

**International Stock Markets Linkages
and
Arbitrage between Futures and Spot
Markets**

by

Jędrzej Białkowski

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Supervisor: Prof. Dr. Martin T. Bohl

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First Supervisor Prof. Dr. Martin Bohl

Second Supervisor Prof. Dr. Alireza Tourani-Rad

to my

MOTHER and FATHER

with love

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Introduction

Over recent years a substantial amount of research has focussed on linkages between stock markets. As a result of the increasing globalisation the world economies have become interdependent and stock markets have been linked with each other. The abolition of foreign exchanges controls in both mature and emerging markets, liberalisation of capital markets, and the technological developments in communication and trading system further facilitate creation of tight linkages between stock markets.

There are at least two reasons, why the understanding of the nature and degree of international stock markets integration is so important for scholars and practitioners. Firstly, during the last decade stock markets have witnessed several financial crises, and as a result of the increasing market integration even a financial distress in a minor market is presently capable of shaking the largest world markets. If the stock markets linkages had been better known before those crises took place, it would have been possible to avoid or at least reduce the result of contagion effect. Secondly, according to the finance theory there are potential gains from international portfolio diversification, in case when different stock markets are not perfectly correlated and correlation structures are constant in time. At the same time, investing into assets from foreign stock markets entails new risks, specific for each market. Therefore, the knowledge of time dependent linkages among stock markets has crucial meaning for international investors. Various methods can be used to examine international stock market linkages (see Bailey and Choi (2003)). The simplest one exploits the idea that if stock markets were perfectly integrated, then risk-adjust returns would be equal across markets. In turn the most popular method of testing international linkages focuses on an examination of correlation structures. If correlation structures were constant in time, it would be probably the commonly accepted approach. However, the investigation of dependencies between stock markets shows that stock market returns are more correlated with each other during turbulent periods compared to tranquil periods. Another method assumes

that linkages between stock markets can be described by a static linear or non-linear set of equations. Nevertheless, a structure of stock markets linkages is far too complex and time dependable to a priori assume a form of relationship between markets. The advantageous alternative for the above mentioned techniques is a model of stock market linkages based on the Markov Switching framework. Such approach does not impose a priori a particular form of relationship between markets, also there is no need of an ad hoc identification of crisis periods. Furthermore, by the definition of a Markov switching model, the linkages during the turbulent period are different from dependencies during calm periods (see Ang and Bekaert (2002), Sola, Spagnolo, and Spagnolo (2002), Ravn and Sola (1996), Philips (1991)).

This thesis contributes to the existing literature in at least two ways. The framework of inter-market dependencies is extended by introducing the concept of Granger causality into the Markov switching models of stock index returns. In contrast to previous studies, the explicit definition of contagion, independence, and causality between capital markets is formulated. Furthermore, in this work a procedure to estimate the Markov switching model under the no-causality hypothesis is developed. In addition, the application of Markov Switching framework allows for calculation of the probabilities of crisis and calm regimes for each market, dependent upon different information sets. These probabilities could be important for international investors and capital market authorities interested in avoiding financial turmoil on the local market when a crisis hits elsewhere. The proposed econometrical methods are employed to investigate the long term and short term stock market linkages. For the purpose of explaining stock market linkages during a crisis time the relationship between the Japanese and the Hong Kong stock markets during the Asian crisis was examined. The analysis of long term stock markets was carried out by inspection of an influence of the US capital market on the UK, Japanese, and German stock markets for the period of almost twenty years.

In addition to the problem of modelling stock markets linkages, the question of the importance of an arbitrage between spot and futures markets has been put forward.

Since the introduction of financial futures contracts in the US markets, researchers and

market participants have been interested in the relationship between futures and spot market. In particular, the question of a potential arbitrage opportunity has attracted the greatest attention. According to the definition, the arbitrage between spot and futures market is a strategy designed to profit from temporary discrepancies between the price of the underlying and the price of a futures contract. From the theoretical point of view the profit from the arbitrage is attainable without exposure to any risk and requires zero initial wealth. Therefore, the arbitrage has seemed to be the holy grail of investing for risk-averse investors. The further motivation for research comes from the fact that an arbitrage has proved useful for valuating derivative securities. Because the price of a new financial instrument can be replicated by combining existing instruments whose individual values are known, the value of the new instrument must be equal to that of the replicating strategy. If it is not equal, an arbitrage opportunity exists. Thus, the optimal price of instrument precludes an arbitrage activity.

This thesis focuses on the examination of the index arbitrage opportunity and on the pricing of derivatives securities on zero-coupon bonds. The index arbitrage has been subject of numerous empirical studies (detail literature review Sutcliffe (1993)). According to the previous studies the mispricing defined as the difference between the market futures price and the theoretical price is a decreasing function of the time to maturity of a futures contract. There is a decline in the number of signals to the index arbitrage for succeeding series of contracts, what is the evidence of the maturing market. The signal for the arbitrage disappears rapidly once the opportunity becomes well-known and many investors act on it. From previous studies it is also known that the index arbitrage is not risk free. Only around 80% of signals for the arbitrage results in profits. Most of the above mentioned observations were made exclusively for mature markets. A natural question arises what results could be expected from emerging markets, characterized by properties absent in case of previous studies.

This thesis contributes to the arbitrage literature by providing the outcomes of arbitrage examination from the fast growing Polish futures market. This market was ranked at the

third place in Europe with respect to the volume growth in year 2003. The growth was equal to 36% in comparison to the previous year. Furthermore this study explains the impact of factors such as a limited access to short sale, irregular dividend payments, and a fluctuation of interest rates, on the profitability of arbitrage.

Along with a close examinations of the arbitrage opportunity between the futures and spot market on the index WIG 20, this work analyses the methods of valuation of forward and future contracts on zero-coupon bonds. Therefore, the thesis contributes to the discussion on reliable methods of pricing derivatives on interest rate derivatives. The remainder of this thesis comprises of four papers. Two of them focus on the modelling relationship between financial markets during crisis and calm periods. The third paper demonstrates the analysis of the arbitrage opportunity between the spot and futures market on the index WIG 20. The last paper presents the derivation of a closed form expression for the price and value of forward and future contracts on a zero-coupon bond.

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Chapter 1

Financial Contagion, Spillovers, and Causality in the Markov Switching Framework

In this paper, we introduce the concept of causality in the Markov switching framework into the analysis of financial inter-market dependencies. We extend the methodology of testing for financial spillovers between capital markets by explicitly defining contagion, spillovers, and independence, and providing statistics to test for the existence of causality. We apply the methodology to stock index returns on the Japanese (Nikkei 225) and the Hong Kong (HSI) markets during the Asian crisis and find no evidence of contagion between the markets, but strong evidence of feedback spillovers between them.

1.1 Introduction

The Markov switching framework enables the construction of models of stock index returns that switch between multiple regimes. The empirical literature suggests that such models outperform their one-regime counterparts in explaining the movements of asset prices (Cecchetti, Lam, and Mark (1990), Turner, Stratz, and Nelson (1990), Rydén, Teräsvirta and Åsbrink (1998), Timmermann (2000)). Recently, Markov switching models have been employed to analyze the inter-market dependencies during calm and tumultuous periods (Ang and Bekaert (2002), Sola, Spagnolo, and Spagnolo (2002)). In this context, Sola, Spagnolo, and Spagnolo (2002) have introduced the idea of independence and contagion (contagious

volatility spillovers) as types of relationships between capital markets in calm and crisis regimes.

Typically the ideas of spillovers or dependencies between financial markets are related to an instantaneous inter-market relationship (e.g. King and Wadhvani (1990), Forbes and Rigobon (2002), Hartmann, Straetmans, and de Vries (2004)). Nevertheless, there exists a significant number of studies covering inter-market spillovers understood as stock returns or volatility on one market causing the specific behavior of returns or volatility on the second market in subsequent periods (Eun and Shim (1989), Malliaris and Urrutia (1992), Karolyi (1995), Cheung and Ng (1996), Booth, Martikainen, and Tse (1997), Climent and Meneu (2003), Sander and Kleimeier (2003) among others). In our paper we follow this latter branch of literature where causality is interpreted as the evidence of inter-market spillovers.

In this paper we augment the framework of inter-market dependencies by introducing the concept of Granger causality into the Markov switching models of stock index returns (see also Granger (1969, 1980), Psardakis, Ravn, and Sola (2003)). The notion of one market Granger-causing the other market can be interpreted as evidence of information or capital flows between the markets. In contrast to previous studies, we explicitly define contagion, independence, and causality between capital markets. Furthermore, we develop a procedure to estimate the Markov switching model under the no-causality hypothesis and propose a statistic to test the null hypothesis of no-causality against the alternative of causality between stock index returns on two markets. In addition, we calculate the probabilities of crisis and calm regimes for each market, dependent upon different information sets. These probabilities could be important for international investors and capital market authorities interested in avoiding financial turmoil on the local market when a crisis hits elsewhere.

Finally, we present an empirical example of the relationship between the Japanese and Hong Kong markets during the 1997 Asian crisis. We find evidence of feedback spillovers between the markets. The volatility on both markets and the correlation between stock index returns increase when both markets enter the crisis regime.

In the next section we describe the Markov switching model, the estimation procedure

and tests for the hypotheses of contagion, independence, and causality. In the third section, we present an empirical analysis of the relationship between the Japanese and Hong Kong markets. The fourth section concludes.

1.2 Methodology

In this section we present the general framework for testing inter-market dependencies based on the idea of Markov switching models. We introduce the definition of causality into the Markov switching models of financial markets and describe the definitions of contagion and independence in the form of mathematical expressions. Although mathematical formulations were avoided in earlier descriptions of contagion and spillover, they are especially useful in clarifying statistical assumptions underlying these economic phenomena. We use the definitions of causality and no-causality to construct a test of inter-market spillover. In addition, we present the formulas for the probability of a market entering crisis or calm regimes, dependent upon different information sets which are useful for forecasting the future state of a market. Finally, we apply the results from our general framework to the Markov switching mixture of normal distributions with constraints on means, extending the models of Phillips (1991), Ravn and Sola (1995), and Sola, Spagnolo, and Spagnolo (2002). We show the estimation procedure of our model under the assumption of no-causality, which is different from the method presented by Phillips (1991) to estimate models under the contagion and independence hypotheses.

1.2.1 The Markov switching framework

Define two time series R^X and R^Y describing daily index returns on two separate markets X and Y . Each of the markets is allowed to switch between two regimes denoted by l and h (e.g. calm and turmoil). The regimes correspond to the states of two hidden processes, S_t^X and S_t^Y , respectively. Both of them have the same state spaces described by the set $A = \{h, l\}$.

In order to examine the relationship between two markets, we construct the Markov chain S_t with its state space defined by the set $K = \{(i, j) : i, j \in A\}$. By definition:

$$S_t = \begin{cases} 1 & S_t^X = l \wedge S_t^Y = l, \\ 2 & S_t^X = l \wedge S_t^Y = h, \\ 3 & S_t^X = h \wedge S_t^Y = l, \\ 4 & S_t^X = h \wedge S_t^Y = h. \end{cases} \quad (1.1)$$

The transition matrix assigned to the process S_t is given by:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}, \quad (1.2)$$

where p_{ij} denotes the probability of entering the state j from state i ; $i, j = 1 \dots 4$.

In the context of financial markets undergoing calm and turbulent periods, we are able to distinguish four different states that the two markets can enter. The first state of the process S_t corresponds to the situation where both markets are in the calm regime. In the second state, market X is in the calm regime, while market Y suffers in the crisis regime. In the third state, X is in the crisis regime and Y is in the calm regime. Finally, in the fourth state both markets are in the crisis regime.

The four-state framework has already been employed to investigate dependencies between two macroeconomic or financial variables that are allowed to switch between two alternative regimes (Phillips (1991), Ravn and Sola (1995), Hamilton and Lin (1996), Sola, Spagnolo, and Spagnolo (2002)). These studies typically set restrictions on the transition matrix \mathbf{P} to analyze various types of dependencies between the variables.

1.2.2 Independence, causality, contagion

In our work, we apply restrictions to the transition matrix to define the three types of relationships between capital markets that have recently met an increasing attention in the literature on international finance, namely independence, spillovers, and contagion. In order to analyze these inter-market relationships, we introduce the concept of Granger causality into the Markov switching framework (Granger (1969, 1980)). In the context of financial markets, causality is usually interpreted as evidence that some information or capital flows between capital markets exist that push stock returns on one market to follow returns on the other market with some lag. Our approach to causality is analogous to the definitions presented by Psardakis, Ravn, and Sola (2003), but we distinguish between the lack of causality and independence. In the following definitions we are interested whether there exists causality or contagion from market X to market Y . However, market Y is not restricted from influencing market X . The definitions where causality and contagion from Y to X is considered are analogous.

Definition 1.1 S_t^X causes S_t^Y in the Granger sense if

$$\exists i, j, k \in A \ P(S_t^Y = i | S_{t-1}^X = j, S_{t-1}^Y = k) \neq P(S_t^Y = i | S_{t-1}^Y = k).$$

Definition 1.2 S_t^X does not cause S_t^Y in the Granger sense if $\forall i, j, k \in A$

$$P(S_t^Y = i | S_{t-1}^X = j, S_{t-1}^Y = k) = P(S_t^Y = i | S_{t-1}^Y = k)$$

or equivalently:

$$P(S_t^Y = i | S_{t-1}^X = h, S_{t-1}^Y = h) = P(S_t^Y = i | S_{t-1}^X = l, S_{t-1}^Y = h)$$

$$P(S_t^Y = i | S_{t-1}^X = h, S_{t-1}^Y = l) = P(S_t^Y = i | S_{t-1}^X = l, S_{t-1}^Y = l).$$

The idea of these two alternative definitions is as follows. The market X has an influence on the market Y , when the magnitude of the conditional probability $P(S_t^Y = i | S_{t-1}^X =$

$j, S_{t-1}^X = \cdot$) for all $i, j \in A$, depends on the regime of market X with one lag. The magnitude of this probability should change depending on the state of S_{t-1}^X . If the magnitude of probability remains unchanged independently of the regime of market X , one concludes that market X has no lagged impact on Y . Definition 1.2 implies a set of restrictions on the transition matrix \mathbf{P} :

$$\begin{aligned}
p_{11} + p_{13} &= p_{31} + p_{33}, \\
p_{21} + p_{23} &= p_{41} + p_{43}, \\
p_{22} + p_{24} &= p_{42} + p_{44}, \\
p_{12} + p_{14} &= p_{32} + p_{34}.
\end{aligned} \tag{1.3}$$

Thus, the matrix \mathbf{P} takes on the following form, when S_t^X does not cause S_t^Y :

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{42} + p_{44} - p_{24} & p_{41} + p_{43} - p_{21} & p_{24} \\ p_{31} & p_{12} + p_{14} - p_{34} & p_{11} + p_{13} - p_{31} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}. \tag{1.4}$$

Finally, it is worth mentioning that the set of conditions (1.3) and the conditions obtained for the case where market Y does not cause X are necessary and sufficient for the processes S_t^Y and S_t^X to be first-order Markov chains. This is obtained from a theorem providing a condition for a Markov chain to be lumpable with respect to a partition. For a detailed discussion see Kemeny and Snell (1960).

Definition 1.3 *Contagion from X to Y is present if*

$$\forall k \in A \ P(S_t = (\cdot, k) | S_{t-1} = (k, \cdot)) = 1 \text{ or equivalently } \forall j \in A \ P(S_t^Y = j | S_{t-1}^X = j) = 1.$$

Definition 1.3 describes a situation where the process S_t^Y replicates realizations of the process S_t^X with a one-period delay. If the market X was in the calm (crisis) regime yesterday, the state of the market Y is calm (crisis) today. This definition is less restrictive than the analogous definition presented by Sola, Spagnolo, and Spagnolo (2002), in this sense that it

allows for the influence of market Y on market X , when there is contagion from X to Y . It must be highlighted that contagion is a very restrictive form of inter-market relationships, because it imposes the following constraints on the transition matrix \mathbf{P} :

$$\mathbf{P} = \begin{pmatrix} p_{11} & 0 & p_{13} & 0 \\ p_{21} & 0 & p_{23} & 0 \\ 0 & p_{32} & 0 & p_{34} \\ 0 & p_{42} & 0 & p_{44} \end{pmatrix}. \quad (1.5)$$

Definition 1.4 *Time series R^X is regime-independent of time series R^Y if process S_t^Y is independent of process S_t^X . Therefore, a sufficient condition for regime-independence is $\forall i, j, k, l \in A \quad P(S_t = (i, k) | S_{t-1} = (j, l)) = P(S_t^X = i | S_{t-1}^X = j) P(S_t^Y = k | S_{t-1}^Y = l)$.*

The relationship defined here assumes independence between the processes S_t^X and S_t^Y , but does not exclude dependence between R^X and R^Y . Index returns R^X and R^Y on the two separate markets can be correlated even though the regimes of the markets are independent. Therefore, we call such returns regime-independent. Independence implies the following form of the transition matrix \mathbf{P} :

$$\mathbf{P} = \begin{pmatrix} \pi_{11}^X \pi_{11}^Y & \pi_{11}^X (1 - \pi_{11}^Y) & \pi_{11}^Y (1 - \pi_{11}^X) & (1 - \pi_{11}^Y) (1 - \pi_{11}^X) \\ \pi_{11}^X (1 - \pi_{22}^Y) & \pi_{22}^Y \pi_{11}^X & (1 - \pi_{22}^Y) (1 - \pi_{11}^X) & \pi_{22}^Y (1 - \pi_{11}^X) \\ \pi_{11}^Y (1 - \pi_{22}^X) & (1 - \pi_{22}^X) (1 - \pi_{11}^Y) & \pi_{11}^Y \pi_{22}^X & \pi_{22}^X (1 - \pi_{11}^Y) \\ (1 - \pi_{22}^Y) (1 - \pi_{22}^X) & \pi_{22}^Y (1 - \pi_{22}^X) & \pi_{22}^X (1 - \pi_{22}^Y) & \pi_{22}^X \pi_{22}^Y \end{pmatrix}, \quad (1.6)$$

where π_{ij}^X denotes the probability that the process S_t^X switches from state i to state j and π_{ij}^Y denotes the probability that the process S_t^Y moves from state i to state j , where $i, j \in A$. It is worth noting that the regime-independence described by definition 1.4 is the special case of the no-causality definition 1.2. In order to see this, it is enough to check that the elements of the matrix \mathbf{P} in (1.6) fulfill the set of conditions (1.3).

1.2.3 The probability of crisis and calm regimes

Knowing the parameters in the transition matrix of the process S_t enables us to calculate the probabilities that are especially interesting from the international investor's perspective. We compute the probability that the particular market Y enters a regime of crisis or calm, conditional on the information that this market and the market X were in their respective regimes yesterday. These probabilities can be obtained by summing suitable elements of matrix \mathbf{P} . For example:

$$P(S_t^Y = h | S_{t-1}^X = l \wedge S_{t-1}^Y = l) = p_{12} + p_{14}.$$

Computation of the probability that market Y enters state i , conditional only on the information that the market X was in state j one period earlier may also be of interest to analysts. It shows how the lack of information about the past state of Y influences forecasts of its present state. For example, the probability $P(S_t^Y = h | S_{t-1}^X = l)$ can be found in the following way. Let us notice that:

$$\begin{aligned} & P(S_t^Y = h \wedge S_{t-1}^X = l) = \\ &= P(S_t^Y = h \wedge S_{t-1}^X = l \wedge S_{t-1}^Y = l) + P(S_t^Y = h \wedge S_{t-1}^X = l \wedge S_{t-1}^Y = h) = \\ &= P(S_t^Y = h | S_{t-1} = 1)P(S_{t-1} = 1) + P(S_t^Y = h | S_{t-1} = 2)P(S_{t-1} = 2) = \\ &= \pi_1(p_{12} + p_{14}) + \pi_2(p_{22} + p_{24}), \end{aligned} \quad (1.7)$$

where $\boldsymbol{\pi}' = (\pi_1, \pi_2, \pi_3, \pi_4)$ is a vector of ergodic probabilities for the Markov chain S_t and p_{ij} are elements of the transition matrix \mathbf{P} . Thus,

$$P(S_t^Y = h | S_{t-1}^X = l) = \frac{P(S_t^Y = h \wedge S_{t-1}^X = l)}{P(S_{t-1}^X = l)} = \frac{\pi_2(p_{22} + p_{24}) + \pi_1(p_{12} + p_{14})}{\pi_1 + \pi_2}. \quad (1.8)$$

1.2.4 The specific model

In this subsection, we construct the model of inter-market dependencies, which applies the four-state Markov chain described above, and present tests for restrictions on the transition matrix \mathbf{P} . These restrictions satisfy the conditions for regime-independence, no-causality, and contagion. The unrestricted version of the model corresponds to bi-directional causality between the markets.

Our model is a Markov switching mixture of bivariate normal distributions. It is an extended version of the models examined by Phillips (1991), Sola, Spagnolo, and Spagnolo (2002) in the respect that it imposes fewer restrictions on the means and volatilities of stock index returns in each regime. We consider the Markov switching model with two time series and four states. Each of the states corresponds to one bivariate normal distribution. The only constraints that have to be imposed are the ones that enable us to differentiate between different states. In this study, we propose the following restriction on means:

$$\boldsymbol{\mu}_1 = \begin{pmatrix} \mu_1^X \\ \mu_1^Y \end{pmatrix}, \quad \boldsymbol{\mu}_2 = \begin{pmatrix} \mu_1^X \\ \mu_2^Y \end{pmatrix}, \quad \boldsymbol{\mu}_3 = \begin{pmatrix} \mu_2^X \\ \mu_1^Y \end{pmatrix}, \quad \boldsymbol{\mu}_4 = \begin{pmatrix} \mu_2^X \\ \mu_2^Y \end{pmatrix}. \quad (1.9)$$

Hence, the vectors of means in the second and third states are completely defined after the means in the first and fourth states are estimated. The model takes on the form:

$$\mathbf{y}_t = I_{\{S_t=1\}}\boldsymbol{\mu}_1 + I_{\{S_t=2\}}\boldsymbol{\mu}_2 + I_{\{S_t=3\}}\boldsymbol{\mu}_3 + I_{\{S_t=4\}}\boldsymbol{\mu}_4 + \boldsymbol{\epsilon}_t$$

where $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{\Sigma})$ (1.10)

$$\boldsymbol{\Sigma} = I_{\{S_t=1\}}\boldsymbol{\Sigma}_1 + I_{\{S_t=2\}}\boldsymbol{\Sigma}_2 + I_{\{S_t=3\}}\boldsymbol{\Sigma}_3 + I_{\{S_t=4\}}\boldsymbol{\Sigma}_4$$

and $I_{\{S_t=i\}}$ is defined as:

$$I_{\{S_t=i\}} = \begin{cases} 1 & S_t = i \\ 0 & S_t \neq i \end{cases}$$

The mean and variance parameters are usually not known *a priori*, therefore we outline

the procedure to estimate these parameters using the Maximum Likelihood (ML) approach. The log-likelihood function is given by the formulae:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^T \log(\boldsymbol{\xi}_{t|t-1} \times \mathbf{f}_t), \quad (1.11)$$

where

$$\boldsymbol{\xi}_{t|t-1} = (P(S_t = i | \boldsymbol{\Lambda}_{t-1}; \boldsymbol{\theta}))'_{i \in 4}, \quad \mathbf{f}_t = (f(\mathbf{y}_t | S_t = j, \boldsymbol{\Lambda}_{t-1}; \boldsymbol{\theta}))'_{j \in 4} \quad (1.12)$$

and

$$f(\mathbf{y}_t | S_t = j, \boldsymbol{\Lambda}_{t-1}; \boldsymbol{\theta}) = \frac{1}{2\pi |\boldsymbol{\Sigma}_j|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{y}_t - \boldsymbol{\mu}_j)' \boldsymbol{\Sigma}_j^{-1} (\mathbf{y}_t - \boldsymbol{\mu}_j)\right). \quad (1.13)$$

The symbol $'$ and \times denotes a transposition of a vector, and the scalar product, respectively. $\boldsymbol{\theta}$ is the vector of all unknown parameters in the model, $\boldsymbol{\Lambda}_T = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_T\}$, and T is the sample size.

