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The Design and Construction of the Vaults in Notre-Dame in Paris and the Development of Medieval Vaulting and Stereotomy: Surveys, Analyses and Experiments

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Abstract

Among the many innovative church constructions in the Île-de-France during the twelfth century that immediately followed the ground-breaking new building of the abbey church of Saint-Denis, the high vaults in the cathedral of Notre-Dame in Paris mark a turning point – both in sheer dimensions and in their quality of execution as stone constructions. On the other hand, some solutions in design and construction that can be observed in these vaults remain unique within the dynamic development of Gothic vaulting in the great cathedrals built in the following years. On closer view it turns out that the general design of the vault as well as its construction details are hardly understood until now, mainly because they have been overlooked in a unifying perspective on Gothic architecture coined by the idealistic view of the Gothic Revival in the nineteenth century. Further, the particular relation between vault design and stereotomy needs to be clarified, and its significance needs to be understood.

Fig. 1: Experiments on the construction of the vaults in Notre-Dame in scale 1:3 (January 2022).



Introduction

After the severe damage and partial destruction of the high vaults in Notre-Dame in Paris in the devastating fire in April 2019, in view to the upcoming interventions a thorough and detailed understanding of this structure is necessary – defining the significance and historical value in regard to the particular design, building technology, and relation between form and construction. The questions arising in regard to consolidation, repair and partial reconstruction can be expected to be far more detailed and precise than usual in research on construction history or even in building archaeology. On the other hand, the damaged state gives way to some observations that wouldn't be possible on intact buildings. Moreover, already in preliminary studies carried out so far, a number of open research questions have emerged, regarding in particular the developments in the geometric design of vaults and in stone planning during the middle-ages in Central Europe, as well as issues of knowledge transfer in a broader geographic context.

Our research in progress¹ addresses the issues of design, planning, stonemasonry and building by means of practical experiments carried out on large scale (Fig. 1). These are based on the available information and preliminary observations on the building,² and the results will have to be cross-linked with archaeological investigations. In the following, we will discuss some correlations with the interpretation of the few existing sources and the developing knowledge on practices of design and planning in medieval architecture and vault construction.

In the development of the Gothic rib vault regarding architectural design, construction system and geometric concept, the first essential steps are attributed to Norman church buildings of the eleventh century, where the cross vaults in the aisles in addition to the transversal arches that separate the bays were provided with stone ribs along the diagonal groins.³ In the second quarter of the twelfth century, vaults with major spans were built over the naves of some larger churches with rib systems that combined the scheme of transversal arches with the figure of the cross vault: uniting two bays of diagonal arches within one compartment of transverse arches in Durham Cathedral, or spanning transverse arches also in the middle of the bay in the sexpartite vaults of the abbey churches in Caen. In the geometry of the vault, the ribs form the primary feature: by principle, they describe circle segments in vertical planes, and the surfaces of the vault are adapted to these curves.⁴

Immediately in the succession of these prominent high vaults of the Norman realm, the new parts of the abbey church of Saint-Denis were built, starting with the western narthex around 1135.⁵ In this innovative architecture with outstanding programmatic claim, the vaults play an essential role. While the construction of the ribs is similar to that in the previous examples – instead of arch voussoirs, rectangular blocks are used that are profiled on the intrados – the new conspicuous feature is the exposed

¹ D. Wendland, “La construction et la stéréotomie des voutes médiévales de Notre-Dame : Les procédures pour la coupe de pierre dans les coques de double courbure au contexte de la histoire de la géométrie appliquée”, proposed in the framework of the *Chantier scientifique CNRS Notre-Dame*.

² D. Wendland, M. Gielen and V. Korensky, “The construction and Stereotomy of the medieval vaults in Notre-Dame: Planning, stone-cutting and building of the double-curved shells”, vol.2 pp.333-340 in J. Mascarenhas-Mateus and A. Paula Pires (eds), *History of Construction Cultures*, Leiden: Balkema, 2021.

³ P. Frankl, *Gothic Architecture* (1962), rev. by P. Crossley, New Haven: Yale U Pr., 2001; N. Nußbaum and S. Lepsky, *Das gotische Gewölbe: Eine Geschichte seiner Form und Konstruktion*, Darmstadt: WBG, 1999.

⁴ This basic principle was already described by the scholars who in the nineteenth century established the modern scientific research of medieval architecture, in particular in the seminal essay by Robert Willis: “On the construction of the vaults in the middle-ages”, *Transactions of the Royal Institute of British Architects*, vol.1, 1842, pp.1-69. See also W. Müller, *Grundlagen gotischer Bautechnik*, München: Deutscher Kunstverlag, 1990.

⁵ S. M. Crosby, *The Royal Abbey of Saint-Denis from Its Beginnings to the Death of Suger, 475-1151*. Yale U Pr., 1987. D. Kimpel and R. Suckale, *Die gotische Architektur in Frankreich 1130-1270*, München: Hirmer, [1985] 1995.

