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An advancement of the gravimetric isotope mixture method rendering the knowledge of the spike purity superfluous

Lukas Flierl¹ · Olaf Rienitz¹ · Jochen Vogl² · Axel Pramann¹

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Abstract

The gravimetric isotope mixture method is the primary method to determine absolute isotope ratios. This method, however, depends on the existence of suitable spike materials and knowledge of their purities. Determining the purity of the spikes can be tedious and labour-intensive. In this publication, an advancement of the gravimetric isotope mixture method, rendering the determination of the purity of the spike materials unnecessary, is presented. The advancement combines mass spectrometry and ion chromatography leading to an approach being independent of the purity of the spike materials. In the manuscript the mathematical background and the basic idea of the novel approach are described using a two-isotope system like copper or lithium.

Keywords Isotope amount ratios · Metrology · Mass spectrometry · Ion chromatography

Introduction

Isotope amount ratios *R* are useful tools in many scientific areas, ranging from archaeology [1] to zoology [2]. Mass spectrometry is usually the method of choice for the isotope analysis of samples. With mass spectrometry excellent precision can be obtained, but isotope amount ratios (unit mol mol⁻¹) are not directly available. Users obtain ion intensity ratios *r* (unit V / V, A / A or s⁻¹ / s⁻¹), which are numerically different from the isotope amount ratios, in some cases the difference is up to 25% [3–6]. This difference is due to the so-called instrumental isotopic fractionation (IIF) [7] or mass bias. The second term is rather colloquial and does not

Lukas Flierl and Olaf Rienitz contributed equally to this work.

☑ Olaf Rienitz olaf.rienitz@ptb.de Lukas Flierl lukas.flierl@ptb.de

Jochen Vogl jochen.vogl@bam.de Axel Pramann

axel.pramann@ptb.de

 Physikalisch-Technische Bundesanstalt, Bundesallee 100, Braunschweig 38116, Germany

² Bundesanstalt für Materialforschung und -prüfung, Richard-Willstätter-Straße 11, Berlin 12489, Germany describe the chemical and physical causes adequately. Both terms are collective terms for all possible effects which lead to the difference between the isotope amount ratio and the ion intensity ratio, for example different ionization probabilities, space charge effects or detection efficiencies. These effects can be minimized, but not totally eliminated, and therefore correction is needed, see Eq. 1.

$$R_{i} = \frac{n_{i}}{n_{1}} = K_{i}r_{i} = K_{i}\frac{I_{i}}{I_{1}}$$
(1)

 R_i is the *i*th isotope ratio, n_i is the amount of substance of the i^{th} isotope, n_1 is the amount of substance of the socalled reference isotope, in most cases the most abundant isotope of the element under investigation. r_i is the ratio of the measured ion intensities of the *i*th isotope and the reference isotope. I_i and I_1 are the measured ion intensities of the i^{th} isotope and the reference isotope, respectively. K_i is the so-called K-factor, which transforms the measured intensity ratio r into the isotope amount ratio R or in other words corrects for the mass bias. The K-factor can be determined by using a certified isotope reference material, which is traceable to the International System of Units (SI) and chemically as close as possible to the sample. The sample and reference need to be measured within the same campaign, since the IIF varies over time. With the knowledge of the certified isotope amount ratio and the measured ion intensity ratio of the reference material the K-factor can be determined by

applying Eq. 1. This K-factor can also be used for the correction of the measured ion intensity ratio of the sample. This simple correction scheme requires that a suitable reference material exists. Since absolute isotope amount ratios are not directly available via mass spectrometry, the question arises how the isotopic composition of the reference material can be determined in the first place. This situation is quite similar to the famous chicken or the egg causality dilemma. Without knowing R, K cannot be determined and vice versa a classical catch-22 situation. But there is a way out of this dilemma and it is known as the gravimetric isotope mixtures (GIM) method [8]. The GIM procedure is a primary method for the determination of SI-traceable isotope ratios. Since it is a primary method no prior knowledge of the true isotope ratio is required. Only measured quantities (not having the unit mol mol^{-1}) are needed. In brief, the basic idea to derive the K-factors for a system with N_{iso} isotopes is to have N_{iso} spike materials (each enriched in one of the isotopes) and to prepare at least $N_{\rm iso} - 1$ binary blends of these spike (or parent) materials under gravimetric control. In the following steps the ion intensity ratios of the parent materials and the binary blends are measured. The isotope amount ratios of the blends can be expressed as a function of the isotope amount fractions of the corresponding parent materials. These functions form a system of linear equations, which can be solved for the wanted K-factors. More detailed information about the basic idea and the underlying mathematics can be found in the literature [9–12]. In case of a two-isotope system (e.g. lithium or copper) the K-factor can be expressed as [13, 14]:

$$K_{2} = \frac{M_{1}}{M_{2}} \frac{w_{A}m_{AB} (r_{A,2} - r_{AB,2}) + w_{B}m_{BA} (r_{B,2} - r_{AB,2})}{w_{A}m_{AB} (r_{AB,2} - r_{A,2}) r_{B,2} + w_{B}m_{BA} (r_{AB,2} - r_{B,2}) r_{A,2}}$$
(2)

 M_1 is the atomic weight of the reference isotope (in the case of lithium ⁷Li), M_2 is the atomic weight of the other isotope. The small r's are the measured ion intensity ratios of the two parent materials (A and B) and the blend AB. m_{AB} and m_{BA} are the masses of the parent materials A and B, respectively, used for the preparation of the blend AB. $w_{\rm A}$ and $w_{\rm B}$ are the mass fractions of the element of interest in the parent materials, or in other words the purity of the parent materials. Since w_A and w_B are needed, these have to be determined as well to derive the K-factor and finally the absolute isotope ratio R. The established primary method to determine the analyte content w is the so-called isotope dilution mass spectrometry (IDMS). The basic principles of the IDMS have been reviewed by Heumann [15] and Vogl et al. [16] and the underlying mathematics have been generalized by Ouerdane et al. [17]. Besides the fact, that the IDMS is both highly precise and accurate, it has several other advantages. For instance, once the sample has been well mixed with the spike material, loss of the mixture does not alter the result since only ratios (being intensive quantities) are measured. Another advantage is that it is less time consuming than standard addition [18]. For the determination of the analyte content, a reference material Z is needed, with a known content w_Z . Material Z is mixed with the sample/spike to be analysed (in this case A or B) and finally the isotope ratios in the parent materials and the blends are measured. Following the aforementioned procedure, the analyte content w of material A and material B can be expressed as:

$$w_{\rm A} = w_{\rm Z} \frac{m_{\rm ZA}}{m_{\rm AZ}} \frac{r_{\rm Z,2} - r_{\rm AZ,2}}{r_{\rm AZ,2} - r_{\rm A,2}} \frac{M_1 + K_2 r_{\rm A,2} M_2}{M_1 + K_2 r_{\rm Z,2} M_2}$$
(3)

and

$$w_{\rm B} = w_{\rm Z} \frac{m_{\rm ZB}}{m_{\rm BZ}} \frac{r_{\rm Z,2} - r_{\rm BZ,2}}{r_{\rm BZ,2} - r_{\rm B,2}} \frac{M_1 + K_2 r_{\rm B,2} M_2}{M_1 + K_2 r_{\rm Z,2} M_2}.$$
 (4)

The three equations (Eqs. 2 to 4) form a system of nonlinear equations. Although there are as many equations as unknowns (three equations and the three unknowns K, w_A and $w_{\rm B}$), there is no solution with a physical meaning. Solving this system leads to solutions where either w_A or w_B are zero and K is negative. The solutions are given in the supplement of this manuscript. A possible explanation could be that there is not enough information to describe the system mathematically correctly. While there are primary methods to determine absolute isotope ratios and the analyte content (GIM and IDMS) they cannot be combined to one method. Such a method would allow the determination of both quantities, due to the lack of additional mathematical information. Hence, it seems that it is impossible to determine absolute isotope amount ratios, without knowing the purity of the spike materials. A potential, but only theoretical, solution would be to have a second material Z2, with a known content. Instead of blend BZ, blend BZ2 is prepared and Eq. 4 changes accordingly. Since there are now three truly independent equations a unique solution can be found. But this approach has no practical meaning, since two conditions must be fulfilled. Firstly, there must be a second material Z2, which is hardly the case and secondly, the isotopic composition of Z2 must be significantly different from Z. If the latter condition is not fulfilled the correct K-factor can be calculated but the relative uncertainty associated with it increases dramatically the more similar the isotopic composition of Z and Z2 are. In the supplement, a simulation showing the described effect can be found as well as the algebraic solutions. This publication moreover presents a possible solution to this issue. Since the method is based on the GIM method and additional ion chromatography measurements, it is called ICeGIM, which is short for Ion Chromatography enhanced Gravimetric Isotope Mixtures.