The parameters are computed using the Expectation-Maximization (EM) algorithm (Hamilton (1990), Kim (1994)). In the $l+1$ -th step of iteration, the following maximum likelihood estimators are used.

$$\hat{p}_{ij}^{l+1} = \frac{\sum_{t=2}^T P(S_t = j, S_{t-1} = i | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)}{\sum_{t=2}^T P(S_{t-1} = i | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)} \quad (1.14)$$

is the approximation of the p_{ij} parameter in the transition matrix \mathbf{P} . The estimator given by (1.14) is defined in terms of smoothed probabilities $P(S_t = i | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)$ where $i \in \{1, 2, 3, 4\}$. In order to identify and examine persistence of regimes in a Markov-switching framework, it is enough to plot smoothed probabilities against time.

The estimates of the vectors $\boldsymbol{\mu}_j$ in the $l+1$ -th step, for $j = 1$ and 4 , are given by:

$$\hat{\boldsymbol{\mu}}_j^{l+1} = \frac{\sum_{t=1}^T \mathbf{y}_t \cdot P(S_t = j | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)}{\sum_{t=1}^T P(S_t = j | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)} = \begin{pmatrix} \hat{\boldsymbol{\mu}}_j^X \\ \hat{\boldsymbol{\mu}}_j^Y \end{pmatrix}, \quad (1.15)$$

and for $j = 2, 3$:

$$\hat{\boldsymbol{\mu}}_2^{l+1} = \begin{pmatrix} \hat{\mu}_1^X \\ \hat{\mu}_4^Y \end{pmatrix}, \quad \hat{\boldsymbol{\mu}}_3^{l+1} = \begin{pmatrix} \hat{\mu}_4^X \\ \hat{\mu}_1^Y \end{pmatrix}. \quad (1.16)$$

The estimators of covariance matrices $\boldsymbol{\Sigma}_j$ for each state of the hidden Markov chain ($j = 1, 2, 3, 4$) are given by:

$$\hat{\boldsymbol{\Sigma}}_j = \frac{\sum_{t=1}^T (\mathbf{y}_t - \hat{\boldsymbol{\mu}}_j^{l+1})(\mathbf{y}_t - \hat{\boldsymbol{\mu}}_j^{l+1})' \cdot P(S_t = j | \mathbf{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)}{\sum_{t=1}^T P(S_t = j | \mathbf{\Lambda}_T; \hat{\boldsymbol{\theta}}^l)}. \quad (1.17)$$

The iteration procedure begins with choosing random starting values for all parameters and continues computing approximations of the ML estimates until $\|\hat{\boldsymbol{\theta}}^l - \hat{\boldsymbol{\theta}}^{l+1}\| > 10^{-8}$. Then, the EM procedure is repeated a large number of times (e.g. 200) to ensure that the local maximum of the likelihood function is a global one.

However, estimation of the model with the constrained transition matrix is different for each type of the relationship between the markets. For example in case of contagion (definition 1.3), Phillips (1991) argues that it is enough to set the starting values of the respective parameters in \mathbf{P} to zero to receive the valid ML estimates of the contagion model. One property of the EM algorithm described by Hamilton (1990) and Kim (1994) is that once transition probabilities are set to zero they remain equal to zero through all iterations.

In this study, we derive the ML estimators for the case where no-causality restrictions are imposed on transition matrix \mathbf{P} , as in equation (1.4). In this restricted case, one assumes that some elements of the matrix \mathbf{P} are functions of the other parameters collected in vector γ , where γ is defined as $(p_{11}, p_{12}, p_{13}, p_{21}, p_{24}, p_{31}, p_{34}, p_{41}, p_{42}, p_{43})$. The method of determining the ML estimators when elements of the transition matrix are a function of the other parameters was developed by Phillips (1991). This is the method we have chosen to use. Therefore, in order to find the maximum value of the likelihood function under the no-causality constraints, we construct the Lagrangian:

$$\Gamma = L(\boldsymbol{\theta}) + \sum_{i=1}^4 \lambda_i \left(1 - \sum_{j=1}^4 p_{ij} \right) \quad (1.18)$$

The first order condition for maximizing the Lagrangian with respect to the k-th element of the vector γ is

$$\frac{\partial \Gamma}{\partial \gamma_k} = \sum_{p_{ij}} \frac{\partial \Gamma}{\partial p_{ij}} \frac{\partial p_{ij}}{\partial \gamma_k} = \sum_{p_{ij}} \left[\frac{\partial L(\boldsymbol{\theta})}{\partial p_{ij}} - \lambda_i \right] \frac{\partial p_{ij}}{\partial \gamma_k} = 0. \quad (1.19)$$

From the derivation in Phillips (1991), it follows that condition (1.19) is equivalent to the following one:

$$\sum_{p_{ij}} \frac{N_{ij}}{p_{ij}} \frac{\partial p_{ij}}{\partial \gamma_k} = \sum_{p_{ij}} D_i \frac{\partial p_{ij}}{\partial \gamma_k}, \quad (1.20)$$

where

$$N_{ij} = \sum_{t=2}^T P(S_t = j, S_{t-1} = i | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}) \quad \lambda_i = D_i = \sum_{t=2}^T P(S_{t-1} = i | \boldsymbol{\Lambda}_T; \hat{\boldsymbol{\theta}}).$$

By finding the estimators of the parameters included in vector γ one can determine all other elements of matrix \mathbf{P} in the form (1.4). One has to consider the first-order conditions for the following subsets of elements of vector γ : $\{p_{13}, p_{31}, p_{11}\}$, $\{p_{12}, p_{14}, p_{34}\}$, $\{p_{41}, p_{43}, p_{21}\}$, $\{p_{42}, p_{44}, p_{24}\}$. For example, by applying the first-order condition (1.20) to subset $\{p_{13}, p_{31}, p_{11}\}$, we receive the following system of equations:

$$\left\{ \begin{array}{l} \frac{N_{11}}{p_{11}} + \frac{N_{33}}{p_{11}+p_{13}-p_{31}} = D_1 + D_3 \\ \frac{N_{13}}{p_{13}} + \frac{N_{33}}{p_{11}+p_{13}-p_{31}} = D_1 + D_3 \\ \frac{N_{31}}{p_{31}} - \frac{N_{33}}{p_{11}+p_{13}-p_{31}} = 0. \end{array} \right.$$

As a solution to the above system we obtain the estimators:

$$\begin{aligned} \hat{p}_{13} &= \frac{N_{13}}{D_1 + D_3} + \frac{N_{13}(N_{33} + N_{31})}{(D_1 + D_3)(N_{13} + N_{11})}, & \hat{p}_{31} &= \frac{N_{31}(\hat{p}_{11} + \hat{p}_{13})}{N_{33} + N_{31}}, \\ \hat{p}_{11} &= \frac{N_{11}}{D_1 + D_3} + \frac{N_{11}(N_{33} + N_{31})}{(D_1 + D_3)(N_{13} + N_{11})}. \end{aligned} \quad (1.21)$$

By solving the corresponding systems of equations for the other subsets of vector γ we find:

$$\begin{aligned}\hat{p}_{12} &= \frac{N_{12}}{D_1 + D_3} + \frac{N_{12}(N_{32} + N_{34})}{(D_1 + D_3)(N_{14} + N_{12})}, \\ \hat{p}_{14} &= \frac{N_{14}}{D_1 + D_3} + \frac{N_{14}(N_{32} + N_{34})}{(D_1 + D_3)(N_{12} + N_{14})}, \\ \hat{p}_{41} &= \frac{N_{41}}{D_4 + D_2} + \frac{N_{41}(N_{23} + N_{21})}{(D_4 + D_2)(N_{43} + N_{41})},\end{aligned}$$

and

$$\begin{aligned}\hat{p}_{43} &= \frac{N_{43}}{D_4 + D_2} + \frac{N_{43}(N_{23} + N_{21})}{(D_4 + D_2)(N_{43} + N_{41})}, \\ \hat{p}_{42} &= \frac{N_{42}}{D_4 + D_2} + \frac{N_{42}(N_{22} + N_{24})}{(D_4 + D_2)(N_{44} + N_{42})}, \\ \hat{p}_{44} &= \frac{N_{44}}{D_4 + D_2} + \frac{N_{44}(N_{22} + N_{24})}{(D_4 + D_2)(N_{44} + N_{42})}, \\ \hat{p}_{34} &= \frac{N_{34}(\hat{p}_{12} + \hat{p}_{14})}{N_{32} + N_{34}}, \quad \hat{p}_{21} = \frac{N_{21}(\hat{p}_{41} + \hat{p}_{43})}{N_{21} + N_{23}}, \quad \hat{p}_{24} = \frac{N_{24}(\hat{p}_{42} + \hat{p}_{44})}{N_{22} + N_{24}}.\end{aligned}$$

The likelihood ratio (LR) tests have usually been used to test for the existence of restrictions on transition matrix \mathbf{P} (Phillips (1991), Raven and Sola (1995), Sola, Spagnolo, and Spagnolo (2002)). Phillips (1991) uses LR tests to check the null hypothesis of contagion given by definition 1.3 against the alternative hypothesis of no restrictions on the transition matrix (i.e. causality between markets) and to test the null hypothesis of regime independence given by definition 1.4, against the alternative of no restrictions on transition matrix \mathbf{P} . Similarly, we propose using the LR statistic to test the null hypothesis of no-causality from X to Y , described in definition 1.2, against the alternative hypothesis of causality between the markets.

The LR statistic for testing the contagion hypothesis can be approximated by $\chi^2(8)$ distribution, because eight elements have to be equal to zero in matrix \mathbf{P} under the null hypothesis. The statistic for testing regime-independence is asymptotically $\chi^2(12)$ distributed, because twelve constraints are imposed on transition matrix \mathbf{P} in the form (1.6). The test statistic for no-causality effect has an asymptotic $\chi^2(2)$ distribution, because two elements

of the transition matrix are subject to constraints.

1.3 Empirical example

In order to present empirical results using our approach, we investigate the relationship between the Japanese and Hong Kong capital markets during the East Asian crisis in 1997. The Asian crisis is well-suited to our multi-regime framework presented above, because it provides an excellent economic interpretation for the crisis regime in the Markov switching model⁷.

During the crisis in October, 1997, the Hong Kong market underwent one of the most significant declines among the Asian markets. In the six-month period following the beginning of July, Hong Kong lost over 30% of its stock market value in both dollar and local terms (Chakrabarti and Roll (2002)). After the crash in Hong Kong, events in Asia became headline news and the spread of the crisis to the markets worldwide, i.e. contagion, was discussed (Forbes and Rigobon (2002)).

The Japanese economy also suffered from the crisis. The Nikkei 225 index fell by 26% in the second half of 1997, and by 8% in October 1997, when the crisis in Hong Kong erupted. Corsetti, Pesenti, and Roubini (1999) argue that the Japanese macroeconomic conditions were still deteriorating in September 1998.

In our study we employ the main indices from the Hong Kong and Japanese markets, namely the HSI and the Nikkei 225. The series are daily returns of these indices covering the three-year period from June 1, 1995 to May 30, 1998. Since we want to avoid the influence of other turmoil events on the relationship between the two markets, we set the sample period to start after the Mexican crisis of 1994 and to end before the Russian crisis in the summer of 1998.

We estimate the bivariate Markov switching model in several versions. The first one is the unconstrained model fulfilling the hypothesis of causality between capital markets. The

⁷Corsetti, Pesenti, and Roubini (1999) provide an extensive description of the dependencies between financial markets during the Asian crisis.

Table 1.1: Test of linkages between markets.

Test	Nikkei225 (X) and HSI (Y)	HSI (X) and Nikkei225 (Y)
Independence	134.56**	134.56**
No spillovers	44.28**	62.74**
Contagion	24.43**	18.10*

Note: * and ** denote rejection of the null hypothesis at 5% and 1% levels, respectively.

second version, with constrained parameter space in transition matrix \mathbf{P} , assumes that the states of both markets are independent, i.e. the markets are regime-independent. Next, we set constraints on the transition matrix in order to estimate the models satisfying the hypotheses that HSI does not lead Nikkei 225 and that Nikkei 225 does not lead HSI, respectively. Finally, we separately estimate the models under the hypotheses of contagion from Nikkei 225 to HSI and contagion from HSI to Nikkei 225. We compare all constrained models with the unconstrained version using the LR tests. The results are presented in Table 1.1.

Our first result suggests that the hypothesis of regime-independence between the two markets is much too restrictive in comparison to the general hypothesis allowing for bi-directional spillovers between the Hong Kong and Japanese markets. The LR test strongly rejects the independence hypothesis at a 1% level of significance, which can be interpreted as evidence of dependence between the regimes on the two markets. We also find that the no-causality hypothesis is rejected in each case. This result indicates that causality from the Hong Kong to the Japanese market and from the Japanese to the Hong Kong market takes place. Granger (1969, 1980) calls such a bi-directional leading relationship a feedback causality. Similarly, the hypotheses of contagion from the Japanese market and from the Hong Kong market are rejected at a 5% level. Forbes and Rigobon (2002) using a correlation

approach also find no evidence of contagion from Hong Kong to Japan during the Asian crisis. However, we observe that the statistic for contagion hypothesis is higher in the case of contagion to the Hong Kong market. This result may indicate that causality from Hong Kong to Japan is more significant than causality in the opposite direction. This assumption is backed by the result from the no-causality tests, where the no-causality hypothesis from the Hong Kong market is more strongly rejected than the hypothesis of no-causality from Japan.

Table 1.2: Correlation matrix decomposition of intraday volume for CAC40 index stocks.

States		Parameters					Transition matrix P				
S_t	S_t^X	S_t^Y	μ_1^X	σ_X^2	μ^Y	σ_Y^2	ρ^{XY}	(l,l)	(l,h)	(h,l)	(h,h)
1	calm(l)	calm(l)	0.049 (0.005)	1.021 (0.128)	0.123 (0.009)	0.874 (0.094)	0.194 (0.066)	0.969	0.031	0.000	0.000
2	calm(l)	crisis(h)	0.049 (0.005)	0.849 (0.081)	-0.466 (0.101)	2.438 (0.394)	0.381 (0.083)	0.000	0.174	0.826	0.000
3	crisis(h)	calm(l)	-0.055 (0.008)	1.798 (0.201)	0.123 (0.009)	1.137 (0.121)	0.239 (0.025)	0.072	0.294	0.620	0.014
4	crisis(h)	crisis(h)	-0.055 (0.008)	2.469 (0.237)	-0.466 (0.101)	4.814 (0.315)	0.476 (0.051)	0.000	0.032	0.000	0.968

Standard errors in parentheses.

In Table 1.2 we present the estimated parameters from the model that satisfies the hypothesis of feedback causality between capital markets. The means on both markets are positive in the calm regime and they are negative in the crisis regime. This suggests that the market indices were falling on average during the crisis and growing during the calm periods. Similarly, the volatility of both index returns is the highest when both markets are in a crisis regime and the volatility is always higher on the specific market when the particular market enters a crisis regime. It must be noted that the latter result was obtained despite the lack of any constraints on the variance parameters in our model.

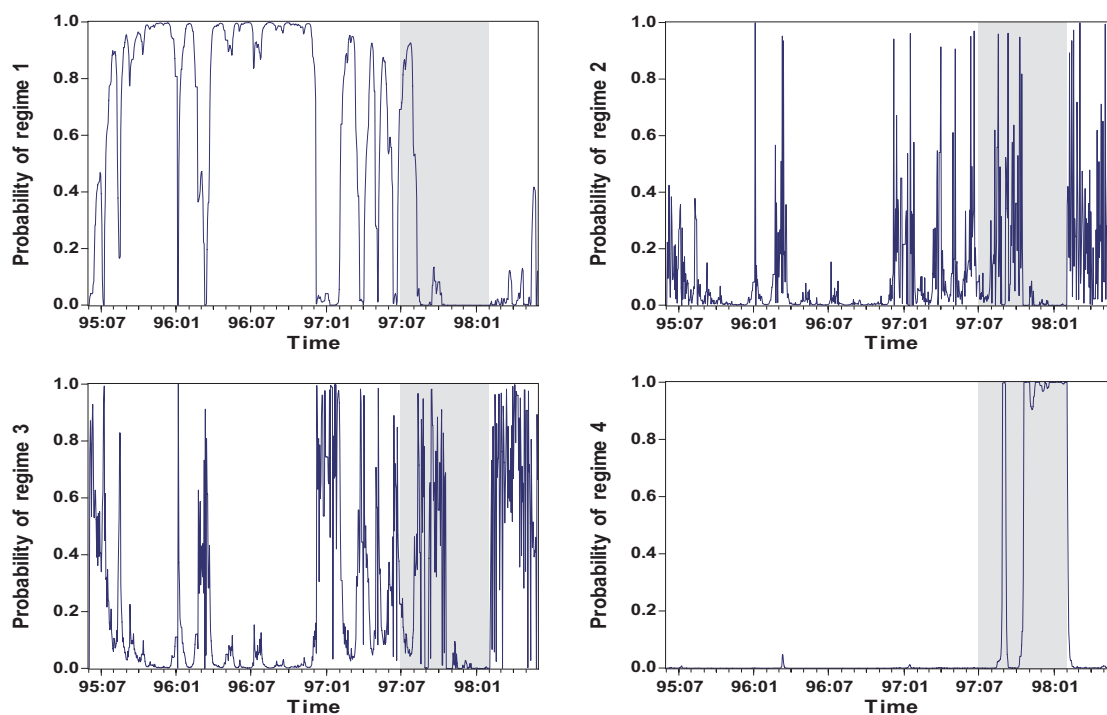


Figure 1.1: Smoothed probabilities of being in the specific regime by the pair of markets. The shaded area includes the period from July 1, 1997, to January 31, 1998.

From the estimated parameters in the transition matrix one can infer that regimes where both markets are in the same state of calm or crisis are very persistent. Once the markets

enter one of these regimes, they stay there for a longer period. The other two regimes, 2 and 3, are less persistent. Figure 1.1 confirms these observations. It presents the smoothed probabilities of being in the particular state conditional on information from the entire sample. State 1, where both markets are in the calm regime, dominates in the period from June, 1995 till the end of 1996. State 4 is persistent in the period from July, 1997 to January, 1998, which corresponds to the crisis period described in the literature (e.g. Corsetti, Pesenti, and Roubini (1999)). In contrast, the states 2 and 3 are not stable, but are frequently visited and left in the period from the beginning of 1997 till the period from the beginning of 1997 till the beginning of the crisis in July 1997 and after the crisis in 1998.

All regimes together reveal some interesting patterns of shock transmission between the markets. First, when both markets are in a calm regime, they will either stay in this regime ($p_{11} = 0.969$) or switch to a regime where Japan is still in a calm state and Hong Kong is in a state of turmoil ($p_{12} = 0.031$). This suggests that Hong Kong enters the crisis first and it is followed by the Japanese market on the next day. Then, Hong Kong sometimes stays in the crisis ($p_{22} = 0.174$), but usually it switches to a calm state while Japan replicates the move of Hong Kong from the preceding day and enters a crisis ($p_{23} = 0.826$). Japan usually stays longer in the crisis ($p_{33} = 0.620$), sometimes exchanges regimes with Hong Kong ($p_{32} = 0.294$), or follows Hong Kong into a calm regime ($p_{31} = 0.072$). It is rare that Hong Kong accompanies Japan into crisis when Japan is already there ($p_{34} = 0.014$), but it is the only way for both markets to get into the crisis ($p_{14} = p_{24} = 0.000$). Once the markets are in a crisis regime, they will, with a high probability, stay there till the next period ($p_{44} = 0.968$) or the Japanese market will leave first ($p_{42} = 0.032$).

Generally, the Hong Kong market enters a crisis first, but it often switches between calm and crisis. Japan follows Hong Kong into crisis and remains in this state much longer. Eventually, both markets are in crisis and stay there until Japan leaves it first. These findings show that the behavior of both markets during the Asian crisis was more complicated than

the contagion hypothesis assumes (Sola, Spagnolo, and Spagnolo (2002)).

Financial investors are interested in the probability of one market entering crisis when the other market was there one period earlier. In Table 1.3 we present such conditional probabilities. We observe that the markets tend to replicate the regimes of the other market from the preceding period. It seems that the Japanese market more often avoids entering a crisis regime despite the other market being there one period earlier. Similarly, there is a higher probability that the Japanese market enters a crisis than that the Hong Kong market enters the crisis conditional on the information that the other market was in a calm regime one day earlier. However, most market participants additionally possess the information about the state of the domestic market from one day earlier, which can dramatically change the estimates of the conditional probabilities. We present the calculations in Table 1.4.

Based on these results, we can more precisely forecast the future state of each market. The Japanese market will almost never enter a crisis when both markets are in a calm state the day before. The Hong Kong market will enter a crisis with the probability 0.031. Analogously, both markets will with a very high probability remain in crisis regimes provided that they were in crisis regimes in the preceding period. Generally, the probabilities from Table 1.4 differ significantly from those presented in Table 1.3, which suggests that the information about the past states of the local market is important for forecasting its future state. Nevertheless, the information about the performance of the Japanese market today is relevant for the state of the Hong Kong market tomorrow and the state of the Hong Kong market today determines the future state of the Japanese market.

In this study we assume that all information is transferred between markets at the latest on the next working day, which is reasonable at the time when even emerging markets are fully computerized and most public information is available immediately. However, in general it is possible that one market is followed by another market with a lag of two days or more.

Table 1.3:

Probabilities conditional on information from the foreign market.

Conditional probabilities			
HSI on Nikkei225		Nikkei225 on HSI	
$P(S_t^Y = l S_{t-1}^X = h)$	0.105	$P(S_t^X = l S_{t-1}^Y = h)$	0.482
$P(S_t^Y = h S_{t-1}^X = h)$	0.850	$P(S_t^X = h S_{t-1}^Y = h)$	0.518
$P(S_t^Y = h S_{t-1}^X = l)$	0.055	$P(S_t^X = h S_{t-1}^Y = l)$	0.191
$P(S_t^Y = l S_{t-1}^X = l)$	0.945	$P(S_t^X = l S_{t-1}^Y = l)$	0.809

Table 1.4: Probabilities conditional on information from the foreign market.

Conditional probabilities			
HSI on Nikkei225		Nikkei225 on HSI	
$P(S_t^Y = h S_{t-1}^X = h \wedge S_{t-1}^Y = h)$	1.000	$P(S_t^X = h S_{t-1}^X = h \wedge S_{t-1}^Y = h)$	0.968
$P(S_t^Y = h S_{t-1}^X = l \wedge S_{t-1}^Y = h)$	0.174	$P(S_t^X = h S_{t-1}^X = h \wedge S_{t-1}^Y = l)$	0.634
$P(S_t^Y = h S_{t-1}^X = h \wedge S_{t-1}^Y = l)$	0.308	$P(S_t^X = h S_{t-1}^X = l \wedge S_{t-1}^Y = h)$	0.826
$P(S_t^Y = h S_{t-1}^X = l \wedge S_{t-1}^Y = l)$	0.031	$P(S_t^X = h S_{t-1}^X = l \wedge S_{t-1}^Y = l)$	0.000
$P(S_t^Y = l S_{t-1}^X = l \wedge S_{t-1}^Y = l)$	0.969	$P(S_t^X = l S_{t-1}^X = l \wedge S_{t-1}^Y = l)$	1.000
$P(S_t^Y = l S_{t-1}^X = h \wedge S_{t-1}^Y = l)$	0.692	$P(S_t^X = l S_{t-1}^X = l \wedge S_{t-1}^Y = h)$	0.174
$P(S_t^Y = l S_{t-1}^X = l \wedge S_{t-1}^Y = h)$	0.826	$P(S_t^X = l S_{t-1}^X = h \wedge S_{t-1}^Y = l)$	0.366
$P(S_t^Y = l S_{t-1}^X = h \wedge S_{t-1}^Y = h)$	0.000	$P(S_t^X = l S_{t-1}^X = h \wedge S_{t-1}^Y = h)$	0.032

Hamilton and Lin (1996) and Sola, Spagnolo, and Spagnolo (2002) propose specifications which allow for testing independence and contagion with two lags. However, these methods require a transition matrix to be twice as large or a more complicated set of restrictions. Therefore, we leave development of tests for causality at higher lags for future research.

1.4 Conclusions

In this article, we present a methodology to construct different types of relationships between financial markets using a bivariate Markov switching model. We explicitly define the hypotheses of causality, regime-independence, and contagion, describe estimation of the model, and present likelihood ratio tests for the hypotheses of causality, regime-independence, and contagion. In this way we introduce the Granger causality approach to the Markov switching model of asset returns, which is related to the methodology of Psardakis, Ravn and Sola (2003).