Fig. 2: Abbey church Saint-Denis, vault in the narthex.



stone masonry of the shells built with a carefully devised pattern. The courses are arranged parallel to the ridge, or to be more precise, the bed joints describe lines between the ribs that are equidistant to the straight ridge line of every shell (Fig. 2). Only in the lower portions, the masonry courses deviate from this regular pattern.

The high vaults of Saint-Denis from the twelfth century, supposedly sexpartite vaults with the same regular pattern of exposed stone in the webs, no longer exist, neither do their immediate successors in the cathedrals of Sens and Noyon.⁶ A significant further step of innovation can be seen in the vaults of the nave of Saint-Maurice in Angers,⁷ 1149-53, where the shells are also built in fine dressed stone blocks, while presenting a concave shape with pronounced double curvature. These “domical vaults” are cross vaults on large square bays with ridges that, like the ribs, describe vertical circle segments – and so do the bed joints of the vault masonry. The perfection of the stonework in these curved surfaces built with perpendicular blocks is remarkable.

In the cathedrals of Laon and Paris, both started in the 1160s, the high vaults once more combine the motif of concave shells in perfectly dressed exposed stonework with the pattern of the sexpartite vault. The effort of creating curved surfaces that display perfect regularity in the stonemasonry was obviously of special importance to the builders. A challenging question for us is how this design could be defined and translated to working instructions, how the planning and assembly was accomplished. This regards issues of technical knowledge and on the historical development of applied geometry. A deeper insight will be essential also for gaining a thorough understanding of the significance of this architecture.

The high vaults in Notre-Dame

Within this dynamic development of increasing effort in geometric complexity along with perfection of the stonemasonry in the vaults, Notre-Dame in Paris stands out also for its sheer dimensions: the high vaults have a clear height of ca. 33 metres and a span of ca. 12 metres. The vaults in the choir, the inner bays of the transept arms, and the nave are formed as sexpartite vaults with bays of an approximately square plan, each corresponding with two bays of the aisles. The choir was vaulted before 1186, the eastern part of the nave before 1200, the western bays in the early thirteenth century.⁸ There are some

⁶ Kimpel and Suckale (Note 5); Nußbaum and Lepsky (Note 3).

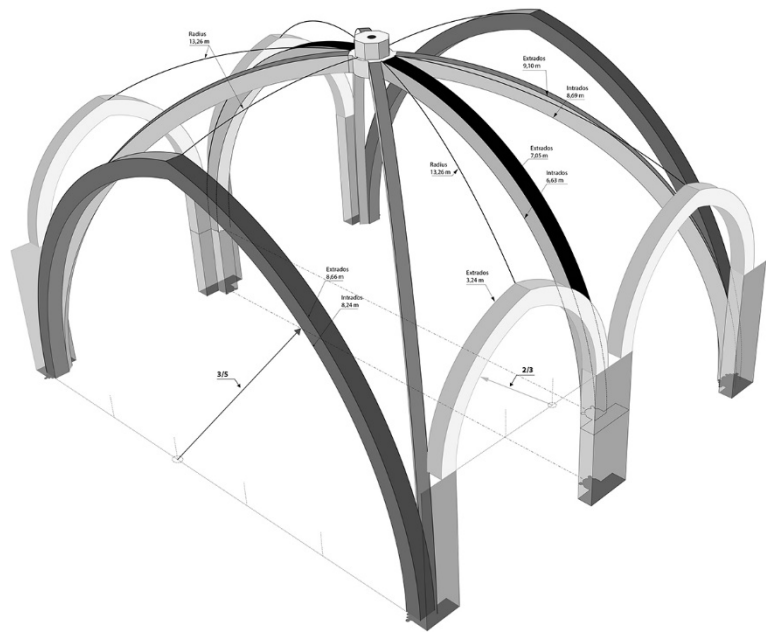
⁷ Frankl, *Gothic Architecture* (Note 3); Nußbaum and Lepsky, (Note 3) p.109.

⁸ C. Bruzelius, “The Construction of Notre-Dame in Paris”, *The Art Bulletin* vol.69, n.4, 1987. pp.540-569; A. Erlande-Brandenburg, *Notre-Dame in Paris: Geschichte, Architektur, Skulptur*. Freiburg/Basel/Wien: Herder, 1992 [1991]; Nußbaum, Lepsky, (Note 3) p.70.

Fig. 3: The high vaults in Notre-Dame in Paris: eastern bays of the nave (2009).



