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# The geometric design of the “Guarinesque” vaults in Banz and Vierzehnheiligen in relation to the treatises of stereotomy

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**ABSTRACT:** Johann Dientzenhofer’s Banz Abbey Church (1710-1718) and Balthasar Neumann’s church of Vierzehnheiligen (1743-1772), both located in Upper Franconia (Germany), are characterised by their masonry vaults, sequences of oval domes separated by double-curved arches. This Late Baroque architecture has been called “Guarinesque”; however, the conceptual connection between the vaults and Guarino Guarini’s architecture remains to be thoroughly investigated. This paper resumes the discussion about the geometric definitions of these vaults using the modern methods of geometric analysis and reverse geometrical engineering, based on 3D laser scanning. The analysis of these scans reveals a design process based on plane circle segments and ovals, while more complex geometric procedures are not necessary to describe the vaults. Further, the relationship between these designs and the procedures of geometric design described in the treatises of stereotomy since the 16<sup>th</sup> century can be shown. On this background, the relation to Guarini’s *Architettura Civile* (1737) is discussed.

## 1 APPROACHING GUARINESQUE VAULT DESIGN WITH NEW METHODS

### 1.1 *The vaults of Banz and Vierzehnheiligen as examples of “Guarinesque” vaults*

Lying near the town of Bad Staffelstein in Upper Franconia (Germany) and within sight of each other, two churches dominate the landscape aptly referred to

as “god’s garden”. They are Banz Abbey Church (1710-1718), designed by Johann Dientzenhofer, and the pilgrimage church of Vierzehnheiligen (Fourteen Holy Helpers, 1743-1772), designed by Balthasar Neumann. Both churches are prominent examples of an emblematic group of sacral buildings that emerged in the central European regions of Franconia and Bohemia (Czech Republic) during the Late Baroque period.

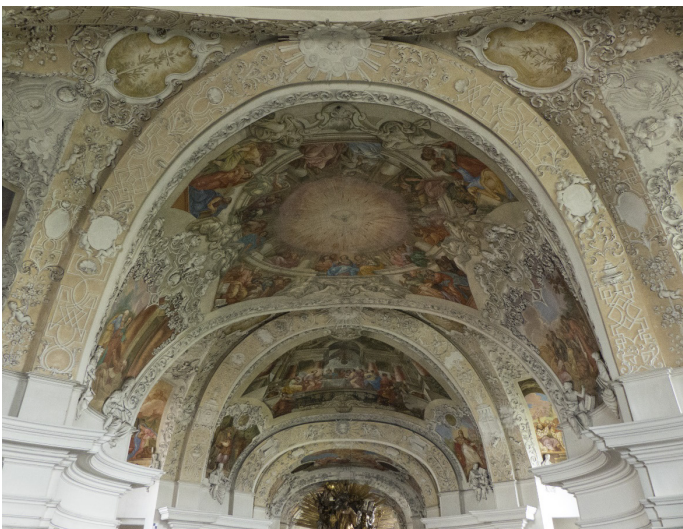


Figure 1. The vaulted ceiling of Johann Dientzenhofer’s Banz Abbey Church (1710-1718).



Figure 2. The vaulted ceiling of Balthasar Neumann’s church of Vierzehnheiligen (1743-1772).

The interiors of these churches have undulating walls covered with vaults. Their complex geometries create extraordinary spaces. This kind of architecture was employed by a small group of people, including members of the Dientzenhofer family of master builders and the famous architect Balthasar Neumann. Due to a perceived conceptual connection to the work of the Piedmontese architect, mathematician, and Jesuit priest Guarino Guarini (1624-1683), the architectural style of these buildings has been described as “Guarinesque” (Brinckmann 1932).

An especially interesting aspect of these churches are their vaulted ceilings built in masonry. Their shape is determined by sequences of oval domes, separated, and enclosed by boundary arches with spatial curvature. Analyzing these double-curved arches, also referred to as warped ribs, may help to identify some characteristics of the geometric concepts and procedures that were used by the architects designing and planning these complex vaults (Figs 1, 2).

### 1.2 Previous research on “Guarinesque” vaults

The sacral architecture created by Johann Dientzenhofer and Balthasar Neumann, especially the vaults and double-curved arches, have been a subject of scholarly debate for a long time.