This methodology has several advantages over other approaches used to estimate links between the markets. First, it allows testing of various hypotheses of dependencies between financial markets. The models do not assume any specific linear or nonlinear links between stock index returns. All hypotheses of causality, contagion, and independence are defined in relation to probability measures. Second, this approach differentiates between calm and crisis periods, which are modelled as multiple random events rather than the dates assumed to be known *a priori* or structural changes taking place in the sample (Psardakis, Ravn, and Sola (2003)). The causality patterns are allowed to be asymmetrical with respect to states of calm and crisis, as argued by Sola, Spagnolo, and Spagnolo (2002). Using the Markov switching model, we are able to calculate the probability that one market enters a particular regime conditional on information about the past states of this and the other market. Finally, the testing procedure enables us to differentiate between extreme types of inter-market de-

dependencies (independence, contagion) and more frequently observed relationships (causality, feedback causality, dependence without causality).

Naturally, the presented model provides a simplified description of dependencies between financial markets. There are surely other factors that impact both of the investigated markets and influence the inter-market relationship (Frankel and Rose (1996), Portes and Rey (1999), Billio and Pelizzon (2003), Wälti (2003) among others). For example, if one of the investigated markets absorbs some external information more quickly than the second market, one could wrongly conclude that the first market leads the second. Adding a variable representing common shocks to both markets, e.g. a third market or macroeconomic policy variable, to our model is possible, but increases the number of parameters and the size of the transition matrix. It also complicates the construction of restrictions in the transition matrix imposing causality or contagion. We leave this issue for further research.

As an empirical application, we model the relationship between the Japanese and Hong Kong markets during the Asian crisis in 1997. We find evidence of feedback causality between stock index returns on these markets, but we reject the hypotheses of contagion as defined by Sola, Spagnolo, and Spagnolo (2002). The characteristics of index returns on both markets are found to be typical for calm and crisis regimes. Lower index returns, higher volatility of returns, and higher correlation between markets are often observed during international financial crises and tumultuous periods (King and Wadhawani (1990), Longin and Solnik (2001), Forbes and Rigobon (2002)).

In contrast to previous studies, our clinical examination of the Asian crisis enables us to investigate the sequence of market crisis entrance. Additionally, for each market we calculate the probability of entering the specific regime conditional on the past performance of this and the other market. We note that estimated probabilities allow for an accurate prediction of the future states of the markets.

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Chapter 2

Testing for Financial Spillovers in Calm and Turmoil Periods

In this paper, we investigate financial spillovers between stock markets during calm and turbulent times. We explicitly define financial spillovers and financial contagion in accordance with the economic literature and construct statistical models corresponding to these definitions in a Markov switching framework. Applying the new testing methodology based on transition matrices, we find that spillovers from the US stock market to the UK, Japanese, and German markets are more frequent when the latter markets are in the crisis regime. However, we reject the hypothesis of strong financial contagion from the US market to the other markets.

2.1 Introduction

The importance of cross-market linkages and spillovers between international stock markets is well established. The literature on this issue allows to draw at least two main conclusions. First, the empirical studies find that the US stock market is the dominant capital market influencing other mature and developing stock markets (Eun and Shim (1989), Hamao, Masulis, and Ng (1990), Lin, Engle, and Ito (1994), Peiró, Quesada, and Uriel (1998), Ng (2000)). International stock markets are strongly correlated with the US market and past US stock returns affect present returns on other markets. Lagged spillovers are particularly

interesting to investigate, because stock markets with some delay assimilate important news from other markets. The most likely reasons may be inefficiencies of international stock markets, different opening hours on those markets, and non-synchronous trading (Cheung and Ng (1996), Peiró, Quesada, and Uriel (1998)). Analyzing lead-lag effects enables investors to learn about the structure and direction of financial spillovers, which is important for effective portfolio allocation and risk management (e.g., Ang and Bekaert (2002, 2003)).

Second, investigations in the field of stock market linkages suggest that stock returns are more volatile and more correlated with each other during turbulent periods compared to tranquil periods (King and Wadhvani (1990), Karolyi and Stulz (1996), Longin and Solnik (2001), Forbes and Rigobon (2002)). A rising positive correlation may suggest a decrease of capital diversification opportunities across markets during financial crises (Ang and Bekaert (2002), Bekaert and Harvey (2003)). The differences in financial spillovers during calm and turmoil periods are of special interest to agents who want to learn about the chance of having a crisis at the home market today, when there was a negative shock to another market yesterday. International investors can adjust their portfolio strategies to a changing structure of spillovers in different regimes. Moreover, financial market regulators are concerned about the vulnerability of home capital markets to international crises.

Despite the importance of both aspects only a few studies investigate changes in lead-lag effects of financial spillovers during calm periods and financial crises. The scarce findings suggest that spillovers from one market to other markets are found to be stronger when the former market is hit by some negative shock (Malliaris and Urrutia (1992), Sola, Spagnolo, and Spagnolo (2002), Chen, Chiang, and So (2003), Climent and Meneu (2003), Sander and Kleimeier (2003)). However, whether stock markets undergoing financial distress are still vulnerable to spillovers from other markets is an open question. Finding an answer to this issue may help in analyzing sources of financial crises. Stronger spillovers to turmoil stock markets could point to contagion as the main source of crises, while weaker spillovers could

suggest an individual character of financial distress. We attempt to answer this question in this paper. Most studies analyzing spillovers between stock markets during tranquil and crisis times do not take into account that the two analyzed markets can be in two different regimes of crisis or calm, i.e., for example, the stock market following the other market can be in the state of crisis independently of the state of the leading market. Another drawback of some studies is the ad hoc method used to identify crisis and calm periods (e.g., Malliaris and Urrutia (1992), Forbes and Rigobon (2002), Dungey and Zhumabekova (2001)). For example, in Chen, Chiang, and So (2003) the two regimes are explicitly defined as past stock returns exceeding (or falling below) an estimated threshold level. Moreover, earlier studies usually concentrate on specific events.

In this paper, we consider spillover effects from the US stock market to three major markets in Japan, the United Kingdom, and Germany over the period from 1984 to 2003 as well as sub-samples. We compare spillover effects during tranquil and turbulent periods and address the problems expressed above by extending the Markov switching model proposed by Phillips (1991). Phillips developed a bivariate Markov switching model to evaluate the transmission of business cycles between countries. Sola, Spagnolo, and Spagnolo (2002) applied this approach in the framework of financial markets to test their specific hypothesis of contagion across stock markets during the Asian crisis in 1997. Edwards and Susmel (2001) added lagged returns and conditional autoregressive heteroscedasticity into the model specification and investigated tests of independence and co-movements between international emerging stock markets in 1990s.

We construct a model of stock index returns for two markets analogous to the one proposed by Sola, Spagnolo, and Spagnolo (2002) and develop a test to investigate the hypotheses that, first, one market leads the other in both turmoil and tranquil periods and, second, one market leads the other only when the latter is already in a turmoil (calm) period. In this way we extend the methodology proposed by Edwards and Susmel (2001) and Sola, Span-

golo, and Spagnolo (2002), used to test financial contagion and independence by applying tests for financial spillovers in a Markov switching framework (see also Ravn and Sola (1995), Hamilton and Lin (1996), Psaradakis, Ravn, and Sola (2004)).

Our testing procedure has several advantages over other approaches to analyze the transmission of spillovers across stock markets. First, for each stock market it differentiates between calm and turbulent regimes. Thus, the method allows for a measurement of spillovers depending on the state of the market. The empirical literature suggests that multi-regime switching models of stock returns perform better than one-regime models (Cecchetti, Lam, and Mark (1990), Turner, Stratz, and Nelson (1990), Rydén, Teräsvirta, and Åsbrink (1998), Ang and Bekaert (2002)). Second, our procedure does not require an ad hoc identification of periods to examine spillovers between stock markets. Instead it estimates the probabilities of being in the crisis in a joint framework with all parameters of the model. Third, correlation and regression measures often fail to explore non-linear relations between variables. We offer a test on cross-market spillovers which does not depend on a specific linear or non-linear structure of linkages between stock returns. Fourth, Sola, Spagnolo, and Spagnolo (2002) provide a test of extreme spillovers, which they call a test of contagion. Our test is more flexible than the one applied there, since it examines a wider range of possible spillovers between the stock markets.

Finally, as an additional characteristics, most of the studies do not explicitly define spillovers between stock markets. In this paper, we provide a definition of one market leading other market that allows for distinguishing between lead-lag relations in calm and turbulent periods. This definition is consistent with the notion of causality, while in the context of financial crises it suits well the concept of contagion. To distinguish between extreme cases of spillovers we provide explicit definitions of independence (no spillovers) and contagion, which are in line with Sola, Spagnolo, and Spagnolo (2002), and compare the empirical results for tests based on those definitions.

The remainder of this paper is organized as follows. In the next section we describe the model based on the idea of Phillips (1991) to estimate stock index returns on two markets. Section 3 discusses our definitions of financial spillovers and discusses the tests for dependencies between the markets. Data and empirical results on spillovers from the US stock market to the Japanese, British, and German stock market are presented in section 4. Section 5 summarizes and concludes.

2.2 Modeling index returns on two markets

Our econometrical starting point is a Markov switching model of index returns on two markets. Let Z be the vector $[X, Y]'$, where $X = \{x_t, t \in N\}$ and $Y = \{y_t, t \in N\}$ are the two time series that can be interpreted as stock market index returns on two separate markets. Both index returns are allowed to enter one of the two complementary states of "crisis" and "calm" periods. Using all four combinations of these states we construct a Markov process with four regimes and we use the index to denote these regimes. "X and Y are in the calm states" defines the first regime ($s = 1$). "X is in the calm state and Y is in the crisis state" denotes the second one ($s = 2$). The third regime indicates that "X is in the crisis state and Y is in the calm state" ($s = 3$). "X and Y are in the crisis states" defines the fourth regime ($s = 4$). At each point in time, the state s is determined by an unobservable Markov chain. The dynamics of the Markov chain are described by a 4×4 transition matrix P :

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}, \quad (2.1)$$

where p_{ij} denotes the probability of changing the state from i to j . Assume that the 2×1

vector $z_t = [x_t, y_t]$ is driven by the four-state regime switching process:

$$z_t = \mu_s + \Theta_s u_t. \quad (2.2)$$

where u_t is a Gaussian process with zero mean and positive-definite covariance matrix Σ . The vector z_t is generated by the mixture of normal distributions with the mean μ_s and the covariance matrix Σ_s , both depending on the state :

$$z_t | (s_t = s) \sim N(\mu_s, \sigma_s^2). \quad (2.3)$$

and:

$$\Sigma_s = \Theta_s' \Sigma \Theta_s. \quad (2.4)$$

for $s = 1, 2, 3, 4$. Thus, the model is called a (four-state) Markov switching mixture of normal distributions and it consists of 32 independent parameters, namely two parameters of means for each state, three independent parameters from Σ_s for each state, and twelve independent parameters from the transition matrix P . In this model no constraints are imposed on the parameters of means, variances, correlations, and parameters from the transition matrix P . Economists highlight the significance of changes in return volatility during crisis periods. The high variance of index returns characterizes turmoil periods and the low variance characterizes tranquil periods. Additionally, the correlation coefficients between returns on different markets tend to increase when one of the markets enters the crisis regime (e.g. King and Wadhvani (1990), Karolyi and Stulz (1996), Longin and Solnik (2001), Forbes and Rigobon (2002)). However, some authors define crisis regimes as low average returns observed over longer periods or appearance of unusually low returns (Longin and Solnik (2001), Chen, Chiang, and So (2003), Mishkin and White (2003), Hartmann, Straetmans, and de Vries (2004)). Therefore, in our paper we highlight the importance of

changes in the variance and correlation by allowing them to take different values in all four regimes. Moreover, we restrict the parameter space by assuming that the mean of returns on each market switches between its high and low value depending on the state of this market. The high value of mean describes a market in the calm regime and the low value of mean describes a market in the tranquil regime. We expect low mean returns, high variances, and high correlation when both markets are in the crisis regime and high means, low variances, and low correlation when both markets are in the tranquil regime. The parameter space for means, variances, and correlations between returns on the two markets is defined as follows:

$$\boldsymbol{\mu} = \left(\boldsymbol{\mu}_{s=1} = \begin{pmatrix} \boldsymbol{\mu}_T^X \\ \boldsymbol{\mu}_T^Y \end{pmatrix}, \boldsymbol{\mu}_{s=2} = \begin{pmatrix} \boldsymbol{\mu}_T^X \\ \boldsymbol{\mu}_C^Y \end{pmatrix}, \boldsymbol{\mu}_{s=3} = \begin{pmatrix} \boldsymbol{\mu}_C^X \\ \boldsymbol{\mu}_T^Y \end{pmatrix}, \boldsymbol{\mu}_{s=4} = \begin{pmatrix} \boldsymbol{\mu}_C^X \\ \boldsymbol{\mu}_C^Y \end{pmatrix} \right). \quad (2.5)$$

and

$$\boldsymbol{\sigma} = \left(\boldsymbol{\sigma}_{s=1} = \begin{pmatrix} \boldsymbol{\sigma}_T^X \\ \boldsymbol{\sigma}_T^Y \end{pmatrix}, \boldsymbol{\sigma}_{s=2} = \begin{pmatrix} \boldsymbol{\sigma}_T^X \\ \boldsymbol{\sigma}_C^Y \end{pmatrix}, \boldsymbol{\sigma}_{s=3} = \begin{pmatrix} \boldsymbol{\sigma}_C^X \\ \boldsymbol{\sigma}_T^Y \end{pmatrix}, \boldsymbol{\sigma}_{s=4} = \begin{pmatrix} \boldsymbol{\sigma}_C^X \\ \boldsymbol{\sigma}_C^Y \end{pmatrix} \right). \quad (2.6)$$

$$\boldsymbol{\rho} = (\rho_{s=1} = \rho_{TT}^{XY}, \rho_{s=2} = \rho_{TC}^{XY}, \rho_{s=3} = \rho_{CT}^{XY}, \rho_{s=4} = \rho_{CC}^{XY}) \quad (2.7)$$

Symbols T_1 , T_2 , and T_3 denote the state of tranquillity on the respective market (the numbers are to distinguish between different values of a particular parameter in different regimes). Symbols C_1 , C_2 , and C_3 denote the crisis state. The transition matrix remains unconstrained, therefore we call this model a "general" or "unconstrained" model. In order to examine how our model fits the data we use several tests proposed by Breunig, Najarian, and Pagan (2003). We compare the means, variances, and peaks of the empirical distributions of the original data and the data simulated from our model. Additionally, we investigate

a "leverage effect" for both sets of data. The leverage effect is a common feature of stock returns indicating higher volatility of returns when past returns are negative (e.g. Black (1976), Engle and Ng (1993)). We find that our models are consistent with the original data in all cases and for all tests. Detailed results are presented in Table 2.6 after references.

2.2.1 Independence, Spillovers, and Contagion

In addition to the Markov switching model we need definitions of regime-independence, contagion and spillovers. These definitions enable us to assess the strength of shock transmission between the markets during stable and turmoil periods. Moreover, the definitions provide us the basis to distinguish between spillovers when one of the markets is in the crisis or in the calm state. We also describe the tests for no spillovers and contagion and propose our testing procedures to analyze the hypotheses of, first, one market leading the other during calm periods and, second, one market leading the other during crisis periods. The null hypothesis is that a spillover effect exists between the markets in both periods.

Definition 2.1 *Let $Z = [X, Y]$ be described by the Markov switching model introduced above. Y is said to be "regime-independent" of X if the event that Y enters the state i at time t is independent of the present and past states of X , where i is the crisis or calm regime in our Markov switching model.*

Sola, Spagnolo, and Spangolo (2002) employ the definition of regime-independence of X and Y to test for contagious spillovers between financial markets. In case Y and X are

regime-independent the following restrictions are imposed on the transition matrix P :

$$\mathbf{P} = \begin{pmatrix} \pi_{11}^X \pi_{11}^Y & \pi_{11}^X (1 - \pi_{11}^Y) & \pi_{11}^Y (1 - \pi_{11}^X) & (1 - \pi_{11}^Y)(1 - \pi_{11}^X) \\ \pi_{11}^X (1 - \pi_{22}^Y) & \pi_{22}^Y \pi_{11}^X & (1 - \pi_{22}^Y)(1 - \pi_{11}^X) & \pi_{22}^Y (1 - \pi_{11}^X) \\ \pi_{11}^Y (1 - \pi_{22}^X) & (1 - \pi_{22}^X)(1 - \pi_{11}^Y) & \pi_{11}^Y \pi_{22}^X & \pi_{22}^X (1 - \pi_{11}^Y) \\ (1 - \pi_{22}^Y)(1 - \pi_{22}^X) & \pi_{22}^Y (1 - \pi_{22}^X) & \pi_{22}^X (1 - \pi_{22}^Y) & \pi_{22}^X \pi_{22}^Y \end{pmatrix}, \quad (2.8)$$

where π_{ij}^Q denotes the probability of entering the state by the time series at time when it was in the state at time $t - 1$. $Q \in \{X, Y\}$, $i, j \in \{T, C\}$, and T and C denote the calm and crisis regimes, respectively. It should be noted that regime-independence does not imply independence of X and Y , since they are still allowed to be correlated with each other.

Definition 2.2 *Contagion from X to Y is present when the probability that Y enters the state i at time t conditional on the information that X was in this state at time $t - 1$ is equal one, where i denotes the crisis or calm regime in our Markov switching model.*

According to this definition the stock index return Y has to enter a specific regime, e.g. the crisis regime, if the stock index return X was there one period earlier. Thus, the sum of conditional probabilities p_{11} and p_{13} in the transition matrix P can be formulated as:

$$p_{11} + p_{13} = 1 \quad (2.9)$$

Calm and crisis are complementary events and we can express the sum of the probabilities as (l-calm, h-crisis):

$$p_{11} + p_{13} = P(Y_t = l | X_{t-1} = l \wedge X_{t-1} = l)$$

because:

$$p_{11} = P(X_t = l \wedge Y_t = l | X_{t-1} = l \wedge X_{t-1} = l)$$

$$p_{13} = P(X_t = h \wedge Y_t = l | X_{t-1} = l \wedge X_{t-1} = l)$$

Analogously, the other constraints on the transition matrix are:

$$p_{11} + p_{13} = 1 \tag{2.10}$$

$$p_{11} + p_{13} = 1 \tag{2.11}$$

$$p_{11} + p_{13} = 1 \tag{2.12}$$

and the transition matrix P takes on the form:

$$\mathbf{P} = \begin{pmatrix} p_{11} & 0 & p_{13} & 0 \\ p_{21} & 0 & p_{23} & 0 \\ 0 & p_{32} & 0 & p_{34} \\ 0 & p_{42} & 0 & p_{44} \end{pmatrix}. \tag{2.13}$$

Our definition of contagion is a less restrictive version of the one put forward by Sola, Spagnolo, and Spagnolo (2002) and is inspired by the work of Phillips (1991). Sola, Spagnolo, and Spagnolo set additional constraints assuming that $p_{11} = p_{21}$ and $p_{32} = p_{42}$, i.e. the probability that both markets, X and Y , enter the crisis or the calm regime does not depend on the regime of Y in the previous period. Thus, the past realizations of Y do not influence X when there is contagion from X to Y . Such an additional restriction has been criticized in the financial contagion literature due to the possibility of an estimation bias coming from overlooking the bi-directional transmission of shocks between the markets (Forbes and Rigobon (2002), Billio and Pelizzon (2003), Moser (2003), Rigobon (2003)). Additionally,

the idea of contagion is usually associated with financial crises spilling over from one market to other markets. One can expect that one market infects the other market only when it is in the crisis regime. Such a definition of "contagion in the crisis regime" corresponds to the transition matrix:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{13} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ 0 & p_{32} & 0 & p_{34} \\ 0 & p_{42} & 0 & p_{44} \end{pmatrix}. \quad (2.14)$$

The above definitions are closely related to the original definition of contagion discussed in Eichengreen, Rose, and Wyplosz (1996), Pericoli and Sbracia (2003), Hartmann, Straetmans, and de Vries (2004) among others. Contagion is defined there as "a significant increase in the probability of a crisis in one country, conditional on a crisis occurring in another country". The important characteristics of our definitions are identification of direction of contagion and financial spillovers from one market to another occurring with a lag, which allows for identification of delays in information or capital flows between markets (Climent and Meneu (2003), Sander and Kleimeier (2003)).¹ Other definitions of contagion and their applications are surveyed in Dornbusch, Park, and Claessens (2000), Claessens and Forbes (2001), Billio and Pelizzon (2003), Karolyi (2003), Moser (2003), Pericoli and Sbracia (2003). Our contagion definitions in the spirit of Sola, Spagnolo, and Spagnolo (2002) are very restrictive in comparison with the original definition of contagion presented above. Even rejecting them does not imply that one market does not lead the other (Ravn and Sola (1995)). Therefore, we propose a weaker form of inter-market dependency that fits well the idea of increased probability of a crisis at home, given the crisis occurred abroad and is based

¹Moreover, multiple regimes in our model enable testing changes in the correlation structure between returns on different markets during crisis periods, i.e. "shift-contagion" hypothesis, introduced by Forbes and Rigobon (2002). Nevertheless, we concentrate on the tests of financial spillovers based on the probability measures in this paper

on the notion of financial spillovers and causality (e.g., Geweke (1984)).

Definition 2.3 *X leads Y by one period if the magnitude of the probability that Y enters the state i at time t depends on whether X was in the state j at time t – 1, where i and j are allowed to be the crisis or calm regimes in our Markov switching model.*

We understand dependence as evidence of the difference in conditional probabilities of Y entering the state i, when X was in the calm state or in the crisis state at time t – 1, respectively. The case of X leading Y is interpreted in the context of inter-market linkages as a presence of financial spillovers from one market to the other. For example, the definition of spillovers comprises the situation when the probability of one market entering the crisis regime depends not only on whether this market was in the state of crisis one period earlier, but also on whether the other market was there in the previous period (l-calm, h-crisis):

$$P(X_t = h|X_{t-1} = l \wedge X_{t-1} = h) \neq P(X_t = h|X_{t-1} = h \wedge X_{t-1} = h) \quad (2.15)$$

which can be expressed in terms of parameters from the transition matrix P as:

$$p_{11} + p_{13} \neq p_{31} + p_{33} \quad (2.16)$$

Analogously, the following inequalities must be valid if X leads Y in all regimes:

$$\begin{aligned} p_{11} + p_{13} &\neq p_{31} + p_{33}, \\ p_{21} + p_{23} &\neq p_{41} + p_{43}, \\ p_{22} + p_{24} &\neq p_{42} + p_{44}, \\ p_{12} + p_{14} &\neq p_{32} + p_{34}. \end{aligned} \quad (2.17)$$

If one assumes that no spillovers exist between the markets in any regimes, the inequalities (2.17) become equalities and then the transition matrix is defined as:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{42} + p_{44} - p_{24} & p_{41} + p_{43} - p_{21} & p_{24} \\ p_{31} & p_{12} + p_{14} - p_{34} & p_{11} + p_{13} - p_{31} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}. \quad (2.18)$$

It can be shown that the constraint $p_{22} = p_{42} + p_{44} - p_{24}$ is equivalent to $p_{23} = p_{41} + p_{43} - p_{21}$ and that the constraint $p_{32} = p_{12} + p_{14} - p_{34}$ is equivalent to $p_{33} = p_{11} + p_{13} - p_{31}$. Therefore, the parameters and can be set unconstrained in the estimation process. Additionally, one can assume that no spillovers from X to Y will be present at time $t + 1$ in case Y is in the crisis state at time t . For example, the influence of the US market on the Japanese market could strongly diminish, when the Japanese market is hit by the strong internal crisis. In this case the transition matrix P will be defined as follows:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{12} + p_{14} - p_{34} & p_{11} + p_{13} - p_{31} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}. \quad (2.19)$$

Alternatively, the opposite hypothesis of no spillovers from X to Y when Y is in the calm regime may be denoted by:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{42} + p_{44} - p_{24} & p_{41} + p_{43} - p_{21} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}. \quad (2.20)$$

Generally, the Markov switching approach fits well the idea of investigating spillovers and contagion between the markets during stable and turmoil periods. Analyzing differences between spillovers to calm and crisis markets is made possible by setting the suitable restrictions on parameters from the transition matrix. Thus, one does not need to assume any specific linear or nonlinear structure of spillovers between the markets, like in autoregressive and ARCH models, since both contagion and spillovers are introduced directly through the probability measures. The definitions introduced above are helpful in building tests of financial contagion, spillovers, and independence between the markets. The general model, described by the equations (2.1) to (2.7), imposes no restrictions on the transition matrix P and assumes financial spillovers in both stable and turbulent regimes. It can be estimated using the standard Expectation-Maximization (EM) algorithm, similarly to Hamilton (1989, 1990). A similar technique is applied to estimate the contagion models, employing equations (2.1) to (2.7) and transition matrices (2.13) and (2.14). The models assuming regime-independence (no spillovers), no spillovers in crisis periods, no spillovers in calm periods, and no spillovers in any regime use the transition matrices (2.8), (2.19), (2.20), and (2.18), respectively. These models with constrained transition matrices are estimated using an algorithm analogous to the one described by Phillips (1991). Details are available upon request. The log-likelihood values corresponding to the estimates are denoted by $L_{SPILLOVERS}$ for the general model with no constraint on the transition matrix P , $L_{INDEPENDENCE}$ for the regime-independence model, $L_{CONTAGION}$ for the contagion model, $L_{CONTAGION-IN-CRISIS}$ for the "contagion during crises" model, and $L_{NO-SPILLOVERS}$, $L_{NO-SPILLOVERS-IN-CRISIS}$, $L_{NO-SPILLOVERS-IN-CALM}$ for the no-spillover models with the transition matrices (2.18), (2.19), and (2.20), respectively. We describe now our testing procedure used to explore possible interdependencies between capital markets. In Figure 2.1 the testing hypotheses are ordered in the general-to-specific sequence. Exceptions are Hypotheses 3a and 3b, which are not nested in Hypothesis 2. We start with testing the null hypothesis assuming that there

is contagion from to when both markets are in the crisis regime (Hypothesis 1) against the alternative of no contagion. Under the null hypothesis, the likelihood ratio statistic:

$$LR = 2(L_{SPILLOVERS} - L_{CONTAGION-IN-CRISIS}) \sim \chi^2(4) \quad (2.21)$$

has the standard asymptotic χ^2 distribution with four degrees of freedom. If the null hypothesis can be accepted, we continue with testing the hypothesis that contagion exists in both calm and crisis regimes (Hypothesis 2). We use the likelihood ratio statistic:

$$LR = 2(L_{SPILLOVERS} - L_{CONTAGION}) \sim \chi^2(8) \quad (2.22)$$

which has the asymptotic χ^2 distribution with eight degrees of freedom (Sola, Spagnolo, and Spagnolo (2002)).