Fig. 4: Geometric design of the rib system in the high vaults in Notre-Dame, eastern part of the nave.



differences in the design among these three building phases; our study focuses on the eastern bays of the nave.⁹

The sexpartite vault combines a system of transverse arches corresponding to the arcade of the aisles with the scheme of the cross vault, where the diagonal arches spanning two compartments create the double-size bays of the nave (Fig. 4). Although the design of all ribs is similar, the transverse ribs have a greater section – suggesting that they were conceived as being the principal structural feature of the vault. The geometric design of the vault, in contrast, is in first place based on the cross ribs, which have

⁹ The considerations on the vault design are based on publications by Rocío Maira, who carried out a laser scan in this area, while arriving at a different interpretation: R. Maira, “Bóvedas sexpartitas. Los orígenes del gótico”, Dissertation, Universidad Politécnica de Madrid, 2016; R. Maira, “The Evolution of the Knowledge of Geometry in Early Gothic Construction: The Development of the Sexpartite Vault in Europe”, *International Journal of Architectural Heritage*, vol.11, 2017, pp.1005-25; and elsewhere. Moreover, we recur to the drawings prior to the transformations in the 19th century in Erlande-Brandenburg 1992 (Note 8).

a semi-circular elevation. This leads to a significant increase of the rise of the diagonal arches and determines the level of the central keystone in the summit. The transverse arch in the middle of the bay also has a semi-circular elevation, but is stilted in order to reach the central keystone: above the abaci at the impost level, the rib first rises vertically over little less than 2 metres until the curvature starts. The other transverse arches that separate the bays start at the common springing level: their elevation is defined as autonomous two-centre pointed arches, apparently with a determined ratio 3/5 of radius to span. The formeret or wall ribs seem to be determined by the radius to span ratio of 2/3; although strongly stilted, their summits remain far below the level of the vertex of the bay. The ridges of the shells of the vault are *a priori* defined by circle segments as well, which like the curves of the arches are inscribed in vertical planes, with a common radius presumably devised to be equal to the width of the nave. In direction towards the clerestories, their curvature is continuous: the centre is located in the middle of the bay. Descending from the summit, they determine the level of the ridges of the formerets, thus having an important impact in the design of the clerestory. In longitudinal direction, the ridge curves, having the same curvature, are inserted between the given endpoints at the central keystone and the summits of the transversal arches between the bays, resulting in a discontinuous course longitudinal to the nave.

All this means before all, that the superelevation of the summit of the bay due to the semi-circular diagonal ribs, is a main determinant of the shape of the vault and directs most features of the geometric design. Hence, as opposed to a regular sequence of transverse arches, which is still present in the plan, the spatial curve system of the vault is in first place determined by the high rising diagonal arches: the *ogives*. This term, which denotes the cross ribs and apparently refers to the matter of “rising”, first appears around 1230 in Villard de Honnecourt’s “lodge book” and in several sources thereafter.¹⁰

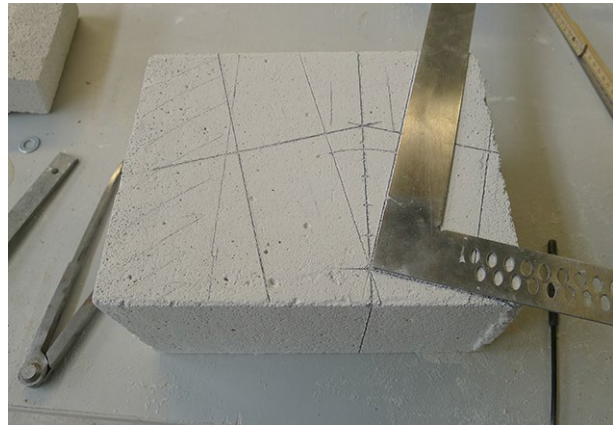
Construction details, planning and building

The ribs are built from stone elements that comprise only the visible part under the vault surface, hence, they are connected with the masonry shells only by a continuous mortar joint. These elements are not carved as arch voussoirs, but as rectangular blocks – unlike the common image of Gothic rib vaults, but similar to the earlier predecessors mentioned above (Fig. 2) and contemporary examples. The carved profile on the intrados still resembles very much the original envelope block, and the curvature of the arch is produced only within the mortar joints. For the construction process and planning, this means that the rib elements could be produced in series without taking into account the exact curvature of the rib. Although not uniform in height, due the standard profile they could be assembled directly from the stack like masonry blocks. Only the last piece of every rib directly under the keystone had to be cut *ad hoc* to the right length.

In contrast, the keystones are elements that needed to be specifically carved according to the general design of the vault. The central keystones are very large and heavy workpieces that, unlike the ribs, are embedded in the shell of the vault and emerge on the extrados. Their volume therefore includes in vertical direction the thickness of the shell, the height of the rib profile, plus the sculptural element on the lower side, and in the horizontal dimensions the branches of the connecting ribs: the envelope block must have measured over 1 metre in height and ca. 1.26 metres in both horizontal edges. Planning and

¹⁰ For *ogive*, occasionally also spelled *augive*, cf. H. Hahnloser, *Villard de Honnecourt: kritische Gesamtausgabe des Bauhüttenbuches ms. fr 19093 der Pariser Nationalbibliothek* (2nd rev. and augm. ed.), Graz: Akad. Druck-u. Verlagsanstalt, 1972. pp.170-71, 380. We find the reference to lat. *augere* by far the most convincing explanation of this term. In the medieval sources, the term refers solely to the diagonal ribs of vaults, while its use for “pointed arch” is inexistent at least until the sixteenth century. The same etymology but a slightly different interpretation of the term is given by P. Frankl, *The Gothic. Literary Sources and Interpretations through Eight Centuries*, Princeton NJ, Princeton U Pr., 1960. p.154.