It has been suggested that the designs for the churches and their vaults were based on bent or deformed rectangular bodies (Franz 1987) or on intersecting geometrical figures, such as rotation bodies (Reuther 1953, 1960, Holst 1981). On this background, a close connection to Guarini’s designs and ideas has usually been recognized (Brinckmann 1932, Norberg-Schulz 1993, Müller 2002b). Because the original drawings of these vaults are often inaccurate or partially missing, they offer little insight into the concepts developed by the architects. However, it has recently become possible to carry out analyses on the buildings themselves; these analyses can be used to ascertain the geometric principles behind the designs (Wiesneth 2011). Consequently, the design principles of the Guarinesque churches in Franconia and Bohemia have received renewed scientific interest (Compán et al. 2015).

Nonetheless, the conceptual connection between these Guarinesque vaults and the architectural

designs and geometric solutions developed by Guarino Guarini, which were spread by architects in Bohemia and Franconia and were widely disseminated through his architectural writings and prints, still remains to be understood.

### 1.3 3D laser scanning as a method for geometrical analyses

Using Johann Dientzenhofer’s Banz Abbey Church and Neumann’s church of Vierzehnheiligen as examples of Guarinesque vault architecture, the modern method of geometric analysis based on 3D laser scanning has been used to shine a new light on the vaults’ underlying design principles. In July 2020, both churches were scanned using a Leica RTC360 time-of-flight laser scanner. The scanning data gathered (Fig. 3) has enabled an analysis of the shape of the built objects in their three-dimensional complexity and the reconstruction of their original design.

In contrast to previous research in which the original building plans or plans drawn by conventional building documentation have been utilized, this method enables the researchers to analyze the shape of the structures as they were actually built, instead of having to refer back to the idealized forms of building plans. Using the methodology of reverse geometric engineering, the scans can be compared to three-dimensional models representing hypotheses of the designs, in order to test them. The quantification and visual representation of these comparisons enable critical revision of the analyses and their scientific discussion (Wendland 2012). The high density of the scans also reveals characteristic geometric irregularities that have the potential to provide evidence on the construction process. The analyses presented herein are a preliminary, as the research is in progress.

Discussing these findings and assumptions concerning the design process in relation to the methods described in contemporary treatises on stereotomy and design practices is necessary to provide context and put the results into perspective.

## 2 THE GEOMETRIC DESIGN OF THE VAULTS

### 2.1 Banz Abbey Church

Banz Abbey Church was built between 1710 and 1718 (Hotz 1993, 63–73). Master builder Johann Dientzenhofer, who had previously worked on the Benedictine Collegiate Church in Fulda (Vilímková and Bruckner 1989, 46–47), designed and oversaw the construction of Banz Abbey Church in his role as the court architect of Bamberg.

The building is made up of a high and wide nave, structured by three pairs of wall pillars. A narrow entrance and a narrow, elongated choir are attached on either side of the nave (Figs 4, 5). The main vault is

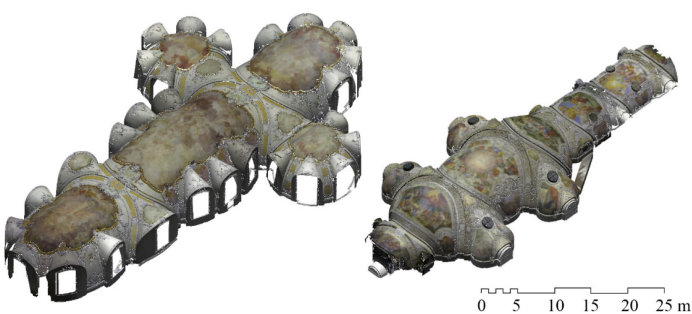


Figure 3. Triangulated scans of the vaults of the church of Vierzehnheiligen (left) and Banz Abbey Church (right); isometric top view.



intersected by transversal lunettes that span between the wall pillars, widening the space.

The vaults are made of brick; there are structural ribs on the extrados. The vaults' surface is composed of a series of domed areas on an oval plan, separated from each other by pairs of double-curved arches. The spatial curves described by these arches, which can be seen in Figures 1, 3 and 4, made the arches the object of research interest for decades.

### 2.1.1 State of research

Reuther described the church's main vault as the fusion of a barrel vault with an oval section and an ellipsoid, trimmed by spherical arches (1954, 362–364). Hotz described the vaults as spherical barrel

vaults, joined by double-curved arches and lunettes (1993, 115). A thorough tachymetric survey of the vaults, including the extrados, has been carried out by Wiesneth. He concluded that the vaults and arches cannot be described by elementary geometrical bodies or rotation surfaces and attempted to trace their shape back to the wooden centering used in their construction process. According to his hypothesis for the construction of the double-curved arches, the first step was the creation of a barrel-shaped wooden formwork. Upon this formwork, circle segments could be plumbed down to draw the curve of the arch (Wiesneth 2011, 40–47). The most recent research on the vaults of Banz Abbey Church by Compán et al. described the design of the ribs as the “intersection of two perpendicular cylinders of different diameters” (2015, 195).