Basic idea and mathematical derivation

In this section a solution to the problem described above is presented. The solution is an advancement of the GIM method. The basic idea of the advancement is schematically depicted for a two-isotope system in Fig. 1, and will be explained step-by-step. In theory the method presented can be used for systems with two or more isotopes, but for the sake of simplicity only a two-isotope system is considered here. The aim of the following mathematical considerations is to eliminate w_A and w_B from Eq. 2. The isotope amount ratio R_{AB} of the blend AB can be expressed as:

$$R_{AB,2} = \frac{n_2}{n_1} = \frac{x_{A,2}n_A + x_{B,2}n_B}{x_{A,1}n_A + x_{B,1}n_B},$$
(5)

where n_A and n_B are the amounts of substance of the materials A and B used for the preparation of blend AB. $x_{A,i}$ is the amount-of-substance fraction of the *i*th isotope in material A, and $x_{B,i}$ is analogously defined.

Next the substance content β is introduced, β is defined as¹:

$$\beta_i = \frac{n_i}{m_i},\tag{6}$$

where m_i is the mass of the respective solution. This mass includes impurities, solvents etc., and n_i is the amount of substance of the analyte.

Every x_i can be expressed as

$$x_{i} = \frac{R_{i}}{\sum_{j=1}^{N_{\rm iso}} R_{j}}.$$
(7)

With $R_i = K_i r_i$, Eq. 5 can be reformulated as:

$$K_{2}r_{AB,2} = \frac{\beta_{A}m_{AB}\frac{K_{2}r_{A,2}}{1+K_{2}r_{A,2}} + \beta_{B}m_{BA}\frac{K_{2}r_{B,2}}{1+K_{2}r_{B,2}}}{\beta_{A}m_{AB}\frac{1}{1+K_{2}r_{A,2}} + \beta_{B}m_{BA}\frac{1}{1+K_{2}r_{B,2}}}$$
(8)

Equation 8 can be solved for K_2 .

$$K_{2} = \frac{\beta_{A}m_{AB}(r_{A,2} - r_{AB,2}) + \beta_{B}m_{BA}(r_{B,2} - r_{AB,2})}{\beta_{A}m_{AB}(r_{B,2}r_{AB,2} - r_{B,2}r_{A,2}) + \beta_{B}m_{BA}(r_{A,2}r_{AB,2} - r_{A,2}r_{B,2})}$$
(9)

Now an expression for β_i must be found since n_i is unknown, and the isotopic composition of material *i* is also

not known. β_i can also be expressed as:

$$\beta_i = \frac{n_i}{m_i} = \frac{w_i}{M_i} \tag{10}$$

By considering Eq. 7, the molar mass M_i can be expressed as:

$$M_i = \frac{1}{1+R_i}M_1 + \frac{R_i}{1+R_i}M_2 \tag{11}$$

Combining Eqs. 10 and 11 and considering $R_i = K_i r_i$ in the case of the reference material Z, leads to:

$$\beta_{Z} = \frac{w_{Z}}{\frac{1}{1+R_{Z,2}}M_{1} + \frac{R_{Z,2}}{1+R_{Z,2}}M_{2}} = \frac{w_{Z}}{\frac{1}{1+R_{Z,2}}(M_{1} + R_{Z,2}M_{2})}$$
$$= \frac{w_{Z}(1+R_{Z,2})}{M_{1}+R_{Z,2}M_{2}} = \frac{w_{Z}(1+K_{2}r_{Z,2})}{M_{1}+K_{2}r_{Z,2}M_{2}}$$
(12)

Now a way has to be found to express β_A and β_B in terms of β_Z , since only w_Z is known. Here, ion chromatography (IC) might be a useful tool. The area *A* under the peak in the chromatogram is proportional to the ratio of the amount of substance of the corresponding ion to the mass of the loaded analyte mass. For this approach another material Y is needed. Material Y is an internal standard, which does not contain the element of the other materials A, B and Z. In the next step three blends are prepared under gravimetric control: ZY, AY and BY, see Fig. 1 lower part. For instance blend ZY is prepared by mixing mass m_{ZY} of material Z with mass m_{YZ} of material Y. AY and BY are prepared analogously. The area A_Z of the peak of the main element of Z in the blend ZY can mathematically be expressed as:

$$A_Z = k_Z \beta_Z \frac{m_{ZY}}{m_{\text{sln},Z}} \tag{13}$$

 k_Z is the sensitivity coefficient. It depends on the specific ion (e.g. Li⁺), the conductivity detection, the ion exchange column and other parameters. *k* is however constant for a specific ion, as long as the samples have a similar chemical composition and the same set-up has been used for the measurement. m_{ZY} is the mass of material Z used to prepare ZY and $m_{sln,Z}$ is the total mass of the solution including m_{ZY} , m_{YZ} and m_{dil} being the mass of the added solvent, if further dilution is necessary. For better clarification, the area A_Z is depicted in the box called ZY in Fig. 1. It is shaded in two colours since it is a result of the two isotopes of the element of interest. In a similar way the area A_{YZ} (blue shaded in box ZY of Fig. 1) can be expressed as:

$$A_{\rm YZ} = k_{\rm YZ} \beta_{\rm Y} \frac{m_{\rm YZ}}{m_{\rm sln,Z}} \tag{14}$$

¹ The International Union of Pure and Applied Chemistry (IUPAC) does not recommend a single symbol for the substance content but n/m [19]. In this publication the symbol β is used for the sake of briefness.



Fig. 1 Schematic presentation of the ICeGIM method. The upper part of the figure shows the mass spectrometry experiment including the measurement of the standard material Z, the two spike materials A and B and the blend AB. The masses m_{AB} and m_{BA} are the amounts of material A and material B used for the preparation of AB, respectively. The lower part shows the ion chromatography experiment. This includes

All quantities occurring in Eq. 14 are analogously defined to those in Eq. 13. The ratio f_Z of these two areas is:

$$f_{\rm Z} = \frac{A_{\rm Z}}{A_{\rm YZ}} = \frac{k_{\rm Z}}{k_{\rm YZ}} \frac{\beta_{\rm Z}}{\beta_{\rm Y}} \frac{m_{\rm ZY}}{m_{\rm YZ}}$$
(15)

In the case of the two other blends (AY and BY) the ratio of the two peak areas can be expressed analogously.

$$f_X = \frac{A_X}{A_{YX}} = \frac{k_X}{k_{YX}} \frac{\beta_X}{\beta_Y} \frac{m_{XY}}{m_{YX}}, X \in \{A, B\}$$
(16)

Since the sensitivity coefficient *k* is constant for a specific ion, the following is valid.

$$\frac{k_{\rm A}}{k_{\rm YA}} = \frac{k_{\rm B}}{k_{\rm YB}} = \frac{k_{\rm Z}}{k_{\rm YZ}} \tag{17}$$

measurements of three blends. These blends consist of the internal standard Y and of the materials A, B or Z. The masses (m_{YX} or m_{XY} with $X \in \{A, B, Z\}$) are the amounts of the corresponding material used to prepare the three blends AY, BY and ZY. All unknown (or wanted) quantities are marked with a question mark

From Eq. 15 follows:

$$\frac{k_{\rm YZ}}{k_{\rm Z}} = \frac{1}{f_{\rm Z}} \frac{\beta_{\rm Z}}{\beta_{\rm Y}} \frac{m_{\rm ZY}}{m_{\rm YZ}} \tag{18}$$

Now combining Eqs. 15 and 18 allows to express β_A and β_B as:

$$\beta_{\rm A} = w_{\rm Z} \frac{1 + K_2 r_{\rm Z,2}}{M_1 + K_2 r_{\rm Z,2} M_2} \frac{m_{\rm ZY}}{m_{\rm YZ}} \frac{m_{\rm YA}}{m_{\rm YZ}} \frac{f_{\rm A}}{f_{\rm Z}}$$
(19)

and

$$\beta_{\rm B} = w_{\rm Z} \frac{1 + K_2 r_{\rm Z,2}}{M_1 + K_2 r_{\rm Z,2} M_2} \frac{m_{\rm ZY}}{m_{\rm YZ}} \frac{m_{\rm YB}}{m_{\rm BY}} \frac{f_{\rm B}}{f_{\rm Z}},\tag{20}$$

respectively. These two expressions (Eqs. 19 and 20) can be inserted into Eq. 9, which then can be solved for K, leading

to:

tive of the isotopic composition.

