following Tax-IntCAPM under deterministic dividend.

$$
E[\tilde{r}_{CG,i,n}] (1 - T_{CG,dt}) + E[\tilde{r}_{CG,i,n}] e \left( A_{W,dt} - ET_{CG,dt}^F \right) - e ET_{CG,dt}^F \\
+ r_{D,i,n} (1 - T_{D,dt}) + r_{D,i,n} e \left( A_{W,dt} - ET_{D,dt}^F \right) \\
- r_{I,n} (1 - T_{I,dt}) - r_{I,n} e \left( A_{W,dt} - ET_{I,dt}^F \right) + e ET_{I,dt}^F \\
= \left( E[\tilde{r}_{CG,m,n}] (1 - T_{CG,dt}) + E[\tilde{r}_{CG,m,n}] e \left( A_{W,dt} - ET_{CG,dt}^F \right) \right) \\
- e ET_{CG,dt}^F + r_{D,i,n} (1 - T_{D,dt}) \\
+ r_{D,i,n} e \left( A_{W,dt} - ET_{D,dt}^F \right) - r_{I,n} (1 - T_{I,dt}) \\
- r_{I,n} e \left( A_{W} - ET_{I,dt}^F \right) + e ET_{I,dt}^F \frac{\text{Cov} \left[ \tilde{r}_{CG,i,n}, \tilde{r}_{CG,m,n} \right]}{\text{Var} \left[ \tilde{r}_{CG,m,n} \right]} 
$$

Apart from the fact that the return of the dividend is secure, the beta factor is independent from tax and exchange rates and determined by the relationship of covariance of capital gains of the asset, world market capital gains return and variance of world market capital gains return.
5.5.2 Portfolio Composition

The Tax-IntCAPM is derived from a model of how the investors construct their portfolios, so the model itself has major implications for the construction of portfolios. Our model is influenced by tax and deviations from Relative PPP, so, domestic and foreign investors have a different composition of portfolios on account of heterogeneous preferences (Long (1977) and Singer (1979)). By recalling investors’ individual optimum expected rate of return for assets (eq. (5.61)) and replacing investors’ terminal real wealth after taxes by the definition of the portfolio (eq. (5.52)), an investor’s optimal asset allocation can be derived. For the domestic investor we have

\[
\frac{1}{A^D} \left( E\left[\bar{r}_{CG_i}^D\right] + r_{D_i}^D - r_i^D \right) = Cov \left[ \sum_{i=1}^{I} \omega_i^D \left( 1 + \tilde{r}_{CG_i}^D + r_{D_i}^D \right) + \omega_0^D \left( 1 + r_i^D \right), \tilde{r}_{CG_i}^D \right] \quad (5.112)
\]

and for the foreign investor the following equation is valid.

\[
\frac{1}{A^F} \left( E\left[\bar{r}_{CG_i}^F\right] + r_{D_i}^F - r_i^F \right) = Cov \left[ \sum_{i=1}^{I} \omega_i^F \left( 1 + \tilde{r}_{CG_i}^F + r_{D_i}^F \right) + \omega_0^F \left( 1 + r_i^F \right), \tilde{r}_{CG_i}^F \right] \quad (5.113)
\]

We separate the tax, inflation and appreciation of exchange rate from the excess return and the covariance, which leads to the following equation for the domestic investor

\[
\sum_{k=1}^{K} \omega_i^D Cov \left[ \tilde{r}_{CG_i,n}, \tilde{r}_{CG_k,n} \right] = \frac{1 + \pi^D}{A^D \left( 1 - t^D_{CG} \right)^2} \left( E\left[\tilde{r}_{CG_i,n}\right] \left( 1 - t^D_{CG,D} \right) + r_{D,n} \left( 1 - t^D_D \right) - r_{I,n} \left( 1 - t^D_I \right) \right) \quad (5.114)
\]

and to the following equation for the foreign investor.

\[
\sum_{k=1}^{K} \omega_i^F Cov \left[ \tilde{r}_{CG_i,n}, \tilde{r}_{CG_k,n} \right] = \frac{1 + \pi^F}{A^F \left( 1 - t^F_{eff,CG,CCG} \right)^2} \left( E\left[\tilde{r}_{CG_i,n}\right] \left( 1 - t^F_{eff,CG,CG} \right) + E\left[\tilde{r}_{CG_i,n}\right] e \left( 1 - t^F_{eff,CG,CG,P,e} \right) - r_{D,n} \left( 1 - t^F_{eff,D,e} \right) - r_{I,n} \left( 1 - t^F_{eff,I,e} \right) + e t^F_{eff,I,P,e} \right) \quad (5.115)
\]
If we assume the existence of one asset, the well-known Samuelson theorem can be derived, that the optimal portfolio weight is separable in risk aversion and market price of risk (Samuelson (1969), Merton (1973) and Gron (2004)). For the domestic investor we have

\[
\omega_i^D = \frac{1 + \pi^D}{A^D (1 - t^D_{CG})^2 \text{Var} [\tilde{r}_{CG,i,n}]} \left( E [\tilde{r}_{CG,i,n}] (1 - t^D_{CG}) + r_{D,i,n} (1 - t^D_D) - r_{I,n} (1 - t^D_I) \right)
\]  

(5.116)

whereas for the foreign investor we have:

\[
\omega_i^F = \frac{(1 + \pi^F)}{A^F ((1 - t^F_{eff,CG}) + \epsilon (1 - t^F_{eff,CG,P,e}))^2 \left( E [\tilde{r}_{CG,i,n}] (1 - t^F_{eff,CG}) + \epsilon \left( 1 - t^F_{eff,CG,P,e} \right) \right) - \epsilon t^F_{eff,CG,P,e} + r_{D,i,n} (1 - t^F_{eff,D}) + r_{D,i,n} \epsilon (1 - t^F_{eff,D,P,e}) - r_{I,n} (1 - t^F_{eff,I}) - r_{I,n} \epsilon (1 - t^F_{eff,I,P,e}) + \epsilon t^F_{eff,I,P,e})}
\]  

(5.117)

The risk aversion coefficient is a scaling constant, consequently an investor’s relative holding of the risky assets is independent from the utility function (Brennan (1970)).

Every investor compiles his optimal portfolio. Based on the separation theorem of Tobin the optimal portfolio consists of the risk-free asset and the tangential portfolio. On account of investor-specific tax, inflation and appreciation of exchange rate, the tangential portfolios are investor-specific (Long (1977), Wiese (2006b) and Mai (2008)). The tangential portfolio of the investors is unequal to the market portfolio (Sharpe (1999)).
6 Tax International Capital Asset Pricing Model under Restrictions

“Things which restrict the common are to be interpreted rigidly”

Latin Proverb

Taxes are largely a source of embarrassment to financial economists, since taxes are regarded as significant, but the equilibrium effect of taxes on international asset pricing is unknown. We derived the Tax-IntCAPM in the framework of equilibrium. It is uncertain whether this equilibrium exists due to the existence of arbitrage. The exclusion of arbitrage is the central theorem in CAPT. Tax rate differentials across income classes in an international framework can generate arbitrage (McDonald (2001)). In order to build a relationship between an arbitrage-free international capital market we analyze constellations leading to the exclusion of global arbitrage.

In models where investors and securities are subject to differential taxation in an international framework, there may be no set of prices that rule out infinite gains to trade - international tax arbitrage -. So the question arises

**What are the implications of excluding global arbitrage opportunities in the Tax-IntCAPM?**

We discuss the implications of the exclusion of tax arbitrage. Having characterized the restrictions that preclude international tax arbitrage, we develop the Tax-IntCAPM under short sale and borrowing restriction concluding with an interpretation of the model.
6.1 Tax Arbitrage

The Tax-IntCAPM is founded on the central theorem of equilibrium, but it is uncertain whether this equilibrium exists due to the existence of tax arbitrage. Tax arbitrage is found where an arbitrage portfolio after tax leads to a semi-positive pay-off (arbitrage opportunity type 1) or where a positive pay-off leads to a non-negative repayment (arbitrage opportunity type 2) (Raab (1993)).

The differential tax treatment of the domestic and foreign investor can lead to varying marginal rates of substitution between investors. Hence, the central assumption of equilibrium is not satisfied, since investors can realize unlimited gains through the substitution of assets. The assumption of short-sale and borrowing restrictions limits these gains (local arbitrage) and satisfies the central theorem of equilibrium despite varying marginal rates of substitution. Schäfer (1982) and Mai (2008) among many others come to the conclusion that in a model with investor-specific tax rates we have to include short sale restrictions and borrowing restrictions in order to exclude unlimited arbitrage opportunities.

6.2 Tax International Capital Asset Pricing Model under Short Sale and Borrowing Restrictions

The Tax-IntCAPM was developed under the assumption that short sales are not excluded and that investors are unlimited in their ability to borrow. We follow the model of Mai (2008) and derive the Tax-IntCAPM under short sale restriction and the restrictions

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206 Semi-positive pay-off implies a positive pay-off in one state.
208 Other possibilities are the formation of an appropriate tax system and identical (marginal) taxation of investors (Schäfer (1982) and Dammon and Green (1987)). Both variants are regarded as irrelevant since taxes are assumed to be given and identical (marginal) taxation of investors is unrealistic (Wiese (2006a)). We ignore the approach of Dammon and Green (1987) who derive the exclusion of tax arbitrage for incomplete markets without short-selling restriction, since incompleteness per se does not guarantee the absence of tax arbitrage when tax schedules are sufficiently heterogeneous. For a critique of the approach of Dammon and Green (1987), cf. Ross (1987).
that the total amount invested in risky assets is nonnegative and implement a borrowing restriction.\textsuperscript{209}

The decision problem of the maximization of the expected utility of end of period real wealth after international tax is extended by the constraint that the total amount invested in risky assets must be positive and that the investor is limited in his ability to borrow. The optimality condition is solved under application of the Lagrange conception. After transformation by application of the definition of the covariance and Stein’s Lemma we derive the individual optimum of the domestic and foreign investor. We aggregate domestic and foreign investors’ demand and define the weighted tax factor on capital gains, dividend and interest, the weighted foreign risk aversion factor and the foreign tax factor on capital gains, dividend and interest exchange gains. We equalize an investor’s aggregated demand to the supply of risky assets and derive the equilibrium pricing relationship of the Tax-IntCAPM under short sale and borrowing restrictions.\textsuperscript{210}

### 6.2.1 Individual Optimum

The representative consumers maximize their expected utility of end of period real wealth after tax and achieve individual optimum. The formulas are constructed in such a way that they are valid for the domestic as well as for the foreign investor (S. chapter 5.4.1).

\[
\begin{align*}
\text{Max} & \quad E \left[ U \left( \tilde{W}_t \right) \right] \\
\text{s.t.} & \quad \sum_{i=0}^{I} \omega_i = 1 \\
\text{s.t.} & \quad \tilde{W}_t = \sum_{i=1}^{I} \omega_i \left( 1 + \tilde{r}_{CG_i} + \tilde{r}_{D_i} \right) + \omega_0 \left( 1 + r_f \right)
\end{align*}
\]

In order to implement short sale and borrowing restrictions into the Tax-IntCAPM we integrate further constraints into the framework.

\textsuperscript{209}S. chapter 3.1.3.
\textsuperscript{210}S. chapter 5.4 for derivation.
The constraints 6.4 and 6.5 are the decisive differences to the prior models. The first constraint 6.4 excludes short sales. The entire amount of money invested in risky stocks is restricted from adopting a negative value; an investor’s portfolio is restricted from short sale.\(^{211}\) The second constraint 6.5 contains the borrowing restriction, incorporating that the borrowed share shall not fall below the investor-specific share \(\omega_{0,\text{min}}\) which is assumed to be negative (Auerbach (1983)).\(^{212}\) Hence, the foreign investor can maximally borrow the investor-specific amount. The inequalities are transformed into equalities by the introduction of the slack variables \(v_1\) and \(v_2\) (Litzenberger and Ramaswamy (1979), König (1990), Wiese (2006b) and Mai (2008)).

\[
\begin{align*}
\sum_{i=1}^{L} \omega_i &\geq 0 \\
\omega_0 - v_1 & \geq -\omega_{0,\text{min}}
\end{align*}
\]  

(6.6)  

\[
\begin{align*}
\sum_{i=1}^{L} \omega_i - v_1 & \leq 0 \\
\omega_0 - v_2 & = -\omega_{0,\text{min}}
\end{align*}
\]  

(6.7)  

The variables are positive if the restrictions are binding; otherwise they are zero (Litzenberger and Ramaswamy (1979) and Mai (2008)). If \(v_1\) is zero, net entire investment in risky stocks is equal to zero and initial wealth is totally invested into the risk-free bond. If \(v_2\) is zero, the investor borrows exactly the investor-specific amount. In order to integrate the value share of risky stocks as the only variable in our maximization problem we remember that the share of the risk-free asset is the opposite of the sum of shares of risky assets (S. chapter 5.4.1).

\(^{211}\) Single assets are not restricted from short-sale, cf. Auerbach (1983).