### 2.1.2 Geometric design

According to the drawings and deviation analyses produced from the laser scan, and corresponding with the most recent research, the geometric design of the vaults of Banz Abbey Church can be described as being based on simple circle segments or, in some cases, oval arches with two radii. As shown by the horizontal section in Figure 4 (A–A), the projections of the double-curved arches on a horizontal plane are inscribed into circle segments. The domed vaults span between these arches. Their shape can be described by circle segments; the main vault is defined by a symmetrical oval.

The cross-section of the main vault (Fig. 5, C–C) is circular in shape: the first deviation analysis carried out (Fig. 6) showed a maximum deviation of ca. 80 mm from a perfect circle. The projection of the double-curved arches on a vertical plane (Fig. 5, D–D) more closely resembles oval arches drawn with two radii. This geometrical feature of the double-curved arches implies that their curves are the primary feature of the design and not simply generated by the intersection of the neighbouring vault surfaces. The arches are determined a priori through the intersection curve of two cylinders, while the surfaces of the adjacent vaults are fitted to the curves described by these arches. The cross-sections of the vault in the

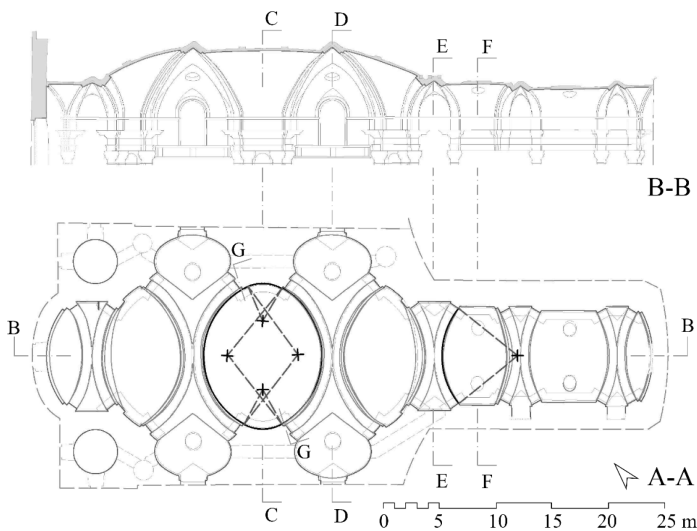


Figure 4. Longitudinal (B–B) and horizontal (A–A) sections of Banz Abbey Church. The crosses mark the centre of the circle sections and the dashed lines mark the radii. The lines indicating the outer edges and extrados are based on Wiesneth 2011. The horizontal section shows a view from above.

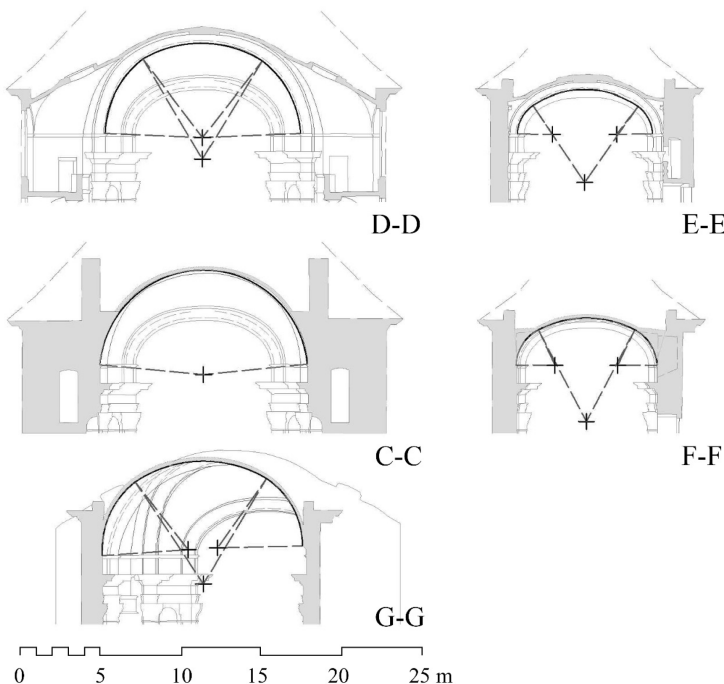


Figure 5. Cross-sections of Banz Abbey Church. Sections of the main double-curved arches (D–D), the double-curved arches in the choir (E–E), main vault (C–C), choir vault (F–F) and the main vault cut diagonally (G–G).