If the Hypothesis 1 is rejected then no contagion exists in any regimes and we follow the procedure by analyzing the hypothesis that no spillovers from X to Y are present in cases Y was in the calm regime at time $t - 1$ (Hypothesis 3a). Alternatively, one can test the hypothesis of no spillovers to Y in case Y was in the crisis regime at time $t - 1$ (Hypothesis 3b). The respective statistics are:

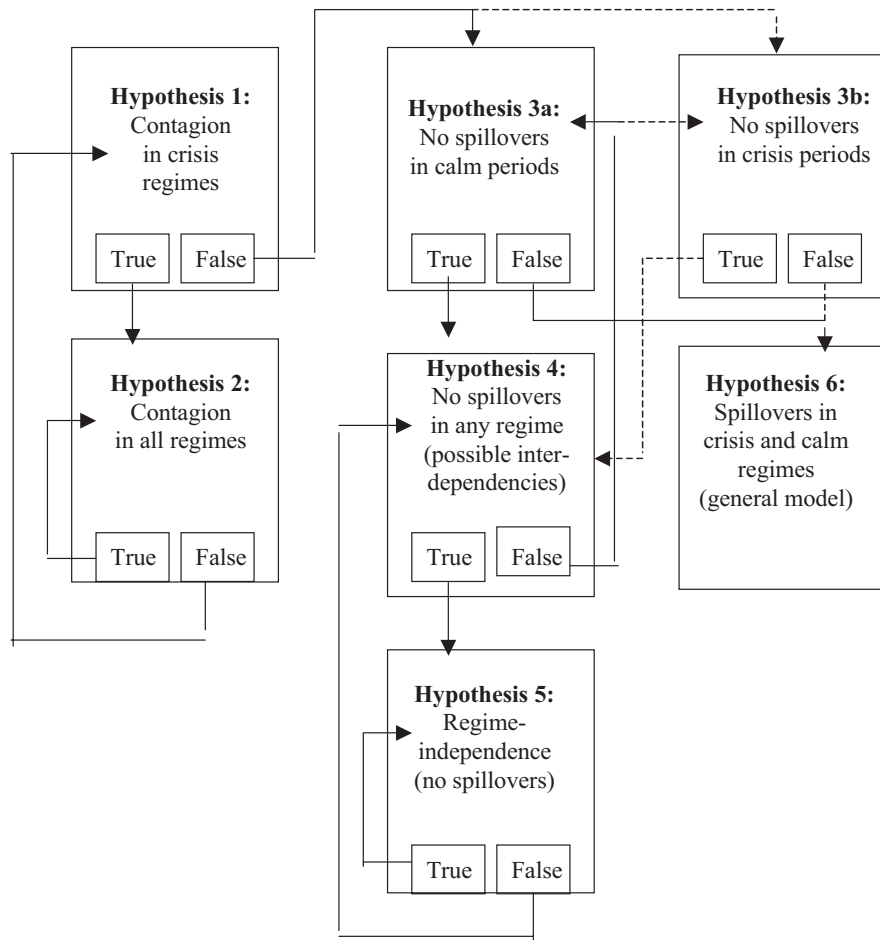
$$LR = 2(L_{SPILLOVERS} - L_{NO-SPILLOVERS-IN-CALM}) \sim \chi^2(1) \quad (2.23)$$

and:

$$LR = 2(L_{SPILLOVERS} - L_{NO-SPILLOVERS-IN-CRISIS}) \sim \chi^2(1) \quad (2.24)$$

If the both hypotheses are rejected, we conclude that financial spillovers from X to Y are present in both regimes (Hypothesis 6) and finish the procedure here. When one of the above hypotheses, 3a or 3b, is accepted, we utilize the following statistic to test the Hypothesis 4

Figure 2.1: The Financial Spillovers Hypotheses and Their Testing Sequence.



of no spillovers between the markets in any regime:

$$LR = 2(L_{SPILLOVERS} - L_{NO-SPILLOVERS}) \sim \chi^2(2) \quad (2.25)$$

When this hypothesis is accepted, we conclude that X does not lead Y by one period, but some interdependencies between stock index returns on both markets, which take place simultaneously (e.g. on the same day) may still be present. The probability of one market entering the crisis or calm regime may still depend on the regime that the other market will enter collaterally. To rule out such dependencies between the markets we test the hypothesis that markets are regime-independent (Hypothesis 5) by applying the following test statistic:

$$LR = 2(L_{SPILLOVERS} - L_{INDEPENDENCE}) \sim \chi^2(12) \quad (2.26)$$

If this hypothesis is accepted, the markets enter any regimes independently of other markets (Phillips (1991), Sola, Spangolo, and Spagnolo (2002)). The flexibility of the test rests on the fact that both markets are still allowed to be correlated in each regime. This characteristic can almost always be observed between financial markets (e.g., Forbes and Rigobon (2002)). The Markov switching models, employed by testing different hypotheses, differ only in parameters of the transition matrix P . In this way we avoid the problem of existence of some nuisance parameters that would be unidentified under the null hypotheses - a typical obstacle in testing multi-regime models. Therefore, our likelihood ratio statistics have their standard asymptotic distributions, as in Phillips (1991), Ravn and Sola (1995), and Sola, Spagnolo, and Spagnolo (2002). The testing procedure outlined here is not meant to compare spillovers between the markets depending on the regime of the leading market. The important feature of the hypotheses 3a, 3b, and 4 is that they enable us to analyze the question raised in the introduction, whether markets undergoing a financial distress are more or less vulnerable to

spillovers from other markets.

2.3 Data and Empirical Results

In this section, we report the results obtained from the testing methodology outlined above and present the calculated probabilities of a crisis on each market when there was a crisis on the US market one day earlier. In our analysis we employed the standard capital market indices from the four largest markets in the world. The *S&P500* index represents the US market, the *NIKKEI 225* is the index for the Japanese market, the *FTSE 100* index corresponds to the UK market, and the *DAX* stands for the German index. The index returns are computed as first differences of logged daily closing prices from the four markets and cover the period from April 3, 1984 to May 30, 2003, which corresponds to 4423 observations. As argued in the introduction, the US is believed to be the dominating market leading other stock markets independently of crisis and calm periods. Therefore, in the empirical analysis we concentrate on spillovers from the US market to the other three markets, although the model applied here complies bi-directional interdependencies. Using the proposed algorithm, we check whether the structure of dependencies of the British, German, and Japanese markets on the US market should be called spillovers or rather contagion. In addition, we test for possible changes in the linkages between the markets during turbulent and calm periods. Next, we present the final models obtained from the testing procedure and compute the probabilities of the potential turmoil on the British, German, and Japanese market individually conditional on the information that the US market was in the turmoil regime one period earlier. In order to analyze whether linkages between the markets have varied over time independently of crisis and calm regimes, we additionally calculate all tests for three non-overlapping sub-periods from April 3, 1984 to December 28, 1988, from January 4, 1989 to December 29, 1995, and from January 4, 1996 to May 30, 2003. The 1996 - 2003 sub-

sample is characterized by a considerable high variance of index returns on all markets in comparison to previous periods, which could eventually influence the general results. We also divide the rest of the time series into the two sub-periods, where the 1989 - 1995 interval is a relatively stable period and the 1984 - 1988 period comprises the great crash of the 1987 that has been found to influence spillovers from the US to other markets (Malliaris and Urrutia (1992)). Each model of the bilateral linkages between the US market and the other market is estimated in seven different versions. The first version corresponds to the general model with no restrictions on the transition matrix , which allows for potential spillovers between the markets. The second model assumes that both markets are regime-independent from each other and the third one assumes no spillovers from the US market to the other market. The fourth model is estimated under the constraint that no spillovers exist when the dependent market is in the state of crisis and the fifth one assumes no spillovers when the dependent market is in the calm regime. The sixth and seventh cases are the models of contagion from the US to the other market and contagion only in the crisis periods, respectively. In Table 2.1 the log-likelihood values from the estimated models are presented. The general model has the highest likelihood value for each pair of markets, since all other models are restricted versions of the general model. Additionally, the "regime-independence" models are special cases of the "no-spillovers" models, which in turn set additional constraints in comparison to the "no-spillovers in crisis" and "no-spillovers in calm periods" models. Finally, the both "contagion" models are restricted forms of the general model.

To distinguish which models are statistically justified and which are too restrictive we employ the likelihood ratio statistics described in the previous section. All the results from our testing procedure are presented in Table 2.2. For all pairs of markets, the hypotheses of contagion and a weaker hypothesis of contagion in the crisis regime is rejected, which corresponds to the result of Sola, Spagnolo, and Spangolo (2002). Hence, we continue the

Table 2.1:

Log-likelihood Values of the Estimated Markov Switching Models.

Log-likelihood	<i>S&P500</i> and <i>Nikkei225</i>	<i>S&P500</i> and <i>FTSE100</i>	<i>S&P500</i> and <i>DAX</i>
$L_{SPILLOVERS}$	-13222.11	-11780.0	-12957.88
$L_{NS-IN-CRISIS}$	-13238.43	-11834.5	-12988.96
$L_{NS-IN-PROSPERITY}$	-13235.73	-11821.0	-12979.5
L_{NS}	-13247.70	-11863.07	-13006.06
$L_{INDEPENDENCE}$	-13390.40	-11858.6	-13062.95
$L_{CONTAGION}$	-13386.40	-11840.58	-13069.87
$L_{CONTAGION-IN-CRISIS}$	-13331.62	-11816.13	-13003.45

Note: The log-likelihood values corresponding with the estimates are denoted by $L_{SPILLOVERS}$ for the general model, $L_{INDEPENDENCE}$ for the independence model, $L_{CONTAGION}$ for the contagion model, $L_{NS-IN-CRISIS}$ for the "contagion during crises" model, and L_{NS} , $L_{NS-IN-CRISIS}$, $L_{NS-IN-PROSPERITY}$ for the no-spillover models with the transition matrices (2.18), (2.19), and (2.20), respectively.

procedure by testing the null hypotheses of no spillovers in crisis periods, no spillovers in calm periods, and no spillovers in any regimes. All of them are also rejected and we interpret these results as existence of spillovers from the US to the Japanese, British, and German markets independently of whether these latter markets are in crisis or calm regimes.

It is interesting to note that the test statistics for the hypothesis of no spillovers during crises always have higher values than the statistics for the hypothesis of no spillovers during calm periods. Assuming no spillovers when the Japanese, British, and German markets are in crisis regimes would be a more likely choice than assuming no spillovers in calm regimes.

Table 2.2: Tests of Linkages between the Markets.

Null hypothesis	<i>S&P500</i> and Nikkei225	<i>S&P500</i> and FTSE100	<i>S&P500</i> and DAX
Regime-independence	336.58**	157.20**	210.14**
No spillovers during calm	26.64**	82.00**	43.24**
No spillovers during crises	33.24**	109.00**	62.16**
No spillovers at any regime	51.18**	127.19**	96.36**
Contagion	327.78**	121.16**	223.98**
Contagion during crises	219.02**	72.26**	91.14**

Note: * and ** denote rejection of the null hypothesis at the 5% and 1% levels, respectively.

However, these both hypotheses, and models, are rejected as too restrictive. Finally, the regime-independence is also rejected in all cases, which confirms that some interdependencies are present between the US and other markets. According to our results the best models of dependencies between the markets are the general unconstrained models allowing for spillovers in all regimes, but not restricting these spillovers only to contagion effects. We present the parameters of these final models in Table 2.3. It is important that all the models match the main empirical patterns found on international capital markets. First, the regime with low average index returns on both markets is characterized by higher volatility of index returns than the regime with both markets in calm periods. It is interesting to note that the highest (lowest) volatilities are always obtained in the same regime for both markets. Moreover, in each model the regime with highest volatilities on the two markets is the one with one market in the state of crisis and the other market in the state of calm.

Table 2.3: Summary Parameters Estimation for Models of Dependencies between the Financial Markets

States		Parameters					Transition matrix P			
S_t^X	S_t^Y	μ_1^X	σ_X^2	μ^Y	σ_Y^2	ρ^{XY}	(l,l)	(l,h)	(h,l)	(h,h)
<i>S&P500</i>	<i>DAX</i>									
calm(l)	calm(l)	0.080 (0.007)	0.670 (0.023)	0.107 (0.013)	0.845 (0.035)	0.175 (0.066)	0.982	0.002	0.001	0.015
calm(l)	crisis(h)	0.080 (0.007)	5.944 (1.891)	0.019 (0.001)	4.994 (1.044)	0.315 (0.092)	0	0.566	0.043	0.391
crisis(h)	calm(l)	0.037 (0.004)	1.842 (0.495)	0.107 (0.013)	3.007 (0.747)	0.684 (0.171)	0	0	0.972	0.028
crisis(h)	crisis(h)	0.037 (0.004)	1.127 (0.297)	0.019 (0.001)	1.493 (0.444)	0.378 (0.092)	0.025	0.007	0.005	0.962
<i>S&P500</i>	<i>FTSE100</i>									
calm(l)	calm(l)	0.082 (0.006)	0.666 (0.084)	0.065 (0.005)	0.732 (0.091)	0.315 (0.024)	0.978	0	0.022	0
calm(l)	crisis(h)	0.082 (0.006)	8.717 (2.110)	-0.088 (0.007)	5.250 (1.417)	0.493 (0.067)	0	0.513	0.382	0.105
crisis(h)	calm(l)	-0.064 (0.009)	1.222 (0.195)	0.065 (0.005)	1.138 (0.214)	0.444 (0.072)	0.035	0.005	0.953	0.007
crisis(h)	crisis(h)	-0.064 (0.009)	1.946 (0.280)	-0.088 (0.007)	2.174 (0.540)	0.517 (0.091)	0	0	0.037	0.963
<i>S&P500</i>	<i>NIKKEI225</i>									
calm(l)	calm(l)	0.099 (0.010)	0.738 (0.135)	0.099 (0.095)	0.640 (0.140)	0.066 (0.009)	0.97	0.014	0	0.016
calm(l)	crisis(h)	0.099 (0.010)	0.637 (0.102)	-0.027 (0.012)	1.659 (0.342)	0.154 (0.033)	0.016	0.969	0.008	0.007
crisis(h)	calm(l)	-0.005 (0.002)	3.066 (0.980)	0.099 (0.095)	3.289 (0.917)	0.142 (0.024)	0	0.057	0.85	0.092
crisis(h)	crisis(h)	-0.005 (0.002)	1.234 (0.123)	-0.027 (0.012)	1.363 (0.202)	0.176 (0.053)	0.017	0	0.018	0.965

Standard errors in parentheses.

Second, when both markets are in the crisis regime they become more correlated with each other than when they are in their calm regimes (e.g., Longin and Solnik (2001)). Thus, using our framework it would be possible to compute the tests of contagion in the spirit of Forbes and Rigobon (2002), but without using any ad hoc procedures to identify crisis and calm sub-periods. However, this is beyond the scope of the paper. Finally, from the elements of the transition matrices it can be observed that the probability of staying in the same regime is always highest for all regimes and all estimated models. This result can be interpreted as evidence of persistence of high (low) volatility in stock market index returns and evidence of autocorrelation in index returns due to high (low) returns following past high (low) returns. This finding is in line with the well-known characteristics of autocorrelation and conditional heteroscedasticity in stock index returns. Moreover, comparing the estimated transition matrices in Table 2.3 with constraints proposed in equations (14) and (15) leads to the conclusion that the high values of the parameters α_2 and α_3 , which can be interpreted as indicators of persistence of the states 2 and 3, are main reasons for rejecting both contagion hypotheses in the spirit of Sola, Spagnolo, and Spagnolo (2002). Our results, suggesting that the spillovers hypothesis is true, are consistent with the literature defining contagion as an increase in the probability of having a crisis at home when there is a crisis on the other market. Eichegreen, Rose, and Wyplosz (1996) and Hartmann, Streatmans, and de Vries (2004) also find evidence of contagion when they apply the same definition of contagion. Having estimated transition matrices for each model we are able to compute the probabilities of some market entering the state of crisis or calm, conditional on the information that this market and the US market were in their respective states yesterday. These results are of special importance for international investors and the great advantage of the model is that they can be obtained directly using standard computations on the elements of the transition matrix. We additionally provide results on the probability of one market entering the crisis (calm) regime conditional on the state of the US market one day earlier. The results are

Table 2.4: Probability of a Crisis or Calm Today Conditional on the Information from Yesterday.

X represents the <i>S&P500</i> index returns and Y represents:	Nikkei225	FTSE100	DAX
Probabilities conditional on the information from X_{t-1} and Y_{t-1} .			
$P(Y_t = l Y_{t-1} = l \wedge X_{t-1} = l)$	0.970	1.000	0.983
$P(Y_t = l Y_{t-1} = l \wedge X_{t-1} = h)$	0.850	0.988	0.972
$P(Y_t = l Y_{t-1} = h \wedge X_{t-1} = l)$	0.024	0.382	0.043
$P(Y_t = l Y_{t-1} = h \wedge X_{t-1} = h)$	0.035	0.037	0.030
$P(Y_t = h Y_{t-1} = l \wedge X_{t-1} = l)$	0.030	0.000	0.017
$P(Y_t = h Y_{t-1} = l \wedge X_{t-1} = h)$	0.150	0.012	0.028
$P(Y_t = h Y_{t-1} = h \wedge X_{t-1} = l)$	0.976	0.618	0.957
$P(Y_t = h Y_{t-1} = h \wedge X_{t-1} = h)$	0.965	0.963	0.970
Probabilities conditional only on the information from X_{t-1}			
$P(Y_t = l X_{t-1} = l)$	0.564	0.929	0.967
$P(Y_t = l X_{t-1} = h)$	0.149	0.206	0.231
$P(Y_t = h X_{t-1} = l)$	0.436	0.071	0.033
$P(Y_t = h X_{t-1} = h)$	0.851	0.794	0.769

Note: l, h denote calm, crisis respectively. For further explanations see text.

presented in Table 2.4.

The main conclusion from the calculated probabilities is that entering one regime by the market is most likely and even close to one when this market and the US market were in the same regime one period earlier. If the US market was not in that regime one period earlier then the probability of entering the regime by the other market drops in almost all cases. The probability is close to zero that the market enters the state of calm (crisis) when the US market and this respective market were in the opposite regime one period earlier. This finding illustrates how the past information about the US market spills over to other mature markets on the next day. Furthermore, we are able to forecast the future state of the market more accurately having the information about the present state of both markets rather than having the information only about the US market. This in turn explains why

the hypothesis of contagion is rejected in our analysis. The past information about each market is significant for its present performance. We continue the analysis with studying the relations between the markets in the selected three non-overlapping sub-samples to learn how the dependencies between international capital markets change over time. The results from testing all hypotheses of contagion, spillovers, and regime-independence are presented in Table 2.5. The general findings from this exercise are that the US leads Japan, the UK and Germany, but the patterns of spillovers from the US to those markets vary over time.

Some evidence of asymmetry in spillover effects between calm and crisis regimes is present the investigated sub-samples. In the 1984 - 1988 period we can accept the hypothesis that the S&P500 index returns do not lead the DAX and NIKKEI 225 index returns when the latter indices are in the calm regimes. Similarly, from 1996 to 2003 returns on the Japanese market follow the US market returns only in the state of crisis and any spillover effects to Japan are quite weak in this period. The lack of spillovers in any regime to the UK is accepted in the 1989 - 1995 sub-sample. Since regime-independence is also rejected there, we interpret this result as evidence of the inter-dependencies between the US and UK capital markets, which take place without delay. One possible explanation for the lack of spillovers to the UK from the US could be the ERM currency crisis of 1992 that affected most strongly the British market. In the most recent period 1996 - 2003, *S&P500* index returns lead very strongly the DAX returns and one can observe the contagion effect when the German index is in the crisis regime. Likely reasons for this contagion effect could be recent shocks which took place on the US market and spread to other markets after the terrorist attack on September 11 and after the burst of the "dot.com" bubble. In all other cases there are significant spillovers from the US to the other markets independently of the crisis and calm regimes. From Table 5 one can observe that spillovers between capital markets evolve over time independently of changing regimes. There are naturally some factors other than changing states of the markets which

Table 2.5:
Tests of Linkages between the Markets in Sub-Samples

Null hypothesis	<i>S&P500</i> and Nikkei225	<i>S&P500</i> and FTSE100	<i>S&P500</i> DAX
from April 3, 1984 to December 28, 1988			
Regime-independence	93.18**	53.12**	63.24**
No spillovers during calm	3.20	7.23**	0.94**
No spillovers during crises	7.02**	15.77**	4.28**
No spillovers at any regime	53.24**	25.52**	4.52**
Contagion	105.38**	87.82**	66.42**
Contagion during crises	22.58**	71.88**	61.88**
from January 4, 1989 to December 29, 1995			
Regime-independence	61.98**	56.96**	65.00**
No spillovers during calm	8.96**	1.16	9.42**
No spillovers during crises	19.14**	2.76	24.04**
No spillovers at any regime	25.98**	5.84	37.42**
Contagion	56.14**	71.30**	61.24**
Contagion during crises	40.58**	56.02**	31.52**
from January 4, 1996 to May 30, 2003			
Regime-independence	94.56**	112.26**	95.46**
No spillovers during calm	3.00	12.48**	8.82**
No spillovers during crises	5.86*	23.04**	11.80**
No spillovers at any regime	7.76*	56.64**	20.90**
Contagion	41.94**	42.68**	24.04**
Contagion during crises	28.96**	27.30**	9.14**

Note: * and ** denote rejection of the null hypothesis at the 5% and 1% levels, respectively.

can influence the strength of spillovers and future applications may extend the proposed models by introducing additional elements or varying parameters. Nevertheless, our results show that spillovers between the four big stock capital markets exist in all periods. There is less evidence of spillovers to the markets in the calm regime than to the stock markets which are in the crisis regime in the sub-periods. This finding could indicate that the market not involved in some international crash often remains resistant to spillovers from the US stock market. As soon as it allows for the high volatility regime at home it becomes more vulnerable to the influence of the US market, because concerned investors observe more carefully the performance of the US market in the context of the international turmoil. This could also suggest that in some periods the analyzed markets are robust to any contagion from the US market, because they enter crisis regimes independently of the US market or simultaneously with the US market. If the latter case was true, then the direction of contagion would be toward the US market rather than from the US market due to possible crises on other not investigated markets that could cause the US market and other analyzed markets to enter the crisis regime in the same time. Additionally, the US market has less influence on the European and Asian markets on the same day because of different trading hours on the stock exchanges in Asia, Europe, and America. American stock markets open and close after the European and Asian markets each day, although some trading hours overlap. In contrast, European and Asian stock index returns may influence the American index returns on the same day (e.g., Cheung and Ng (1996)).