\(^{212}\) Litzenberger and Ramaswamy (1979), König (1990) and Wiese (2006b) integrate two different kinds of borrowing restrictions into the Tax-CAPM. The first restriction incorporates that the borrowed amount shall not exceed a specific share invested in risky assets, the other one that the debt interest shall not exceed deterministic dividend. Since we assume stochastic dividend the second restrictions reveals not to be plausible to be incorporated in our model (Mai (2008)).
\[ \omega_0 = 1 - \sum_{i=1}^{I} \omega_i \]  

(6.8)

We integrate the above expression into constraint 6.7, so the reformulated constraint becomes

\[ 1 - \sum_{i=1}^{I} \omega_i - v_2 + \omega_{0,\text{min}} = 0. \]  

(6.9)

Apart from the constraints, that the value shares sum to unity (eq. 6.2) and that terminal real wealth after tax is set up of the return after tax of the portfolio, consisting of risky and risk-free assets (eq. 6.3), the short-sale and borrowing restrictions (eq. (6.6) and eq. (6.9)) are integrated into the investor’s maximization problem. We expand and rearrange the optimization equation and formulate the Lagrange equation (Litzenberger and Ramaswamy (1979), Auerbach (1983), König (1990), Wiese (2006b) and Mai (2008)).

\[
\frac{\partial L}{\partial \omega_i} = E \left[ U \left( (1 + r_I) + \sum_{i=1}^{I} \omega_i (\tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I) \right) \right] - \lambda_1 \left( \sum_{i=1}^{I} \omega_i - v_1 \right) - \lambda_2 \left( 1 - \sum_{i=1}^{I} \omega_i - v_2 + \omega_{0,\text{min}} \right)
\]

(6.10)

The Lagrangean multipliers \( \lambda \) represent the shadow prices of the short-selling and borrowing restrictions. The Lagrangean multiplier \( \lambda_1 \) (\( \lambda_2 \)) is the shadow price for the regrouping of the entire invested amount from risky (risk-free) assets to the risk-free (risky) asset, (Mai (2008)). We differentiate partially in respect of the shares by application of the chain rule and set the derivative equal to zero.

\[
E \left[ \frac{\partial U \left( (1 + r_I) + \sum_{i=1}^{I} \omega_i (\tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I) \right)}{\partial \omega_i} (\tilde{r}_{CG_i} + \tilde{r}_{D_i} - r_I) \right] - \lambda_1 + \lambda_2 = 0
\]

(6.11)

for \( i \in 1..I. \)
We replace the variable of the utility function by terminal real wealth after tax.

\[
E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} (\tilde{r}_{CG} + \tilde{r}_{D_i} - r_I) \right] - \lambda_1 + \lambda_2 = 0 
\]  
\text{(6.12)}

The application of the definition of the covariance leads to the following expression.

\[
E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} \right] E [\tilde{r}_{CG} + \tilde{r}_{D_i} - r_I] + \text{Cov} \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i}, \tilde{r}_{CG} + \tilde{r}_{D_i} - r_I \right] - \lambda_1 + \lambda_2 = 0 
\]  
\text{(6.13)}

After application of Stein’s Lemma we have

\[
E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} \right] E [\tilde{r}_{CG} + \tilde{r}_{D_i} - r_I] + E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \tilde{W}_t \partial \omega^i} \right] \text{Cov} [\tilde{W}_t, \tilde{r}_{CG} + \tilde{r}_{D_i}] - \lambda_1 + \lambda_2 = 0 
\]  
\text{(6.14)}

We divide by the term \( E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} \right] \) and consider the definition of the expected coefficient of global risk aversion \( A = - \frac{E \left[ \frac{\partial^2 U (\tilde{W}_t)}{\partial \omega^i \partial \tilde{W}_t} \right]}{E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} \right]} \) (S. eq. 5.60 in chapter 5.4.1), so we have the following equation.

\[
E [\tilde{r}_{CG} + \tilde{r}_{D_i} - r_I] - A \text{Cov} [\tilde{W}_t, \tilde{r}_{CG} + \tilde{r}_{D_i}] - \frac{\lambda_1 + \lambda_2}{E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} \right]} = 0 
\]  
\text{(6.15)}

We define the term \( \frac{\lambda_1 + \lambda_2}{E \left[ \frac{\partial U (\tilde{W}_t)}{\partial \omega^i} \right]} \) as \( SB \) - the short sale and borrowing restriction factor - and insert it into the above equation.
Equation 6.16 represents an investor’s individual optimum under short sale and borrowing restrictions. If the investor does not short sell his portfolio and the borrowed amount is less than the borrowing restriction, the short sale and borrowing restriction factor is equal to zero. If the investor could enhance utility through further short selling, we have $\lambda_1 > 0, \lambda_2 = 0$ or through further borrowing, we have $\lambda_1 = 0, \lambda_2 > 0$ and the short sale and borrowing restriction factor is positive.\(^{213}\) The multipliers can be interpreted as a lowest extra dividend that, when added to the expected return, would have eliminated the incentive to short sale the portfolio or to exceed the allowed borrowing share (Serçu (2001)).

The domestic and foreign investors’ individual optimum is reformulated to express the excess returns. We separate the risk aversion factor $A$ and the inflation rates from the covariance. We aggregate the tax system in order to derive an expression depending solely on the nominal capital gains and dividend return by application of the linearity property of the covariance, which leads to domestic investor’s individual optimum (S. eq. (5.69)).

\[
\frac{(1 + \pi^D)^2}{A^D} \left( E[\tilde{r}^D_{CGi} + \tilde{r}^D_{Di}] - r^I \right) - \frac{(1 + \pi^D)^2}{A^D} SB^D = \text{Cov} \left[ \sum_{i=1}^{l} \tilde{r}^D_{CGi,n} \omega_i^D (1 - t_{CG}^D)^2, \tilde{r}^D_{CGi,n} \right] + \text{Cov} \left[ \sum_{i=1}^{l} \tilde{r}^D_{Di,n} \omega_i^D (1 - t_{CG}^D) (1 - t_{D}^D), \tilde{r}^D_{CGi,n} \right] + \text{Cov} \left[ \sum_{i=1}^{l} \tilde{r}^D_{Di,n} \omega_i^D (1 - t_{D}^D)^2, \tilde{r}^D_{Di,n} \right] + \text{Cov} \left[ \sum_{i=1}^{l} \tilde{r}^D_{Di,n} \omega_i^D (1 - t_{D}^D), \tilde{r}^D_{Di,n} \right] \quad (6.17)
\]

Having derived the pricing relationship term of the domestic investor, we derive analogously the equilibrium pricing relationship for the foreign investor (See appendix C.3.2 and eq (5.70)).

\[
\begin{align*}
\frac{(1 + \pi_F)^2}{A_F} \left( E[\tilde{r}_{CG,i}^F + \tilde{r}_{D,i}^F] - r_F^F \right) - \frac{(1 + \pi_F)^2}{A_F} SB^F \\
= \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} \omega_i^F \left( 1 - t_{eff,CG}^F + e \left( 1 - t_{eff,CG,P,e}^F \right) \right)^2, \tilde{r}_{CG,i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{D,i,n} \omega_i^F \left( 1 - t_{eff,D}^F + e \left( 1 - t_{eff,D,e}^F \right) \right)^2, \tilde{r}_{D,i,n} \right] \\
\end{align*}
\]

\[(6.18)\]

### 6.2.2 Market Equilibrium

By aggregating domestic and foreign investors’ demand in a state of individual optimum and equalization to the supply of risky assets, the state of equilibrium of the capital market is expressed. Equivalently to the derivation of the Tax-IntCAPM, we first consider the left hand side and decompose the return of the asset to formulate an expression of excess return comprising the return of capital gains, exchange gains on capital gains and principal, dividend, exchange gains on dividend, interest and exchange gains on interest and principal after international tax. After some mathematical transformation and the definition of domestic and foreign risk aversion factor under inflation, the world aggregate risk aversion factor under inflation, the weighted tax factor on capital gains, dividend and interest, weighted foreign risk aversion factor and the foreign exchange gains tax factor on capital gains, dividend and interest, the following formulation for the excess return
can be derived (S. chapter 5.4.2 and appendix C.2.1).

\[
E[\tilde{r}_{CG,i,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e \left( A_W^F - E T_{CG}^F \right) - e E T_{CG}^F \\
+ E[\tilde{r}_{D,i,n}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e \left( A_W^F - E T_D^F \right) \\
- r_{I,n} (1 - T_I) - r_{I,n} e \left( A_W^F - E T_I^F \right) + e E T_I^F - S B_{aggr}
\]  

(6.19)

We define the aggregate short sale and borrowing restriction factor as

\[
\frac{(1+\pi_D)^2}{A_D^0} S B_D^D + \frac{(1+\pi_F)^2}{A_F^0} S B_F^F = S B_{aggr}.
\]

(6.20)

Having derived the excess return, we consider the right hand side and can derive the following formulation (See chapter 5.4.2).

\[
\frac{M_0}{P_0^D A_W^0} \left( C o v \left[ \tilde{r}_{t,aggr}^{CG,CG} + \tilde{r}_{t,aggr}^{D,CG}, \tilde{r}_{CG,i,n} \right] + C o v \left[ \tilde{r}_{t,aggr}^{CG,D} + \tilde{r}_{t,aggr}^{D,D}, \tilde{r}_{D,i,n} \right] \right)
\]

(6.21)

If we put the derived formulations of the left hand side 6.19 and right hand side 6.21 together, we have

\[
E[\tilde{r}_{CG,i,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e \left( A_W^F - E T_{CG}^F \right) - e E T_{CG}^F \\
+ E[\tilde{r}_{D,i,n}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e \left( A_W^F - E T_D^F \right) \\
- r_{I,n} (1 - T_I) - r_{I,n} e \left( A_W^F - E T_I^F \right) + e E T_I^F - S B_{aggr}
\]

(6.22)

= \frac{M_0}{P_0^D A_W^0} \left( C o v \left[ \tilde{r}_{t,aggr}^{CG,CG} + \tilde{r}_{t,aggr}^{D,CG}, \tilde{r}_{CG,i,n} \right] + C o v \left[ \tilde{r}_{t,aggr}^{CG,D} + \tilde{r}_{t,aggr}^{D,D}, \tilde{r}_{D,i,n} \right] \right)

As CAPT prices every risky asset, the world market portfolio can be applied. This benchmark is perfectly correlated with the world market return. Equivalently to the derivation of the Tax-IntCAPM, we multiply the above equation with the share of each asset in the market portfolio \( \frac{n_{t,k,0}}{M_0} \) and extend it with the market equity portfolio.
\[ E[\tilde{r}_{CG,m,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,m,n}] e (A^F_W - ET^F_{CG}) - eET^F_{CG} \\
+ E[\tilde{r}_{D,m,n}] (1 - T_D) + E[\tilde{r}_{D,m,n}] e (A^F_W - ET^F_D) \\
- r_{I,n} (1 - T_I) - r_{I,n} e (A^F_W - ET^F_I) + eET^F_I - SB_{aggr} \\
= \frac{M_0}{P_0A^F_W} \left( Cov \left[ \tilde{r}_{CG,\text{aggr}} \tilde{r}_{D,\text{aggr}}, \tilde{r}_{CG,m,n} \right] + Cov \left[ \tilde{r}_{CG,\text{aggr}} \tilde{r}_{D,\text{aggr}}, \tilde{r}_{D,m,n} \right] \right). \] (6.23)

We divide equation 6.22 by equation 6.23, so we have the Tax-IntCAPM under short sale and borrowing restrictions.

\[
\begin{aligned}
E[\tilde{r}_{CG,m,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,m,n}] e (A^F_W - ET^F_{CG}) - eET^F_{CG} \\
+ E[\tilde{r}_{D,m,n}] (1 - T_D) + E[\tilde{r}_{D,m,n}] e (A^F_W - ET^F_D) \\
- r_{I,n} (1 - T_I) - r_{I,n} e (A^F_W - ET^F_I) + eET^F_I - SB_{aggr} \\
= \left( E[\tilde{r}_{CG,m,n}] (1 - T^D_{CG}) + E[\tilde{r}_{CG,m,n}] e (A^F_W - ET^F_{CG}) - eET^F_{CG} \\
+ E[\tilde{r}_{D,m,n}] (1 - T^D_D) + E[\tilde{r}_{D,m,n}] e (A^F_W - ET^F_D) \\
- r_{I,n} (1 - T^D_I) - r_{I,n} e (A^F_W - ET^F_I) + eET^F_I - SB_{aggr} \right) \\
\left( Cov \left[ \tilde{r}_{CG,\text{aggr}} \tilde{r}_{D,\text{aggr}}, \tilde{r}_{CG,m,n} \right] + Cov \left[ \tilde{r}_{CG,\text{aggr}} \tilde{r}_{D,\text{aggr}}, \tilde{r}_{D,m,n} \right] \right) \\
= \left( Cov \left[ \tilde{r}_{CG,\text{aggr}} \tilde{r}_{D,\text{aggr}}, \tilde{r}_{CG,m,n} \right] + Cov \left[ \tilde{r}_{CG,\text{aggr}} \tilde{r}_{D,\text{aggr}}, \tilde{r}_{D,m,n} \right] \right).
\end{aligned}
\]

6.2.3 Interpretation

The derivation of the Tax-IntCAPM under short sale and borrowing restrictions leads to the integration of the aggregate short sale and borrowing factor. The structure of the equilibrium pricing relationship with adapted borrowing and portfolio short sale restrictions is comparable to the one without restrictions, since it is solely adapted by the term $SB_{aggr}$. The factor is reduced from the excess return and can be interpreted as the world market (extra) payment of short sale and borrowing restrictions. In contrast to the Tax-CAPM of Mai (2008) the factor is influenced by international tax, inflation and exchange rates.
In the case of the Tax-IntCAPM under deterministic dividend the tax factors can be eliminated from the covariance term and the equilibrium pricing relationship is equivalently adapted by the term $SB_{aggr, dt}$ which is defined as

$$SB_{aggr, dt} = \frac{(1 + \pi_D)^2}{A^D(1 - t_D CG)} SB^D + \frac{(1 + \pi_F)^2}{A^F(1 - t_F eff,CG + e(1 - t_F eff,CG,P,e))} SB^F. \quad (6.24)$$

Finally we analyze constellations leading to the irrelevance of $SB_{aggr} = 0$.\(^{214}\) This is the case if the restrictions are binding for neither the domestic investor nor for the foreign investor. Furthermore it could be possible that the restrictions are binding for the domestic and foreign investor but neutralized in aggregate. This can be the case if the shadow prices for short sale and borrowing are neutralized in aggregate.\(^{215}\)


\(^{215}\)Under the assumption that the investor’s tangential portfolio is equal to the market portfolio, Mai (2008) draws further conclusions from the Tax-CAPM under short sale and borrowing restrictions. We disregard this aspect, since Mai (2006a) and Mai (2008) admit that simplifying transformations of the $\beta$-factor are required which are formally not proved, s. chapter 3.1.2.
7 Tax International Capital Asset Pricing Model with Homogeneous Expectations

“But in this world nothing can be said to be certain, except death and taxes.”

Benjamin Franklin, author, diplomat, inventor, physicist, politician and printer (1706 – 1790)

The Tax-IntCAPM under differential taxation of stochastic capital gains and dividends leads to a CAPM which is difficult to implement on account of unknown risk parameters and the complicated structure of the beta-factor.\footnote{Domestic and foreign investors' risk aversion factor under inflation $A_{\pi_D}$ and $A_{\pi_F}$ are empirically not observable, as for that purpose a simultaneous determination of inflation rates and risk tolerances of all market participants would be required. The beta-factor is determined by a complex term incorporating the stochastic relationship of dividend and capital gains (exchange gains) return. It is dependent on national and international taxation and exchange rates as well as the value weighted share of each asset and the world market portfolio in the beginning of the period, which makes empirical studies difficult (Wiese (2006a), Wiese (2006a) and Dausend and Schmitt (2007)).} In order to enable the implementation of the model, it is necessary to introduce an additional restriction. The model assumes simplistically that there is solely a foreign representative investor. So the question arises

How is the Tax-IntCAPM affected under the assumption of a representative foreign investor?
The solution of the model will allow us to determine the pricing relationship under market equilibrium for international assets between expected return and risk. An intensive interpretation and example elaborates the economic conceptions and implications.