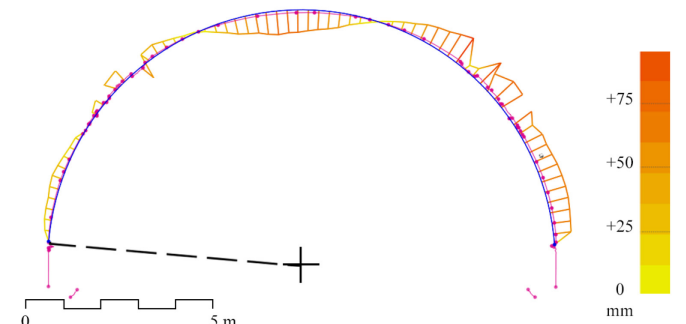


Figure 6. Cross-section of the main vault in Banz Abbey Church. The maximum deviation of the scan data (pink line) from a fitted circle segment (blue line) is 80mm. Stucco decoration disturbs the findings in the area showing the maximum deviation.

choir (E–E, F–F) are based on oval arches with a significantly flatter curve and reduced height. A diagonal cross-section through the main vault (G–G) reveals a form based on circle segments. Sections drawn in radial direction in respect to the oval plan are defined by just two different radii.

## 2.2 The pilgrimage church of Vierzehnheiligen

Vierzehnheiligen was built as a pilgrimage church over a period of almost 30 years between 1743 and 1772. The renowned architect Balthasar Neumann had to drastically alter his designs after construction had begun because executing builder Krohne had made changes to the plans. These changes resulted in a “displaced” crossing (Ruderich 2000, 131–158). At the time of Neumann’s death in 1753, the bare structure had only been completed up to the springing of the vaults, built in brick masonry (Ruderich 2000, 171–174). Even though Neumann did not personally oversee the construction of the vaults, it can reasonably be assumed that his successor, the local builder Nißler, refrained from making major alterations to the great architect’s designs. Nevertheless, the construction of the high vaults is substantially different from Neumann’s other vaults, and from those of the Dientzenhofers. Vierzehnheiligen’s vaults are built with limestone tufa instead of brick and have no ribs on the extrados (Wiesneth 2011, 162–168).

The church has a Latin cross plan created by oval and circular shapes (Figs 7, 8). Two ovals in longitudinal direction make up the nave, the third oval marks the choir. Two circular transepts are located to the North and South of the main crossing.

At the level of the ceiling these same ovals and circles form the bases of gently domed vaults. The vaults

are separated by two pairs of double-curved arches, forming two crossings. The main surfaces of the vaulted ceiling are interrupted by lunettes for the clerestory windows.

### 2.2.1 State of research

In terms of their geometric design, the vaults of the church of Vierzehnheiligen have been described by Teufel as approximating ellipsoid shells with blurred edges separated by helical ribs (1939, 103). Reuther described them similarly as a mutual penetration of ellipses, spherical triangles within double-curved arches, and circular transept vaults (1954, 362; 1983, 80). In his comprehensive monograph on Vierzehnheiligen, Ruderich pointed out that ellipses were rarely used in architectural design and that oval constructions were important to Neumann’s design. He claimed that the form of the vaults is not based on geometrical bodies, such as rotation surfaces. Therefore, he considered analyses of the geometrical bodies misleading and preferred two dimensional geometric constructions based on the building plans (Ruderich 2000, 150-195). Müller explained that the design of the double-curved arches of Vierzehnheiligen was the result of the intersection of a vertical cylinder and a horizontal cone (2002b, 44–48). Most recently, Compán et. al. described the projection of the double-curved arches on a vertical plane as composed of circle segments with three different radii (2015, 200–202).