2.4 Conclusions

In this article, we investigate international financial spillovers from the US stock market to the Japanese, British, and German markets. We introduce a statistical framework to deal with the problem of asymmetries in financial spillovers in calm and turbulent regimes. Spillovers

and contagion to stock markets during crisis and calm periods are explicitly defined and new tests are proposed to distinguish between financial spillovers in crisis and calm regimes. Our testing framework is capable of distinguishing between different types of relations connecting two markets, i.e., contagion, spillovers, and independence. Thus, we compare the results from testing financial spillovers with outcomes from the tests of contagion and independence and obtain evidence that the Japanese, UK, and German stock markets are dependent on the past performance of the US market, but encounter almost no indication of contagion in the spirit of Sola, Spagnolo, and Spangolo (2002). We find that spillovers taking place when the dependent markets are in the crisis regime are more frequent than spillovers to the markets in the state of calm, which is in line with the results of Chen, Chiang, and So (2003). This result suggests that financial crashes on the US market do not always directly cause turmoil on the Japanese, UK, and German markets. However, the crashes on the US market increase the probability of a crisis on the three other mature markets, which is in line with the hypothesis of contagious crises introduced by Eichengreen, Rose, and Wyplosz (1996). Additionally, we present the probabilities for the Japanese, UK, and German stock markets individually entering the states of calm and crisis periods, conditional on the information about the past performance of those markets and the US market. Information from both markets is found to be relevant for efficient forecasting of future stock market index returns on those markets, therefore further research could incorporate our framework in testing for diversification benefits from asset allocation on international markets, as in Ang and Bekaert (2002, 2003).

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Table 2.6:

Appendix: Specification Tests for the Estimated Markov Switching Models.

Null hypothesis	<i>S&P500</i> and Nikkei225	<i>S&P500</i> and FTSE100	<i>S&P500</i> and DAX
$\mu_S^X = \mu_D^X$	0.122 [0.9030]	-0.653 [0.5138]	-0.293 [0.7694]
$\mu_S^Y = \mu_D^Y$	-0.420 [0.6748]	0.670 [0.5026]	-0.498 [0.6182]
$\sigma_S^X = \sigma_D^X$	1.01 [0.5191]	0.99 [0.7629]	1.02 [0.6949]
$\sigma_S^Y = \sigma_D^Y$	0.98 [0.5769]	01. Feb [0.4323]	0.99 [0.6369]
$Leverage_S^X = Leverage_D^X$	0.044 [0.9647]	0.358 [0.7203]	0.167 [0.8677]
$Leverage_S^Y = Leverage_D^Y$	0.588 [0.5568]	0.820 [0.4121]	0.067 [0.9469]
$Peak_S^X = Peak_D^X$	-1.036 [0.3003]	0.316 [0.7523]	-0.940 [0.3473]
$Peak_S^Y = Peak_D^Y$	-0.765 [0.4445]	-0.518 [0.6046]	-0.936 [0.3493]

Note: The symbols D and S denote the original and simulated data, respectively. P-values are presented in squared parentheses under the values of test statistics. * and ** denote rejection of the null hypothesis at the 10% and 5% levels, respectively.

Chapter 3

The Test of Market Efficiency and Index Arbitrage Profitability on Emerging Polish Stock and Futures Index Markets

The efficiency of the market for stock index futures and profitability of arbitrage for contracts on the Warsaw Stock Exchange Index WIG20 is studied in this paper. The Polish market has unique attributes, namely in relatively short time the risk-free interest rate decreased significantly, short sale cannot be used to construct arbitrage position by institutional investors and the dividends are small and paid in irregular manner. Examining intraday transaction data shows that ex post and ex ante violations for short arbitrage reveal almost all properties of mature market. Nonetheless, findings for long arbitrage indicate inefficiency of the market.

3.1 Introduction

The relationship between stock index futures market and stock index market has been subject of numerous empirical studies. A large part of them concentrate on examining an opportunity of index arbitrage. From the theoretical point of view, existence of an arbitrage strategy

violates assumptions of the efficiency of the market, thus studies in this field have fundamental character. In turn, brokerage houses, mutual funds, large investors etc. seek profits from the spread between prices on the spot and futures markets. Therefore for practitioners, the analysis of the magnitude and frequency of mismatching of these prices is a subject of vital interest.

The idea of the classical approach to the problem of examining the arbitrage opportunity is based on the cost-of-carry model proposed by Cornell and French (1983). According to assumptions of the model an investor has an opportunity to make an arbitrage trade if the mispricing series is not in the corridor with boundaries determined by transaction costs. The mispricing is defined as the difference between the market futures price and the theoretical price of a contract, where theoretical price is given by the cost-of-carry formula (which gives the fair price of the forward contract). The sources and attributes of mispricing series were analyzed in the number of papers. Cornell and French (1983) postulated that mispricing is the result of different taxation between stocks and futures contracts. The mispricing is also caused by another factors like the lack of knowledge about the nature of the market Figlewski (1984), divided uncertainty Peter (1984), unequal borrowing and lending rates Gould (1988), imperfect replication of underlying index MacKinlay and Ramaswamy (1988). It is difficult to evaluate importance of each of the factors which are supposed to have influence on mispricing (see Modest (1984), Cornell (1985), Yadev and Pope (1990)). There are much less controversies about the properties of the mispricing series. The results of different investigations have showed that average mispricing series is usually negative, absolute level of the mispricing increases with time to maturity, and mispricing times series are mean reversal processes (e.g. Kempf (1998), Miller, Muthuswamy and Whaley (1994), Puttonen (1993), MacKinlay and Ramaswamy (1988)).

Also the examination of properties of potential arbitrage profits signaled by violation of boundaries is the subject of numbers of studies. MacKinlay and Ramaswamy (1988), Chung

(1991), Klemkosky and Lee (1991), Neal (1996) examined these properties for US market, Brenner et al. (1990) and Lim (1992) for Japanese market. In the case of European markets, Yadev and Pope (1990) analyzed properties of mispricing and arbitrage before and after Big Bang in U.K., Bühler and Kempf (1995) studied them in detail for Germany market, Stulz, Wasserfallen and Stucki (1990) for Swiss market, Puttonen (1993) for Finnish and Berglund and Kabir (2003) for Dutch market. The results indicated that usually mispricing series are path-dependent, the frequency and an average magnitude of the mispricing increased with time to delivery of contract. Furthermore, the arbitrageurs who follow the signals generated by the cost-of-carry model are able to obtain the significant profit. Its size depends on transaction costs, type of arbitrage strategy and institutional regulations, especially in the case of short sale. For example the percentage of signals, which lead to profit, in case of well developed markets reach the value of 95% for Germany and 92% for US market. Nevertheless the index arbitrage should not be understood as the risk-free returns. The factors which influence the size and frequency of arbitrage profit were pointed out in Kawaller (1987) paper. These were tracking risk, margin variation risk, dividend uncertainty, transaction costs and interest rate risk.

The purpose of this study is to examine the attributes of intraday ex post and ex ante profitability of index arbitrage for Polish stock market and compare them with those observed on other markets. The motivation of this investigation is the fact, that the Polish market has some specific properties, which make this study unique in comparison to the previous ones.

A risk free rate, namely interest rate for 3 months WIBOR¹ rate, decreased during investigated period from 19.62% to 6.88%. The changes of 3 month WIBOR and WIBID

¹WIBOR and WIBID are the average interest rate on interbank deposit market in Poland. Since January 2001 the new fixing rule were adopted allowing for 10 domestic banks or branches of foreign banks for participation in WIBOR. The reference WIBOR rates are set for the following maturities: O/N, T/N, SW, 1 month, 3, 6, 9 and 12 months. WIBOR and WIBID rate are calculated as the arithmetic mean of the rates quoted by the participants, after rejecting two outliers (the highest and lowest quote).

Table 3.1: The changes of 3 months WIBOR and WIBID rate during three months periods between deliveries of futures contracts on WIG20.

The period between auctions	18.12 -19.03	19.03 -18.06	18.06 -24.09	24.09 -27.12	27.12 -18.03	28.03 -24.06	24.06 -23.09	23.09 -16.12
The changes of yield in 3-month WIBOR rate	-1.56	-1.20	-2.53	-2.48	-1.45	-1.40	-1.15	-0.96
The changes of yield in 3-month WIBID rate	-1.53	-1.32	-2.59	-2.47	-1.46	-1.06	-1.26	-0.95

rate during three months period are presented in Table 3.1. The time intervals shown in Table 3.1 corresponded with periods, when futures contracts under consideration were nearest to delivery.

Comparatively, Klemkosky and Lee (1991) used yield on the US Treasury bills as estimation of the risk-free rate. During examination period, around four years, the yield on 3-months Treasury bills ranged from 8.66 to 5.96%. Yadev and Pope (1990) considered yield on 3-months Treasury bills for United Kingdom, it fluctuated from 11.4 to 9.71% in period between the middle of 1984 to the end of 1988. The rate selected to represent risk-free rate for Japan lied in the interval 5 – 6% during the period of one year. In turn, during the two years period examined by Bühler and Kempf (1995) the 3-month FIBOR dropped from 9.21% to 9.00%. Finally, the change of 3-month AIBOR from 9.42% to 5.60% in 3-year period and the increase of 3-month HELIBOR from 9.47% to 14.3% in period from the middle of 1988 to the end of 1990 (see Berglund and Kabir (2003) and Puttonen (1993)) cannot be compared with the change observed on Polish market.

The downtrend of the risk-free interest rate in Poland is the source of investor's uncertainty about the profitability of arbitrage. It is known, according to literature on the relationship between the monetary policy and asset price, that on an efficient market the

decline of risk-free rate increases stock returns. It applies especially to large companies (see Rigobon and Sack (2004), Thorbecke (1997), Gertler and Gilchrist (1994)). In the index WIG20 only blue chips companies are listed, so there is potential risk for arbitrageurs that prices react positively on changing risk-free interest rate. An increase of price on spot market is undesirable in the case of investor who opened a short position on that market. Moreover, there is other effect of uncertainty about risk-free rate on profitability of arbitrage, namely an investor is uncertain of the rate on which the potential losses from marking-to-market procedure will be covered. However, this effect seems to be a minor one. The period selected for this study was characterized by low inflation so the market participants expected that the Monetary Policy Council would decide to reduce the interest rates or at least keep them on the same level.

Further unique attribute of examined Polish market are unusual constraints to access to short sale of stocks. The large number of papers have stressed the importance of short sale as a financial instrument, and discussed the implication of constraints imposed on it. Figlewski (1981) claims that limited access to short sale has positive impact on the market. It eliminates uninformed investors from pool of short sellers, because only investor with a strong belief about future of the particular company intends to use the short sale. In turn, Diamond and Verrecchia (1987) proposed the theoretical model of short sale. According to the model one can conclude that restriction on short sale has impact on efficiency of market and speed of adjustment of stocks price to bad news. These results were empirically confirmed by Senchack and Starks (1993). Karpoff (1988) put forward the hypothesis that higher correlation between positive returns and volume on US market can be explained by constraints in access to short sale. The problem of influence of such constraints on the size of mispricing and profitability of arbitrage has been also discussed in literature. In the work of Kempf (1998), the restriction in short sale was pointed out as the reason for the tendency of mispricing to remain negative for contracts on DAX. The relationship between

futures and spot market on index FOX was examined in the paper of Puttonen (1993) . It was characteristic for this study that during analyzed period there was no institutional framework for short sale on Finnish market. The obtained results suggest that constraints on short sale reduces efficiency of market, and have significant influence on negative mispricing.

On Polish market, short sale is allowed by Warsaw Stock Exchange (WSE) according to regulations introduced in 1999. It is hardly used by investors, however it does not mean that on emerging Polish market there are not enough investors with strong beliefs about future development of companies quoted at WSE. Common investor's point of view is that using short sale is too expensive and in the result short sale is not popular. The potential large lenders of stocks like pensions funds, mutual funds cannot supply the market with stocks, because there is no executory provision, which would encourage them to do so. The shortage in supply of stocks leads to the the high costs of short sale, costs which often exceed the risk free rate (see Maciejewski and Mejszutowicz (2003)). Additionally, restriction in using short selling for arbitrage trades is due to the fact that not for all companies listed in WIG20² short sale can be conducted. As the result, the share of short sale in the turnover of shares trading on Warsaw Stock Exchange (WSE) is very low in the analyzed period. This observation applies to all companies included in blue chips index WIG20 (see Table 3.2).

Therefore an institutional investor, who has strong beliefs about future development of situation on the market and does not own shares, cannot treat short sale as reliable instrument. It is not known, how many stocks investors will be able to sell short and it is very difficult to forecast the cost of this operation. As the result, an institutional investor has to find another way of realizing her strong beliefs. The most popular methods are: opening short position in futures contracts on individual stocks, and "stock replacement" in case of index arbitrage. The second method was discussed in detail by Chung (1991). Surprisingly,

²18-19 of all 20 assets included in the WIG20 index may be short sold.

Table 3.2:

The short sale in the overall sale volume in the period from February to December 2002.

AGORA	N/A ¹	KGHM	N/A
BIG	N/A	PEKAO	0.13%
BPHPBK	1.01%	PKNORLEN	N/A
BRE	0.12%	PROKOM	0.03%
ELEKTRIM	N/A	TPSA	0.01%

Source: Rynek Terminowy Vol. 19, 2003.

¹ N/A denotes that data were not available

this adaptation of investor to rules of short selling seems to be one of the reasons of the delay in introducing the new regulations improving functioning of short sale, because investors do not exert enough pressure on WSE regulators (see Maciejewski and Mejszutowicz (2003)).

Moreover, Polish stock market has another unique attribute, namely the percentage of companies paying dividends have varied between 14% and 23% during the last three years, and values of dividends have fluctuated around 1%. Polish companies pay dividends, with small exception, once per year, and in contrast to Finnish market the dates of payment are not clustered in particular month of a year (cf. Puttonen (1993)). Therefore, dividends on Polish stock market are small and paid in irregular way. It is a priori not obvious what kind of influence on the mispricing and the arbitrage this unique attribute of Polish market has.

It should be also stressed that this study analyzes arbitrage opportunity on Polish market in the period of time which was the first phase of development of the market after introducing the continuous quotation system WARSET in November 2000³.

The results of this study are interesting for investors, especially nowadays, when one can observe the increasing interest in the financial markets of Central European countries, including Poland, which will soon become a member of the European Union.

³The value of futures trading exceeded turnover on the equity market in the last quarter of the year 2000.

To summarize, the contribution of this study is that it examined the arbitrage opportunity on emerging Polish market in the way that takes notice of unique characteristics of the market, especially significant change of risk free interest rate, limited access to the short sale and irregular payments of dividends. It should be highlighted that to carry out this examination while taking into account the particular characteristics of emerging Polish capital market, the classical cost-of-carry model has to be modified.

The paper is organized as follows. Section 2 contains data description and discusses trading rules for futures on the WIG20 index. Section 3 provides a description of the cost-of-carry model with modification imposed by the feature of Polish market. The obtained results are compared with the conclusions reported for other markets in section 4. Finally, section 5 presents resume.

3.2 Data

The contract on WIG20 started trading at the Warsaw Stock Exchange on January 16, 1998. The dynamic development of the market began in the year 2000. The total turnover in that year was equal to 57 390 millions PLN, in 2001 it reached the value of 95 932 millions PLN, and in 2002 it was equal to 75 246 millions PLN⁴. The average daily number of transactions increased from 2101 in the year 2000 to about 3874 in 2002⁵. This number is comparable with those recorded for the Singapore International Monetary Exchange (SIMEX) at the end of the eighties (cf. Lim (1992)).

The price of the WIG20 index measures the performance of twenty blue chips stock listed on the main market of the WSE. The WIG20 index base value is 1000 points. The WIG20 is a price index, which means that only price changes for its constituent shares are taken into

⁴The average exchange rate of USD in year 2000 was equal to 4.35 PLN, in 2001 - 4.09 PLN, and in 2002 - 4.08 PLN.

⁵Further information and statistics are available on web site *www.wse.com.pl*.

account in the calculations of its value. The WIG20 value is published every 30 seconds, based on the most recent transaction prices, with accuracy of 0.01 point. Changes in the WIG20 portfolio take place every quarter, according to the ranking of listed companies by turnover and markets values. The weight average of these two magnitudes is calculated for each company considered to be included in the WIG20 index. The weight for turnover amounts to 0.6, and the weight for market value is equal to 0.4. Additionally, no more than five companies from any sector can participate in the WIG20 index.

The contract trade follows the March-June-September-December cycle. The symbol of each contract consists of three parts: FW20 is in general the name of all contracts on index WIG20, the next letter denotes the month of maturing (H-March, M-June, U-September, Z-December), and the last digit corresponds to maturity year. For example, FW20H2 is a contract which expired in March 2002. The last trading day for a contract is the third Friday of the contract month. The contracts on the WIG20 are settled at 10 PLN times value of the WIG20 index. The minimum price fluctuation is equal to one index point.

Since the moment of the introduction of the quotation system WARSET, the trading takes place in accordance with the following schedule. The futures contracts are continuously traded between 9:00 A.M and 4:00 P.M. At 4.10 P.M the closing price is determined. In contrast to futures index market, continuous trade of assets included in WIG20 starts at 10:00 A.M and ends at 4:00 P.M. In this study the arbitrage opportunity between 10:02 A.M and 4:00 P.M is the subject of examination.

The data set used in this paper contains the reported value of the WIG20 index at one minute interval and transaction prices for eight futures contracts on this index traded on the floor of the Warsaw Stock Exchange (WSE) for the period from December 2000 to December 2002. The futures prices are adjusted to 1 minute interval, by taking the last transaction price in a given minute of session. For each contract a three month period before maturity is considered. This is a period, when contracts are traded most frequently.

The daily quotations of the Warsaw Inter-Bank Offer Rate WIBOR rate are used as the risk free rate for Poland, what is consistent with previous studies of European markets where also interbank interest rate was selected (see Bühler and Kempf (1995), Puttonen and Martikainen (1991)).

As it was mentioned in the introduction, dividends are paid at Polish stock market only by around 25% of companies. The average value of dividend is small in comparison to those paid on developed markets. Likewise, investors paid tax equal to 20% in 2001, and 15% in 2002 of dividend's value. For the examined period, dividends for WIG20 portfolio adjusted for tax payment are equal to 0.9% in 2001 and 1.27% in 2002.

The existing of marking-to-market procedure imposes financial obligations on an investor. It is assumed that investor has no financial means to cover the loss, so it is covered by a loan. The profits have to be invested on risk-free rate. So, profits are invested on interest rate equal to the WIBID, and loss is covered by a loan with interest rate equal to the WIBOR on daily basis.

3.3 Methodology

The theoretical framework of this work is consistent with early studies, among others Cornell and French (1983), MacKinlay and Ramaswamy (1988), Chung (1991), Klemkosky and Lee (1991). Nevertheless, because of some characteristics of Polish market, this methodology has to be modified. One of them are short sale constraints, which have significant influence on long arbitrage opportunity. On Polish stock market appropriate legal regulations of short sale were adopted in December 1999. However, institutional investors are unwilling to use short sale. The main reasons of this aversion is shortage in supplies of stocks, which can be subject of short sale, what in turn implicate high cost of application of this instrument. Both factors lead to the situation in which a share of short sale in the overall sale volume

amounts to just 0.1%. So, it is irrational to expect that an institutional investor who possesses very unfavorable information or strong signal to long arbitrage, is going to use short sale. It would be an inefficient way to use investor's knowledge (see Figlewski (1981), Miller (1977)).

Therefore for the purpose of this study, it is necessary to limit the examination to investors, who possess the portfolio replicating the value of the index WIG20. An investor sells this portfolio after obtaining a signal to arbitrage implying short sale. This strategy is often called "stock replacement" (see Chung (1991)). Hence, this analysis concentrates on index arbitrage available for financial institutions (mutual funds, pension funds) which possess index portfolio of WIG20.

Further assumption is that investor executes her order in futures and spot markets completely. It is a realistic assumption because of the high and comparable turnover on both markets. The arbitrage positions after opening are kept till maturity of contract. This assumption is consistent with previous studies⁶. The positions opened by investors are named after positions opened on futures market. So, if market participant opens long arbitrage position, she sells portfolio of shares included in WIG20 and opens long position in futures contract on index. Buying the portfolio of assets from the WIG20 index and selling futures contract on this index is referred to as a short arbitrage position⁷. In this work tax effect is not considered, since in Poland similarly to Netherlands, pension funds and mutual funds are not the subject of capital gain tax at all (cf. Berglund and Kabir (2003)).

For presenting the model the following notation is introduced:

T - the date of maturity of contract.

i - the minute of session, where $i = 3, \dots, 360$.

t_i - the time of opening position on both markets, t -date, and i minute of session.

S_{t_i} - the actual reported value of the WIG20 index.

⁶In contrast to the above mentioned papers, the study of Brennan and Schwartz (1990) admits a possibility to liquidate arbitrage position before expiration of the futures contract.

⁷The reverse notation is also popular. It was used by Chung (1991).

f_{t_i} - the actual price of futures contract on the WIG20 index.

f_t - the settlement price of futures contract on the WIG20 index.

d - the annual rate of dividends from WIG20 portfolio adjusted for tax payment.

r_t^{wibor} - the PLN interbank loan interest rate (WIBOR).

r_t^{wibid} - the PLN interbank deposit interest rate (WIBID).

C_s - the commission for a stock in percentage of transaction value at time T.

C_f - the round-trip commission for futures on a per-index basis at time T⁸.

F_{t_i} - the theoretical price of futures contract is given by the cost-of-carry formula

$$F_{t_i} = S_{t_i} e^{(r_t^{wibor} - d)(T-t)/365} . \quad (3.1)$$

The hypothesis of efficiency of Polish market is verified by examining the following hypotheses (cf. MacKinley and Ramaswamy (1988), Klemkosky and Lee (1991), Chung (1991)):

1. *The relative mispricing $R_{t_i} = 100 * (f_{t_i} - F_{t_i})/S_{t_i}$ lie within non-arbitrage pricing boundaries, given by transaction costs u , thus $-u < R_{t_i} < u$.*
2. *The $|R_t|$ is decreasing function of time till maturity of contract.*
3. *The arbitrage profit from an arbitrage position held to an expiration of contract is equal to zero.*

If the first and the third of the above hypotheses are not fulfilled, there is an arbitrage opportunity on the market. The violation of non-arbitrage pricing boundaries, mentioned in Hypothesis 1, is called ex post violation. So, an investor receives a signal to open the arbitrage position when the value of the difference between the actual price, and its theoretical price of contract, given by cost-of-carry formula (3.1), all normalized by the index value and multiply

⁸The commission for opening a position in futures markets ranges from 13 to 18 PLN. In this study the average commission is equal to 1.5 (= 15 PLN/10 PLN) on per-index basis.

by 100, lie outside the interval determined by transaction costs u . Those of ex post violations which lead to profit are called ex ante violations.

The ingredients of transaction costs and action made by investor after ex post signal are described in the next part of this section. This would be important for testing Hypothesis 1 and 3. It is assumed that the barrier cost u contains market impact cost, round-trip commission for a futures market and round-trip commission for spot market. u is the cost at the time of maturity of contract. It is assumed that transaction costs u takes one of values 0.9%, 0.6%, 0.3%. Each of these values corresponds to a different group of institutional investors. The transaction costs and market impacts vary over these groups. For example, the magnitude 0.9% corresponds to transaction costs of small institutional investor, because its round-trip commission is equal to 0.8%, and opening position in futures and impact on market is approximated by 0.1%. The costs on such levels can be attained even by individual investor in one of the most popular brokers house. So, the market impact for such investor is around zero, what is consistent with reality.

It is worthwhile to emphasize that the above approach to transaction costs has surely advantages. First of all it gives a possibility to institutional investor ⁹ to asses how much she could earn from arbitrage trade, when she reduces her transaction costs to one of the mentioned levels. Moreover, this approach allows to avoid the difficulties in evaluating market impact (cf. Puttonen (1993), Stulz, Wasserfallen and Stucki (1990), Yadev and Pope (1990)).