7.1 Model Solution

The derivation of an investor’s individual optimum forms the basis for the Tax-IntCAPM. The foreign consumer maximizes his expected utility of end of period real wealth after tax and hence, achieves individual optimum (S. eq. (5.61)).

$$E \left[ \tilde{r}_{CG_i}^F + \tilde{r}_{D_i}^F \right] = r_I^F + A^F \text{Cov} \left[ \tilde{r}_{CG_i}^F, \tilde{r}_{D_i}^F, \tilde{W}_i^F \right]$$ (7.1)

The capital market is in a state of equilibrium - the foreign investor maximizes expected utility and an exogenous quantity of each asset equals the foreign investor’s demand for the asset (S. eq. (5.72)).

$$n_i^F = \frac{\omega_i^F P_0^F}{V_{i,0}} = n_k^0$$ (7.2)

$$n_i^F V_{i,0} = \omega_i^F = n_k^0 \frac{V_{k,0}}{P_0^F}$$ (7.3)

The world market capital gains and dividend rate of return can be expressed by the following equation (S. eq. (5.84)).

$$\tilde{r}_{CG,m,n} + \tilde{r}_{D,m,n} = \sum_{k=1}^{K} n_k^0 \left( \frac{\bar{C}_G_k + \bar{D}_k}{M_0} \right) - 1$$

$$= \sum_{k=1}^{K} \left( \left( \frac{n_k^0 \left( \frac{\bar{C}_G_k + \bar{D}_k}{n_k^0 V_{k,0}} \right) - 1}{M_0} \right) \frac{n_k^0 V_{k,0}}{M_0} \right)$$ (7.4)

$$= \sum_{k=1}^{K} (\tilde{r}_{CG,k,n} + \tilde{r}_{D,k,n}) \frac{n_k^0 V_{k,0}}{M_0}$$

\(^{217}\)For mathematical purposes we redefine the index, s. footnote 204.
In order to derive the pricing relationship under market equilibrium, equation (7.1) has to be formulated in nominal terms and separated by the influences of tax, inflation and appreciation of exchange rate. By considering the foreign investor’s real capital gains and dividend rate of return (eq. (5.31)) and replacing the investor’s terminal real wealth after tax by the definition of the portfolio (eq. (5.52)), we can derive the equilibrium pricing relationship for the foreign investor (See appendix C.3.2).

\[
\left(1 + \pi_F^F\right)^2 \frac{A_F}{F} \left( E \left[ r_{CG,i}^F + r_{Di}^F \right] - r_f^F \right) \\
= \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} \omega_i^F \left( 1 - t_{eff,CG}^F + e \left( 1 - t_{eff,CG,P,e}^F \right) \right)^2, \tilde{r}_{CG,i,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{Di,n} \omega_i^F \left( 1 - t_{eff,CG}^F + e \left( 1 - t_{eff,CG,P,e}^F \right) \right), \tilde{r}_{CG,i,n} \right] \\
= \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{CG,i,n} \omega_i^F \left( 1 - t_{eff,CG}^F + e \left( 1 - t_{eff,CG,P,e}^F \right) \right) \left( 1 - t_{eff,D}^F + e \left( 1 - t_{eff,D,e}^F \right) \right), \tilde{r}_{Di,n} \right] \\
+ \text{Cov} \left[ \sum_{i=1}^{I} \tilde{r}_{Di,n} \omega_i^F \left( 1 - t_{eff,D}^F + e \left( 1 - t_{eff,D,e}^F \right) \right)^2, \tilde{r}_{Di,n} \right].
\] (7.5)

We define the terms

\[
t_{seCG}^F \equiv t_{eff,CG}^F + e \left( 1 - t_{eff,CG,P,e}^F \right) \quad t_{seD}^F \equiv t_{eff,D}^F + e \left( 1 - t_{eff,D,e}^F \right)
\] (7.6)
We consider the definition of the foreign investor’s value weighted share and expand the above equation by the definition of the world market portfolio (eq. (5.81)).
Market equilibrium implies that the demand equals an exogenous quantity of each asset (eq. (7.2)).

\[
\frac{(1 + \pi^F)^2}{A^F} \frac{M_0}{M_0} (E [\bar{r}_{CG_i} + \bar{r}_{D_i}] - r_f^F)
\]

\[
= (1 - tse_{CG}^F)^2 Cov \left[ \sum_{k=1}^{K} \bar{r}_{CG_i,n} \frac{V_k,0_0}{M_0}, \bar{r}_{CG_i,n} \right] \frac{M_0}{P_0^F}
\]

\[+ (1 - tse_{D}^F) (1 - tse_{CG}^F) Cov \left[ \sum_{k=1}^{K} \bar{r}_{D_i,n} \frac{V_k,0_0}{M_0}, \bar{r}_{CG_i,n} \right] \frac{M_0}{P_0^F}
\]

\[+ (1 - tse_{D}^F) (1 - tse_{CG}^F) Cov \left[ \sum_{k=1}^{K} \bar{r}_{CG_i,n} \frac{V_k,0_0}{M_0}, \bar{r}_{D_i,n} \right] \frac{M_0}{P_0^F}
\]

\[+ (1 - tse_{D}^F)^2 Cov \left[ \sum_{k=1}^{K} \bar{r}_{D_i,n} \frac{V_k,0_0}{M_0}, \bar{r}_{D_i,n} \right] \frac{M_0}{P_0^F}
\]

We can insert the world capital gains and dividend rate of return (eq. (7.4)).

\[
\frac{(1 + \pi^F)^2}{A^F} (E [\bar{r}_{CG_i} + \bar{r}_{D_i}] - r_f^F)
\]

\[= \left( (1 - tse_{CG}^F)^2 Cov [\bar{r}_{CG_i,n}, \bar{r}_{CG_m,n}] + (1 - tse_{D}^F) (1 - tse_{CG}^F) Cov [\bar{r}_{CG_i,n}, \bar{r}_{D_m,n}] \right) \frac{M_0}{P_0^F}
\]

\[+ (1 - tse_{D}^F) (1 - tse_{CG}^F) Cov [\bar{r}_{D_i,n}, \bar{r}_{CG_m,n}] + (1 - tse_{D}^F)^2 Cov [\bar{r}_{D_i,n}, \bar{r}_{D_m,n}] \right) \frac{M_0}{P_0^F}
\]

Equivalently, we separate the investor’s tax, inflation and appreciation of exchange rate from the excess rate of return in order to install the nominal rate of return before tax by considering the definition of foreign real asset rate of returns after tax (eq. (5.31) and eq. (5.35)). We reduce the inflation rate and sum the terms in the bracket, so we can derive the following equation.
\[ \frac{1 + \pi^F}{A^F} \left( E[\tilde{r}_{CG, i, n}] (1 - t^F_{eff, CG}) + E[\tilde{r}_{CG, i, n}] e (1 - t^F_{eff, CG, P, e}) - \epsilon t^F_{eff, D, e} \right) + E[\tilde{r}_{DI, i, n}] (1 - t^F_{eff, D}) + E[\tilde{r}_{DI, i, n}] e (1 - t^F_{eff, D, e}) - \epsilon \right) 

\] 

\begin{align*}
&= \left( (1 - tse^F_{CG})^2 \text{Cov} [\tilde{r}_{CG, i, n}, \tilde{r}_{CG, m, n}] + (1 - tse^F_{CG}) (1 - tse^F_{CG}) \text{Cov} [\tilde{r}_{CG, i, n}, \tilde{r}_{D, m, n}] \\
&\quad + (1 - tse^F_{D}) (1 - tse^F_{CG}) \text{Cov} [\tilde{r}_{DI, i, n}, \tilde{r}_{CG, m, n}] + (1 - tse^F_{D})^2 \text{Cov} [\tilde{r}_{DI, i, n}, \tilde{r}_{D, m, n}] \right) \frac{M_0}{P_0}^F \\
\end{align*}

CAPT prices every risky asset allowing the world market portfolio to be applied. This benchmark is perfectly correlated with the world market capital gains and dividend rate of return. The expected capital gains and dividend rate of return on the world market portfolio is endogenously determined, since they are weighted averages of the expected rate of returns of the individual assets. We can prove the application of the world market portfolio to equation (7.11) by multiplying with the share of each asset in the nominal world market portfolio \( \frac{n^V_k M_0}{M_0} \) and aggregating over all risky assets (König (1990) and Mai (2008)). We recall that the world market portfolio capital gains and dividend return rate of is the sum of value weighted rate of return of each risky asset (eq. (7.4)) and consider that the aggregated quotient \( \sum_{k=1}^{K} \frac{n^V_k \tilde{r}_{k}}{M_0} \) equals unity. The elimination of the term \( \frac{1 + \pi^F}{A^F} \) and the real value of the world market portfolio \( \frac{M_0}{P_0} \) produces the following equilibrium pricing relationship (S. appendix D).
Based on equilibrium on the capital market we developed the Tax International Capital Asset Pricing Model with homogeneous expectations.\footnote{Based on equilibrium on the capital market we developed the Tax International Capital Asset Pricing Model with homogeneous expectations.}

\[ E[\tilde{r}_{CG,i,n}] (1 - t_{eff,CG}) + E[\tilde{r}_{CG,m,n}] e (1 - t_{eff,CG,P,e}) - et_{eff,CG,P,e} \\
+ E[\tilde{r}_{D,i,n}] (1 - t_{eff,D}) + E[\tilde{r}_{D,m,n}] e (1 - t_{eff,D,e}) \\
- r_{I,n} (1 - t_{eff,I}) - r_{I,n} e (1 - t_{eff,I,P,e}) + et_{eff,I,P,e} \\
= (E[\tilde{r}_{CG,m,n}] (1 - t_{eff,CG}) + E[\tilde{r}_{CG,m,n}] e (1 - t_{eff,CG,P,e}) - et_{eff,CG,P,e} \\
+ E[\tilde{r}_{D,m,n}] (1 - t_{eff,D}) + E[\tilde{r}_{D,m,n}] e (1 - t_{eff,D,e}) \\
- r_{I,n} (1 - t_{eff,I}) - r_{I,n} e (1 - t_{eff,I,P,e}) + et_{eff,I,P,e}) \beta_{hE} \\
\]

\[
\beta_{hE} = \frac{(1-t_{se,F})^{2} \text{Cov}[\tilde{r}_{CG,i,n}, \tilde{r}_{CG,m,n}] + (1-t_{se,F}) (1-t_{se,D}) \text{Cov}[\tilde{r}_{CG,i,n}, \tilde{r}_{D,m,n}] + \text{Cov}[\tilde{r}_{D,i,n}, \tilde{r}_{CG,m,n}) + (1-t_{se,F})^{2} \text{Cov}[\tilde{r}_{D,i,n}, \tilde{r}_{D,m,n})}{(1-t_{se,F})^{2} \text{Var}[\tilde{r}_{CG,m,n}] + (1-t_{se,F}) (1-t_{se,D}) \text{Var}[\tilde{r}_{CG,m,n}] + (1-t_{se,F})^{2} \text{Var}[\tilde{r}_{D,m,n})}
\]

\[ 7.2 \text{ Interpretation} \]

The derived version of the Tax-IntCAPM is independent from investors’ preferences, since all investors reside in the foreign state and have homogeneous expectations. The investors have homogenous expectations and hold the same shares of assets in their tangential portfolio, the tangential portfolio is the market portfolio.\footnote{Chapter 5.3.} In contrast to the Tax-IntCAPM, the excess rate of return and the risk premium are not influenced by domestic and foreign investors’ tax factors but adapted to foreign investor’s tax and exchange rates. Equivalently to the Tax-IntCAPM, the $\beta_{hE}$ factor is influenced by tax and exchange rates which is due to differential taxation of stochastic dividend and capital gains.\footnote{Cf. Mai (2008).} Since the foreign 

\[ \text{The derived Tax-IntCAPM is plausible. By ignoring the domestic investor in the Tax-IntCAPM the tax factors turn to tax rates and the foreign risk aversion factor becomes one. The $\beta$-factor of the Tax-IntCAPM is equal to $\beta_{hE}$. By ignoring tax rates and the appreciation of nominal exchange rate, we have the nominal IntCAPM.} \]
An empirical version of the Tax-IntCAPM with homogenous expectations can be developed to test the explanatory power of the Tax-IntCAPM.\textsuperscript{221}

Figure 7.1 represents the Tax-IntCAPM with homogenous expectations.\textsuperscript{222} Any investor determines the expected return of an asset by the nominal risk-free rate and a world market risk premium, composed of the product of the beta-factor and the nominal expected return on the world market portfolio in excess of the nominal risk-free rate.

The above securities market line represents the IntCAPM under exchange gains. The expected return of the world market portfolio, which is composed of world market dividend, world market exchange gains on dividend, world market capital gains and world market exchange gains on capital gains before tax, is the sum of interest, exchange gains on interest and world market risk premium before tax. The middle securities market line is equivalent to the IntCAPM under PPP as derived in chapter 3.2.2. The lower securities market line represents the Tax-IntCAPM with homogeneous expectations. The expected return of the world market portfolio, which is composed of world market dividend, world market exchange gains on dividend, world market capital gains and world market exchange gains on capital gains after tax, is the sum of interest, exchange gains on interest and world market risk premium after tax.

The effects of an increase in the effective exchange gains tax rate on interest and principal depend on the exchange gains on interest and principal and on the beta-factor. If the beta-factor is higher than unity, an increase in the effective exchange gains tax rate on interest and principal will lead to a higher expected stock return and vice versa.\textsuperscript{223} The effects of a change in the effective exchange gains tax rate on capital gains and principal

\textsuperscript{221}Due to the complicated structure of the $\beta_{FE}$, Mai (2006b) considers an equivalent version of the Tax-CAPM as impracticable. Every variable of the Tax-IntCAPM is empirically observable, so there are no objections to implementing and testing the derived empirical version of Tax-IntCAPM. Roll (1977) believes that the CAPM cannot be tested empirically at all due to the non-observability of the market portfolio; nevertheless, countless empirical studies exist (Dausend and Schmitt (2007)). The testing of an empirical version of the Tax-IntCAPM faces certain obstacles: Dividends are distributed annually, so there is a certain tension between the amount of data, the prevalence of the tax system and the nature of the stock.