$$K_{2} = \frac{\frac{m_{YA}}{m_{AY}} f_{A} m_{AB} \left(r_{A,2} - r_{AB,2} \right) + \frac{m_{YB}}{m_{BY}} f_{B} m_{BA} \left(r_{B,2} - r_{AB,2} \right)}{\frac{m_{YA}}{m_{AY}} f_{A} m_{AB} \left(r_{B,2} - r_{B,2} r_{A,2} \right) + \frac{m_{YB}}{m_{BY}} f_{B} m_{BA} \left(r_{A,2} r_{AB,2} - r_{A,2} r_{B,2} \right)}$$
(21)

All steps leading to Eq. 21 are shown in the electronic supplement of this publication. With the knowledge of K_2 also w_A and w_B can be calculated, by using Eqs. 3 and 4. This applies, in the case that additional mixtures of Z + A and Z + B are prepared. At this point it should be added, that w_A and w_B can also be expressed only in terms of measured quantities (so without directly using K_2). But since these expressions are quite long they are only shown in the electronic supplement. In short: Eqs. 19 and 20 can be rearranged yielding:

$$w_{\rm A} = w_{\rm Z} \frac{1 + Kr_{\rm Z}}{1 + Kr_{\rm A}} \frac{M_1 + Kr_{\rm A}M_2}{M_1 + Kr_{\rm Z}M_2} \frac{m_{\rm ZY}}{m_{\rm YZ}} \frac{m_{\rm YA}}{m_{\rm AY}} \frac{f_{\rm A}}{f_{\rm Z}}$$
(22)

and

$$w_{\rm B} = w_{\rm Z} \frac{1 + Kr_{\rm Z}}{1 + Kr_{\rm B}} \frac{M_1 + Kr_{\rm B}M_2}{M_1 + Kr_{\rm Z}M_2} \frac{m_{\rm ZY}}{m_{\rm YZ}} \frac{m_{\rm YB}}{m_{\rm BY}} \frac{f_{\rm B}}{f_{\rm Z}}$$
(23)

Inserting Eq. 21 into the last two equations yields equations S.43 and S.44 in the supplement. From Eq. 21 it can easily be seen that ICeGIM allows the determination of absolute isotope ratios without knowing the purity of the spike materials and that therefore their associated uncertainties do not contribute to the uncertainties of the absolute isotope ratios. This feature is especially useful if the spike material is not a high purity metal but a chemical compound with unknown stoichiometry and molar mass (e.g. Li₂CO₃). Actually, even knowledge of w_Z is not needed for the determination of absolute isotope ratios. Hence, material Z can be any material with a similar chemical composition, but it is an important feature of the ICeGIM method that if you are only interested in the isotope ratios alone, no material Z is needed (see Eq. 21). Also the molar masses of the materials A and B (M_A, M_B) are not needed. Equation 2 shows another characteristic of ICeGIM. Unlike GIM, ICeGIM does not require knowledge of the atomic masses of the isotopes. This is due to the usage of ion chromatography, since here all isotopes are detected as one signal. Besides ion chromatography, other analytical techniques could also be used. These might be inductively coupled plasma atomic emission spectrometry, conductometry, quantitative nuclear magnetic resonance (qNMR) or any other method which delivers signals being proportional to the substance content of the analyte irrespec-

Simulation and uncertainty consideration

To test the advancement of the GIM method presented above, a simulation has been performed. The simulation can be found in the electronic supplement of this publication. The purpose of this simulation was to demonstrate the principal of ICeGIM and also to assess the achievable uncertainties. The supplement enables potential users to perform their own calculations using their own data. In this simulation copper was chosen as the possible two-isotope system. Real values and realistic estimates of the uncertainties associated with the input quantities were used to assess the achievable uncertainties. For example, in the simulation the NIST material 3114 [20] was used as the certified reference material Z. In Table 1, the relative uncertainties used in this simulation are listed. These uncertainties are typical for each of the input quantities. Since ICeGIM depends on IC measurements the achievable precision of IC is crucial. Brennan et al. [21] showed that by applying internal standardization analyte anion mass fractions can be determined with relative expanded uncertainties as low as 0.2%. This value can be seen as upper limit, since the peak area ratio is only one part of the uncertainty budget reported by Brennan et al. Also Röhker et al. [22] reported similar relative uncertainties for Li, Na, K, Mg and Ca. Therefore, 0.1% is a reasonable but conservative

 Table 1
 Relative uncertainties of all input quantities used in this simulation

relative uncertainty	value in %
$\frac{1}{u(f_{\rm A})}$	0.10
$u(f_{\rm B})$	0.10
$u(m_{AB})$	0.022
$u(m_{\rm BA})$	0.019
$u(m_{\rm YA})$	0.028
$u(m_{\rm AY})$	0.053
$u(m_{\rm YB})$	0.029
$u(m_{\rm BY})$	0.057
$u(r_{\rm A,2})$	0.010
$u(r_{\mathrm{B},2})$	0.010
$u(r_{AB,2})$	0.0010

estimate for the relative uncertainty associated with f_A or f_B . The whole data set can be found in the supplement part 2.