Fig. 5: Instruction for carving the keystone of an arch with defined ratio of length and span, in Villard de Honnecourt's "lodge book" (Paris, BNF, ms. fr 19093); application in a rib keystone and in a ridge stone of the vault.



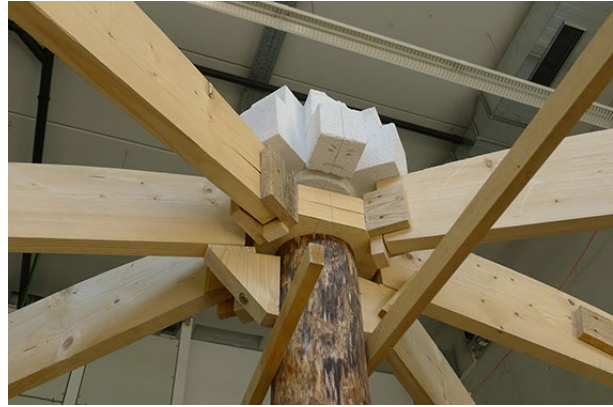
carving of these stone elements not only included the intersection of the rib profiles in their approach to the central ring, but also their directions in plan which is not exactly square but rectangular. They are stereotomic items that could be produced only on the basis of detailed information upon the general design of the vault – produced in a full-scale drawing and transmitted by means of templates.¹¹

The keystones of the single arches also had to be carved with the radial beds that for their definition referred to the general design. However, this could be done even without needing a full-scale drawing, if the arch was designed with a defined ratio of span and radius. For this, a procedure requiring only a square, carried out directly on the workpiece, is shown in Villard de Honnecourt's "lodge book". This instruction belongs to a passage in the book where procedures of setting-out as well as planning media like templates are described. In our experimental reconstruction, the procedure turned out to be practical and reliable (Fig. 5). Such possibility of simplifying the stone-planning may have motivated the use of arches with defined ratio, as we suppose in the reconstruction of the vault design (Fig. 4).

In the construction process the ribs and keystones had to be assembled upon a temporary supporting structure, designed according to the general project of the vault by means of appropriate planning media. There is no comprehensive information about these structures from sources or from archaeological

¹¹ For the procedure in complex late Gothic stone elements see D. Wendland and F. Degenève, "How to order fitting components for looping ribs: Design procedures for the stone members of complex Late Gothic vaults", pp.159-170 in J. Campbell et al. (eds), *Building Histories: The Proceedings of the Fourth Conference of the Construction History Society*, Cambridge 2017; D. Wendland, *Steinerne Ranken, wunderbare Maschinen: Entwurf und Planung spätgotischer Gewölbe und ihrer Einzelteile*. With contributions by M. Aranda Alonso, A. Kobe and M. J. Ventas Sierra, Petersberg: Imhof, 2019.

Fig. 6: Experiments on the construction process: For the assembly of the ribs, first the keystone is positioned, then the centring is adjusted, enabling a precise connection of the ribs.



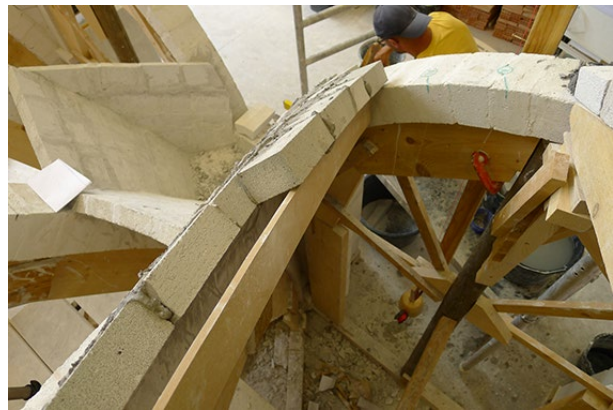
evidence. Nevertheless, from previous research on late Gothic structures¹² it is possible to develop hypotheses that correlate well with the design principles of the vault: In the experiments, a system of vertical posts under the keystones to which the centring arches made of boards cut according to the curvature of the ribs were mounted, turned out to be practical. This structure, erected on a temporary floor at the springing level on which the geometric system could be established *in situ* in full scale, also facilitates the information flow from design to execution.

The starting points of the ribs were already fixed when the clerestories were built. Hence, the assembly starts with positioning the keystones according to the centre of the bay defined within the real boundary conditions of the building (Fig. 6). Then the centring arches can be adjusted precisely to the rib branches of the keystone. After that, the ribs can be built on the centring arches.

As the ridges of the shells appear straight in plan and describe the defined curvature in the elevation, it must be supposed that they were directed by centring arches similar to those of the ribs, produced according to the design described above (Fig. 7, top). For building the masonry shell of the vault, geometric guidance and support was needed, although a complete formwork was probably not necessary and could have even caused difficulties in the execution of the exposed masonry. In the vaults of Saint-Denis (Fig. 2), the linear courses could be built on straight laths fixed to the ribs – this support for every course had to be maintained until the vault was closed. In the curved courses of the concave shells in Notre-Dame, instead of laths, centring arches had to be used for every single course. Here, due to the curvature, they were only needed to support the unfinished courses and could then be taken out (Fig. 7).