\textsuperscript{222}Cf. Institut Der Wirtschaftsprüfer In Deutschland E.V. (2008) for a comparable illustration.

\textsuperscript{223}The equivalent conclusion we can take for the effective exchange gains tax rate on capital gains and principal and dividend for deterministic dividend.
expected return

World market risk premium

Exchange gain on interest

Interest

0

1

β

β_{\text{hE}}

Figure 7.1: Tax-IntCAPM with Homogeneous Expectations

as well as on dividends are hardly interpretable since these tax rates are an integral part of the beta-factor.

The Tax-IntCAPM with homogeneous expectations allows us to derive hypotheses for
international stock returns.\footnote{These hypotheses need to be tested in an empirical version of the Tax-IntCAPM with homogeneous expectations.}

**Hypothesis 1** International taxation of dividend, capital and exchange gains and interest

*In international stocks the source state’s tax and the residence state’s tax incorporating the methods of reducing and avoiding double taxation on dividend, capital and exchange gains and interest are priced.*

The tax-clientele theory suggests that higher (lower) tax-rate investors should, ceteris paribus, concentrate their portfolios in tax-favored assets (Forbes and Benek \(1988\)) and Collins and Murphy \(1995\)).

**Hypothesis 2** International tax clientele

*The differential taxation of dividend, capital and exchange gains as well as interest lead to international tax clienteles.*

**Hypothesis 3** Exchange gains

*In international stocks’ the non-linear deterministic behavior of exchange gains is priced.*

**Hypothesis 4** Exchange gains taxation

*In the pricing of international stocks exchange gains taxation can lead to home bias.*

### 7.3 Implementation

Based on chapter 5.2.5 we implement the Tax-IntCAPM with homogeneous expectations. The beta-factor \(\beta_{hE}\) involves the estimation of the nominal world market capital gains and dividend rate of return, the market shall comprise all equities in the world \(Lally \ (1996)\)). As the risky asset we consider agains the K+S AG stock and as the world market portfolio
we consider the Morgan Stanley Capital International All Country World Index (MSCI ACWI).\textsuperscript{225} In November 2007 the K+S AG stock became part of the MSCI ACWI.\textsuperscript{226}

Example 6 Tax-IntCAPM

The expected nominal world capital gains rate of return $E[\tilde{r}_{CG,m,n}]$ is -16.06\% whereas the expected nominal world dividend rate of return $E[\tilde{r}_{D,m,n}]$ is 0.5\%. Equivalently to the determination of the rate of return of the K+S AG stock we applied the logarithm and annualized the return. As in Schmid and Trede (2006) we annualize the variances and covariances. The calculation beta-factor of the Tax-IntCAPM with homogeneous expectations consists of several components, these are presented in the following table.

<table>
<thead>
<tr>
<th>Components of $\beta_{hE}$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tse_{CG}$</td>
<td>0.0518</td>
</tr>
<tr>
<td>$tse_{D}$</td>
<td>0.0736</td>
</tr>
<tr>
<td>$Cov[\tilde{r}<em>{CG,K+S,n}, \tilde{r}</em>{CG,m,n}]$</td>
<td>0.0479</td>
</tr>
<tr>
<td>$Cov[\tilde{r}<em>{CG,K+S,n}, \tilde{r}</em>{D,m,n}]$</td>
<td>-0.0473</td>
</tr>
<tr>
<td>$Cov[\tilde{r}<em>{D,K+S,n}, \tilde{r}</em>{CG,m,n}]$</td>
<td>-0.0048</td>
</tr>
<tr>
<td>$Cov[\tilde{r}<em>{D,K+S,n}, \tilde{r}</em>{D,m,n}]$</td>
<td>-0.0046</td>
</tr>
<tr>
<td>$Cov[\tilde{r}<em>{CG,m,n}, \tilde{r}</em>{D,m,n}]$</td>
<td>0.0447</td>
</tr>
<tr>
<td>$Var[\tilde{r}_{CG,m,n}]$</td>
<td>0.0457</td>
</tr>
<tr>
<td>$Var[\tilde{r}_{D,m,n}]$</td>
<td>0.0478</td>
</tr>
</tbody>
</table>

Table 7.1: Calculation of Components of $\beta_{hE}$

\[ -0.0412 = \frac{(1-0.0518)^2 0.0479+(1-0.0518)(1-0.0736)(-0.0473+(-0.0048))+(1-0.0736)^2(-0.0046)}{(1-0.0518)^2 0.0457+2(1-0.0518)(1-0.0736)0.0447+(1-0.0736)^2 0.0478} \]

The low beta-factor might be due to the low empirical data of dividend. Under deterministic dividend the beta-factor would be 1.0481.

\textsuperscript{225}The MSCI ACWI is a market capitalization weighted index that is designed to measure the equity market performance of developed and emerging markets. As of May 2010, the index comprised 45 country indices, including Germany and Russia. The index measures the market performance, including both price performance and income from (gross) dividend payments, s. www.msci.com/products/indices/equity/definitions. The MSCI ACWI FM comprises 71 countries, but the starting day was November 30, 2007 and hence the index does not include enough data.

\textsuperscript{226}S. financial report 2007 of the K+S AG.
\[ E[\tilde{r}_{CG,i,n}] (1 - 0.13) + E[\tilde{r}_{CG,i,n}]( -0.0899)(1 - 0.13) - (-0.0899)0.13 \\
+ E[\tilde{r}_{D,i,n}](1 - 0.15) + E[\tilde{r}_{D,i,n}]( -0.0899)(1 - 0.15) \\
- 0.0252(1 - 0.13) - 0.0252(-0.0899)(1 - 0.13) + (-0.0899)0.13 \\
= (-0.1606(1 - 0.13) + (-0.1606)(-0.0899)(1 - 0.13) - (-0.0899)0.13 \\
+ 0.005(1 - 0.15) + 0.005(-0.0899)(1 - 0.15) \\
- 0.0252(1 - 0.13) - 0.0252(-0.0899)(1 - 0.13) + (-0.0899)0.13)(-0.0412) \]

Foreign investor’s expected nominal capital gains rate of return of the K+S stock amounts to 1.54% if the expected nominal dividend rate of return is 1.77%. If we assume that the nominal capital gains rate of return is 47.31%, than the expected dividend rate of return would be -45.08%.

We compare the Tax-IntCAPM with homogenous expectations with its counterpart where exchange gains are not subject to taxation (S. appendix D). Under the assumption of irrelevance of exchange gains taxation the expected nominal capital gains rate of return would be 1.55% if the nominal dividend rate of return is 1.77%. If we assume that the nominal capital gains rate of return is 47.31%, than the expected dividend rate of return would be -45.19% under irrelevance of exchange gains taxation.

The derivation of the Tax-IntCAPM gives the ball back to the empiricists, we need to conduct empirical investigations to study the explanatory power of this new Tax-IntCAPM and its hypotheses.

\[^{227}\text{Cf. Lally (1996).}\]
8 Critique

“Criticism is prejudice made plausible.”

Henry Louis Mencken, US editor (1880 – 1956)

It is evident that certain concepts do not seem to apply in the real world. Certain assumptions do not coincide with the conditions of reality, and as a consequence certain phenomena in reality are unexplainable by the model. The assumptions underlying the Tax-IntCAPM are very restrictive. The restrictions of unlimited short-sale and borrowing have been lifted and integrated into the Tax-IntCAPM. Nevertheless, further concepts are to be reviewed. In this critique, we address the answers to the questions

How far is reality distorted by making certain assumptions and to what kind of conclusions do they lead?

On account of the fact that the author’s aim is to integrate taxation into IntCAPT, we regard critically explicitly those concepts that impact the framework of international taxation. We study the distortion of the framework of international taxation of these assumptions and work out the results of an approach to reality of these assumptions.

We used formal models which are based on definitions and simplifying assumptions deemed appropriate. On this basis we used logical operations to draw conclusions whose validity can be checked by an expert third person at any time. Assuming that we did not make mistakes during the logical operations, our results can only be legitimately criticized by referring to a possible use of inappropriate assumptions (Kruschwitz and Löffler (2010)).
The critique of our models can be categorized into four dimensions: Incomplete international taxation, incomplete exchange rate modeling, missing risk factors and the question of investor's rationality.

Assumption 6 implies that gains and losses are equally taxed. Clark (2007) derives a CAPM with asymmetric taxation of gains and losses. In general, capital and exchange gains and losses are asymmetrically taxed and tax systems incorporate limited loss offset. Gains can be taxed at statutory rates while losses can be carried back to obtain refunds of taxes paid in prior years or carried forward to be offset against future taxes payable (Long (1977) and Shevlin (1990)). Such a conception would imply the derivation of the Tax-IntCAPT to a multiperiod context. Fama (1970b) discusses conditions for the development of the single period CAPM to a multiperiod context. Wiese (2006a) assumes that the extension to a multiperiod framework leads to a CAPM where investors hedge against the risks of insecure dividends.

In assumption 7 we excluded the existence of progressive tax rates. If the tax takes a progressive curve further income influences the tax payments (Mai (2008)). Litzenberger and Ramaswamy (1979), Litzenberger and Ramaswamy (1980), König (1990) and Wiese (2006b) integrated progressive taxes for deterministic income in Tax-CAPT while Lai (1989) and Wang (1995) apply progressive tax rates for stochastic income in the framework of the CCAPM.228 Progressive tax rates are implemented in many countries and they furthermore reduce arbitrage opportunities.229 Progressive tax rates imply stochastic tax rates, which is in the vein of tax policy resulting from tax reforms and different interpretations of tax laws by investors, fiscal authorities and tax courts (Singer (1979), Niemann (2004) and Niemann (2007)).

In assumption 17 and in chapter 5.2.3 we lay the foundation for the non-linear model of exchange rates.

We assumed that the trade balance is determined by exchange rates (assumption 23). Other factors such as the cost of production in the exporting economy in relation to those in the importing economy and taxes or restrictions on trade can also influence the trade balance (Ostry and Rose (1992) and Siebert (2006)). In order to assess the domestic costs

\footnote{As far as the critique of the CCAPM is concerned, s. chapter 3.2.}
\footnote{S. footnote 22.}
of production in relation to those of the foreign state, the concept of real exchange rates should be applied. However, the empirical evidence of the relationship between exchange rates and trade balance remains mixed. While the view that devaluation improves the trade balance is empirically supported by Gylfason and Risager (1984), Bahmani-Oskooee (1985) and Shirvani and Wilbratte (1997), Rose and Yellen (1989) and Rose (1991) concluded that there is no significant relationship between the trade balance and the real exchange rate. Ostry and Rose (1992) conclude that there is no clear conclusion about the effect of a tariff change on the trade balance. However, empirical studies support the argument that exchange rates are likely to have a strong influence on the trade balance (Prasad and Gable (1997)).

The model requires the determination of several sensitivity parameters. These parameters must be determined in an estimation model and are found to have a highly unstable impact on the exchange rate in the case of a slight change.

Our models explain the expected returns only by a single variable – the risk of an asset relative to the market. It is reasonable to assume that there exist other dimensions of risk in the world financial market that are sufficiently systematic to receive a non-zero reward. Bodnar (2003) among many others introduce with country, industry, political and liquidity further dimensions of international risk. Dimensions of risk must be recognized in such a way that there are traded securities capable of indicating the price of each risk dimension. From an empirical point of view it seems to be necessary to perform some audacious transposition, so that a particular venture can be priced, even if only approximately.

We assumed the normal distribution for our model (assumption 4). The normal distribution is not descriptive for capital asset pricing because of limited liability: In the context of the prevailing legal system, all stock holders are protected by limited liability so their rates of return are bounded below by -100% and empirical distributions of returns have tails that are leptokurtic (Balvers (2001)). Furthermore, many commonly used utility functions are not defined at negative values and hence cannot be applied in the framework of normal distribution (Balvers (2001)). The CAPM should apply to all asset classes. However, under the assumption of normal return distributions the pricing of options can

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be excluded, since option returns are non-normally distributed. However, the multivariate normality is solely a sufficient and not a necessary condition for investors to choose mean-variance efficient portfolios (Balvers (2001)). Elliptical distributions including normal distributions better describe asset return behavior (Balvers (2001) and Hamada and Valdez (2004)).231 Owen and Rabinovitch (1983) and Ingersoll (1987) integrate elliptical distributions into the CAPM. Hamada and Valdez (2004) derive call option prices when the underlying is elliptically distributed.

For the Tax-IntCAPM we pursued the EMH (assumption 12). The validity of the EMH has been questioned by critics. Many financial economists and statisticians began to believe that stock prices are at least partially predictable. The assumption of capital market equilibrium is a central theorem in CAPT (assumption 14). Disequilibrium theory provides a sound theoretical reason why price trends are possible as markets adjust to information shocks. The main thrust of this theory is that prices do not quickly adjust to information shocks and therefore markets are in short-run disequilibrium (Beja and Goldman (1980)). The adjustment period to a new equilibrium is slowed by factors such as transaction costs and the cost of acquiring and evaluating information (Lukac (1988)). In the vein of the neoclassical finance paradigm, we assume that investors are rational (assumption 21). They should act in an unbiased fashion and make decisions by maximizing their self-interests. In essence, the economic concept of rationality means that economic agents make the best choices possible for themselves. Although appealing, this concept entails strong and unrealistic assumptions about human behavior and the functioning of financial markets. We assumed that economic agents process new information correctly and make decisions that are normatively acceptable (Barberis and Thaler (2002)) and that they are capable of integrating and considering many different pieces of information and must fully understand the future consequences of all their actions. Moreover, financial markets must be frictionless, such that security prices reflect their fundamental value and the influence of irrational market participants is corrected by rational traders. Unfortunately, we as human beings do not possess all of these capabilities and characteristics. We fail to update beliefs correctly, have limitations in the processing of information, and only have certain capabilities to solve complex problems (Baltussen (2009)).

Behavioral economists attribute the imperfections in financial markets to a combination of cognitive biases such as overconfidence, overreaction, representative bias, information bias, and various other predictable human errors in reasoning and information processing. The new breed of economists emphasized psychological and behavioral elements of stock-price determination, and came to believe that future stock prices are somewhat predictable on the basis of past stock price patterns as well as certain “fundamental” valuation metrics. Moreover, many of these economists were even making the far more controversial claim that these predictable patterns enable investors to earn excess risk-adjusted rates of return (Hawawini and Keim (1994) and Malkiel (2003)).