The calculation of the associated uncertainty of K_2 was done by a Monte Carlo simulation following internationally accepted rules and recommendations [23–25]. The result obtained using 1 000 000 trials is shown in Fig. 2 and $K_2 = 0.9455(33)^{\text{mol V/mol V}}$, with k = 2, was determined. The relative expanded uncertainty U_{rel} is 0.35%, which is system. This approach can be generalized for systems with an arbitrary number of isotopes N_{iso} . The above-described mathematical problem can be transformed into a matrix equation. The whole transformation is given in the supplement.

$$\mathbf{A}\,\mathbf{k} = \mathbf{b} \tag{24}$$

Matrix A is defined as:

$$A_{i,j} = (r_{AX(i+1),i+1} \cdot r_{A,j+1} - r_{X(i),i+1} \cdot r_{A,j+1}) \cdot m_{YX(i+1)} \cdot m_{AY} \cdot m_{X(i+1)A} \cdot f_{X(i+1)} - (r_{AX(i+1),i+1} \cdot r_{X(j),i+1} - r_{A,i+1} \cdot r_{X(j),i+1}) \cdot m_{X(i+1)Y} \cdot m_{YA} \cdot m_{AX(i+1)} \cdot f_{A}$$
(25)

in the same order of magnitude if compared with previous two-isotope GIM experiments, such as Malinovsky et al. (carbon, $U_{rel} = 0.22\%$) [26], or De Bièvre et al. (boron, $U_{rel} = 0.26\%$) [27]. At this point it must be stressed, that such a comparison of different isotope systems can only be a general orientation. Nevertheless the simulation shows that with ICeGIM comparable uncertainties could be obtained. It should be added that correlation was not considered in the simulation. Therefore, it is likely that lower uncertainties can be achieved.

i and *j* are running variables (from 1 to $N_{iso} - 1$), X(i) is a function returning the *i*th letter of the alphabet (e.g. X(2)=B), and all other quantities are analogously defined as in the above-mentioned two-isotope system.

The vector \mathbf{k} contains all the *K*-factors, therefore it is defined as:

$$k_i = K_{i+1}, i \in \{1, N_{\text{iso}} - 1\}$$
(26)

And finally, vector **b** is defined as:

$$b_{i} = -\left[\begin{pmatrix} r_{AX(i),i} - r_{X(i),i+1} \end{pmatrix} \cdot m_{YX(i+1)} \cdot m_{AY} \cdot m_{X(i+1)Y} \cdot f_{X(i+1)} + \\ \left(r_{AX(i+1),i+1} - r_{A,i+1} \right) \cdot m_{X(i+1)Y} \cdot m_{YA} \cdot m_{AX(i+1)} \cdot f_{A} \end{bmatrix}$$
(27)

Generalization

In the second section of this publication, the mathematics of the ICeGIM method have been derived for a two-isotope



By multiplying Eq. 24 with A^{-1} (the inverse of **A**), the wanted *K*-factors can be calculated. Note, inverting a matrix is computationally expensive and for larger systems it is advisable to use methods like Gaussian elimination, Cholesky decomposition or LU decomposition. The latter is implemented in the accompanying supplement to this paper.



Applying Cramer's rule allows a generic solution for the K-factors to be formulated. It is noteworthy that Ouerdane et al. [17] also employed Cramer's rule to solve IDMS equations.

$$K_{i+1} = \frac{\det\left(\mathbf{A}_{i}\right)}{\det\left(\mathbf{A}\right)}$$
(28)

 A_i can be formed by replacing the *i*th column with the vector **b**. The above equations allow to calculate the wanted *K*-factors for systems with an arbitrary number of isotopes and therefore also the uncertainties associated with them.