¹² Wendland, *Steinerne Ranken*, (Note 11). See also D. Wendland and M. J. Ventas Sierra, “Zum Bau figurierter Gewölbe – eine Anleitung im Werkmeisterbuch des Rodrigo Gil de Hontañón”, pp.244-72 in S. Bürger, B. Klein and K. Schröck (eds), *Werkmeister der Spätgotik: Personen, Amt und Image*, Darmstadt: WBG, 2010.

Fig. 7: Experiments on the construction process: Top, application of guiding marks for the masonry pattern. Centre, building of the courses upon single centring. Bottom, double centring for the last course and the ridge stones – here a different procedure from the one shown in figure 1 is experimented.



We discussed the geometric properties of the masonry pattern elsewhere.¹³ By principle, the courses must be aligned according to the curve of the ridge. This requires guiding their position – however, an absolute definition cannot be provided due to the changing heights of the courses and also due to the cumulating errors when the masonry is built up from the bottom to the top. In the experiments we used a system of marks based on equidistant lines that was established starting from the ridge line downwards and gave indication on the general direction of the courses in every portion of the shell (Fig. 7, top).

While the shells, despite their double curvature and with all the perfection of the joint pattern, are executed as block masonry with serial prismatic elements, the ridges are made with special stones that have the shape of keystones: stereotomic elements that have to be carved according to their position. These ridge stones are a rather singular phenomenon: While the linear ridges of the vaults in the narthex of Saint-Denis simply show a joint, special rib stones appear in the concave shells of the vaults in Saint-Maurice in Angers – along with the increasing perfection of the dressed stone surface and the introduction of the arched curvature of the ridges and the single courses. After Notre-Dame, these special stone elements disappear: the ridges in the vaults of the Cathedral of Reims are built with special elements of a different type, and in the later development these elements seem to disappear altogether.

Although these special stones had to be produced upon precise instructions drawn from the design of the vault, we suppose that they were standardized and could be carved according to the keystones of the transversal and formeret arches. Here again, the effort of planning could be considerably reduced by using the procedure described in Villard's "lodge book" (Fig. 5). In any case, this element gives strong emphasis to the aim of perfection in the stone surfaces of the vault.

Some considerations on the development of structural, geometric and architectural design of vaults in the context of early Gothic

For a thorough understanding of the design, planning and construction of the high vaults of Notre-Dame, two aspects are of particular importance. One is the development of stone planning, taking into account also the information transfer from design to construction and the use of media related to it. While the built object is the principal information source on this matter, the observations made can be correlated with very few written or graphic sources that can be critically revised upon this research. The other aspect is the development of the geometric and structural design of vaults.

As to the geometric design of vaults, there is a fundamental difference between the cross vault as it developed in the Roman Antiquity, which is defined by intersecting cylindrical surfaces creating groins that describe elliptic curves, and the Gothic vault which is determined by the curves that are defined *a priori* as circle segments, to which the surfaces of the vault are adapted.¹⁴ The first concept has the advantage of geometrically defined surfaces that facilitate the planning particularly in ashlar vaults, but it also has important disadvantages: the difficulties related to the elliptic arches and, foremost, the necessity of having a square plan in order to avoid spatial curves at the cross groins. In the second case, the arches due to their elementary geometric description bring no difficulty at all for planning and for communicating the design specifications, and there are no limitations to the plan of the vault. The only

¹³ Wendland, Gielen and Korensky (Note 2); D. Wendland, *Lassaulx und der Gewölbebau mit selbsttragenden Mauerschichten. Neumittelalterliche Architektur um 1825-1848*. Petersberg: Imhof, 2008.

¹⁴ See Note 4. On the research history Wendland, *Lassaulx* (Note 13). Among the early scholars, in particular Ungewitter emphasizes that the geometric concept of Gothic vaults is not necessarily linked to the architectural style and pointed arches, but can be perfectly associated with semi-circular arches: G. G. Ungewitter, *Lehrbuch der gothischen Constructionen*, Leipzig: Weigel, 1859-1864.

Fig. 8: Vault in the aisle of Speyer Cathedral, first half of eleventh century. The cross vaults on rectangular plan are designed with defined curves over the groins, like in Gothic vaults.



inconvenience is the greater geometric complexity in the surfaces, which however can be overcome in practice by rather simple means, for instance with ruled surfaces spanned between the arches.

The ground-breaking invention of this new concept of vault geometry is usually attributed to the early rib vaults, from which, as mentioned above, a direct succession can be drawn to the evolution of the prominent early Gothic vaults in the Île-de-France. It appears plausible to assume that in a rib vault all the effort of planning as well as the main temporary support during construction is necessarily focused on the stone ribs. However, this long-established notion is now challenged by accurate surveys and geometrical analyses of the shape of some vaults of earlier date.