The field of behavioral finance argues that behavior that differs from that of the rational investor can cause prices in financial markets to deviate from their fundamental value. By applying insights from behavioral sciences, the field of behavioral finance tries to improve our understanding of financial decisions and their affect on market prices.

Behavioral tax research addresses the topics of tax compliance, tax knowledge acquisition and transmission, tax professionals’ judgement and decision making, and behavioral changes induced by tax incentives (Outslay (1995) and Kirchler (2007)). The impact of taxes on international asset pricing by use of experimental economics as initiated by Rolfe and White (1997), Collins and Murphy (1995) and Anderson and Butler (1997) opens a new field of tax research in finance.

The thesis intends to outline and inspire future research on taxation in IntCAPT. When one considers all these extension possibilities of the model, it is obvious that the model analyzed here is an elementary one. It provided some insights into the effects of taxation on international asset pricing under varying market structures and is expected to build up the means of moving to a more general and thus realistic, setting for a better understanding of taxation in international asset pricing, exchange rate modeling, the integration of further risk components and market participants’ behavior.
9 Conclusions

"People do not like to think. If one thinks, one must reach conclusions."
Helen Keller, US blind and deaf educator (1880 – 1968)

The dissertation is inspired by the compelling theoretical and empirical arguments that the benefit of international diversification is distorted due to the existence of home bias. The delineation of national groups of investors by deviation from Relative PPP and international taxation is evidence of the international market puzzle of why domestic assets can offer a superior risk and return relationship to foreign assets. The research problem of the dissertation was the pricing of international assets under barriers in the form of taxes. We raised the research question “What are the Impacts of Taxation on the International Capital Asset Pricing Model?” and hypothesized that the integration of international taxation leads to new models of IntCAPT. By deriving and interpreting the Tax-IntCAPM we gain new insights into the framework of taxation in IntCAPT.

By modeling the dividend, capital gains, interest and exchange gains tax system as well as the methods of double taxation reduction in chapter 4, we presented a model for a general integrated international tax framework which lays the foundation for the theoretical exposition in order to derive the Tax-IntCAPM in chapter 5. The heterogeneity of international market participants was elaborated on by integrating a tax system differing between the domestic and foreign states and the entire income classes in IntCAPT (capital gains, dividends, interest and exchange gains), the concept of deviation from Relative PPP and the integration of non-linear behavior of exchange rates.
We analyzed certain tax constellations in which the decision problem of the Tax-IntCAPM would be equivalent to the one of the Tax-CAPM.

Through the integration of a non-linear deterministic exchange rate process, characterized by a market approach in which demand for foreign currency equals the supply, we pursued the new approach of Peters (1996) to regard the capital and currency markets as complex, interdependent systems which are characterized by the coexistence of randomness and determination. The equilibrium pricing relationship of the Tax-IntCAPM is in line with the concept of IntCAPT, which says that the expected international return is composed of the risk-free rate and a world risk premium. The equilibrium pricing relationship incorporates the taxation of all income types in international asset pricing - dividends, interest, capital and exchange gains - leading to factors which are the equilibrium market gains (costs) of international taxes and deviation from Relative PPP. In contrast to Lally (1996), differential inflation rates are considered. The value of the factors depends decisively on the features of international taxation and the assignment of the right of taxation.

In contrast to the Tax-IntCAPM of Lally (1996), it was found to be important to consider the features of exchange gains taxation and the characteristic features of the exchange gains and losses tax system: The raise of exchange gains taxes in dependence of the recognition, character, nature, source and hedging. Exchange gains are differently taxed from the other income types in IntCAPT which leads to the integration of the foreign tax factor on exchange gains of capital gains, dividends and interest in the excess return.

In chapter 6 we realize that tax rate differentials across income classes in an international framework can generate arbitrage. In order to exclude international unlimited arbitrage opportunities we have to include short sale and borrowing restrictions into the Tax-IntCAPM. The imposition of an amounted short sale and borrowing restriction leads to a Tax-IntCAPM which is adapted by the aggregated short sale and borrowing factor, which can be interpreted as the market value of short sale and borrowing restrictions and influenced by international taxation and exchange gains.

We operationalize the Tax-IntCAPM in chapter 7 through the assumption of homogeneous foreign market participants and the Tax-IntCAPM with homogeneous expectations which is not influenced by domestic and foreign investors’ tax and exchange gains factors but adapted by foreign investor’s tax and exchange gains rates.
The critique in chapter 8 presents the boundaries of the model in order to elaborate on the phenomena of reality which are not able to be explained by the model. We come to the conclusion that stochastic tax rates, an advanced modeling of exchange rates, the integration of further risk components, and experiments studying market participants’ behavior are ideas which should be considered in new approaches of Tax-IntCAPM.

The research in this thesis has been conducted with the hope that it will shed light on a deeper understanding of the integration of taxation in IntCAPM. In the light of the preceding chapters, the dissertation has been successful in answering the research question “What are the Impacts of Taxation on the International Capital Asset Pricing Model?” by deriving and providing perspectives on novel IntCAPMs incorporating the framework of international taxation.
A Taylor Series Approximation of the Utility Function

The Taylor series is a series expansion of a function about a point; it is a representation of a function as an infinite sum of terms calculated from the values of its derivatives at a single point. The value of the utility function can be approximated by terms related to the expected value of the random input under the assumption that the random input is close to its means. Thus, the expected utility of wealth can be presented as the desirability of the expected return plus the rate of change in the utility returns times the variance of returns plus some smaller size terms (Huang and Litzenberger (1988) and Kruschwitz (2007)).

\[
U \left( \tilde{W} \right) = U \left( E \left[ \tilde{W} \right] \right) + \frac{\partial U \left( E \left[ \tilde{W} \right] \right) \left( \tilde{W} - E \left[ \tilde{W} \right] \right)}{1!} + \frac{\partial^2 U \left( E \left[ \tilde{W} \right] \right) \left( \tilde{W} - E \left[ \tilde{W} \right] \right)^2}{2!} + \frac{\partial^3 U \left( E \left[ \tilde{W} \right] \right) \left( \tilde{W} - E \left[ \tilde{W} \right] \right)^3}{3!} + \ldots
\]  

(A.1)

The utility function \( U \left( \tilde{W} \right) \) is monotonically increasing and concave in its argument \( \tilde{W} \) (implying desirability, non-satiation and variance aversion) and the wealth \( \tilde{W} \) represents stochastic end-of-period wealth with mean of \( E \left[ \tilde{W} \right] \). Taking expectations produces:
A Taylor Series Approximation of the Utility Function

\[ E \left[ U \left( \tilde{W} \right) \right] = U \left( E \left[ \tilde{W} \right] \right) + \frac{\partial U \left( E \left[ \tilde{W} \right] \right)}{\partial \tilde{W}} E \left[ \left( \tilde{W} - E \left[ \tilde{W} \right] \right) \right] + \frac{\partial^2 U \left( E \left[ \tilde{W} \right] \right)}{\partial \tilde{W}} E \left[ \left( \tilde{W} - E \left[ \tilde{W} \right] \right)^2 \right] + \frac{\partial^3 U \left( E \left[ \tilde{W} \right] \right)}{\partial \tilde{W}} E \left[ \left( \tilde{W} - E \left[ \tilde{W} \right] \right)^3 \right] + \frac{\partial^4 U \left( E \left[ \tilde{W} \right] \right)}{\partial \tilde{W}} E \left[ \left( \tilde{W} - E \left[ \tilde{W} \right] \right)^4 \right] + \cdots \] (A.2)

The above equation indicates a preference for expected wealth and an aversion to variance of wealth for investors with increasing and strictly concave utility functions. We cannot define the expected utility over the expected value and the variance of wealth for arbitrary distributions and preferences, since the last term of the above equation involves higher order moments. We assume that the returns are multivariate normally distributed; the normal distribution can be completely described by its mean and variance. Normal distribution also has the advantage of being stable; e.g. the return of a portfolio consisting of the sum of multivariate normally distributed assets is also normally distributed. The assumption of normal distribution of wealth leads to:

\[ E \left[ \left( \tilde{W} - \mu \right)^k \right] = \begin{cases} 0 & \text{if } k \text{ is impair} \\ 1 \cdot 3 \cdot 5 \cdots \cdot (k - 1) \sigma^2 & \text{if } k \text{ is pair} \end{cases} \quad \text{(A.3)} \]

with \( \mu \) expected value of normal distribution
\( \sigma \) variance of normal distribution

Due to this rule every impair row member is void. The row can be expressed by a term of mean and variance.

\[ E \left[ U \left( \tilde{W} \right) \right] = U \left( \mu \right) + \frac{\partial^2 U \left( \mu \right)}{\partial \tilde{W}} E \left[ \left( \tilde{W} - \mu \right)^2 \right] + \frac{\partial^4 U \left( \mu \right)}{\partial \tilde{W}} E \left[ \left( \tilde{W} - \mu \right)^4 \right] + \cdots \] (A.4)
A Taylor Series Approximation of the Utility Function

\[
E \left[ U \left( \tilde{W} \right) \right] = U \left( \mu \right) + \frac{\partial^2 U \left( \mu \right) 1 \ast \sigma^2}{\partial \tilde{W}} \frac{1}{2!} + \frac{\partial^4 U \left( \mu \right) 1 \ast 3 \ast \sigma^4}{\partial \tilde{W}} \frac{1}{4!} + ... \quad (A.5)
\]

Under the assumption of normal distribution of wealth the expected utility depends solely on the parameters \( \mu \) and \( \sigma \) irrespective of the type of utility function (Balvers (2001) and Kruschwitz (2007)).
B Stein’s Lemma

Stein’s Lemma provides a linearization result for covariances of normally distributed variables of which one argument is a (possible) non-linear, differentiable function.\(^{232}\) It says that the covariance of a normally distributed variable and the differentiable function of a normally distributed variable is equal to the expected function of the normally distributed variable and the covariance of the variables.

\[
\text{Cov} \left[ \tilde{X}, h \left( \tilde{Y} \right) \right] = E \left[ \frac{\partial h \left( \tilde{Y} \right)}{\partial \tilde{Y}} \right] \text{Cov} \left[ \tilde{X}, \tilde{Y} \right] \tag{B.1}
\]

with \(\tilde{X}\) normally distributed variable

\(\tilde{Y}\) normally distributed variable

\(h \left( \tilde{Y} \right)\) differentiable function

Ingersoll’s proof of Stein’s Lemma is given below. The standard decomposition for any two random variables according to Ordinary Least Squares (OLS) regression is used:

\[
\tilde{X} = \alpha + \frac{\text{Cov} \left[ \tilde{X}, \tilde{Y} \right]}{\text{Var} \left[ \tilde{Y} \right]} \tilde{Y} + \bar{\epsilon}. \tag{B.2}
\]

with \(\alpha\) parameter

\(\bar{\epsilon}\) error variable

It is assumed that the expected error variable is equal to zero.

\[ E [ \tilde{\epsilon} ] = E [ \epsilon \tilde{Y} ] = 0 \]  

We insert the standard composition into the left hand side of equation (B.1) which leads to

\[
\text{Cov} \left[ \alpha + \frac{\text{Cov} [ \tilde{X}, \tilde{Y} ]}{\text{Var} [ \tilde{Y} ]} \tilde{Y} + \tilde{\epsilon}, h ( \tilde{Y} ) \right].
\]  

From the linearity property of covariances

\[
\text{Cov} \left[ a \tilde{X}_1 + b \tilde{X}_2, \tilde{Y} \right] = a \text{Cov} \left[ \tilde{X}_1, \tilde{Y} \right] + b \text{Cov} \left[ \tilde{X}_2, \tilde{Y} \right]
\]  

follows for normal distributions.

\[
\frac{\text{Cov} [ \tilde{X}, \tilde{Y} ]}{\text{Var} [ \tilde{Y} ]} \text{Cov} \left[ \tilde{Y}, h ( \tilde{Y} ) \right] + \text{Cov} \left[ \tilde{\epsilon}, h ( \tilde{Y} ) \right].
\]  

On account of the independence of \( \tilde{\epsilon} \) and \( Y \) the covariance between these arguments is zero, which simplifies our equation to:

\[
\frac{\text{Cov} [ \tilde{X}, \tilde{Y} ]}{\text{Var} [ \tilde{Y} ]} \text{Cov} \left[ \tilde{Y}, h ( \tilde{Y} ) \right].
\]  

Given \( \text{Cov} [ \tilde{X}, \tilde{Y} ] = E \left[ ( \tilde{X} - \mu_x ) ( \tilde{Y} - \mu_y ) \right] \) for a continuous distribution and since we know that \( E \left[ ( \tilde{Y} - \mu_y ) E \left[ h ( \tilde{Y} ) \right] \right] \) is equal to zero, the covariance between \( \tilde{Y} \) and \( h ( \tilde{Y} ) \) can be written as follows:

\[
\text{Cov} \left[ \tilde{Y}, h ( \tilde{Y} ) \right] = \int_{-\infty}^{\infty} ( \tilde{Y} - \mu_y ) h ( \tilde{Y} ) f ( \tilde{Y} ) d\tilde{Y}
\]  

\( f ( \tilde{Y} ) \) is a normal density function given as
\[ f(\tilde{Y}) = \frac{1}{\sqrt{\text{Var}[\tilde{Y}] 2\pi}} e^{\left(-\frac{(\tilde{Y} - \mu_Y)^2}{2\text{Var}[\tilde{Y}]}\right)} \]  \hspace{1cm} (B.9)

The differentiation of the normal density function yields

\[ \frac{\partial f(\tilde{Y})}{\partial Y} = -\frac{\tilde{Y} - \mu_Y}{\text{Var}[\tilde{Y}]} f(\tilde{Y}) \]  \hspace{1cm} (B.10)

Substituting the above term into equation

\[ \text{Cov}[\tilde{Y}, h(\tilde{Y})] = -\text{Var}[\tilde{Y}] \int_{-\infty}^{\infty} h(\tilde{Y}) df(\tilde{Y}). \]  \hspace{1cm} (B.11)

The right hand side of the above equation is integrated by parts.

\[ \text{Cov}[\tilde{Y}, h(\tilde{Y})] = -\text{Var}[\tilde{Y}] \left( \int_{-\infty}^{\infty} h(\tilde{Y}) d(\tilde{Y}) + h(\tilde{Y}) d(\tilde{Y}) \int_{-\infty}^{\infty} \right) \]  \hspace{1cm} (B.12)

The last term vanishes at both limits, since the normal density converges to zero at the limits, as long as \( h(\tilde{Y}) \) does not go to infinity too quickly; or, in other words, as long as \( h(\tilde{Y}) = 0 e^{[\tilde{Y}^2]} \). Thus, we have

\[ \text{Cov}[\tilde{X}, h(\tilde{Y})] = \frac{\text{Cov}[\tilde{X}, \tilde{Y}]}{\text{Var}[\tilde{Y}]} \text{Cov}[\tilde{Y}, h(\tilde{Y})] = E \left[ \frac{\partial h(\tilde{Y})}{\partial \tilde{Y}} \right] \text{Cov}[\tilde{X}, \tilde{Y}]. \]  \hspace{1cm} (B.13)
C Derivation of Tax International Capital Asset Pricing Model

C.1 Non-linear Behavior of Exchange Rate

The value table of example 4 is given. It contains the time series of the non-linear behavior of exchange rate with $\alpha = 5$.