### 2.2.2 Geometric design

Just like the vaults in Banz, the vaults of the church of Vierzehnheiligen were designed using circle segments, belonging to single circles or oval arches with two radii. As previous research has already made clear, the horizontal section (Fig. 7, A–A) reveals a

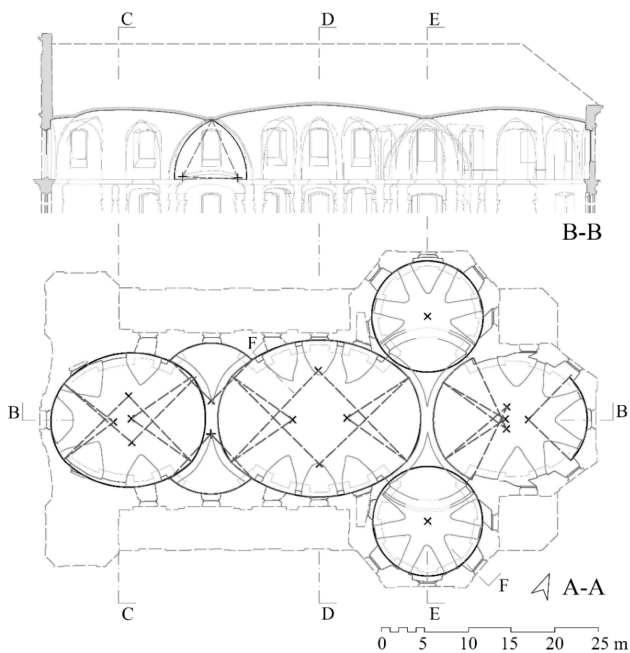


Figure 7. Longitudinal (B–B) and horizontal (A–A) sections of the church of Vierzehnheiligen. The lines indicating the outer edges and extrados are based on the documentation by Albertini and Roth in 1913. The horizontal section shows a view from above.

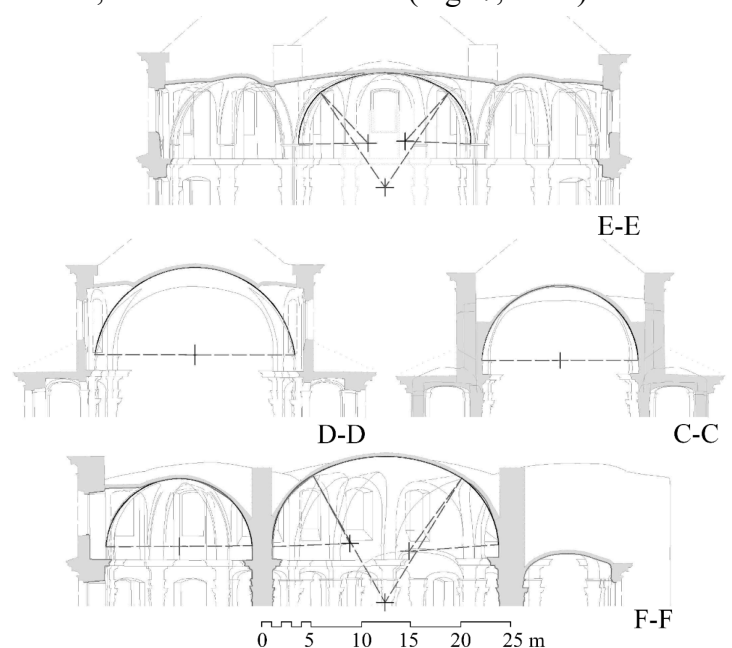


Figure 8. Cross-sections of the church of Vierzehnheiligen. Sections of the crossing’s double-curved arches (E–E), the main vault (D–D), the organ vault (C–C) and the main vault and transept vault cut diagonally (F–F).

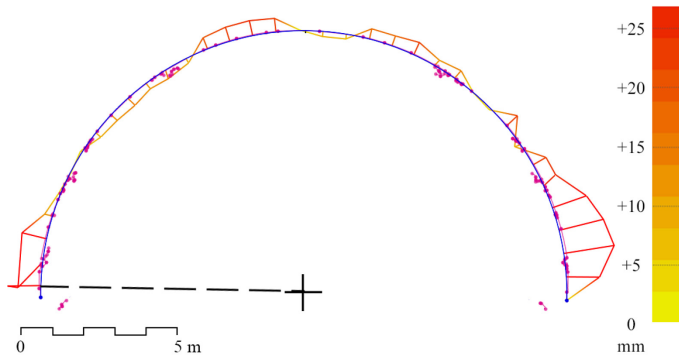


Figure 9. Cross-section of the church of Vierzehnheiligen's main vault. The maximum deviation of the scan data (pink line) from a fitted circle segment (blue line) is 25mm.

design based on the use of the compass. Circles are used for the transepts and ovals of different orientation and sizes are used for the main nave and choir. Unlike the projections of the double-curved arches of Banz on a horizontal plane, the projections here are drawn with two different radii, resulting in a “flatter” curve. The double-curved arches are based on circle segments in the longitudinal section (B–B).