In fact, recent analyses carried out in the church of St. Pantaleon at Cologne by U. Heckner and H. Meyer¹⁵ showed that in the cross vaults in the western part of the structure, dating from the tenth century, the curves defining the boundary arches as well as those of the diagonal arches all describe circle segments. Direct evidence shows that the vault was built on a formwork of wooden planks, which according to the shape of the vault surface must have been fixed upon centrings erected along the walls and the diagonal groins.

Our surveys in Speyer Cathedral give a similar picture of the geometric features of curves and surfaces.¹⁶ The groin vaults in the aisles, dating from the mid-eleventh century, were the first cross-vaults built in monumental scale in Central Europe after the fall of the Roman empire (Fig. 8): they span 8.46 metres and reach a clear height of 14.46 metres. Already the fact that the plan of the bays is not square but rectangular shows that their geometric concept cannot be based on the antique scheme of intersecting cylindrical surfaces.¹⁷ The geometric analyses of the scan data confirm that the design of these vaults actually corresponds with the concept we associate with Gothic vaulting (Fig. 9).

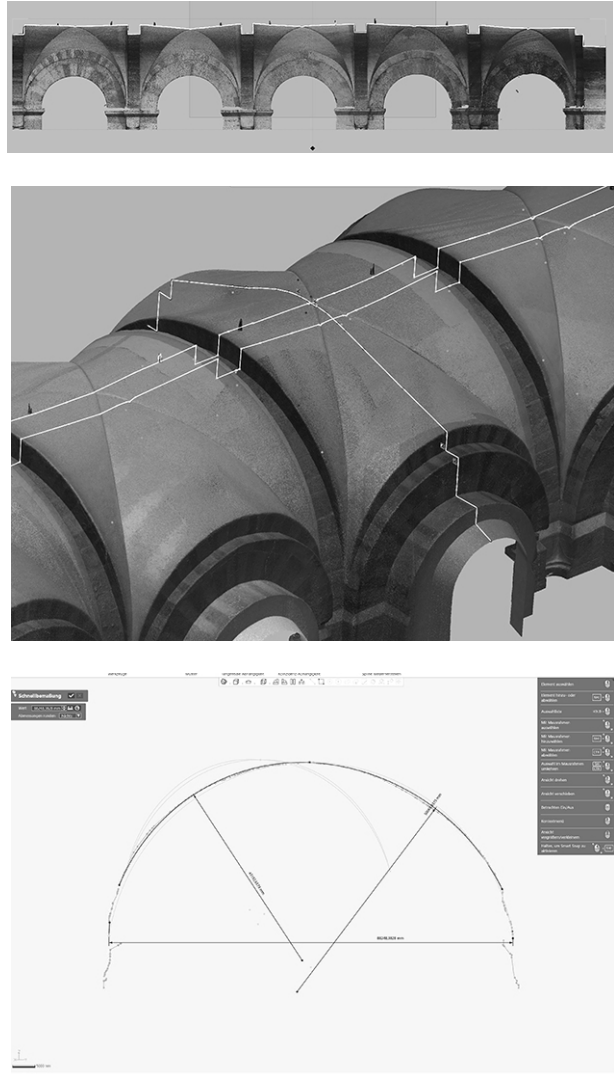
The bays are defined by semi-circular arches in the four sides upon a common springing level: transverse arches separating the bays and lesenes integrated in the arcade and outer wall, all in exposed stone. The transverse arches are higher than the wall arches due to their greater span. The boundaries of the vault describe semi-circles with the same radii as the stone arches but are slightly elevated, leaving their flanks visible in a crescent shape. The surfaces of the vault are plastered – in particular the extra plaster applied

¹⁵ U. Heckner and H. Meyer, “Was macht das Holz im Gewölbe? Spuren von Bau- und Lehrgerüsten am Westbau von St. Pantaleon in Köln”, lecture at the symposium *Lehrgerüste und Gewölbe-Bau* (C. Voigts, S. Holzer), ETH Zurich, 4 October 2021. U. Heckner, “Über 1000 Jahre altes Bauholz im Westbau von St. Pantaleon entdeckt.” *Denkmalpflege im Rheinland* vol.38, n.2, 2021, pp.1-8.

¹⁶ The survey of the remaining medieval vaults in Speyer by laser-scanning was carried out in autumn 2021. The vaults are described in the comprehensive archaeological study: H.E. Kubach and W. Haas, *Der Dom zu Speyer*, München: Dt. Kunstverlag, 1972.

¹⁷ The idea that the design of these vaults is based on cylinders has been repeatedly proposed in the scientific literature, although it is clearly visible from the shape of the groins that this cannot be true.