<table>
<thead>
<tr>
<th>Day</th>
<th>Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>1.4430</td>
</tr>
<tr>
<td>3</td>
<td>2.0189</td>
</tr>
<tr>
<td>5</td>
<td>0.3041</td>
</tr>
<tr>
<td>6</td>
<td>1.0830</td>
</tr>
<tr>
<td>7</td>
<td>0.8120</td>
</tr>
<tr>
<td>8</td>
<td>1.3387</td>
</tr>
<tr>
<td>9</td>
<td>0.5121</td>
</tr>
<tr>
<td>10</td>
<td>1.9894</td>
</tr>
<tr>
<td>11</td>
<td>0.2144</td>
</tr>
<tr>
<td>12</td>
<td>1.1406</td>
</tr>
<tr>
<td>13</td>
<td>0.3597</td>
</tr>
<tr>
<td>14</td>
<td>1.5556</td>
</tr>
<tr>
<td>15</td>
<td>0.3867</td>
</tr>
<tr>
<td>16</td>
<td>1.9375</td>
</tr>
<tr>
<td>17</td>
<td>0.2258</td>
</tr>
<tr>
<td>18</td>
<td>1.2451</td>
</tr>
<tr>
<td>19</td>
<td>0.6402</td>
</tr>
<tr>
<td>20</td>
<td>1.8565</td>
</tr>
</tbody>
</table>

Table C.1: Value Table of Time Series of Exchange Rate

The graph of the non-linear behavior of an exchange rate with $\alpha$ equal to 4 is given. If the parameter $\alpha$ is 4, the exchange rate adopts a totally different path. The maximum value is 2.2634, whereas the minimum value is 0.1377 for the first 25 days. The value table of example C.1 is given. It contains the time series of the non-linear behavior of exchange rate with $\alpha = 4$. The return of the nominal exchange rate is -0.5935%.
C Derivation of Tax International Capital Asset Pricing Model

C.2 Expected Return

In the following, we derive the expected return of the Tax-IntCAPM. First of all, we derive the expected return under stochastic dividend. The derivation of the expected return under deterministic dividend is equivalent, so we present the differing factors.
C.2.1 Stochastic Dividends

We consider solely the excess return of the individual optimum and sum them over the domestic and foreign investors.

\[
\frac{(1 + \pi_D)^2}{A_D} E[\tilde{r}_{CG,i} + \tilde{r}_{Di}] - r_D^I + \frac{(1 + \pi_F)^2}{A_F} E[\tilde{r}_{CG,i} + \tilde{r}_{Di}] - r_F^I
\]  

(C.1)

We decompose the return of the asset to formulate the nominal return of capital gains, dividend and interest. We consider the return of the risky and risk-free asset of the domestic investor (eq. (5.2) and eq. (5.4)) and of the foreign investor (eq. (5.31) and eq. (5.35)). The investor’s rate of returns is inserted into the excess return and we reduce the inflation rate.

\[
\frac{1 + \pi_D}{A_D} E[\tilde{r}_{CG,i,n}] (1 - t_D) + \frac{1 + \pi_D}{A_D} E[\tilde{r}_{Di,n}] (1 - t_D) 
- \frac{1 + \pi_F}{A_F} E[\tilde{r}_{CG,i,n}] (1 - t_{eff,CG}) + \frac{1 + \pi_F}{A_F} E[\tilde{r}_{CG,i,n}] e (1 - t_{eff,CG,P,e}) 
+ \frac{1 + \pi_F}{A_F} (1 + e) - \frac{1 + \pi_F}{A_F} e t_{eff,CG,P,e} 
+ \frac{1 + \pi_F}{A_F} E[\tilde{r}_{Di,n}] (1 - t_{eff,D}) + \frac{1 + \pi_F}{A_F} E[\tilde{r}_{Di,n}] e (1 - t_{eff,D,e}) 
- \frac{1 + \pi_F}{A_F} E[\tilde{r}_{Di,n}] (1 - t_{eff,I}) - \frac{1 + \pi_F}{A_F} E[\tilde{r}_{Di,n}] e (1 - t_{eff,I,P,e}) 
- \frac{1 + \pi_F}{A_F} (1 + e) + \frac{1 + \pi_F}{A_F} e t_{eff,I,P,e}
\]  

(C.2)

The excess return consists of an expression of expected capital gains, expected dividend, exchange gains and risk-free return. Each income type is differently taxed. We define the domestic and foreign risk aversion factor under inflation.

\[
A_D^{\pi_D} = \frac{1 + \pi_D}{A_D} \quad A_F^{\pi_F} = \frac{1 + \pi_F}{A_F}
\]  

(C.3)

with \( A_D^{\pi_D} \) domestic investor’s risk aversion factor under inflation

\( A_F^{\pi_F} \) foreign investor’s risk aversion factor under inflation
Having defined the terms we insert them into the expression (C.2) and summarize it.

\[
\begin{align*}
A^D_{\pi D} E[\bar{r}_{CG,n}] (1 - t^D_{CG}) + A^D_{\pi D} E[\bar{r}_{D,n}] (1 - t^D_{D}) - A^D_{\pi D} r_{I,n} (1 - t^D_{I}) \\
+ A^F_{\pi F} E[\bar{r}_{CG,n}] (1 - t^F_{eff,CG}) + A^F_{\pi F} E[\bar{r}_{CG,n}] e (1 - t^F_{eff,CG,P,e}) - A^F_{\pi F} e t^F_{CG,e} \\
+ A^F_{\pi F} E[\bar{r}_{D,n}] (1 - t^F_{eff,D}) + A^F_{\pi F} E[\bar{r}_{D,n}] e (1 - t^F_{eff,D,e}) \\
- A^F_{\pi F} r_{I,n} (1 - t^F_{eff,I}) - A^F_{\pi F} r_{I,n} e (1 - t^F_{eff,I,P,e}) + A^F_{\pi F} e t^F_{eff,I,P,e}
\end{align*}
\]

We multiply the bracket by the defined terms \( A^D_{\pi D} \) and \( A^F_{\pi F} \), so we have

\[
E[\bar{r}_{CG,n}] (A^D_{\pi D} - t^D_{CG} A^D_{\pi D}) + E[\bar{r}_{D,n}] (A^D_{\pi D} - t^D_{D} A^D_{\pi D}) - r_{I,n} (A^D_{\pi D} - t^D_{I} A^D_{\pi D}) \\
+ E[\bar{r}_{CG,n}] (A^F_{\pi F} - t^F_{eff,CG} A^F_{\pi F}) + E[\bar{r}_{CG,n}] e (A^F_{\pi F} - t^F_{eff,CG,P,e} A^F_{\pi F}) - A^F_{\pi F} e t^F_{CG,e} \\
+ E[\bar{r}_{D,n}] (A^F_{\pi F} - t^F_{eff,D} A^F_{\pi F}) + E[\bar{r}_{D,n}] e (A^F_{\pi F} - t^F_{eff,D,e} A^F_{\pi F}) \\
- r_{I,n} (A^F_{\pi F} - t^F_{eff,I} A^F_{\pi F}) - r_{I,n} e (A^F_{\pi F} - t^F_{eff,I,P,e} A^F_{\pi F}) + A^F_{\pi F} e t^F_{eff,I,P,e}
\]

We define the world aggregate risk aversion factor under inflation \( A^W_{\pi} \).

\[
A^W_{\pi} = A^D_{\pi D} + A^F_{\pi F}
\]

with \( A^W_{\pi} \) world aggregate risk aversion factor under inflation

and sum foreign and domestic investors’ nominal returns of capital and exchange gains, dividends and interest.

\[
E[\bar{r}_{CG,n}] (A^W_{\pi} - t^D_{CG} A^D_{\pi D} - t^F_{eff,CG} A^F_{\pi F}) + E[\bar{r}_{CG,n}] e (A^F_{\pi F} - t^F_{eff,CG,P,e} A^F_{\pi F}) \\
- e t^F_{eff,CG,P,e} A^F_{\pi F} + E[\bar{r}_{D,n}] (A^W_{\pi} - t^D_{D} A^D_{\pi D} - t^F_{eff,D} A^F_{\pi F}) \\
+ E[\bar{r}_{D,n}] e (A^F_{\pi F} - t^F_{eff,D,e} A^F_{\pi F}) \\
- r_{I,n} (A^W_{\pi} - t^D_{I} A^D_{\pi D} - t^F_{eff,I} A^F_{\pi F}) \\
- r_{I,n} e (A^F_{\pi F} - t^F_{eff,I,P,e} A^F_{\pi F}) + e t^F_{eff,I,P,e} A^F_{\pi F}
\]
We divide by the world aggregate risk aversion factor under inflation.

\[
E[\tilde{r}_{CG_i,n}] \left(1 - \frac{t_{CG}^D A_{CG}^D}{A_n^W} - \frac{t_{eff,CG}^F A_{π}^F}{A_n^W}\right) + E[\tilde{r}_{CG_i,n}] e \left(\frac{A_{π,F}^F}{A_n^W} - \frac{t_{eff,CG,π,F}^F A_{π}^F}{A_n^W}\right)
\]

\[
- e \frac{t_{eff,CG,π,F}^F A_{π}^F}{A_n^W}
\]

\[
+E[\tilde{r}_{Di,n}] \left(1 - \frac{t_{D}^D A_{D}^D}{A_n^W} - \frac{t_{eff,D}^F A_{π}^F}{A_n^W}\right) + E[\tilde{r}_{Di,n}] e \left(\frac{A_{π,F}^F}{A_n^W} - \frac{t_{eff,D,π,F}^F A_{π}^F}{A_n^W}\right)
\]

\[
- r_{I,n} \left(1 - \frac{t_{I}^D A_{D}^D}{A_n^W} - \frac{t_{eff,I}^F A_{π}^F}{A_n^W}\right) - r_{I,n} e \left(\frac{A_{π,F}^F}{A_n^W} - \frac{t_{eff,I,π,F}^F A_{π}^F}{A_n^W}\right) + e \frac{t_{eff,I,π,F}^F A_{π}^F}{A_n^W}
\]

We define the weighted tax factor on capital gains, dividends and interest.

\[
T_{CG} = \frac{t_{CG}^D A_{CG}^D + t_{eff,CG}^F A_{π}^F}{A_n^W}
\]

\[
T_{D} = \frac{t_{D}^D A_{D}^D + t_{eff,D}^F A_{π}^F}{A_n^W}
\]

\[
T_{I} = \frac{t_{I}^D A_{D}^D + t_{eff,I}^F A_{π}^F}{A_n^W}
\]

We define the weighted foreign risk aversion factor as

\[
A_{W}^F = \frac{A_{π,F}^F}{A_n^W}
\]

We define the foreign tax factor on capital gains, dividends and interest exchange gains.

\[
ET_{CG}^F = \frac{t_{eff,CG,π,F}^F A_{π}^F}{A_n^W}
\]

\[
ET_{D}^F = \frac{t_{eff,D,π,F}^F A_{π}^F}{A_n^W}
\]

\[
ET_{I}^F = \frac{t_{eff,I,π,F}^F A_{π}^F}{A_n^W}
\]

Finally, we insert the defined factors into equation (C.8); the derived excess return for the Tax-IntCAPM is:

\[
E[\tilde{r}_{CG_i,n}] (1 - T_{CG}) + E[\tilde{r}_{CG_i,n}] e \left(A_{W}^F - ET_{CG}^F\right) - e ET_{CG}^F
\]

\[
+ E[\tilde{r}_{Di,n}] (1 - T_{D}) + E[\tilde{r}_{Di,n}] e \left(A_{W}^F - ET_{D}^F\right)
\]

\[
- r_{I,n} (1 - T_{I}) - r_{I,n} e \left(A_{W}^F - ET_{I}^F\right) - r_{I,n} e \left(A_{W}^F - ET_{I}^F\right) + e ET_{I}^F
\]
C Derivation of Tax International Capital Asset Pricing Model

C.2.2 Deterministic Dividends

The derivation of the excess return is equivalent to the one of the Tax-IntCAPM under stochastic dividend. Apart from the deterministic dividends; the Tax-IntCAPM under deterministic dividend differs in respect of the factors which are weighted by different risk aversion terms.

We define the domestic and foreign risk aversion terms as follows:

\[
\frac{1 + \pi^D}{A^D t_D^{CG}} = A_{\pi^D,t_D^{CG}}^D \quad 1 + \pi^F \quad \frac{1 + \pi^F}{A^F (1 - t_{eff,CG}^F + e (1 - t_{eff,CG,P,e}^F))} = A_{\pi,F,tse_CG}^F.
\] (C.13)

We define the world aggregate risk aversion factor under deterministic dividend as

\[ A^{W}_{dt} = A_{\pi^D,t_D^{CG}}^D + A_{\pi,F,tse_CG}^F. \] (C.14)

We define the average tax factor on capital gains, dividends and interest under deterministic dividend.

\[
T_{CG,dt} = \frac{t_{CG}^{D} A_{\pi^D,t_D^{CG}}^D + t_{eff,C} A_{\pi,F,tse_CG}^F}{A_{dt}^W} \quad T_{D,dt} = \frac{t_{D}^{D} A_{\pi^D,t_D^{CG}}^D + t_{eff,D} A_{\pi,F,tse_CG}^F}{A_{dt}^W} \quad T_{I,dt} = \frac{t_{I}^{D} A_{\pi^D,t_D^{CG}}^D + t_{eff,I} A_{\pi,F,tse_CG}^F}{A_{dt}^W}
\] (C.15)

We define the weighted foreign risk aversion factor under deterministic dividend as

\[ A_{W,dt}^F = \frac{A_{\pi,F,tse_CG}^F}{A_{dt}^W}. \] (C.16)

We define the foreign exchange gains tax factor on capital gains, dividends and interest
Finally, we derive excess return for the Tax-IntCAPM under deterministic dividend:

\[
E[r_{CG,i,n}] (1 - T_{CG,dt}) + E[r_{CG,i,n}] e \left( A_{W,dt}^F - ET_{CG,dt}^F \right) - e ET_{CG,dt}^F \\
+ r_{D,n} (1 - T_{D,dt}) + r_{D,n} e \left( A_{W,dt}^F - ET_{D,dt}^F \right) \\
- r_{I,n} (1 - T_{I,dt}) - r_{I,n} e \left( A_{W,dt}^F - T_{I,dt} \right) + e ET_{I,dt}^F
\]  

(C.18)

### C.3 Risk Premium

Having derived the excess return, we derive the covariance terms of the risk premiums. First of all we regard only the covariance of the domestic investor. Having derived the domestic investor’s covariance of the risk premium, we derive analogously the foreign investor’s covariance of the risk premium. Finally, the aggregate covariance terms are derived.