As the cross-sections in Figure 8 show, the double-curved arches' projections on a vertical plane (E–E) are based on oval arches with two different radii. The cross-sections through the middle of the vaults themselves (main vault D–D, organ vault C–C, and choir vault), are based on half circles, just as they are in Banz Abbey Church. The deviation analysis revealed the maximum deviation from a perfect circle to be only ca. 25 mm (Fig. 9). The diagonal cross-section through the main vault (Fig. 8, F–F) revealed an oval arch similar to that of the diagonal section of Banz.

### 2.3 Vault design based on plane geometry

The survey and geometric analyses of the vaults of Banz Abbey Church and the church of Vierzehnheiligen revealed that the designs of the two churches' vaults, despite the 30 years that divide them, show many similarities: the most obvious similarity being the use of circles, circle segments, and circle segments combined to form ovals. The preliminary deformation analyses (Figs 6, 9) support this finding, showing a maximum local deviation from the ideal circle of 8 cm (further analyses of this kind are in progress). Such deviations can be explained by typical vault deformations: the sagging of the summit, the



Figure 10. Triangulated scans of the double-curved arches of the church of Vierzehnheiligen (left) and Banz Abbey Church (right); isometric top view.

numerous uneven layers of plaster, the stucco decorations that disturb the scan, and the inaccuracies of the building process. It must also be considered that the surveyed surface is the secondary plaster surface. The masonry that was defined by planning lies beneath the plaster. Nevertheless, the deviations are small enough for the results to back up the assumption that circles formed the basis of the design process.

The surveys of both churches revealed that the main vaults follow oval shapes and have circular cross-sections. The double-curved arches of both churches are oval arches in the projections on a vertical plane, but the projections on a horizontal plane revealed that the church of Vierzehnheiligen has oval arches and Banz Abbey Church has arches based on circle segments.

The designs of the vaults of both churches are based on the ovals of the floor plan and the circle segments of longitudinal- and cross-sections. The vault surfaces are simply connecting these geometrically defined curves. The double-curved arches (Fig. 10) were not created to function as section curves of the neighbouring vault surfaces but were planned geometric elements that defined the design in their own right. Their form can be described by the intersection line of the extrusions of the projections on horizontal and vertical planes (Fig. 11). The vertical extrusion is based on the oval shape (consisting of two by two different circle segments) of the projection on a horizontal plane. The horizontal extrusion is based on the oval arch (consisting of two different circle segments) of the projection on a vertical plane.

There is no part of the vault, neither the double-curved arches nor the vault surfaces, that cannot be defined with simple curves such as circle segments with one or two different radii. There are no curves that derive from intersections, even when cut diagonally. The geometric conception is based solely on geometric construction in the plane. By no means was the use of surfaces of revolution or other more complex surfaces, as some authors have suggested, needed to define the vaults' geometry, and no evidence of such forms has been found in the analyses.

Dientzenhofer's and Neumann's reliance on ovals and circle segments is also apparent from the

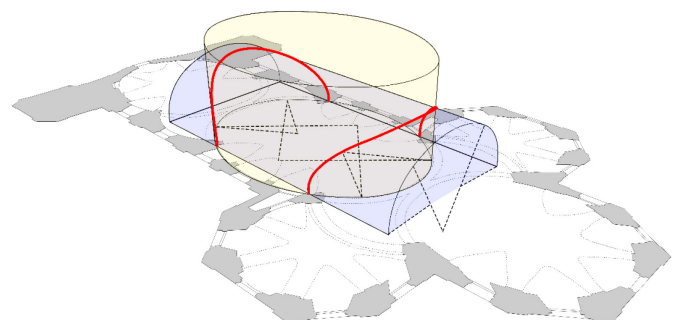


Figure 11. The form of the double-curved arches of the church of Vierzehnheiligen (pictured) and Banz Abbey Church can be described as the intersection curve (red) of the extrusions of the oval of the horizontal projection with the oval arch of the vertical projection. Visualisation by Mark Gielen.

surviving building plans, which show the holes left by their compasses (Ruderich 2000, 51–158, Wiesneth 2011, 41–42).

That the designers took this approach to the geometric design of the complex shapes in the vaults is more than plausible. One can suppose that the construction and form control of the vaults was entirely based on wooden centring arches erected in longitudinal, transversal, and radial directions – a feature easily determinable in an oval plan. These centring arches could easily be traced and built as plane oval arches – a geometric description which any skilled mason or carpenter could work with. In these elementary geometric terms, the specification for production of the centering arches could be easily formulated, passed on, and controlled. One can conclude that the astonishingly complex shapes in the vaulted ceilings could be produced by simple geometric control and the use of temporary supporting structures, which were easy, false-proof, and conventional.