Fig. 9: Geometric analysis of the cross vault in the aisle of Speyer Cathedral: the longitudinal ridges are straight and in most cases horizontal; the surfaces of the transverse shells are also defined by straight generating lines, avoiding the geometric problems that would occur near the summit; the diagonal groins are defined by circle segments in vertical planes.



to highlight the groins creates some problems for the analysis of the survey. Nevertheless it is clear that the groins describe circle segments in vertical planes. The design is rather straight-forward: the centres of the bays are placed on the same level as the boundaries in longitudinal direction (only in some cases slightly lower), and the elevations of the four diagonal arches are defined by the upper end point with horizontal tangent and the starting point in the corner, creating surbased segment arches (Fig. 9). The vault surfaces can be in principle described as being generated with straight lines tended between the curves at the groins and boundaries – hence, it can be supposed that the shells were built on a wooden formwork. We may assume that also in this case the temporary supporting structure consisted of a central pole erected on a wooden platform with diagonal centrings on which the planks of the formwork were mounted.

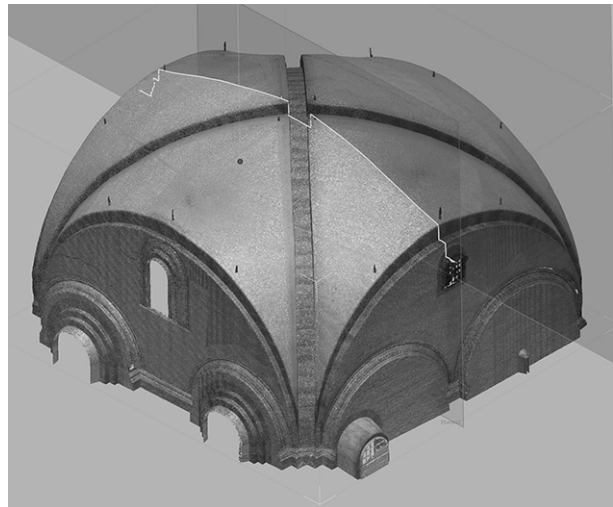
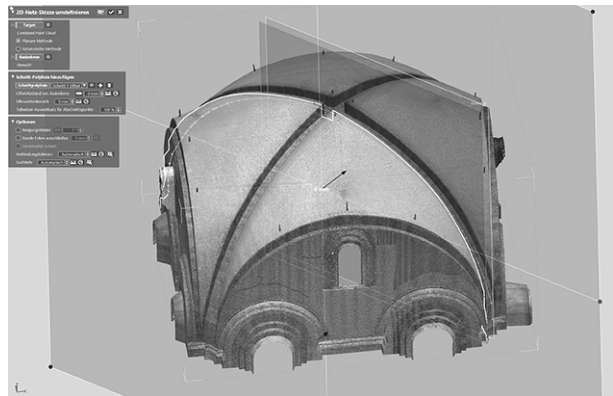
Analysing the shape of the vault surface, the straight lines in the shells in longitudinal direction can easily be observed. Particularly interesting is the profile of the ridges in transverse direction, which descend from the centre to the lower arches on the sides (Fig. 9, centre): while completely straight in the middle and lower portions, they show a concave curvature in the vicinity of the summit. In fact, if the ridge lines described straight lines descending directly from the summit to the boundary arches, near the summit they would remain below the level of the diagonal groins, resulting convex surfaces in the uppermost parts of the ridges – a problem that was described by scholars in the nineteenth century.¹⁸ It

¹⁸ First by J.C. von Lassaulx in 1847, later by K. Mohrmann. Wendland, *Lassaulx* (Note 13). G. G. Ungewitter and K. Mohrmann, *Lehrbuch der gotischen Konstruktionen*, Leipzig: Weigel, 1890, pp.9-11.

Fig. 10:
Speyer Cathedral, rib vault in the transept



Fig. 11:
Geometric analyses of the vault in the transept, late eleventh century: the elevation of the ribs as well as of the boundaries describe circle segments and are nearly semi-circular; the surfaces are not spherical, but determined by the boundary curves.



is obvious that the builders of these vaults were aware of this problem and solved it systematically: according to the shape analysis we suppose that an extra linear element was inserted in longitudinal direction between the diagonal centrings near the summit. Hence, not only for their great dimensions and sound construction, but also for the geometric expertise they show, we can conclude that the early cross vaults in Speyer cannot be considered just as some first attempt or experiment: rather, the “Gothic” scheme of vault design was already well established in the eleventh century.

Speyer Cathedral, promoted by the Emperors of the Salian dynasty and in its time the largest church in Central Europe, in the transept built ca. 1080-1105, contains another ground-breaking vault construction:¹⁹ The two bays flanking the crossing, approximately square in plan, each have a ribbed

¹⁹ For the dating and construction details: D. v. Winterfeld, “Die Rippengewölbe des Domes zu Speyer”, *Jahrbuch des Vereins für christliche Kunst in München*, vol.12, 1988, pp.101-112; D. v. Winterfeld, “Offene Fragen der

cross-vaults with the huge span of ca. 14 metres and a height of over 30 metres (Fig. 10). The ribs of dressed stone have rectangular section; there is no keystone, but one rib is continuous while the other two half-ribs are set against it. The elevations of the diagonal ribs clearly describe circle segments (Fig. 11) – they are somewhat surbased, in contrast to the perfectly semi-circular boundary arches, but nevertheless have a considerable rise over their summits. The difference in respect to the vaults in the aisles is more than obvious: instead of defining the central summit level with the transverse arches creating rather flat diagonal groins, here the elevations of the diagonal ribs, in analogy to the other arches in dressed stone, are nearly semi-circular (although actually not determined geometrically, but by an absolute measure given for the height of the summit). Hence, this design decisively gives rise to the diagonal stone rib – according to the idea of *ogive* discussed above.