#### C.3.1 Domestic Investor

The covariance with the capital gains rate of return was derived in chapter 5.4.1. In the following we derive the covariance with the dividend rate of return.

\[
Cov \left[ \tilde{W}_t^D, \tilde{r}_D^D \right]
\]  

(C.19)

By replacing the investor’s terminal real wealth after taxes by the definition of the portfolio, the following expression can be derived.
\[ Cov \left[ \sum_{i=1}^{I} \omega_i D \left( 1 + \tilde{r}_{CG, i} + \tilde{r}_{D, i} \right) \right] + \omega_0 D \left( 1 + r_{I} \right), \tilde{r}_D \]  \hspace{1cm} (C.20)

Since the covariance with the risk-free asset is zero, we can derive the following term.

\[ Cov \left[ \sum_{i=1}^{I} \omega_i D \left( 1 + \tilde{r}_{CG, i} + \tilde{r}_{D, i} \right), \tilde{r}_D \right] \]  \hspace{1cm} (C.21)

We apply the linearity property of covariances.

\[ Cov \left[ \sum_{i=1}^{I} \omega_i D \tilde{r}_{CG, i} \tilde{r}_D \right] + Cov \left[ \sum_{i=1}^{I} \omega_i D \tilde{r}_{D, i} \tilde{r}_D \right]. \]  \hspace{1cm} (C.22)

The first covariance describes the value weighted real capital gains rate of return with the real dividend rate of return after national tax, and the other one the value weighted real dividend rate of return with the real dividend rate of return after national tax. By considering the definition of nominal capital gains and dividend rate of return (eq. (5.1)), we can separate the tax and inflation rate from the risky real returns of the covariance.

\[ Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG, i} \omega_i D \left( 1 - \frac{t_D}{1 + \pi_D} \right), \tilde{r}_{D, i} \right] \]  \hspace{1cm} (C.23)
Now, we can derive the domestic investor’s individual optimum.

\[
\frac{(1 + \pi^D)^2}{A^D} (E[\tilde{r}_{CG_i} + \tilde{r}_{D_i}] - \tilde{r}^D_I)
\]

\[
= Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG_i} \omega_i^D (1 - t_{CG}^D) \tilde{r}_{CG_i,n} \right]
\]

\[
+ Cov \left[ \sum_{i=1}^{I} \tilde{r}_{D_i} \omega_i^D (1 - t_{D}^D) (1 - t_{CG}^D) \tilde{r}_{D_i,n} \right]
\]

\[
Cov \left[ \sum_{i=1}^{I} \tilde{r}_{CG_i} \omega_i^D (1 - t_{CG}^D) (1 - t_{D}^D) \tilde{r}_{D_i,n} \right]
\]

\[
+ Cov \left[ \sum_{i=1}^{I} \tilde{r}_{D_i} \omega_i^D (1 - t_{D}^D)^2 \tilde{r}_{D_i,n} \right]
\]

(C.24)

**C.3.2 Foreign Investor**

For the foreign investor we can express the following individual optimum relationship.

\[
E[\tilde{r}_{CG_i} + \tilde{r}_{D_i}] - \tilde{r}^F_I = A^F Cov \left[ \tilde{W}_t^F, \tilde{r}_{CG_i} + \tilde{r}_{D_i} \right]
\]

(C.25)

Having derived the domestic investor’s term in chapter 5.4.1 and in appendix C.3.1 we derive analogously the foreign investor’s covariance pricing relationship term. We decompose the covariance term by application of the linearity property.

\[
Cov \left[ \tilde{W}_t^F, \tilde{r}_{CG_i} \right] + Cov \left[ \tilde{W}_t^F, \tilde{r}_{D_i} \right]
\]

(C.26)

We consider the first covariance term. By replacing the foreign investor’s terminal real wealth after tax by the definition of the portfolio, the following expression can be derived.
\[ \text{Cov} \left[ \sum_{i=1}^{l} \omega_i \left( 1 + \tilde{r}_{CGi} + \tilde{r}_{Di} \right) + \omega_0 \left( 1 + r_F^r \right), \tilde{r}_{CG} \right] \]  
(C.27)

Since the covariance with the risk-free asset is zero, we can derive the following term

\[ \text{Cov} \left[ \sum_{i=1}^{l} \omega_i \left( 1 + \tilde{r}_{CGi} + \tilde{r}_{Di} \right), \tilde{r}_{CG} \right] \]  
(C.28)

and decompose the covariance of the foreign investor into two terms:

\[ \text{Cov} \left[ \sum_{i=1}^{l} \omega_i \tilde{r}_{CGi}, \tilde{r}_{CG} \right] + \text{Cov} \left[ \sum_{i=1}^{l} \omega_i \tilde{r}_{Di}, \tilde{r}_{CG} \right]. \]  
(C.29)

One covariance describes the value weighted real rate of capital gains return with the real capital gains rate of return, and the other one the value weighted real dividend rate of return with the real capital gains rate of return. By considering the foreign investor’s capital gains and dividend rate of return after tax (eq. (5.31)), we can separate the tax, inflation and appreciation of exchange rates from the real capital gains and dividend rate of return of the covariance.

\[ \text{Cov} \left[ \sum_{i=1}^{l} \tilde{r}_{CGi,n} \omega_i \frac{1 - t^F \epsilon_{CG} + e \left( 1 - t^F \epsilon_{CG,P,e} \right)}{1 + \pi^F}, \tilde{r}_{CG,n} \right], \]
\[ + \text{Cov} \left[ \sum_{i=1}^{l} \tilde{r}_{Di,n} \omega_i \frac{1 - t^F \epsilon_{D} + e \left( 1 - t^F \epsilon_{D,e} \right)}{1 + \pi^F}, \tilde{r}_{CG,n} \right] \]  
(C.30)
The covariance terms incorporate the stochastic relationship of capital gains and dividends. Having derived the covariance with the capital gains rate of return we consider the covariance with the dividend rate of return.

\[
Cov \left[ \tilde{W}_t^F, \tilde{r}_{D_i}^F \right]
\]  \hspace{1cm} (C.31)

By replacing the foreign investor’s terminal real wealth after tax with the definition of the portfolio, the following expression can be derived.

\[
Cov \left[ \sum_{i=1}^{I} \omega_i^F \left( 1 + \tilde{r}_{CG_i}^F + \tilde{r}_{D_i}^F \right) + \omega_0^F \left( 1 + r_i^F \right), \tilde{r}_{D}^F \right]
\]  \hspace{1cm} (C.32)

Since the covariance with the risk-free asset is zero, we can derive the following term.

\[
Cov \left[ \sum_{i=1}^{I} \omega_i^F \left( 1 + \tilde{r}_{CG_i}^F + \tilde{r}_{D_i}^F \right), \tilde{r}_{D}^F \right]
\]  \hspace{1cm} (C.33)

We apply the linearity property of covariances:

\[
Cov \left[ \sum_{i=1}^{I} \omega_i^F \tilde{r}_{CG_i}^F, \tilde{r}_{D}^F \right] + Cov \left[ \sum_{i=1}^{I} \omega_i^F \tilde{r}_{D_i}^F, \tilde{r}_{D}^F \right].
\]  \hspace{1cm} (C.34)

One covariance describes the value weighted real capital gains rate of return with the real dividend rate of return, and the other one the value weighted real dividend rate of return with the real dividend rate of return. By considering the foreign investor’s real return after tax (eq. (5.31)), we can separate the tax, inflation and appreciation of the exchange rate.
C Derivation of Tax International Capital Asset Pricing Model

\[
Cov \left[ \sum_{i=1}^{I} \tilde{\omega}_{CG,i} \tilde{\omega}_{CG,i} (1 - t_{eff,CG} + e \left( 1 - t_{eff,CG,e} \right)) \frac{1}{1 + \pi^F} \right] 
\]

\[
+ Cov \left[ \sum_{i=1}^{I} \tilde{\omega}_{D,i} (1 - t_{eff,D} + e \left( 1 - t_{eff,D,e} \right)) \frac{1}{1 + \pi^F} \right] 
\]

\[
\left( 1 - t_{eff,D} + e \left( 1 - t_{eff,D,e} \right) \right) \frac{1}{1 + \pi^F} \right] 
\]

By dividing by the foreign risk aversion factor and multiplying by the inflation rate, the left hand side of equation C.25 and sum of the terms C.30 and C.35 lead to the foreign investor’s individual optimum.

\[
\frac{(1 + \pi^F)^2}{A^F} \left( E[\tilde{\omega}_{CG,i}^F + \tilde{\omega}_{D,i}^F] - r_I^F \right) 
\]

\[
= Cov \left[ \sum_{i=1}^{I} \tilde{\omega}_{CG,i} (1 - t_{eff,CG} + e \left( 1 - t_{eff,CG,e} \right))^2 ; \tilde{\omega}_{CG,i} \right] 
\]

\[
+ Cov \left[ \sum_{i=1}^{I} \tilde{\omega}_{D,i} (1 - t_{eff,D} + e \left( 1 - t_{eff,D,e} \right))^2 ; \tilde{\omega}_{D,i} \right] 
\]

\[
\left( 1 - t_{eff,CG} + e \left( 1 - t_{eff,CG,e} \right) \right) ; \tilde{\omega}_{CG,i} \right] 
\]

\[
\left( 1 - t_{eff,D} + e \left( 1 - t_{eff,D,e} \right) \right) ; \tilde{\omega}_{D,i} \right] 
\]

By dividing by the foreign risk aversion factor and multiplying by the inflation rate, the left hand side of equation C.25 and sum of the terms C.30 and C.35 lead to the foreign investor’s individual optimum.
C.3.3 aggregate Covariance Term

We aggregate the other covariance terms. The aggregate covariance of foreign and domestic investor’s second capital gains covariance term is defined as follows (Eq. (5.85) and (5.87)):

\[
\sum_{i=1}^{I} \bar{r}_{D,i,n} D V_{i,0} M_{0} (1 - t_{D}^{D}) (1 - t_{CG}^{D})
\]

\[
+ \sum_{i=1}^{I} \bar{r}_{D,i,n} F V_{i,0} M_{0} (1 - t_{eff,CG}^{F} + e (1 - t_{eff,CG,P,e}^{F})) \]

\[
(1 - t_{eff,D}^{F} + e (1 - t_{eff,D,e}^{F})), \bar{r}_{CG,i,n} \].
\]

The definition for the second aggregate capital gains rate of return is:

\[
\sum_{i=1}^{I} \bar{r}_{D,i,n} D V_{i,0} M_{0} (1 - t_{D}^{D}) (1 - t_{CG}^{D})
\]

\[
+ \sum_{i=1}^{I} \bar{r}_{D,i,n} F V_{i,0} M_{0} (1 - t_{eff,CG}^{F} + e (1 - t_{eff,CG,P,e}^{F})) \]

\[
(1 - t_{eff,D}^{F} + e (1 - t_{eff,D,e}^{F})), \bar{r}_{CG,D}^{D}. \]

The aggregation of the investors’ first dividend covariance term leads to the following expression (Eq. (5.86) and (5.88)).
The definition for the first aggregate dividend rate of return is:

\[
\tilde{r}_{D,CG}^{\tau,aggr} = \sum_{i=1}^{I} \tilde{r}_{CG,i,n} n_i D V_{i,0} \left( 1 - t_D^D \right) \left( 1 - t_{CG}^D \right) + \sum_{i=1}^{I} \tilde{r}_{CG,i,n} n_i F V_{i,0} \left( (1 - t_{eff,D}^F) + e \left( 1 - t_{eff,D,e}^F \right) \right) \left( (1 - t_{eff,D}^F) + e \left( 1 - t_{eff,D,e}^F \right) \right), \tilde{r}_{D,i,n}
\]

what leads to the following covariance term.

\[
\frac{M_0}{P_0^D} Cov \left[ \tilde{r}_{D,CG}^{\tau,aggr}, \tilde{r}_{D,i,n} \right].
\]
The second aggregate dividend rate of return is defined as

\[
\tilde{r}_{D,D}^{t,aggr} \equiv \sum_{i=1}^{I} \tilde{r}_{D,n}^{i} \frac{V_{i,0}}{M_0} \left(1 - t_D^D\right)^2
\]

\[+ \sum_{i=1}^{I} \tilde{r}_{D,n}^{F} \frac{V_{i,0}}{M_0} \left((1 - t_{eff,D}) + e \left(1 - t_{eff,D,e}\right)\right)^2.
\]

so, our covariance term is

\[
\frac{M_0}{P_0^D} \text{Cov} \left[\tilde{r}_{D,D}^{t,aggr}, \tilde{r}_{D,n}^i\right].
\]

### C.3.4 Pricing Relationship

We put the derived formulations of the excess return (5.80) and covariance terms (5.98) together and consider that we divided the excess return by the world aggregate risk aversion factor under inflation \(A^W\) (See appendix C.2.1).

\[
E[\tilde{r}_{CG,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,n}] e(A^F_{W} - ET_{CG}) - eET_{CG}
\]

\[+ E[\tilde{r}_{D,n}] (1 - T_{D}) + E[\tilde{r}_{D,n}] e(A^F_{W} - ET_{D})
\]

\[ - r_{I,n} (1 - T_{I}) - r_{I,n} e(A^F_{W} - ET_{I}) + eET_{I}
\]

\[= \frac{M_0}{P_0^D A^W_0} \left( \text{Cov} \left[\tilde{r}_{CG,CG}^{t,aggr} + \tilde{r}_{D,CG}^{t,aggr}, \tilde{r}_{CG,n}\right] + \text{Cov} \left[\tilde{r}_{D,CG}^{t,aggr} + \tilde{r}_{D,D}^{t,aggr}, \tilde{r}_{D,n}\right] \right)
\]