Conclusion and prospects

For the determination of absolute isotope ratios, knowledge of the purities of the spike materials used for the preparation of the gravimetric mixtures was needed. In this publication, an advancement of the primary method "gravimetric isotope mixtures" is presented. The method described above combines mass spectrometry and ion chromatography measurements. This approach has a distinct advantage over the classical GIM method. The purities of the spike materials are not needed any longer and therefore also do not contribute to the uncertainty of the absolute isotope ratios. This is a

big advantage since ICeGIM allows to determine absolute isotope ratios in cases where no certified content reference material exists. This is a huge advantage especially in cases where no high purity metals are available but only salts with unknown stoichiometry and molar mass. In principle, the purities of the spike materials could be very low and do not have to be known with high accuracy. This is the case as long as measurements are not biased by it. For example, if the matrices (impurities) of the spike materials are very different from the sample this would lead to quite different conditions in the argon plasma and, therefore leading to a different mass bias. The mathematical background of ICeGIM for a two-isotope system has been presented in detail. An initial simulation with realistic data demonstrated that, in terms of achievable uncertainties. ICeGIM is a reasonable alternative, when determining the spike purity is not possible. Moreover, it is worth further developing and testing this alternative method. In the last section the mathematical ansatz was generalized, so that the ICeGIM approach can be applied to any number of isotopes. First practical tests will be presented in a follow-up publication.

Appendix. List of quantities

 Table 2
 Quantities used in the GIM method

Symbol	Unit	Quantity	Property
K	mol V/mol V	Mass bias correction factor	Result
$w_{ m A}$	g/g	Mass fraction of analyte element in enriched parent solution A	Unknown
w_{B}	g/g	Mass fraction of analyte element in enriched parent solution B	Unknown
wz	g/g	Mass fraction of analyte element in natural standard solution Z	certified
β_{A}	mol/g	Substance content of enriched analyte element in parent solution A, equal to w_A/M_A	Unknown
$\beta_{ m B}$	mol/g	Substance content of enriched analyte element in parent solution B, equal to $w_{\rm B}/M_{\rm B}$	Unknown
R _i	mol/mol	Amount ratio of spike isotope (2) over reference isotope (1) in sample, standard, parent or mixture <i>i</i> with $i \in \{A, B, Z, AB\}$ and $R = Kr$	Unknown
r _i	V/V	Signal intensity ratio of spike isotope (2) over reference isotope (1) in sample, standard, parent or mixture <i>i</i> with $i \in \{A, B, Z, AB\}$ and $R = Kr$	Measured (MS)
$m_{\rm AB}$	g	Mass of parent solution A blended with parent solution B to yield mixture AB	Measured (balance)
m _{BA}	g	Mass of parent solution B blended with parent solution A to yield mixture AB	Measured (balance)

MS is short for mass spectrometry

Symbol	Unit	Quantity	Property
m _{AY}	g	Mass of parent solution A blended with internal standard solution Y to yield mixture AY	Measured (balance)
$m_{\rm YA}$	g	Mass of internal standard solution Y blended with parent solution A to yield mixture AY	Measured (balance)
$m_{\rm BY}$	g	Mass of parent solution B blended with internal standard solution Y to yield mixture BY	Measured (balance)
$m_{\rm YB}$	g	Mass of internal standard solution Y blended with parent solution B to yield mixture BY	Measured (balance)
mZY	g	Mass of standard solution Z blended with internal standard solution Y to yield mixture ZY	Measured (balance)
$m_{\rm YZ}$	g	Mass of internal standard solution Y blended with standard solution Z to yield mixture ZY	Measured (balance)

Table 3 Quantities (masses) used in the preparation of the blend needed for the IC measurements

Table 4 Quantities obtained by IC

Symbol	Unit	Quantity	Property
f _A	1	Ratio of the area of the chromatographic peak of the analyte element over the area of the chromatographic peak of the internal standard element (ion chromatography) in the mixture AY of parent solution A and internal standard solution Y	Measured (IC)
$f_{\rm B}$	1	Ratio of the area of the chromatographic peak of the analyte element over the area of the chromatographic peak of the internal standard element (ion chromatography) in the mixture BY of parent solution B and internal standard solution Y	Measured (IC)
fz	1	Ratio of the area of the chromatographic peak of the analyte element over the area of the chromatographic peak of the internal standard element (ion chromatography) in the mixture ZY of standard solution Z and internal standard solution Y	Measured (IC)

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Declarations

Conflict of interest The authors declare no competing interests.

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