Next, the action of an investor after ex post violations is presented. Let the value R_{t_i} corresponding to minute i of the day t , lie outside boundaries. If $R_{t_i} < -u$ (futures contract is underpricing), an investor sells assets included in WIG20, which will have to be bought back after the delivery date of contract. In this way she collects capital S_{t_i} and it is assumed that she can invest this capital at interest rate r_t^{wibid} . At the same moment, she opens long

⁹There is no available data on the cost, it is even difficult to determine how many of mutual funds seek profit from arbitrage. Nonetheless, since the beginning of 2002, at least one mutual fund concentrates on possible arbitrage on floor of WSE.

position in futures contract on the WIG20 index. The price of contract at this moment is equal to f_{t_i} . Next, the position in spot market and futures is kept till maturity of contract. The marking-to-market procedure takes place every trading day. The profits are invested at the rate equal to the WIBID. The losses are settled by loan at interest rate equal to the WIBOR. Finally, on maturity day the last marking-to-market procedure takes place, investor buys the WIG20 index for the price S_T .

If $R_{t_i} > u$ (futures contract is overpricing) an investor borrows capital S_{t_i} and buys the WIG20 index. Simultaneously, investor opens short position in futures contract on the WIG20 index. The price of contract at this moment is equal to f_{t_i} . The capital invested in the index is obtained by a loan at interest rate r_t^{wibor} . Next, the position in spot market and futures market is kept till maturity of contract. As in the case of underpricing of contract, the profits are invested at the rate equal to the WIBID. The losses are paid by loan at the rate of WIBOR. Finally, on maturity day when the last marking-to-market procedure takes place, an investor sells the WIG20 index for the price S_T .

The costs of the whole arbitrage trade in both cases (under- and overpricing of contract) are determined by the commission in percentage of transaction value from spot market and by a round-trip commission for a futures contract.

The whole profit or loss obtained by investor after ex post signal may be expressed in terms of loss-profit function. The loss-profit function $z_l(\cdot)$ for a contract with delivery date T after the signal at time $t_i < T$ for the long arbitrage position in case of instantaneous execution of order after a signal is given by formula

$$\begin{aligned}
z_l(t_i) = & (f_t - f_{t_i})I(f_t - f_{t_i}, t) + \sum_{\tau=t+1}^T (f_\tau - f_{\tau-1})I(f_\tau - f_{\tau-1}, \tau) \\
& + (S_{t_i} e^{(r_i^{wibid} - d)(T-t)/365} - S_T) - C_s(S_{t_i} + S_T) - C_f, \tag{3.2}
\end{aligned}$$

where

$$I(x, t) = e^{r_t^{wibid}(T-t)/365} \mathbf{1}_{(0, \infty)}(x) + e^{r_t^{wibor}(T-t)/365} \mathbf{1}_{(-\infty, 0]}(x). \quad (3.3)$$

The loss-profit function $z_s(\cdot)$ in the case of the short arbitrage position is given, for $t_i < T$, by

$$\begin{aligned} z_s(t_i) &= (f_{t_i} - f_t)I(f_{t_i} - f_t, t) + \sum_{\tau=t+1}^T (f_{\tau-1} - f_\tau)I(f_{\tau-1} - f_\tau, \tau) + \\ &+ (S_T - S_{t_i} e^{(r_t^{wibor}-d)(T-t)/365}) - C_s(S_{t_i} + S_T) - C_f. \end{aligned} \quad (3.4)$$

Additionally, in this study the case of lag execution orders is examined. An investor, after receiving a signal, needs some time to open position. The time between signal and opening arbitrage position is equal to a , and it is measured in minutes. The loss-profit function $\bar{z}_l(\cdot)$ for the long arbitrage position in case of lag execution of order is given by

$$\begin{aligned} \bar{z}_l(t_i) &= (f_t - f_{t_{i+a}})I(f_t - f_{t_{i+a}}, t) + \sum_{\tau=t+1}^T (f_\tau - f_{\tau-1})I(f_\tau - f_{\tau-1}, \tau) \\ &+ (S_{t_{i+a}} e^{(r_t^{wibid}-d)(T-t)/365} - S_T) - C_s(S_{t_{i+a}} + S_T) - C_f, \end{aligned} \quad (3.5)$$

and for the short arbitrage position the loss-profit function $\bar{z}_s(\cdot)$ is given by

$$\begin{aligned} \bar{z}_s(t_i) &= (f_{t_{i+a}} - f_t)I(f_{t_{i+a}} - f_t, t) + \sum_{\tau=t+1}^T (f_{\tau-1} - f_\tau)I(f_{\tau-1} - f_\tau, \tau) \\ &+ (S_T - S_{t_{i+a}} e^{(r_t^{wibor}-d)(T-t)/365}) - C_s(S_{t_{i+a}} + S_T) - C_f, \end{aligned} \quad (3.6)$$

where $I(x, t)$ is defined by (3.3).

Finally, for testing Hypothesis 2 one examines the relationship between the time to delivery and magnitude of mispricing. It will be done by applying the method suggested in the

work of MacKinley and Ramaswamy (1988). The idea is to estimate linear relation between the average absolute mispricing at 1-minute intervals over a given day and the number of days remaining until maturity. So, the regression has the form $v_t = \alpha + \psi(T - t) + \epsilon_t$, where $v_t = |\sum_{i=1}^{N_t} R_{t_i}/N_t|$, and N_t is the number of observations on day t and error terms are independent and have standard normal distribution, namely $\epsilon_t \sim N(0, 1)$.

The motivation to consider the model presented above comes from reasoning that uncertainty about interest rate and size of dividends used in the cost-of-carry model become smaller as the time to maturity decreases. As the result, there is an agreement between market participants regarding the magnitude of relative mispricing. If the market is efficient, then investors act to reduce relative mispricing. Because all investors have the same expectation of relative mispricing size, their actions lead to real reduction of relative mispricing. In such case one can expect a positive coefficient ψ , that is positive correlation of mispricing and time to maturity.

Table 3.3: Summary statistics of ex post violations of futures price boundaries for WIG20 index.

Contract	TC(%) ¹	Violations for Long Arbitrage				Violations for Short Arbitrage				Regression ³		No. of all observations	
		No. of		Average	No. of		No. of		Average	α ⁴	ψ ⁵		F -statistic
		lower bound	crossing	time below	upper bound	crossing	time above						
		violations ²		lower bound	violations ²		upper bound	violations ²					
FW20H1	0.9	2090(9.43)	93	22.2	12408(56.0)	105	118.2	0.66**	0.02	21.1	22160		
	0.6	2976(13.4)	162	18.2	13522(61.0)	98	137.9						
	0.3	4625(20.9)	250	18.4	14444(65.2)	110	131.3						
FW20M1	0.9	7893(36.7)	86	90.7	2914(13.6)	149	19.6	0.74**	0.05	88.4	21476		
	0.6	8883(41.4)	117	75.3	4171(19.4)	169	24.7						
	0.3	10235(47.6)	191	53.3	6663(31.0)	296	22.5						
FW20U1	0.9	19207(77.8)	262	73.0	1(0.00)	1	1	0.73**	0.03	41.6	24678		
	0.6	21135(85.6)	253	83.2	17(0.07)	12	1.42						
	0.3	22684(91.9)	130	173.2	242(0.98)	52	4.65						
FW20Z1	0.9	7044(30.9)	124	56.4	1947(8.57)	254	7.66	-0.52**	0.04	97.2	22806		
	0.6	8082(35.4)	163	49.3	4020(17.6)	420	9.57						
	0.3	9231(40.5)	204	45.0	7483(32.8)	514	14.6						
FW20H2	0.9	13597(68.1)	137	98.5	-	-	-	0.26	0.03	90.9	19967		
	0.6	15043(75.3)	152	98.3	1(0.01)	1	1						
	0.3	16628(83.3)	181	91.4	106(0.53)	34	3.12						
FW20M2	0.9	8906(38.3)	92	95.8	10(0.04)	7	1.42	-0.21	0.02	130.5	23245		
	0.6	9815(42.2)	108	90.0	906(3.90)	181	5.01						
	0.3	11657(50.1)	243	47.7	4212(18.1)	256	16.4						
FW20U2	0.9	10614(46.5)	243	43.5	4(0.00)	3	1.33	0.29	0.02	22.8	22831		
	0.6	13135(57.5)	273	47.9	156(0.68)	61	2.55						
	0.3	15979(69.9)	307	51.8	1457(6.38)	275	5.29						
FW20Z2	0.9	8500(37.8)	239	35.4	137(0.61)	26	5.27	-0.18	0.02	88.7	22482		
	0.6	11283(50.2)	309	36.4	705(3.13)	130	5.42						
	0.3	14414(64.1)	303	47.4	2972(13.2)	270	11.0						
Overall	0.9	9740(43.4)	160	64.4	2488(11.1)	78	22.0	0.22	0.03	72.7	22456		
	0.6	11303(50.3)	191	62.6	2934(13.1)	134	23.4						
	0.3	13191(56.7)	225	66.2	4685(20.9)	225	26.1						

¹ TC(%)=market impact cost (%) + round-trip commission for a futures contract (%) + round-trip commission for spot market(%).

² Ex post violations as a percentage of the all observation of relative mispricing are given in parentheses.

³ Regression of size of relative mispricing d_t which violates boundaries $-u$ or u on time to maturity $T - t$, thus $d_t = \alpha + \psi(T - t) + \epsilon_t$.

⁴ *, **, *** denote significance of coefficient α at the level 1%, 5%, 10%, respectively.

⁵ Coefficient ψ is significant at the level 1%.

Table 3.4: Summary statistics of ex ante violations without execution lag.

Contract	TC(%) ¹	Long Arbitrage						Short Arbitrage						Regression coefficient ⁵	
		Average signal size ²	Average profit ³	STD of profits	Min	Max	Profitable trades ⁴	Average signal size	Average profit	STD of profits	Min	Max	Profitable trades	β	SE(β)
FW20H1	0.9	-1.89	8.40	5.52	0.34	21.9	510(24.4)	2.36	32.9	14.5	2.90	61.8	11911(96.0)	0.71	0.003
	0.6	-1.70	9.55	6.70	0.35	26.0	703(23.6)	2.23	35.2	16.1	2.82	66.4	12845(95.0)	0.75	0.002
	0.3	-1.55	11.6	7.40	0.05	30.0	862(18.6)	2.12	37.7	17.6	2.01	71.1	13288(92.0)	0.79	0.002
FW20M1	0.9	-3.87	27.4	17.4	0.02	72.8	7090(89.8)	-	-	-	-	-	-	-0.66	0.003
	0.6	-3.58	28.5	18.5	0.01	76.8	7875(88.6)	2.80	1.23	0.91	0.82	2.86	5(0.12)	-0.67	0.002
	0.3	-3.26	29.7	19.5	0.02	80.9	8822(86.2)	2.60	2.51	1.48	0.86	7.00	50(0.75)	-0.64	0.002
FW20U1	0.9	-3.45	4.62	3.40	0.02	16.0	1228(6.39)	1.02	6.37	-	6.37	6.37	1(100)	-0.45	0.01
	0.6	-3.13	5.65	4.14	0.06	19.1	1847(8.74)	0.76	6.50	2.65	4.97	9.56	3(17.6)	-0.44	0.008
	0.3	-2.73	5.52	4.79	0.01	22.3	3382(14.9)	0.40	5.52	1.03	4.75	12.7	99(40.9)	-0.30	0.006
FW20Z1	0.9	-	-	-	-	-	--	1.14	5.91	2.18	1.49	14.1	1946(99.9)	1.01	0.005
	0.6	-	-	-	-	-	--	0.93	7.06	3.03	1.42	17.8	4005(99.6)	0.98	0.003
	0.3	-0.93	1.96	-	1.96	1.96	1(0.00)	0.71	8.15	3.79	0.92	21.5	7157(95.6)	0.96	0.002
FW20H2	0.9	-2.97	26.8	9.24	0.05	37.7	401(2.95)	-	-	-	-	-	-	0.90	0.08
	0.6	-3.05	26.2	13.8	0.22	41.9	480(3.20)	0.62	2.96	-	2.96	2.96	1(100)	1.28	0.09
	0.3	-2.97	13.9	16.8	0.01	46.2	1130(6.79)	0.39	3.76	1.20	1.61	7.02	106(100)	-0.18	0.03
FW20M2	0.9	-	-	-	-	-	--	1.03	3.09	3.02	1.13	11.4	10(100)	-	-
	0.6	-	-	-	-	-	--	0.70	2.18	1.23	0.42	15.4	766(84.5)	0.91	0.03
	0.3	-	-	-	-	-	--	0.51	3.35	1.83	0.28	19.3	3948(99.7)	0.96	0.01
FW20U2	0.9	-2.02	2.41	1.73	0.01	8.40	1102(10.3)	0.93	1.83	0.53	1.37	2.38	4(100)	-0.41	0.01
	0.6	-1.88	4.38	2.34	0.11	11.65	1665(12.6)	0.68	3.28	1.13	1.55	6.01	156(100)	-0.11	0.01
	0.3	-1.77	6.70	3.07	0.18	14.99	1987(12.4)	0.44	3.77	1.45	1.58	9.29	1447(99.3)	-0.15	0.003
FW20Z2	0.9	-	-	-	-	-	--	1.28	0.58	0.56	0.012	1.29	5(3.61)	-	-
	0.6	-1.56	0.75	0.50	0.04	1.83	60(0.51)	1.05	1.25	1.12	0.03	4.78	88(12.5)	0.02	0.001
	0.3	-1.18	1.67	1.23	0.01	5.29	568(3.94)	0.85	2.43	1.74	0.01	8.26	345(11.5)	0.03	0.001
Overall	0.9	-2.84	13.9	7.46	0.09	31.4	2066(26.8)	1.30	8.44	3.46	2.21	16.2	2313(83.3)	0.18	0.019
	0.6	-2.48	12.5	7.66	0.13	29.5	2105(22.9)	1.22	7.46	3.27	1.87	15.72	2234(63.6)	0.34	0.018
	0.3	-2.04	10.1	7.54	0.32	28.8	2393(20.5)	1.00	8.40	3.76	1.50	19.5	3305(67.5)	0.18	0.007

¹ TC(%)=market impact cost (%) + round-trip commission for a futures contract (%) + round-trip commission for spot market(%).

² Average signal size given in percentage of index value.

³ Average profit at time T in terms of index points: a full index point is equivalent to 10 PLN per contract.

⁴ The number of ex ante violations (profitable ex post violations). Ex ante violations as a percentage of the all ex post violations (violations for Short Arbitrage+violations for Long Arbitrage) are given in parentheses.

⁵ Regression of profit magnitude z on corresponding ex ante violations represented by relative mispricing R , thus $z_t = R_t\beta + \alpha + \epsilon_t$.

Table 3.5: Summary statistics of ex ante violations with 15 minutes execution lag.

Contract	Long Arbitrage						Short Arbitrage						Regression coefficient ⁵		
	TC(%) ¹	Average signal size ²	Average profit ³	STD of profits	Min	Max	Profitable trades ⁴	Average signal size	Average profit	STD of profits	Min	Max	Profitable trades	β	SE(β)
FW20H1	0.9	-1.91	8.66	5.50	0.35	21.9	488(23.3)	2.38	33.2	14.2	2.91	61.8	11698(94.0)	0.70	0.003
	0.6	-1.72	9.84	6.7	0.35	26.0	668(22.4)	2.25	35.6	15.8	3.10	66.4	12748(94.2)	0.74	0.002
	0.3	-1.57	11.8	7.42	0.05	30.0	827(17.9)	2.14	38.3	17.2	2.01	71.1	13590(94.1)	0.77	0.002
FW20M1	0.9	-3.91	27.6	17.3	0.02	72.8	6706(84.9)	-	-	-	-	-	-	-0.62	0.003
	0.6	-3.61	28.8	18.5	0.01	76.8	7477(84.2)	2.80	1.23	0.91	0.82	2.86	5(0.12)	-0.64	0.003
	0.3	-3.33	30.2	19.4	0.02	80.9	8246(80.6)	2.62	2.78	1.52	0.86	7.00	39(0.58)	-0.62	0.002
FW20U1	0.9	-3.44	4.55	3.32	0.02	13.9	1182(6.15)	-	-	-	-	-	-	-0.50	0.01
	0.6	-3.12	5.58	4.10	0.07	17.1	1784(8.44)	-	-	-	-	-	-	-0.47	0.008
	0.3	-2.73	5.47	4.74	0.01	20.3	3272(14.4)	0.39	5.59	0.95	4.77	8.04	32(13.2)	-0.39	0.006
FW20Z1	0.9	-	-	-	-	-	-	1.17	6.31	2.15	1.91	14.1	1274(65.4)	1.01	0.005
	0.6	-	-	-	-	-	-	0.98	7.61	3.01	1.42	17.8	2874(71.4)	0.98	0.003
	0.3	-	-	-	-	-	-	0.75	8.62	3.73	1.44	21.5	5767(77.0)	0.95	0.002
FW20H2	0.9	-2.86	27.6	8.66	0.05	37.7	377(2.77)	-	-	-	-	-	-	0.89	0.08
	0.6	-3.06	26.9	13.7	0.22	41.9	450(3.01)	-	-	-	-	-	-	1.66	0.09
	0.3	-2.97	13.8	17.0	0.01	46.2	1093(6.57)	0.41	3.95	1.10	1.62	5.62	36(33.9)	-0.10	0.06
FW20M2	0.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0.6	-	-	-	-	-	-	0.71	2.28	1.18	0.42	7.40	450(49.6)	0.94	0.03
	0.3	-	-	-	-	-	-	0.51	3.47	1.81	0.28	11.3	3405(80.8)	0.97	0.01
FW20U2	0.9	-2.02	2.44	1.74	0.01	8.40	1057(9.96)	-	-	-	-	-	-	-0.83	0.01
	0.6	-1.89	4.43	2.35	0.12	11.7	1582(12.0)	0.67	3.40	0.92	1.80	5.01	49(31.4)	-0.21	0.01
	0.3	-1.78	6.74	3.07	0.18	14.9	1882(11.8)	0.45	3.93	1.43	1.58	9.25	836(57.3)	-0.15	0.003
FW20Z2	0.9	-	-	-	-	-	-	1.28	0.58	0.56	0.01	1.29	5(3.6)	-	-
	0.6	-1.56	0.75	0.51	0.04	1.83	59(0.52)	1.04	1.27	1.12	0.04	4.78	86(12.2)	0.02	0.005
	0.3	-1.18	1.66	1.24	0.00	5.30	547(3.79)	0.85	2.47	1.75	0.00	8.26	328(12.0)	0.03	0.004
Overall	0.9	-2.82	14.2	7.3	0.09	30.9	1962(25.4)	1.61	13.40	5.63	1.61	25.7	4326(54.3)	0.11	0.019
	0.6	-2.49	12.7	7.64	0.135	29.2	2003(21.8)	1.41	8.60	3.82	1.26	17.4	2702(43.1)	0.38	0.019
	0.3	-2.26	11.6	8.81	0.05	32.9	2645(22.5)	1.02	8.64	3.69	1.57	17.8	3004(46.1)	0.18	0.01

¹ TC(%)=market impact cost (%) + round-trip commission for a futures contract (%) + round-trip commission for spot market(%).

² Average signal size given in percentage of index value.

³ Average profit at time T in terms of index points: a full index point is equivalent to 10 PLN per contract.

⁴ The number of ex ante violations (profitable ex post violations). Ex ante violations as a percentage of the all ex post violations (violations for Short Arbitrage+violations for Long Arbitrage) are given in parentheses.

⁵ Regression of profit magnitude z on corresponding ex ante violations represented by relative mispricing R , thus $z_t = R_t \beta + \alpha + \epsilon_t$.

Table 3.6: Summary statistics of ex ante violations with 60 minutes execution lag.

Contract	TC(%) ¹	Long Arbitrage						Short Arbitrage						Regression coefficient ⁵	
		Average signal size ²	Average profit ³	STD of profits	Min	Max	Profitable trades ⁴	Average signal size	Average profit	STD of profits	Min	Max	Profitable trades	β	SE(β)
FW20H1	0.9	-1.96	9.26	5.6	0.37	21.9	415(19.8)	2.37	33.2	14.1	2.9	61.8	10086(81.3)	0.67	0.003
	0.6	-1.75	10.3	6.9	0.36	26.0	581(19.5)	2.25	35.6	15.5	3.15	66.4	10956(81.0)	0.71	0.003
	0.3	-1.60	12.2	7.65	0.04	30.0	717(15.5)	2.14	38.3	16.8	2.02	71.1	11705(81.0)	0.75	0.002
FW20M1	0.9	-3.94	27.9	17.3	0.02	72.8	5754(72.9)	-	-	-	-	-	-	-0.66	0.003
	0.6	-3.64	29.0	18.4	0.07	76.8	6416(72.2)	-	-	-	-	-	-	-0.63	0.002
	0.3	-3.36	30.4	19.4	0.03	80.9	7079(69.2)	2.47	0.86	0.00	0.86	0.87	2(0.03)	-0.61	0.002
FW20U1	0.9	-3.42	4.20	3.11	0.02	13.9	994(5.17)	-	-	-	-	-	-	-0.50	0.01
	0.6	-3.11	5.26	3.84	0.06	17.1	1539(7.28)	-	-	-	-	-	-	-0.47	0.008
	0.3	-2.73	5.29	4.51	0.00	20.2	2822(12.4)	0.35	5.10	0.34	4.81	6.04	19(7.85)	-0.43	0.006
FW20Z1	0.9	-	-	-	-	-	-	1.17	6.19	2.21	1.91	14.1	924(47.4)	1.01	0.005
	0.6	-	-	-	-	-	-	0.99	7.68	3.05	1.41	17.8	2020(50.2)	0.98	0.004
	0.3	-	-	-	-	-	-	0.75	8.53	3.81	1.44	21.5	4303(57.5)	0.95	0.003
FW20H2	0.9	-2.98	29.4	6.64	0.05	37.7	311(2.28)	-	-	-	-	-	-	0.89	0.08
	0.6	-3.05	29.2	12.6	0.25	42.0	361(2.40)	-	-	-	-	-	-	1.66	0.09
	0.3	-2.96	13.5	17.2	0.02	46.2	969(5.82)	0.44	4.61	0	4.61	4.61	1(0.94)	-0.16	0.04
FW20M2	0.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0.6	-	-	-	-	-	-	0.7	2.01	0.87	0.53	4.38	248(27.4)	0.65	0.05
	0.3	-	-	-	-	-	-	0.51	3.47	1.82	0.28	11.26	2730(64.8)	0.98	0.01
FW20U2	0.9	-2.02	2.46	1.72	0.01	8.40	922(8.68)	-	-	-	-	-	-	-0.81	0.01
	0.6	-1.89	4.46	2.37	0.13	11.7	1354(10.3)	0.67	3.39	0.64	2.56	3.98	8(5.12)	-0.53	0.01
	0.3	-1.78	6.73	3.11	0.21	14.9	1627(10.2)	0.41	3.62	1.23	1.58	8.82	474(32.5)	-0.19	0.004
FW20Z2	0.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0.6	-1.56	0.75	0.51	0.04	1.83	59(0.52)	1.04	1.27	1.13	0.03	4.77	77(10.9)	0.02	0.003
	0.3	-1.19	1.74	1.32	0.00	5.30	426(2.95)	0.86	2.65	1.77	0.01	8.26	289(9.72)	0.03	0.004
Overall	0.9	-2.86	14.6	6.87	0.09	30.9	1679(21.7)	1.77	19.7	8.15	2.41	37.9	5505(64.3)	0.10	0.02
	0.6	-2.50	13.2	7.44	0.15	29.2	1718(18.7)	1.13	9.90	4.24	1.54	19.5	2662(34.9)	0.30	0.02
	0.3	-2.27	11.6	8.86	0.05	32.9	2273(19.3)	0.99	8.40	3.22	1.95	16.6	2440(31.8)	0.21	0.01

¹ TC(%)=market impact cost (%) + round-trip commission for a futures contract (%) + round-trip commission for spot market(%).

² Average signal size given in percentage of index value.

³ Average profit at time T in terms of index points: a full index point is equivalent to 10 PLN per contract.

⁴ The number of ex ante violations (profitable ex post violations). Ex ante violations as a percentage of the all ex post violations (violations for Short Arbitrage+violations for Long Arbitrage) are given in parentheses.

⁵ Regression of profit magnitude z on corresponding ex ante violations represented by relative mispricing R , thus $z_t = R_t \beta + \alpha + \epsilon_t$.

3.4 Results

In this section the results of testing efficiency of Polish market are presented. Additionally, the obtained results are compared with those reported for indices on other market over the world.

Table 3.3 contains summary statistics of ex post violations of futures price boundaries. That table is divided into three parts. The first one corresponds to violations of lower bound (signals for long arbitrage), the second one contains statistics of violations of upper bound. Both have the following columns: number of lower (resp. upper) bound violations, number of lower (upper) bound crossing, average time below lower (above upper) bound. Moreover, the percentage of ex post violations in all observation is given in parentheses. Bound crossings are defined as those cases for which one observation of relative mispricing lies outside the corridor, and the previous observation lies within the corridor. By violation of lower (upper) bound one means an observation of relative mispricing which lies above or below arbitrage corridor. The values of average time below lower (above upper) bound are calculated as the sum of all length of all violations of lower (upper) bound divided by the number of such cases.