### 3 TREATISES AND DESIGN PRACTICES IN LATE BAROQUE VAULT ARCHITECTURE

The geometric design of Late Baroque vaults must be viewed within the broader context of vault construction in the 17<sup>th</sup> and 18<sup>th</sup> centuries. During this period, theoretical knowledge was conveyed through treatises on architecture. Knowledge concerning issues of geometric design were shared in treatises of stereotomy, illustrated books that describe design procedures for complex stone construction. Additionally, the common practices used in execution planning and craftsmanship in this era should be taken into consideration.

#### 3.1 *The design of arches with spatial curvature in the treatises of stereotomy*

Within the Early Modern treatises on the particular subject of stereotomy, a group of several French illustrated printed books refer to the architectural treatise *Le premier tome de l'architecture* by Philibert de l'Orme, (1567). In this work, large sections are dedicated to design procedures for complex shapes (Pérouse de Montclos 1982), and individual work steps are explained. Some of these procedures are exemplified in de l'Orme's own designs of impressively complex stone structures. There is also explicit reference to an existing practice among stonemasons, and some sort of curriculum of standard exercises in stone planning becomes tangible (Wendland et al. 2019, 46–55). While this book and its later editions have never ceased to be in circulation, several treatises solely dedicated to stereotomy adopted and partially adapted the design exercises coined by de l'Orme. François Derand's *L'Architecture Des Voutes* (1643)

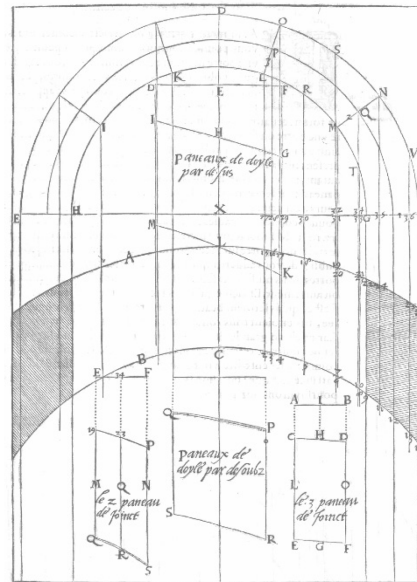


Figure 12. The “tour ronde” in Philibert de l’Orme’s treatise *Le premier tome de l’architecture* (1567, 77); Reproduced from Universitätsbibliothek Bern, MUE Bong IV 783, <https://doi.org/10.3931/e-rara-15161> (public domain).

followed de l’Orme’s path and made an important impact on subsequent influential published architects and mathematicians, such as Dechales (1674) and Frezier (1737). These treatises, aimed at stonemasons and builders, functioned as instruction manuals that detailed how to execute precision carving of exposed stonework (Müller 1968, 202).

A basic exercise contained in all these treatises addresses the particular problem of designing an arch on a curved plan. The exercise describes how to create an arched door in a round tower – “tour ronde” – and how to draw and cut the single voussoirs according to the geometry of the curvilinear arch. This seems to have been a standard exercise that had been in use since the medieval stonemasons’ workshops. A variety of different solutions for the geometric problems of the generation of the curve, shape of the voussoirs, and spatial arrangement of the joints are known (Wendland et al. 2019, 48–51).

According to de l’Orme and the later treatises, the geometric construction of a curvilinear arch is based on the intersection of two cylindrical surfaces: the vertical cylinder constituted by the wall of the round tower and an ideal horizontal half-cylinder, according to a barrel vault, generating the arch (Fig. 12).

It is obvious that the design of double-curved arches proposed in this exercise is based on the same geometric concept that can be encountered in the vaults of Dientzenhofer’s Banz Abbey Church and Neumann’s church of Vierzehnheiligen.