The shape of the vault surface (Fig. 11) shows greater complexity than in the aisles, surely also due to the fact that in the formwork the great distance between the diagonal and boundary arches required some additional centrings. In any case, the difficulties given by the inclination of the ridges rising to the summit of the vault are well mastered, and it can be definitely stated that the surfaces are not spherical but generated between the pre-determined curves of the arches.

The high vaults in the nave of Speyer Cathedral from the late eleventh century unfortunately for the most part no longer exist in the original.²⁰ As rebuilt in the seventeenth century, they are cross vaults with diagonal groins in approximately square bays, each comprising two bays of the aisles, separated by transverse arches like in the vaults over the aisles built some decades earlier. As the remaining original transverse arches show, they surely reproduce the original dimensions: 12 metres free span, 33 metres in height – which probably corresponds to 100 feet height. From a historical point of view, a comparison between the high vaults in Speyer with those in Notre-Dame of Paris, built hundred years later, may be surprising. But it could be more than mere coincidence that the section of the nave in the great cathedral of the Roman-German Emperor and in the cathedral of the King of France have the same dimensions.

Conclusions

For these considerations about the geometric concept of the vaults, which is essential for the architectural and structural design of these buildings and which, as we have seen, must be discussed in a much broader context than it has been done in previous research, the built objects are the only source of information. This underlines the importance of preserving the original structures – in particular in the vaults of Notre-Dame in Paris, all remaining parts must be maintained because they are historical documents of primary value, and reconstruction of the destroyed parts must be kept to the minimum and carried out with the greatest respect for the original. For this, thorough knowledge on the original structure, its design and building process is essential – these processes and practices determined its shape and every detail. It will still require a lot of scientific research.

This applies not only to the shape of the vaults and to the construction details, but also to the characteristics of the stonework that are intrinsically related to the intentions of the builders as well as to the practices of stone planning as application of knowledge in practical mathematics. Again, the built object is literally shaped by historical knowledge and working practice, and at the same time it is a source for understanding this knowledge and these practices in history. Until now, little of all this is

Bauforschung zum Speyerer Dom”, pp.135-157 in M. Müller, M. Untermann and D. v. Winterfeld (eds), *Der Dom zu Speyer. Konstruktion, Funktion und Rezeption zwischen Salierzeit und Historismus*, Darmstadt: WBG, 2012.

²⁰ Possibly the two eastern bays are still original at least in some parts: Kubach and Haas (note 16), pp.575-577. This will be subject of our forthcoming investigations on the building.

Fig. 12:
Advanced stereotomy in the abbey church of
Saint-Martin-des-Champs in Paris, first half
of twelfth century



understood – what has become more than obvious already in the preliminary studies presented here, however, is that many features of the high vaults in Notre-Dame differ substantially from uniform image of the Gothic vault as it has been coined in the modern reception.

While from the preliminary research and from the considerations made above, the high vaults of Notre-Dame are positioned in a broadened context, we can also state a distinctive characteristic feature of the early Gothic church architecture in the Ile-de-France: This can be seen in the intrinsic relation between vault design and stone planning – like for instance in the effort of creating the regular pattern in the shells and of adding the specially tailored ridge stones. The exposed stonework, with the search for perfection or the aspiration for giving notion of perfection in the greatest complexity, is a guiding theme in the regional and geographic context (Fig. 12), and in particular a principal motif in the architecture of Notre-Dame. Elsewhere we sketched a possible interpretation as to the significance of this architecture,²¹ and we could also show the relation of the exercises described in Villard de Honnecourt’s “Lodge Book” – the only known source on the practices of stone planning in the context of Gothic architecture and, as we have seen, correlating with the tasks and solutions we can find on the building – with the contents of the treatises of stereotomy that appear only centuries later.²²

This distinct feature, incidentally, was to remain an essential characteristic and a strong tradition in the architecture of this region way beyond the middle-ages, as the immense efforts of artful stereotomic solutions in the early modern Parisian architecture impressively demonstrate.

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²¹ Wendland, Gielen and Korensky (Note 2).

²² Wendland, *Steinerne Ranken* (Note 11). See also J. Lengenfeld and D. Wendland, “Die Planung der komplexen Werksteinkonstruktion im Bergfried der Schönburg – das Werk eines Naumburger Meisters”, in proceedings *Fünfte Jahrestagung der Gesellschaft für Bautechnikgeschichte Zürich 2021*. Petersberg: Michael Imhof (forthcoming).