The last part of Table 3.3 reports the value of the coefficient of regression of average absolute relative mispricing of 1-minute interval over a given day on the number of days remaining to maturity (compare Figlewski (1984), MacKinley and Ramaswamy (1988)). The idea of this regression is explained in the previous section.

From the results in Table 3.3, it is clear that the frequency of ex post violations for long arbitrage changes rapidly for different contracts with delivery date between March 2001 and December 2002. However, the frequency of ex post violations for short arbitrage decreases for succeeding contracts. For FW20H1 (contract with delivery date March 2001), the ex post violation for short arbitrage are equal to 56.0, 61.0, 65.2 % of all observations with

transaction costs on level 0.9, 0.6, 0.3 %, respectively. But for contract FW20Z2 (contract with delivery date December 2002) these values are reduced to 0.61, 3.13, 13.2 % of all observations.

In the panel "Overall" the average frequency of ex post violations is reported. It is much higher for long than for short arbitrage and for transaction costs on level 0.9, 0.6, and 0.3% it is equal to 43.4, 50.3, and 56.7% respectively, when the frequency of ex post violations for short arbitrage are equal to 11.1, 13.1, 20.9 % of total observations.

Comparatively, the study of Chung (1991) shows that the percentage of ex post violations in all observation decreases from 33.12% in 1984 to 8.73% in 1986 in case of MMI contracts. Similar observation is made by Klemkosky and Lee (1991) in their study about arbitrage on contracts on the S&P 500 index.

Persistence defined as an average time above upper (below lower) bound, is another feature making difference between ex post violations for long and for short arbitrage. The higher the value in this column, the larger is the mispricing persistence for each contract. The persistence of ex post violations for short arbitrage has significantly declined for the succeeding contracts. It is indicated by decrease in the average time above upper bound, for contract FW20H1 it is equal to 118.2, 137.9, and 131.3 minutes for transaction costs on level 0.9%, 0.6%, and 0.3% respectively. But for contract FW20Z2 these numbers are reduced to 5.27, 5.42 and 11.0 minutes. On the contrary, in the persistence of ex post violations for long arbitrage one cannot note any declining trend. The average time below lower bound fluctuates from contract to contract. On the average, as shown in Table 3.3 in the panel "Overall", the persistence of ex post violations for long arbitrage is almost tripled in comparison to short arbitrage, and it is equal to 64.4, 62.6, 66.2 minutes depending on transaction cost. Further argument for persistence of violations for long arbitrage on Polish market is provided by the number of crossings of lower bound. Namely, the overall average number of lower bound crossing ranges from 160 to 225 depending on transaction costs,

which is of much smaller magnitude than the average number of violation which ranges from 9740 to 13191.

The similar results about sign and persistence of mispricing was reported by Bühler and Kempf (1995) in case of German market, Puttonen (1993) for Finnish, Yadev and Pope (1990) for United Kingdom, Stulz, Wasserfallen and Stucki (1990) for Switzerland, and Berglund and Kabir (2003) for Dutch market. In previous studies also the attention was paid to the reasons of negative mispricing and its persistence. Chung (1991) pointed out underestimation of transaction cost, the lack of arbitrage capital and noise. Kempf (1998) and Puttonen and Martikainen (1991) claim that short sale restriction are mainly responsible for it. This factor in case of Polish market seems to play decisive role. But, the underestimation of transaction cost can be excluded since costs at level of 0.9% are available even for individual investors, who set their order via internet.

The results of regression indicate that the magnitude of the relative mispricing is positively correlated with the time until delivery date of contract. Nevertheless, the average coefficient of this regression is very small, it is equal to 0.03. Looking at Figure 3.1, this result is not surprising. From Figure 3.1, it is also clear that in general relative mispricing is not a decreasing function of time until maturity. All values of F -statistic reported in Table 3.3 are greater than critical value at the 0.01 level of significance (which is equal to 6.64), what confirms that all coefficients of these regressions are significantly different from zero. It is worth to confront these results with previous studies. On the one hand, MacKinlay and Rammswamy (1988) reported that on US market there is clear indication that mispricing disappears, with decrease of time to maturity. Similar results were reported by Bühler and Kempf (1995) for Germany, Yadev and Pope (1990) for United Kingdom, Stulz, Wasserfallen and Stucki (1990) for Switzerland. Also Brenner, Subrahmanyam and Uno (1989) obtained analogous results for Japan. On the other hand, Puttonen (1993) claims that reverse situation was observed on Finnish market in the period characterized by lack of institutional

framework for short sale. This suggests that short sale restriction can be a decisive factor for the discussed attribute of mispricing series. However, in case of Polish market it is another factor which can obstruct the relationship between size of mispricing and time to maturity of contract, namely the uncertainty of the level of interest rate. It makes it difficult to identify the theoretical price of futures contract in the unique manner.

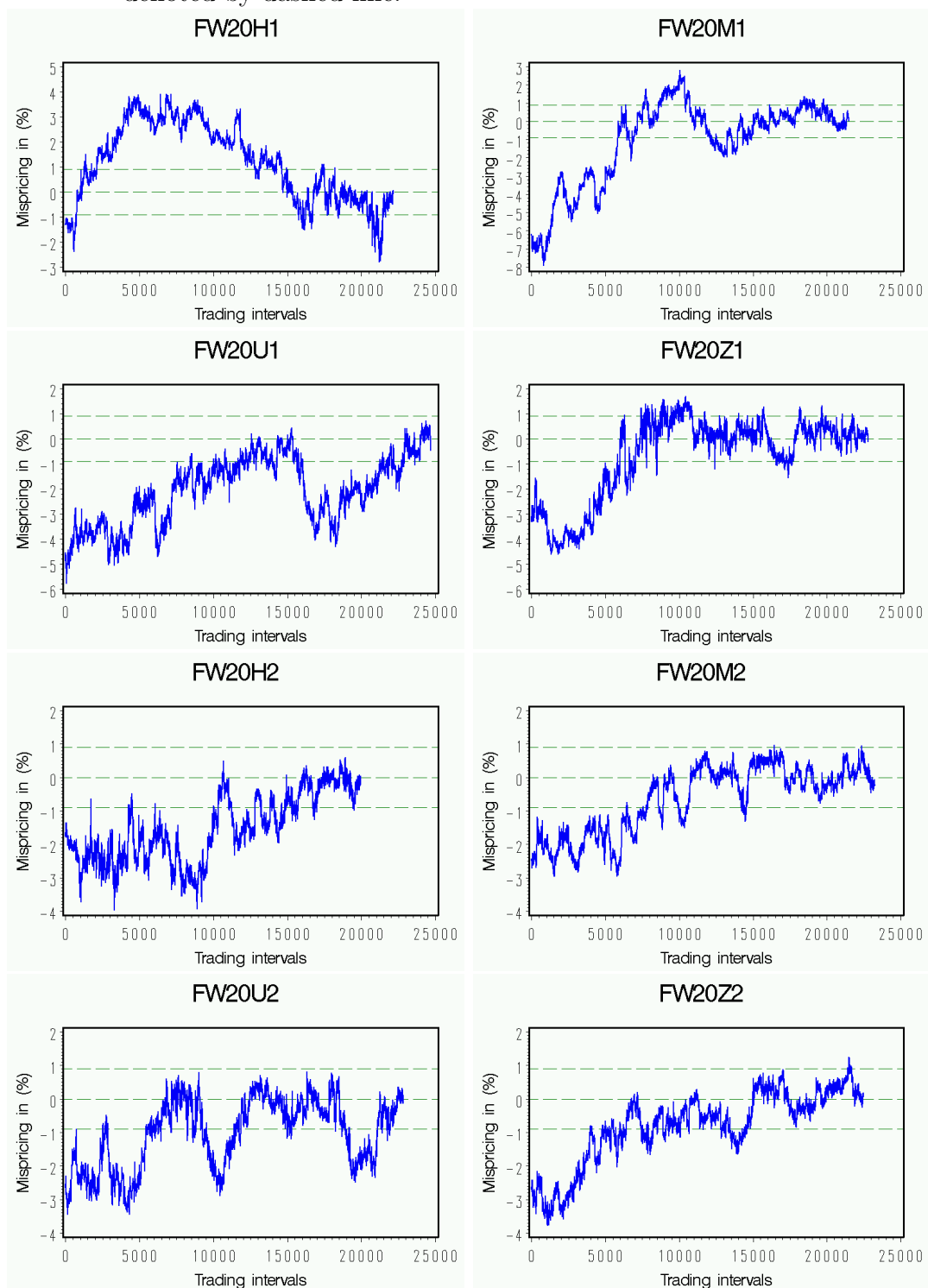
Next, the summary of statistics for ex-ante violations (these of ex post violations which lead to profit, after opening position on both markets) is presented. In this work, six execution lags are considered, namely instantaneous execution with ex post violation and with lag equal to 1, 5, 10, 15 and 60 minutes. The results reported in Tables 3.4, 3.5 and 3.6 correspond to instantaneous execution with ex post violation, with 15 minutes lag and with 60 minutes lag, respectively¹⁰. The results for these lags are most representative. It is worthwhile to highlight that in the case of contracts on the MMI or the S&P 500 index most authors consider smaller delays in execution of orders. For example, Chung carried out a study based on delays equal to 20 seconds, 2 minutes and 5 minutes. In turn, Klemkosky and Lee examined instantaneous execution with ex post violation, and lags equal to 1, 3, 5 and 10 minutes for contracts on the S&P 500 index.

Each of Tables 3.4, 3.5 and 3.6 contains the following columns for long and short arbitrage: average signal size as a percentage of index value, average profit in terms of index points, standard deviation of profit, minimum and maximum of profits (arising from those of ex post violations which lead to positive profit). Finally, the number of profitable trades is reported, and in parentheses the number of ex ante violations as percentage of ex post violations is given.

It is clear from the results reported in Table 3.4 that the average profit from short arbitrage diminishes over time, albeit not constantly. For example, the average profit arising

¹⁰The tables just as 3.4, 3.5, 3.6 for lag equal to 1, 5, 10 minutes are available upon request from the authors

Figure 3.1: The graphs present relative mispricing of contracts on WIG20 for series from the FW20H1 to the FW20Z2; the horizontal axis contains 1 minute trading intervals; the transaction costs boundaries for $u = 0.9\%$ are denoted by dashed line.



from the short arbitrage for FW20H1 contract with transaction costs of 0.3%, 0.6%, 0.9% is equal to 32.9, 35.2, 37.7 points, respectively, and for FW20Z2 contract the average profit is reduced to 0.58, 1.25, 2.43 points. In contrast to short arbitrage one cannot recognize such tendency for long arbitrage. There are two cases of dramatic increase of the average profit, one took place in case of FW20M1 contract, the other one for FW20H2 contract. For comparison the profitability of arbitrage trades were reported by Chung (1991) especially for early series of contract on MMI. Also Bühler and Kempf (1995), Puttonen (1993) pointed out the possible profit to be made from arbitrage on German and Finnish market respectively. Yet, Stulz, Wasserfallen and Stucki (1990) for Swiss and Lim (1992) for Japanese market found that profit from arbitrage is difficult to be obtained. It is due to the high transaction costs.

One of characteristic attributes for mature market is increase of standard deviation of profits for succeeding contracts. Likewise, the means of profits are substantially smaller than corresponding standard deviations (cf. Chung 1991). These theses are supported by the fact that profits from arbitrage become more risky as market matures.

According to the results reported in Table 3.4, Polish market does not reveal such properties. The mean of average profit is higher than standard deviation. This observation applies also to ex ante violations for long and short arbitrage. The maximum of profits obtained after ex post violations for long arbitrage amounts to 80.9 index points, and for short arbitrage it is equal to 71.1. On average, the average profit from long arbitrage varies from 10.1 to 13.9 (see panel "Overall") depending on transaction cost, and for short arbitrage it lies between 7.46 and 8.44 of index points.

Table 3.4 shows also that ex ante profits are smaller than corresponding ex post violations which are the signals to opening position on both markets. For pointing out the above mentioned properties one should recall that the average profit is expressed in terms of index points, and the average signal size in percentage of index value. Therefore, to compare

these magnitudes one should multiply the average signal size by value of index, so at least by 977.9¹¹. The exception to the fact that ex ante profits are smaller than corresponding ex post violations is FW20H1 contract for which, in case of short arbitrage, the ex ante violations range from 2.12 to 2.36% of index value and profit lies within the scope from 32.9 to 37.7 points. Additionally, the closer the delivery date of the given contract to the sample end date (December 2002), the larger is the difference between ex post and corresponding ex ante violations. The above attributes of Polish market coincide with those reported by Chung (1991). Nevertheless, for Germany Bühler and Kempf (1995) reported that average arbitrage profit is larger than the arbitrage signal. It applies also to arbitrage trades with execution lag equal to 15 minutes.

From results collected in Table 3.4, one can evaluate the quality of ex post violation as predictor of profit from arbitrage trade. One striking observation from the column of profitable trades is usefulness of ex post violations as predictor. Ex post violations varies dramatically, depending on contract. It is particularly evident in case of long arbitrage. For example, for FW20M1 contract more than 86% of all ex post violations lead to profit, but for FW20Z2 contract this percentage was reduced to interval from 0.51% to 3.94%. For short arbitrage, ex post violations is a good predictor of arbitrage profit, with the exception of FW20M1 and FW20Z2 contracts. Following the idea of Chung (1991), an examination of the attributes of ex ante violations as predictor of realized profits is carried out. It is done by considering the linear regression of realized profits on the size of relative mispricing that lead to these profits. Table 3.4 reports slope in linear regression and standard error of regression. Results indicate that generally ex ante violations make very poor prediction of profits with average regression coefficient ranging from 0.18 to 0.34 depending on transaction cost. Further argument confirming its unreliability is the fact that for contracts FW20H1,

¹¹The lowest value of index WIG 20 in period December 2000 to December 2002 was equal to 977.9 index points.

FW20Z1 and FW20M2 the coefficient of regression is close to 1, but for contracts FW20M1, FW20U1 and FW20U1 it is negative and relatively large. Therefore, results obtained for contract on WIG20 are consistent with findings of Chung (1991) and Klemkosky and Lee (1991). These works also indicate that ex ante violations cannot be used as reliable predictor of realized profits.

As it was mentioned before, the influence of lag in execution of orders on profit from arbitrage is investigated. It is worth to remind that previous studies show that lags in execution of orders make arbitrage trade more risky. The number of profitable trades is reduced, the standard deviation of profits increase, and extreme value of profits change (cf. eg. Chung (1991) and Klemkosky and Lee (1991), Bühler and Kempf (1995)). Table 3.5 and 3.6 report summary statistics for ex ante violations with 15 minutes and 60 minutes lag. Such a large difference between execution lags used in the previous studies and in this work indicates that investors on Polish market react slower than investors on mature markets. Arbitrage on Polish market is still possible after 60 minutes in the contrary to mature market where it is almost impossible after 5 minutes. In contrast to mature markets, one cannot observe that lag execution has an important influence on maximum and minimum of profits. This result also confirms that violations of non-arbitrage boundaries are persistent. It also supports the hypothesis that the market does not react quickly enough to eliminate arbitrage opportunity. Further surprising result for examined market is that increase of the interval between ex ante violation and opening position, does not result in the significantly higher standard deviation of profits. Nevertheless, there is one property which supports the hypothesis that Polish market is becoming mature. Delays in execution of orders reduce the number of ex ante violations especially in case of short arbitrage. The average percentage of ex ante violations for the short arbitrage (only non-zero factors are taken into account during calculation of average) with 0.9% transaction costs declines from 83.3% to 64.3% of all ex post violations for lag changing from 0 minutes to 60 minutes. For violations of lower

bound (which corresponds to a signal for long arbitrage), this percentage changes from 26.8 to 21.7 %. In turn, for some contracts the average signal size becomes higher with rising of time between signal to arbitrage and execution of orders. For long arbitrage the mean of average signal size increases from 2.04 to 2.27% with transaction costs of 0.3%. In case of short arbitrage this magnitude remains almost unchanged for the same level of transaction costs.

One of the conclusions that might be drawn from the results reported in Table 3.5 and 3.6, is that mean of the average profit increases with delay in execution of order. However it is due to the fact that the number of contracts for which ex ante violations are observed is declining with time (for example: FW20U1, FW20H2, FW20M2).

Finally, the usefulness of ex ante violations as predictor of realized profit is evaluated. The last columns in Table 3.5 and 3.6 report the result of this assessment. The results of regression of profit obtained from lag orders on the size of relative mispricing, confirm that ex ante violations do not make a plausible indicator of future profits. The average of coefficient of this linear regression ranges from 0.11 to 0.38 in the case of lag equal to 15 minutes, and it changes between 0.10 to 0.21 for the lag equal to 60 minutes. These numbers are comparable to those obtained in case of execution orders without delay, which implicates that this is a poor predictor.

3.5 Conclusions

In this study, the relationship between spot market and futures market for contracts on the WIG20 index has been examined with special attention devoted to possibility of arbitrage between these two markets. The results have been obtained by application of the cost-of-carry model with transaction costs and with alternative execution lags. Moreover results are compared to those reported for other markets.

From theoretical point of view this study is interesting at least for the following reasons. Firstly, the period of time under examination was characterized by uncertainty of the size of risk-free rate. Secondly, in spite of existing institutional framework, the short sale is hardly used by investors which is mainly due to insufficient supply of stocks which are subject of short sale and its high costs. Thirdly, dividends are paid rarely and their size in almost all cases fluctuated around 1%.

Based on investigated properties of ex post and ex ante violations for long arbitrage, it is found that Polish market is relatively "arbitrage inefficient". On the one hand, the number of ex post violations did not decrease and the average persistence was equal to about 57 minutes. Further, there was no evidence that profits from arbitrage became more risky for succeeding series of contracts. In turn, the lag in execution of order caused no significant changes in the size of ex ante violations and in profits to which ex ante violations lead. On the other hand, it should be stressed that results obtained for short arbitrage indicate that Polish market has features of mature market. The number of ex post violations decreased for succeeding series of contracts. The average persistence was equal to about 26 minutes, whereas arbitrage profits became less for succeeding series of contracts. Examining of the lag in execution indicates that the number of profitable trades was substantially reduced with increase of lag. So, arbitrage opportunity became rare and profits less probable. The average percentage of profitable trades vary from 67.5 to 83.3% of all ex post violations for short arbitrage (arbitrage strategy are named after the position opened on futures market). For long arbitrage this magnitude ranges from 20.5 to 26.8 %. Therefore, the signals from cost-of-carry formulae are less reliable in comparison to previous studies.

The reason for such significant difference between properties of ex post and ex ante violations for long and short arbitrage are short sale constraints, which make this instrument not useful for institutional investor. Additionally, in the case of long arbitrage investors are exposed to the risk, that the value of index increases as reaction on declining the risk-free

interest rate. As a result the signals for long arbitrage are not followed by sufficient investing activity of markets participants, as it is in case of short arbitrage.

In addition to mentioned result it is worth highlighting that Polish market has similar unusual property like Finnish market, namely the size of relative mispricing does not constantly decrease over time. It seems that it is characteristic for emerging markets with short sale constraints.

The present situation on Polish futures market motivates for its further investigation. It should be investigated, whether stabilization of the risk-free rate influences the size and frequency of arbitrage opportunity. It would be interesting to examine again existence of arbitrage after introducing new regulations of short sales on WSE. Regulations improving functioning of short sales will be established in the nearest future.

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Chapter 4

Forward and Futures Contracts on Zero-coupon Bonds in Cox-Ingersoll-Ross Model

In this paper, a closed form expression for the value of a forward contract on a zero-coupon bond is derived. Also, another proof of the known formula for futures price of a zero-coupon bond which is known from the paper of Cox, Ingersoll and Ross (1981) (where it was proved by arbitrage arguments) is presented. Moreover, it is proved that a hedging portfolio approach analogous to that used in the derivation of the price of many contingent claims (see e.g. Black and Scholes (1973), Black (1976), Rebonato (1996) or Wilmott (1999)), is not rigorous enough from a mathematical point of view in the case of a futures contract on a zero-coupon bond.

4.1 Introduction

Cox, Ingersoll and Ross have introduced the model of the short-term interest rate, $r(t)$, in which $r(t)$ is a strong solution of the Itô equation given by the formula:

$$dr(t) = (a - br(t))dt + \sigma\sqrt{r(t)}dW(t), \quad (4.1)$$

where a, b and σ are constants with $a \geq 0$, $b > 0$, $\sigma > 0$ and $W(t)$ is the Wiener process (see Cox, Ingersoll and Ross (1985)). Thus, the short-term $r(t)$ is modelled by the squared gaussian

process, the properties of which have been examined by Feller (1951).

After the introduction of this model (CIR, for short) the question how to value zero-coupon bonds and derivatives emerged. The closed form of the price of zero-coupon bond can be found in Cox, Ingersoll and Ross (1985). The formulae for the price of a zero-coupon bond in CIR model can also be found in many monographs, including Duffie (1992), Musiela and Rutkowski (1997), Rebonato (1996), Wilmott (1999) and in several papers, for example Elliott and Hoek (2001), Rogers (1995). This formulae has many different proofs. Next, the derivatives of zero-coupon bonds have been considered. The formulae for a bond option were found by Longstaff (1990). The problem of pricing of options on bond futures was considered by Feldman (1993). In turn, a detailed derivation of prices of caps and options inter alia on bonds, bond futures and forward contracts can be found in Jamshidian (1995), (1996). It is worth noticing that in the case of options on interest rate the price of the underlying is χ^2 distributed, which is the core of the Jamshidian approach.

In this paper, focus is put upon pricing of forward and futures contracts on a zero-coupon bond in the CIR model. A formula for the value of a forward contract during its lifetime is contained in Section 2. In Lemma 4.1 today's price of delivery at future moment s ($s < T$) of a zero-coupon bond with maturity T is given. It is also presented that this price solves the partial differential equation.

In section 3, the formula for the futures price of a zero-coupon bond is derived. It is assumed continuous marking to market procedure for the considered futures contract. Therefore, the futures price is equal to the expected value of the price of underlying on the delivery day under the risk-neutral measure, see e.g. Cox, Ingersoll and Ross (1981), Richard and Sundaresan (1981) or Chen (1992). To our best knowledge, the complete proof in such a form is not available in the literature. The approach of Cox, Ingersoll and Ross (1985) is embedded in a general equilibrium framework.

In the Appendix we consider another approach to prove the formula for the futures price. One could expect, that it is possible to obtain the price of futures contract by taking a hedging portfolio analogous to a classical one used in following studies Black and Scholes (1973), Black (1976), Gibson, Lhabitant and Talay (1998), Rebonato (1996) or Wilmott (1999). However, we have proved that

this portfolio in our case is not self-financing. Which is similar to the situation pointed out by Rosu and Stroock (2004) in the case of the Black-Scholes model.

4.2 Forward contract

In this paper, we will use the following notation:

t - the current time,

T - the maturity time of a zero-coupon bond issued at moment 0,

s - the time of delivery of bond, where $t \leq s \leq T$,

$B(r, t)$ - the current price of bond,

$F(r, t)$ - the forward price of bond,

$f(r, t)$ - the futures price of bond.

As usual, the process $B(r, t)$ is a function $B(r, t, T)$ of three arguments r, t, T evaluated at $r(t), t, T$ i.e. $B(r, t) = B(r(t), t, T)$, and analogously for $F(r, t)$ and $f(r, t)$. T is fixed, so we omit T in what follows. We assume that $B(x, t)$, $F(r, t)$ and $f(x, t)$ are functions of class C^2 with respect to x and of class C^1 with respect to t .