#### 3.2 *The theoretical approach of Guarini’s Architettura Civile*

Even though the comprehensive printed edition of Guarino Guarini’s *Architettura Civile* was not published posthumously until 1737, the plates and other



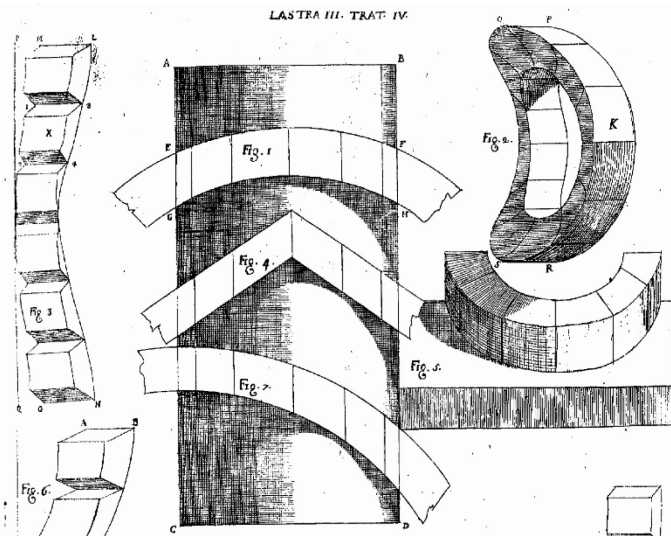


Figure 13. Projections of different forms such as arches on a cylinder, figures 1-6 of table 3 of treatise 3 of Guarino Guarini's *Architettura Civile* (1737, 349).

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parts of the book were already widely known in the architectural circles of Prague around 1700, where they influenced in particular sacral architecture (Hubala 1989, 133). Although Guarini's work shows similarities to Derand's methods (Calvo-López 2020, 96–100), it is not part of the French tradition of knowledge. In contrast to this group, Guarini's book is less concerned with stonemasonry and practical building (Müller 2002a, 80–94). The Italian, who taught and published books on mathematics, produced writing that focused on theoretical questions. Guarini himself suggested that his theoretical ideas about projections and sections are necessary for (mathematically proficient) architects (1737, 191).

*Architettura Civile* is divided into five treatises. The fourth one, *Della Ortografia Gettata* deals with stereometric problems, such as sections of cylinders, spheres, and cones. Figure 13 shows the projection of arches on a cylinder – this is reminiscent of the “tour ronde” (Fig. 12). In his theoretical description, Guarini does not state the practical use for these forms, but their application in a practical building context is apparent.

### 3.3 Double-curved arches as an established building practice

Stereometric problems like the projections on cylinders in Guarini's book must have been very fascinating to well-educated architects with the renown of Johann Dientzenhofer and Balthasar Neumann. However, to recreate Guarini's designs, they had to rely on the established design practices that were available at their Franconian building sites, as his treatise lacked comprehensive instructions. Despite their different creative periods, influences, and backgrounds, when attempting to recreate geometrical ideas such as the double-curved arches made popular

by Guarini, both fell back on very similar design methods for the vaults of Banz Abbey Church and the church of Vierzehnheiligen respectively.

As the descriptions of the “tour ronde” exercise in most contemporary treatises makes clear, the design of double-curved arches from the 15<sup>th</sup> to 18<sup>th</sup> century was a standard task in the training of architects – in France for sure – and was made accessible to erudite architects through written treatises. De'Orme made clear that these exercises were part of the common practice in building crafts. Hence, the design of the great double-curved arches of the churches of Banz Abbey and Vierzehnheiligen was far from being an unusual problem requiring an unusual solution. This was a standard task for architects, planners, and craftsmen. Needless to say, the tracing of oval arches – on which the whole geometry of the concave vault surfaces is based – was general knowledge on the building sites.

The approach to the planning of the vault, which started with creating systems of simple arches, is a sound procedure that goes back to Gothic design practice. In the projects discussed here, Dientzenhofer and Neumann deal with their double-curved arches in the same way. It must be stressed that the design of these extraordinary vaults was achieved in a way that could be realized by means and methods everyone involved in their planning and building would have been familiar with from traditional practices.

## 4 CONCLUSION

At first glance, the designs of the vaults of Banz Abbey Church and the church of Vierzehnheiligen appear to be based on complex geometrical concepts. The survey of the buildings and the subsequent geometrical analysis, however, proved that all the surfaces within the vaults can be traced back to simple geometrical curves consisting of circle segments and ovals with one or two different radii. The double-curved arches are not the result of two intersecting vaults. They were designed by what is essentially the same method used in the “tour ronde”, which was described in the building instructions of the Early Modern French treatises of stereotomy. This concept was widely understood by building professionals in central Europe and was familiar due to the persistence of Late Gothic building traditions. For the purposes of planning, design and construction of these complicated looking vaults, the theoretical descriptions of Guarino Guarini's *Architettura Civile* were neither necessary – nor very helpful. Contemporary mainstream practices were capable of producing the design of the Guarinesque-looking vaults.



## ACKNOWLEDGEMENTS

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