CAPT prices every risky asset, consequently the world market portfolio can be applied. This benchmark is perfectly correlated with the world market return. Equivalently to the procedure in the Tax-IntCAPM, we multiply the above equation with the share of each asset in the market portfolio \(n^f_{V_i} \frac{V_{i,0}}{M_0}\).
We aggregate over all risky assets which leads to:

\[
E \left[ \frac{\tilde{r}_{CG,i,n}}{M_0} \right] (1 - T_{CG}) + E \left[ \frac{\tilde{r}_{CG,i,n}}{M_0} \right] e \left( A^F_{W,x} - ET^F_{CG} \right) - \frac{n^0 V_{i,0}}{M_0} e ET^F_{CG} \\
+ E \left[ \frac{\tilde{r}_{D,i,n}}{M_0} \right] (1 - T_D) + E \left[ \frac{\tilde{r}_{D,i,n}}{M_0} \right] e \left( A^F_{W,F} - ET^F_D \right) \\
- r_{I,n} \frac{n^0 V_{i,0}}{M_0} (1 - T_I) - r_{I,n} e \frac{n^0 V_{i,0}}{M_0} \left( A^F_{W,x} - ET^F_I \right) + \frac{n^0 V_{i,0}}{M_0} e ET^F_I
\]

We recall that the world market portfolio rate of return is the aggregate sum of value weighted capital and dividend rate of return of each risky asset (eq. (5.84)) and consider that the aggregate quotient \( \sum_i \frac{n^0 V_{i,0}}{M_0} \) equals unity. So we have

\[
E[\tilde{r}_{CG,m}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e \left( A^F_{W} - ET^F_{CG} \right) - e ET^F_{CG} \\
+ E[\tilde{r}_{D,m}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e \left( A^F_{W,F} - ET^F_D \right) \\
- r_{I,n} (1 - T_I) - r_{I,n} e \left( A^F_{W} - ET^F_I \right) + e ET^F_I
\]

We divide eq. (C.47) by eq. (C.49) to eliminate the world aggregate risk aversion factor under inflation. Under the framework of international taxation, deviation from Relative PPP and stochastic dividends we developed the Tax International Capital Asset Pricing Model.
\[ C \text{ Derivation of Tax International Capital Asset Pricing Model} \]

\[
E[\tilde{r}_{CG,n}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e (A^F_W - ET^F_{CG}) - eET^F_{CG} \\
+ E[\tilde{r}_{D,i,n}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e (A^F_W - ET^F_D) \\
- r_{I,n} (1 - T_I) - r_{I,n} e (A^F_W - ET^F_I) + eET^F_I
\]

\[ = \left( E[\tilde{r}_{CG,m}] (1 - T_{CG}) + E[\tilde{r}_{CG,i,n}] e (A^F_W - ET^F_{CG}) - eET^F_{CG} \right) \]

\[ + E[\tilde{r}_{D,m}] (1 - T_D) + E[\tilde{r}_{D,i,n}] e (A^F_W - ET^F_D) \]

\[ - r_{I,n} (1 - T_I) - r_{I,n} e (A^F_W - ET^F_I) + eET^F_I \]

\[ \text{Cov}\left[ \tilde{r}_{CG,CG}^{t,aggr} + \tilde{r}_{D,CG}^{t,aggr}, \tilde{r}_{CG,i,n} \right] + \text{Cov}\left[ \tilde{r}_{CG,D}^{t,aggr} + \tilde{r}_{D,D}^{t,aggr}, \tilde{r}_{D,i,n} \right] \]

\[ \text{Cov}\left[ \tilde{r}_{CG,CG}^{t,aggr} + \tilde{r}_{D,CG}^{t,aggr}, \tilde{r}_{CG,m,n} \right] + \text{Cov}\left[ \tilde{r}_{CG,D}^{t,aggr} + \tilde{r}_{D,D}^{t,aggr}, \tilde{r}_{D,m,n} \right] \]
D Tax System International Capital Asset Pricing Model with Homogeneous Expectations

CAPT prices every risky asset, consequently the world market portfolio can be applied. This benchmark is perfectly correlated with the world market capital gains and dividend rate of return. The expected capital gains and dividend rate of return on the world market portfolio is endogenously determined, since they are weighted averages of the expected rate of returns of the individual assets. We can prove the application of the world market portfolio to equation (7.11) by multiplying with the share of each asset in the nominal world market portfolio $\frac{n_k V_k}{M_0}$.

\[
\begin{align*}
&\frac{1 + \pi^F}{A^F} \left( E \left[ \tilde{r}_{CG_k,n} \frac{n_k^0 V_k}{M_0} \right] (1 - t_{eff,CG}^F) ight) \\
&+ E \left[ \tilde{r}_{CG_k,n} \frac{n_k^0 V_k}{M_0} \right] \left(1 - t_{eff,CG,P,e}^F\right) + E \left[ \tilde{r}_{D_k,n} n_k^0 V_k \frac{\theta}{M_0} \right] \left(1 - t_{eff,D,e}^F\right) \\
&- r_{I,n} \frac{n_k^0 V_k}{M_0} (1 - t_{eff,I}) - r_{I,n} n_k^0 V_k \frac{\theta_{D,e}}{M_0} (1 - t_{eff,I,P,e}) + \frac{n_k^0 V_k}{M_0} t_{eff,I,P,e} \\
&= \left(1 - t_{eff,CG}^F\right)^2 \text{Cov} \left[ \tilde{r}_{CG_k,n} \frac{n_k^0 V_k}{M_0}, \tilde{r}_{CG_{m,n}} \right] + \left(1 - t_{eff,D}^F\right)^2 \text{Cov} \left[ \tilde{r}_{D_k,n} \frac{n_k^0 V_k}{M_0}, \tilde{r}_{D_{m,n}} \right] \\
&+ \left(1 - t_{eff,D}^F\right) \left(1 - t_{eff,CG}^F\right) \text{Cov} \left[ \tilde{r}_{D_k,n} \frac{n_k^0 V_k}{M_0}, \tilde{r}_{CG_{m,n}} \right] + \left(1 - t_{eff,D}^F\right)^2 \text{Cov} \left[ \tilde{r}_{D_k,n} \frac{n_k^0 V_k}{M_0}, \tilde{r}_{D_{m,n}} \right]
\end{align*}
\]
and aggregating over all risky assets (König (1990) and Mai (2008)).

\[
\frac{1 + \pi^F}{A^F} \left( E \left[ \sum_{k=1}^{K} \bar{r}_{CG,k,n} \frac{n_k^0 V_{k,0}}{M_0} \right] (1 - \tau_{eff,CG}^F) \right) \\
+ E \left[ \sum_{k=1}^{K} \bar{r}_{CG,k,n} \frac{n_k^0 V_{k,0}}{M_0} \right] e \left( 1 - \tau_{eff,CG}^F \right) - \tau_{eff,CG}^F e \\
+ E \left[ \sum_{k=1}^{K} \bar{r}_{D,k,n} \frac{n_k^0 V_{k,0}}{M_0} \right] \left( 1 - \tau_{eff,D}^F \right) + E \left[ \sum_{k=1}^{K} \bar{r}_{D,k,n} \frac{n_k^0 V_{k,0}}{M_0} \right] e \left( 1 - \tau_{eff,D}^F \right) \\
- \sum_{k=1}^{K} r_{tse,n} \frac{n_k^0 V_{k,0}}{M_0} (1 - \tau_{eff,I}) - \sum_{k=1}^{K} r_{tse,n} \frac{n_k^0 V_{k,0}}{M_0} e (1 - \tau_{eff,I}^F) + \sum_{k=1}^{K} \frac{n_k^0 V_{k,0}}{M_0} \tau_{eff,I}^F e \\
= \left( 1 - \tau_{eff,CG}^F \right)^2 \text{Cov} \left[ \sum_{k=1}^{K} \bar{r}_{CG,k,n} \frac{n_k^0 V_{k,0}}{M_0}, \bar{r}_{CG,m,n} \right] \\
+ \left( 1 - \tau_{eff,CG}^F \right) \left( 1 - \tau_{eff,CG}^F \right) \text{Cov} \left[ \sum_{k=1}^{K} \bar{r}_{CG,k,n} \frac{n_k^0 V_{k,0}}{M_0}, \bar{r}_{D,m,n} \right] \\
+ \left( 1 - \tau_{eff,CG}^F \right) \left( 1 - \tau_{eff,CG}^F \right) \text{Cov} \left[ \sum_{k=1}^{K} \bar{r}_{D,k,n} \frac{n_k^0 V_{k,0}}{M_0}, \bar{r}_{CG,m,n} \right] \\
+ \left( 1 - \tau_{eff,CG}^F \right)^2 \text{Cov} \left[ \sum_{k=1}^{K} \bar{r}_{D,k,n} \frac{n_k^0 V_{k,0}}{M_0}, \bar{r}_{D,m,n} \right] \frac{M_0}{F_0} \\
\text{(D.2)}
\]

We recall that the world market portfolio capital gains and dividend rate of return is the sum of value weighted rate of return of each risky asset (eq. (7.4)) and consider the definition of the nominal value of the world market portfolio in the beginning of period (eq. (5.81)) and that the aggregated quotient \( \sum_{k=1}^{K} \frac{n_k^0 V_{k,0}}{M_0} \) equals unity.

\[
\frac{1 + \pi^F}{A^F} \left( E \left[ \bar{r}_{CG,m,n} \right] (1 - \tau_{eff,CG}^F) + E \left[ \bar{r}_{CG,m,n} \right] \tau_{eff,CG}^F \right) - \tau_{eff,CG}^F e \\
+ E \left[ \bar{r}_{D,m,n} \right] (1 - \tau_{eff,D}^F) + E \left[ \bar{r}_{D,m,n} \right] \tau_{eff,D}^F \\
- r_{tse,n} (1 - \tau_{eff,I}) - r_{tse,n} e (1 - \tau_{eff,I}^F) + \tau_{eff,I}^F e \\
= \left( 1 - \tau_{eff,CG}^F \right)^2 \text{Var} \left[ \bar{r}_{CG,m,n} \right] + \left( 1 - \tau_{eff,CG}^F \right) \text{Cov} \left[ \bar{r}_{CG,m,n}, \bar{r}_{D,m,n} \right] \\
+ \left( 1 - \tau_{eff,CG}^F \right) \left( 1 - \tau_{eff,CG}^F \right) \text{Cov} \left[ \bar{r}_{D,m,n}, \bar{r}_{CG,m,n} \right] + \left( 1 - \tau_{eff,CG}^F \right)^2 \text{Var} \left[ \bar{r}_{D,m,n}, \bar{r}_{D,m,n} \right] \frac{M_0}{F_0} \\
\text{(D.3)}
\]

Dividing equation (7.11) by equation (D.3) to eliminate the term \( \frac{1 + \pi^F}{A^F} \) and the real value
of the world market portfolio $\frac{M}{P_0}$ produces:

\[
E [\tilde{r}_{CG,i,n}] (1 - t_{eff,CG}) + E [\tilde{r}_{CG,i,n}] e (1 - t_{eff,CG,e}) - et_{eff,CG,P,e} + E [\tilde{r}_{D,i,n}] (1 - t_{eff,D}) + E [\tilde{r}_{D,i,n}] e (1 - t_{eff,D,e}) - r_{I,n} (1 - t_{eff,I}) - r_{I,n} e (1 - t_{eff,I,e}) + et_{eff,I,P,e} = (E[\tilde{r}_{CG,m,n}] (1 - t_{eff,CG}) + E [\tilde{r}_{CG,m,n}] e + E [\tilde{r}_{D,m,n}] (1 - t_{eff,D}) + E [\tilde{r}_{D,m,n}] e) (D.4)
\]

\[
\beta_{hE} = \frac{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{CG,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{D,m,n}]}{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{CG,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]} + \frac{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]}{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{CG,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]}\]

Based on equilibrium on the capital market we developed the Tax International Capital Asset Pricing Model with homogeneous expectations. In order to quantify the effect of exchange gains taxation we regard the Tax-IntCAPM with homogeneous expectations where exchange gains taxation is regarded as irrelevant. The Tax-IntCAPM under irrelevance of exchange gains taxation is defined as follows:

\[
E [\tilde{r}_{CG,i,n}] (1 - t_{eff,CG}) + E [\tilde{r}_{CG,i,n}] e + E [\tilde{r}_{D,i,n}] (1 - t_{eff,D}) + E [\tilde{r}_{D,i,n}] e - r_{I,n} (1 - t_{eff,I}) - r_{I,n} e = (E[\tilde{r}_{CG,m,n}] (1 - t_{eff,CG}) + E [\tilde{r}_{CG,m,n}] e + E [\tilde{r}_{D,m,n}] (1 - t_{eff,D}) + E [\tilde{r}_{D,m,n}] e) (D.5)
\]

\[
\beta_{hE_{ext,E}} = \frac{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{CG,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,i,n},\tilde{r}_{D,m,n}]}{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{CG,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]} + \frac{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]}{(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{CG,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]+(1-t_{eff,E})^2 Cov[\tilde{r}_{CG,m,n},\tilde{r}_{D,m,n}]}\]

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We define the terms.

\[ te_{CG}^F \equiv t_{eff,CG}^F + e \quad te_{D}^F \equiv t_{eff,D}^F + e. \]

(D.6)

We implement the Tax-IntCAPM under homogeneous expectations and irrelevance of exchange gains tax.

\[
-0.0419 = \frac{(1-0.0401)^20.0479+(1-0.0401)(1-0.0601)(-0.0473+(-0.0048))+(1-0.0601)^2(-0.0046)}{(1-0.0401)^20.0457+2(1-0.0401)(1-0.0601)0.0447+(1-0.0601)^20.0478}
\]

\[
E[\tilde{r}_{CG_i,n}] (1 - 0.13) + E[\tilde{r}_{CG_i,n}] (-0.0899) \\
+ E[\tilde{r}_{D_i,n}] (1 - 0.15) + E[\tilde{r}_{D_i,n}] (-0.0899) \\
- 0.0252 (1 - 0.13) - 0.0252(-0.0899) \\
= (-0.1606 (1 - 0.13) + (-0.1606)(-0.0899) \\
+ 0.005 (1 - 0.15) + 0.005(-0.0899) \\
- 0.0252 (1 - 0.13) - 0.0252(-0.0899)) (-0.0419) \\
\]
Bibliography


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