As we have mentioned before, the price of the zero-coupon bond is known. Namely, the price $B(r, t)$ of the zero-coupon bond at time t yielding 1 at maturity T on an arbitrage free market equals

$$B(r, t) = e^{m(t) - n(t)r}, \quad (4.2)$$

where

$$n(t) = n(t, T) = \frac{2(e^{\gamma(T-t)} - 1)}{(b + \gamma)(e^{\gamma(T-t)} - 1) + 2\gamma}, \quad (4.3)$$

$$m(t) = m(t, T) = \frac{2a}{\sigma^2} \ln \left(\frac{2\gamma e^{(b+\gamma)(T-t)/2}}{(b + \gamma)(e^{\gamma(T-t)} - 1) + 2\gamma} \right), \quad (4.4)$$

$$\gamma = \sqrt{b^2 + 2\sigma^2}. \quad (4.5)$$

Moreover, the function B is a solution of a partial differential equation:

$$B_t(x, t) + \frac{\sigma^2 x}{2} B_{xx}(x, t) + (a - bx) B_x(x, t) - xB(x, t) = 0, \quad (4.6)$$

$x \geq 0, 0 \leq t < T$ with boundary condition

$$B(x, T) = 1. \quad (4.7)$$

In Cox, Ingersoll and Ross (1985), Elliott and Hoek (2001), Musiela and Rutkowski (1997), Rogers (1995) one can find detailed and different proofs of the above facts. Moreover, it is proved there that $n(t) = n(t, T)$ and $m(t) = m(t, T)$ satisfy equations:

$$\frac{\sigma^2}{2} n^2(t) + bn(t) - n_t(t) - 1 = 0, \quad n(T) = 0, \quad (4.8)$$

$$m_t(t) = an(t), \quad m(T) = 0. \quad (4.9)$$

We consider the problem of pricing of the forward contract, written at moment t_0 , during its lifetime. As it is well known (see Cox, Ingersoll and Ross (1981)), the forward price $F(r, t)$ at time t for delivery at delivery date s of a one zero-coupon bond yielding 1 at maturity T equals

$$F(r, t) = e^{m(t) - \bar{m}(t) - (n(t) - \bar{n}(t))r}, \quad (4.10)$$

where $\bar{n}(t) = n(t, s)$, $\bar{m}(t) = m(t, s)$, $n(t) = n(t, T)$, $m(t) = m(t, T)$ are defined by (4.3) and (4.4) for bonds with maturity date s and T respectively ($s < T$).

In next theorem we look at the value of the forward contract in CIR model.

Theorem 4.1 *If the spot interest rate r is described by the dynamics (4.1), then the value $V(r, t)$ at time $t > t_0$ of a forward contract written at time t_0 for delivery at delivery date s of a one*

zero-coupon bond yielding 1 at maturity T equals

$$V(r, t) = e^{l(t)-k(t)r(t)} - e^{m(t_0)-\bar{m}(t_0)-(n(t_0)-\bar{n}(t_0))r(t_0)+\bar{m}(t)-\bar{n}(t)r(t)}, \quad (4.11)$$

where

$$k(t) = \frac{(b + \gamma)\Psi + 2\theta e^{\gamma(s-t)}}{(b + \gamma)\theta e^{\gamma(s-t)} - \Psi\sigma^2}, \quad (4.12)$$

$$l(t) = m(s) + \frac{2a(s-t)(b\gamma + \gamma^2 - \sigma^2)}{\sigma^2(b + \gamma)} + \frac{2a}{\sigma^2} \ln \left(\frac{(b + \gamma)\theta - \Psi\sigma^2}{(b + \gamma)\theta e^{\gamma(s-t)} - \Psi\sigma^2} \right) \quad (4.13)$$

and

$$\theta = n(s)\sigma^2 + (b + \gamma),$$

$$\Psi = n(s)(b + \gamma) - 2$$

with γ , $n(t)$, $m(t)$ defined by (4.5), (4.3) and (4.4).

Before proving Theorem we formulate and prove lemma which is interesting by itself. It gives a price of delivery at future moment $s < T$ of the one zero-coupon bond with maturity T . We find a partial differential equation, for which that value is a solution.

Lemma 4.1 *If the spot interest rate r is described by the dynamics (4.1), then the price $H(r, t)$ at time $t \leq s$ for delivery at $s < T$ of a one zero-coupon bond yielding 1 at maturity T , equals*

$$H(r, t) = e^{l(t)-k(t)r},$$

where $k(t)$, $l(t)$ are defined by (4.12) and (4.13).

Moreover, the function $H(x, t)$ is a solution of the following partial differential equation

$$H_t(x, t) + \frac{\sigma^2 x}{2} H_{xx}(x, t) + (a - bx)H_x(x, t) - xH(x, t) = 0, \quad (4.14)$$

$x \geq 0, 0 \leq t \leq s$ with boundary condition

$$H(x, s) = B(x, s). \quad (4.15)$$

Proof: From general theory (see e.g. Musiela and Rutkowski (1997)) the price $H(r, t)$ is given by formula

$$H(r, t) = e^{\int_0^t r(u)du} E(B(r, s) e^{-\int_0^s r(u)du} | \mathcal{F}_t) \quad (4.16)$$

under the martingale measure P , where $\mathcal{F}_t = \sigma(W_u : u \leq t)$. By assumption of the CIR model, the dynamics of the short rate r under P is given by (4.1). Hence, by Itô's lemma, we obtain

$$\begin{aligned} d(H(r(t), t) e^{-\int_0^t r(u)du}) &= e^{-\int_0^t r(u)du} \left[H_t(r(t), t) + H_r(r(t), t)(a - br(t)) + \right. \\ &\quad \left. + \frac{\sigma^2 r(t)}{2} H_{rr}(r(t), t) - r(t)H(r(t), t) \right] dt + H_r(r(t), t) \sigma \sqrt{r(t)} dW_t. \end{aligned}$$

Since (4.16) implies that $e^{-\int_0^t r(u)du} H(r, t)$ is a martingale, we have

$$[H_t(r(t), t) + H_r(r(t), t)(a - br(t)) + \frac{\sigma^2 r(t)}{2} H_{rr}(r(t), t) - r(t)H(r(t), t)] dt = 0.$$

Hence, we come to a conclusion that H satisfies partial differential equation (4.14).

We obtain boundary condition using an observation that the price $H(r, s)$ is equal to the market price of the bond, which is subject of this contract.

Now, we find the closed form of the price $H(r, t)$. We consider the price in the form

$$H(r, t) = e^{l(t) - k(t)r} \quad (4.17)$$

This form of price is consistent with the form given in paper by Schlögl and Schlögl (2000) for contingent claims.

By substituting $H(r, t)$ in form given by (4.17) to equation (4.14), the following two differential equations have been obtained:

$$\frac{\sigma^2}{2}k^2(t) + bk(t) - k_t(t) - 1 = 0, \quad k(s) = n(s). \quad (4.18)$$

$$l_t(t) = ak(t), \quad l(s) = m(s). \quad (4.19)$$

Equation (4.18) is a Riccati's equation. In a general case Riccati's differential equation cannot be solved, if we do not know at least one solution (Walter (1998)). However, the above equation (4.18) is a particular case of Riccati's equation for which there is an algorithm of finding the solution, (see e.g. Kamke (1956)).¹ Now, we apply the above mentioned procedure to solve equation (4.18).

Putting into equation (4.18) the function $k(\cdot)$ of the form

$$k(t) = u(t) + \frac{2}{b + \gamma},$$

we obtain

$$u_t(t) - \gamma u(t) - \frac{\sigma^2}{2}u(t)^2 = 0, \quad (4.20)$$

(we recall that $\gamma = \sqrt{b^2 + 2\sigma^2}$). This equation is a Bernoulli equation for which a general solution is known. Therefore, after simple but lengthy calculation we see that a general solution of equation (4.18) is given by

$$k(t) = \frac{2(e^{t\gamma}(b^2 + b\gamma + \sigma^2) + 2\gamma C)}{(b + \gamma)(-\sigma^2 e^{t\gamma} + 2\gamma C)}. \quad (4.21)$$

The constant C is determined by the boundary condition $k(s) = n(s)$ and equals

$$C = \frac{e^{s\gamma}(b + \gamma) [n(s)\sigma^2 + (b + \gamma)]}{2\gamma [n(s)(b + \gamma) - 2]}.$$

Substituting C into equation (4.21), we obtain that $k(t)$ is given by (4.12).

The function $l(\cdot)$ satisfies equation (4.19). Since $k(t)$ is known, by easy calculation one can

¹The Riccati differential equation: $y'(t) = f(t)y^2(t) + g(t)y(t) + h(t)$ can be easily solved, if it is possible to find constants c, d such that: $c^2 f(t) + cdg(t) + d^2 h(t) = 0$, and $|c| + |d| > 0$. If $d = 0$, then $f \equiv 0$, so Riccati's equation is a linear equation. If $d \neq 0$, then it could be simplified to Bernoulli equation by substitution $y(t) = u(t) + \frac{c}{d}$.

obtain that a general solution of (4.19) is as follows

$$l(t) = -\frac{2at(b\gamma + \gamma^2 - \sigma^2)}{\sigma^2(b + \gamma)} - \frac{2a}{\sigma^2} \ln \left((b + \gamma)\theta e^{\gamma(s-t)} - \Psi\sigma^2 \right) + \bar{C}.$$

A constant \bar{C} is determined by the boundary condition $l(s) = m(s)$ and is equal to

$$\bar{C} = m(s) + \frac{2as(b\gamma + \gamma^2 - \sigma^2)}{\sigma^2(b + \gamma)} + \frac{2a}{\sigma^2} \ln \left((b + \gamma)\theta - \Psi\sigma^2 \right).$$

Finally, the solution of (4.19) is of the form (4.13). Moreover, equation (4.18) has a unique solution (see e.g. Walter (1998) p. 56), so the solution given by (4.18) is the only one. Lemma 4.1 is proved.

Now, we can give the proof of Theorem 4.1.

Proof: By general form of price of contingent claim we have

$$\begin{aligned} V(r, t) &= e^{\int_0^t r(u)du} E((B(r, s) - F(r, t_0))e^{-\int_0^s r(u)du} | \mathcal{F}_t) \\ &= H(r, t) - F(r, t_0)e^{\int_0^t r(u)du} E(e^{-\int_0^s r(u)du} | \mathcal{F}_t). \end{aligned}$$

So $V(r, t) = H(r, t) - F(r, t_0)\bar{B}(r, t)$, where $\bar{B}(r, t)$ is the price at time t of the bond yielding 1 at maturity s . The end of proof of Theorem 4.1 follows from Lemma 4.1 and (4.10).

4.3 Futures prices

The next theorem which expresses futures prices in Cox-Ingersoll-Ross model is known from work published in 1981, where it is proved by arbitrage arguments. As has been mentioned in the Introduction, the proof presented below is different from the original presented by authors, and it is not attainable in literature.

Theorem 4.2 *If the spot interest rate r is described by the dynamics (4.1), then the price $f(r, t)$ of a futures contract at time t for delivery at delivery date s of a one zero-coupon bond yielding 1 at maturity T equals*

$$f(r, t) = e^{p(t)-g(t)r}, \tag{4.22}$$

where

$$p(t) = m(s) + \ln \left(\left(\frac{\eta(t)}{n(s) + \eta(t)} \right)^{2a/\sigma^2} \right),$$

$$g(t) = \frac{\eta(t)n(s)e^{b(t-s)}}{\eta(t) + n(s)},$$

$$\eta(t) = \frac{2b}{\sigma^2(1 - e^{-b(s-t)})},$$

with $n(t)$, $m(t)$ defined by (4.3) and (4.4). The function $f(x, t)$ is a solution of the following partial differential equation:

$$f_t(x, t) + \frac{\sigma^2 x}{2} f_{xx}(x, t) + (a - bx) f_x(x, t) = 0, \quad (4.23)$$

$x \geq 0$, $0 \leq t < s$ with boundary condition

$$f(x, s) = B(x, s). \quad (4.24)$$

Proof: As we know from general theory (see e.g. Musiela and Rutkowski (1997)) the price of futures is given by formula

$$f(r, t) = E(B(r, s) | \mathcal{F}_t) \quad (4.25)$$

under the martingale measure P , where $\mathcal{F}_t = \sigma(W_u : u \leq t)$. By assumption of the CIR model the dynamics of the short rate r under P is given by (4.1). Hence, by Itô's lemma, we obtain

$$df(r(t), t) = f_t(r(t), t)dt + f_r(r(t), t)dr(t) + \frac{\sigma^2 r(t)}{2} f_{rr}(r(t), t)dt =$$

$$\left[f_t(r(t), t) + f_r(r(t), t)(a - br(t)) + \frac{\sigma^2 r(t)}{2} f_{rr}(r(t), t) \right] dt + f_r(r(t), t)\sigma\sqrt{r(t)}dW_t.$$

Since (4.25) implies that $f(r, t)$ is a martingale, we have

$$[f_t(r(t), t) + f_r(r(t), t)(a - br(t)) + \frac{\sigma^2 r(t)}{2} f_{rr}(r(t), t)]dt = 0.$$

Hence, we come to a conclusion that f satisfies partial differential equation (4.23). We obtain the boundary condition using an observation that the price of a futures contract is equal to the market

price of the bond, which is subject of this contract, at the moment of delivery.

Now, we find the closed form of the price of a futures contract on a zero-coupon bond. We consider the futures price in the form

$$f(r, t) = e^{p(t) - g(t)r}. \quad (4.26)$$

It is consistent with earlier results in Chen (1992a), (1992b), Cox, Ingersoll and Ross (1981), and French (1983).

Putting representation (4.26) of the futures price into equation (4.23), the following two differential equations have been obtained:

$$\frac{1}{2}\sigma^2 g(t)^2 + bg(t) - g_t(t) = 0, \quad g(s) = n(s) \quad (4.27)$$

and

$$p_t(t) = ag(t), \quad p(s) = m(s). \quad (4.28)$$

Equation (4.27) is a Bernoulli differential equation, so a general solution is given by the formula

$$g(t) = \frac{2be^{bt}}{Cb - \sigma^2 e^{bt}}.$$

The constant C is determined by the boundary condition of equation (4.27)

$$C = \frac{e^{bs}(2b + \sigma^2 n(s))}{bn(s)}$$

and a simple algebra gives the solution

$$g(t) = \frac{\eta(t)n(s)e^{b(t-s)}}{\eta(t) + n(s)},$$

where

$$\eta(t) = \frac{2b}{\sigma^2(1 - e^{-b(s-t)})}.$$

Integration of the function $ag(t)$ gives

$$p(t) = \bar{C} - \frac{2a \ln(2b + \sigma^2 n(s)(1 - e^{b(t-s)}))}{\sigma^2}.$$

Using the boundary condition of (4.28), we get

$$\bar{C} = \frac{2a \ln(2b) + \sigma^2 m(s)}{\sigma^2}.$$

Hence, we conclude that $p(t)$ equals

$$p(t) = m(s) + \ln \left(\left(\frac{\eta(t)}{n(s) + \eta(t)} \right)^{2a/\sigma^2} \right).$$

This ends the proof of Theorem 4.2.

4.4 Appendix

Here, it will be proved that classical approach to find a partial differential equation for the price of derivatives, which is based on constructing of a hedging portfolio as in Black and Scholes (1973) , Black (1976), Gibson et al. (1998), Pelsser (2000), Rebonato (1996), Wilmott (1999) is not suitable for proving Theorem 4.2. The reason is that the portfolio analogous to that, which has been used e.g. in derivation of partial differential equation for the price of zero-coupon in the CIR model (cf. Gibson et al. (1998), Rebonato (1996) , Wilmott (1999), is not self-financing.

To see this, let us consider portfolio $\phi = (\phi_1(r(t), t), \phi_2(r(t), t))$, where functions ϕ_1 and ϕ_2 represent a number of zero-coupon bonds and a number of futures contracts, respectively, and have the form:

$$\phi_1(r(t), t) = -\beta \frac{f_r(r(t), t)}{B_r(r(t), t)}, \quad \phi_2(r(t), t) = \alpha, \quad (4.29)$$

for some constants $\alpha, \beta \in \mathbf{R}$, $\beta \neq 0$ (in classical approach $\alpha = \beta = 1$).

The condition of self-financing in the case of portfolio defined by (4.29), is equivalent to equality

$$d\left(-\beta \frac{f_r(r(t), t)}{B_r(r(t), t)} B(r(t), t)\right) = -\beta \frac{f_r(r(t), t)}{B_r(r(t), t)} dB(r(t), t) + \alpha df(r(t), t). \quad (4.30)$$

Using Itô's lemma, we obtain

$$\begin{aligned} d\left(\frac{f_r(r(t), t)}{B_r(r(t), t)} B(r(t), t)\right) &= \left(\frac{f_r(r(t), t)}{B_r(r(t), t)} B(r(t), t)\right)_t dt \\ &+ \left(\left(\frac{f_r(r, t)}{B_r(r, t)}\right)_r \Big|_{r=r(t)} B(r(t), t) + f_r(r(t), t)\right) dr(t) \\ &+ \frac{\sigma^2 r(t)}{2} \left(\left(\frac{f_r(r, t)}{B_r(r, t)}\right)_r B(r, t) + f_r(r, t)\right)_r \Big|_{r=r(t)} dt, \end{aligned}$$

and

$$\begin{aligned} \alpha df(r(t), t) - \beta \frac{f_r(r(t), t)}{B_r(r(t), t)} dB(r(t), t) &= \\ \alpha \left(f_t(r(t), t) dt + f_r(r(t), t) dr + \frac{\sigma^2 r(t)}{2} f_{rr}(r(t), t) dt \right) & \\ - \beta \frac{f_r(r(t), t)}{B_r(r(t), t)} \left(B_t(r(t), t) dt + B_r(r(t), t) dr(t) + \frac{\sigma^2 r(t)}{2} B_{rr}(r(t), t) dt \right) & \end{aligned}$$

Therefore, if (4.30) is satisfied, then the following condition has to be fulfilled:

$$-\beta \left(\frac{f_r(r, t)}{B_r(r, t)}\right)_r \Big|_{r=r(t)} B(r, t) = \alpha f_r(r(t), t).$$

Hence, using explicit formulae (4.22) and (4.2) for functions f and B we obtain condition

$$\frac{g(t)}{n(t)}(n(t) - g(t)) = \frac{\alpha}{\beta} g(t).$$

So the self-financing of portfolio (4.29) implies that:

$$\forall t \quad \left(1 - \frac{\alpha}{\beta}\right) n(t) = g(t), \quad (4.31)$$

because $g(t) \neq 0$. However,

$$\begin{aligned} & \left(1 - \frac{\alpha}{\beta}\right)n(t) - g(t) = \\ & = \frac{2\left(1 - \frac{\alpha}{\beta}\right)}{\gamma \coth\left(\frac{1}{2}(T-t)\gamma\right) + b} - \frac{2b}{e^{b(s-t)}[(b^2 + \sigma^2) + b\gamma \coth\left(\frac{1}{2}(T-s)\gamma\right)] - \sigma^2} \neq 0, \end{aligned}$$

which contradicts (4.31). Thus, portfolio defined by (4.29) is not self-financing. The lack of equality is not so surprising, when we recall that the function $g(t)$ is the solution of Bernoulli differential equation and the function $n(t)$ is the solution of Riccati differential equation (see Musiela and Rutkowski (1997) or Rogers (1995) for the form of equations).

4.5 Conclusions

The work contains detailed mathematical derivation of formulae for the price of futures contracts and the value of forward contracts on a zero coupon bond. The obtained results are interesting for the investor, who considers reselling a forward contract on zero coupon bond. In turn, the presented derivation of formulae for the price of futures contract is innovative in comparison to the approach known from the previous studies.

Further contribution of this work is that it pointed out the weakness in the sense of mathematical accuracy of some economical reasoning applied in Arbitrage Theory. Namely, it is shown that a hedging portfolio approach analogous to that used in the derivation of the price of many contingent claims (see e.g. Black and Scholes (1973), Black (1976), Rebonato (1996) or Wilmott (1999), is not rigorous the enough from a mathematical point of view in the case of a futures contract on a zero-coupon bond.

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Concluding Remarks

This thesis investigated the stock markets linkages and an arbitrage opportunity between spot and futures markets on an index and zero-coupon bonds. The obtained results here are of potential relevance for international investors, policy makers, and financial mathematicians.

The first and second paper contribute to the literature on stock markets linkages by presenting the methodology to construct different types of relationships between financial markets using a bivariate Markov switching model. Hypotheses of causality, regime-independence, and contagion are explicitly defined. In this work a new element is introduced, namely the concept of Granger causality is incorporated into the Markov switching framework. The proposed methodology is superior as compared to previous approaches to modelling the relationship between financial markets. First, it allows for testing of various hypotheses of dependencies between the markets, without imposing any *a priori* assumption on its form. Secondly, this approach differentiates between calm and crisis periods, which are modelled as multiple random rather than the fixed-date events. The causality patterns are allowed to be asymmetrical with respect to states of tranquillity and crisis, as argued by Sola, Spagnolo, and Spagnolo (2002). For the purpose of examining of the long and short term stock markets linkages two data sets were selected. One of them embraced the period around Asian crisis of 1997 for the Japanese and Hong Kong stock market. The second one of a period of almost twenty years was used to examine the influence of US market on other main world market such as in Germany, United Kingdom, and Japan. For both data sets, similar results were obtained. The independence of performance of German, Japanese and United Kingdom stock market from past performance of US market was rejected at the 1% significance level. Furthermore, spillovers are present in both cases when dependent (non-US market) market is in crisis, or in calm period. Spillover takes place more frequently when the dependent market experiences crisis. It is a clear indication that in the period of the last twenty years, not all financial crashes in the US directly caused turmoil in the Japanese, United Kingdom, and German markets. However, the crashes on

the US market increase the probability of a crisis on the three other mature markets, which is in line with the hypothesis of contagious crises introduced by Eichengreen, Rose, and Wyplosz (1996). Lastly, it is worth noting that the methodology based on the Markov switching framework allows for calculating the probability of entering the specific regime (calm or crisis) conditional on the past performance of the other market. This information has essential meaning for investors who would like to maximize the benefits of international portfolio diversification.

The third paper contributes to the discussion of the profitability of the arbitrage between a spot and futures market on the index. According to the obtained results for the fast growing Polish futures market on the index WIG20 the factors such as a limited access to short sale, irregular dividend payments, and a fluctuation of interest rates have significant impact on the quality of signals to the arbitrage and its profitability. The results indicate that the Polish future market is inefficient in the arbitrage sense. It is mainly due to the fact that the long arbitrage, unlike the short one does not exhibit features observed on well-established financial markets. On the average, the time between appearing of the signal to arbitrage and its disappearing is equal to 57 minutes, what is incomparable to other markets where this period is shorter than 2 minutes. Moreover, there is no decreasing trend in the size of profit for subsequent contract series, which is a further evidence against the efficiency of Polish market. In addition, there is no indication that the lag in execution of arbitrage orders caused significant changes in profits. The signals from cost-of-carry formulae are on average less reliable in comparison to the previous studies.

Finally, the fourth paper focuses on a relationship between the zero-coupon bond futures and spot markets. The new method of finding the value of the forward contract on a zero-coupon bond is derived. The application of this method gives an answer to the question what is a current value of the forward contract on zero-coupon bond in the Arbitrage Pricing framework. Thus, this result is appealing for an investor who considers a reselling of the forward contract on a zero-coupon bond. Additionally, an alternative method of pricing the future contract on a zero-coupon bond has been developed. Finally, it is shown that the formula for the price of a future contract on zero-coupon bond cannot be obtained by using the approach of hedging portfolio. It is a noteworthy result

from the perspective of mathematical finance, because a similar result was obtained in the case of European call option (see Rosu and Stroock 2001).

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Rosu I., Stroock D., (2004), *On the Derivation of the Black-Scholes Formula*, Seminaire de Probabilites 37, 399-414.

Curriculum Vitae

JĘDRZEJ BIAŁKOWSKI

PERSONAL

Citizenship: Polish

Born: January 14, 1977, Warsaw, Poland

EDUCATION

- 2005 Ph.D. in Finance, European University Viadrina Frankfurt(O), Germany Thesis Title: *International Stock Markets Linkages and Arbitrage between Futures and Spot Markets*
- 2001 M.Sc. in Financial Mathematics, University of Warsaw, Poland Thesis Title: *Comparison of Stochastic Models of Pricing Forward and Futures Contracts*
- 1999 B.Sc. in Mathematics, University of Warsaw, Poland
-

ACADEMIC POSITIONS

- 2005–to date Senior Lecturer, Department of Finance, Faculty of Business, Auckland University of Technology, New Zealand
-

ACADEMIC EXPERIENCE

- 2004-2005 Visiting Doctoral Fellow, Laboratory of Finance-Insurance, Center for Research in Economics and Statistics, France
- 2004 Teaching Assistant, European University Viadrina, Module: Asset Pricing
- 2003–2004 Engaged in project "Mathematical Methods in the Analysis and Financial instruments in Poland", supported by KBN grant PBZ-KBN-016/P03/99
- 2001–2004 Ph.D. Student, Postgraduate Research Programme "Capital Markets and Finance in the Enlarged Europe", European University Viadrina Frankfurt(Oder), Germany