

Recommendations on finite element modelling of non-seismic excitation in soil-structure interaction problems

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Abstract. Nowadays geotechnical engineering firms have powerful software tools to extent their consulting business also into dynamic soil-structure interaction, which before has been restricted to a rather small community of specialized experts in this field, and they certainly do. This is particularly true with respect to non-seismic sources, that is all kinds of human induced vibrations. Hence, there is a demand from clients as well as from contractors to have guidance on the requirements as well as the limits of numerical modelling of soil-structure interaction. From the literature as well as from relevant standards, recommendations for the numerical modelling of soil-structure interaction problems involving seismic actions are well known, e. g. ASCE/SEI 4-16. There are, however, some particularities when dealing with human-induced vibrations, which are absent in seismic analyses. For human-induced excitations very little specific guidance has been published in the past. A machine foundation on a homogeneous half space excited by harmonic loads with excitation frequency between 4 Hz and 64 Hz has been analysed by means of several commercially available software packages. Parametric studies have been performed to verify if recommendations for seismic soil-structure analyses are valid for non-seismic analyses as well. This paper provides details on the benchmark example and the most important conclusions from the undertaken parametric studies.

1. Introduction

Within the framework of the DGGT – German Society for Geotechnics the working groups AK 1.4 “Soil Dynamics” and AK 1.6 “Numerical Methods in Geotechnics” realized that there is an increasing demand to the Geotechnical Engineering profession to provide the design of geotechnical structures not only with respect to stationary loads. In urban environments, to give an example, structures next to existing underground rail lines are subjected to vibrational loads, and mitigation measures must be incorporated in the design of foundations. At the same time upgrades of existing rail lines on soft ground require dynamic analyses of the subground’s response to vibrational loads to determine a cost-effective upgrade procedure. Last, but not least, the design of foundations of vibratory machines must be such that the vibratory emissions do not cause any soil compaction in the machine’s surroundings. These demands provided an incentive to manufacturers of geotechnical software products to develop modules



for dynamic soil-structure analyses, which are now widely available. However, a lack of expertise in the geotechnical engineering community on the pitfalls of dynamic soil-structure analyses is well observed.

The advances of computer technology in the last two decades or so let numerical geotechnical analyses mature to the extent that not only 2D non-linear static and dynamic FE analyses can well be considered as the state of practice, but that from a hardware perspective also 3D non-linear static FE analyses are feasible nowadays. Still, contrary to seismic loads, for on- or near-surface vibrating sources the suitability of 2D analyses is rather limited, except for some particularly axisymmetric cases. Hence, a full 3D analysis is mostly required.

Though recommendations for dynamic analyses for seismic loads are readily available, there are some particularities in dynamic soil-structure analyses due to man-made dynamic loads in a built environment which must be addressed. Hence, the two working groups of DGGT joined forces to compile recommendations for numerical dynamic soil-structure analyses subjected to man-made loads, targeted to practitioners using commercially available software. This paper provides details on the benchmark example and the most important conclusions from the parametric studies.

2. Dynamic analysis in a geotechnical context

If loads are variable with respect to time t , so will be displacements $\mathbf{u}(t)$ and their derivatives with respect to time, velocities $\mathbf{v}(t) = \dot{\mathbf{u}}(t)$ and accelerations $\mathbf{a}(t) = \ddot{\mathbf{u}}(t)$. Since material properties of soils are intrinsically non-linear, the equation of motion comprises a system of non-linear ordinary second order differential equations

$$\mathbf{K}(\mathbf{u}, t)\mathbf{u}(t) + \mathbf{M}\ddot{\mathbf{u}}(t) = \mathbf{p}(t), \quad (1)$$

with \mathbf{K} the tangential stiffness matrix, \mathbf{M} mass matrix, and \mathbf{p} the load vector. The solution of equation (1) must be obtained by means of numerical methods.

One peculiarity of soil as a subject of structural analyses is its spatial dimension. Soil, as the bearing medium of a foundation, and even more as a wave transmitting medium can and must be considered as being an infinite or semi-infinite structure. At first hand, Finite Elements, with, as the name points out, it's restriction to "finite" structures seems to be an unfavorable tool for soil dynamic analyses. But due to the tremendous advantages of the Finite Element Method procedures have been developed, first at the end of the 1960s, to extent the applicability of finite elements to infinite structures. However, due to its approximative nature a finite element model of a soil-structure system must be well designed to provide sufficiently accurate results.

A second peculiarity of soil is its non-linearity. It is well known that the dynamic properties of soil strongly depend on stress state, stress history, and load amplitude, amongst other factors. Hence, strictly speaking it is prohibitive to use linear elastic soil properties. But, on the other hand, due to its computational costs a non-linear dynamic analysis of an infinite half-space is prohibitive as well. The compromise is to use non-linear soil models for very simplified problems, like 1D site response analyses, while for very complex and large soil bodies linear elastic soil properties are used, with the applied stiffness properties determined for the stress and deformation level under the true loads. The latter requires an iterative loop, because the final stress level must be estimated prior to the first analysis, and must be adjusted in a later analysis according to the stress and deformation computed in the preceding analysis. In cases at which that assumption is reasonable the equation of motion (1) becomes the system of linear ordinary second order differential equations (2). An additional damping matrix \mathbf{C} is required to account for dissipation of energy which would otherwise be accounted for by the nonlinear constitutive model.

$$\mathbf{K}\mathbf{u}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{M}\ddot{\mathbf{u}}(t) = \mathbf{p}(t) \quad (2)$$

Though the members of the subgroup considered themselves as "advanced" with respect to their knowledge about numerical analysis in soil dynamics, it was the consensus within the group that the application of several commercial software packages to solve the same problem set would not yield identical results, but results within a certain range only, and that a large span of such a range would clearly identify a pitfall of soil dynamic finite element analysis. As the first step the subgroup therefore

agreed upon a real world inspired benchmark problem set to be solved by each of the subgroup members with the software packages available to them.

A description of the benchmark problem set, the results of the analyses, intermediate results of sensitivity studies, as well as some recommendations on the proper design of a Finite Element model of an infinite half-space subjected to human-induced dynamic loading are provided in the subsequent section.

3. Description of the benchmark example

The steady-state response of a homogeneous half-space supporting a machine foundation subjected to a uniform sinusoidal load has been agreed upon as a benchmark problem.

3.1. Geometric details

Dimensions of the foundation, load setting and evaluation point locations are given in figure 1.

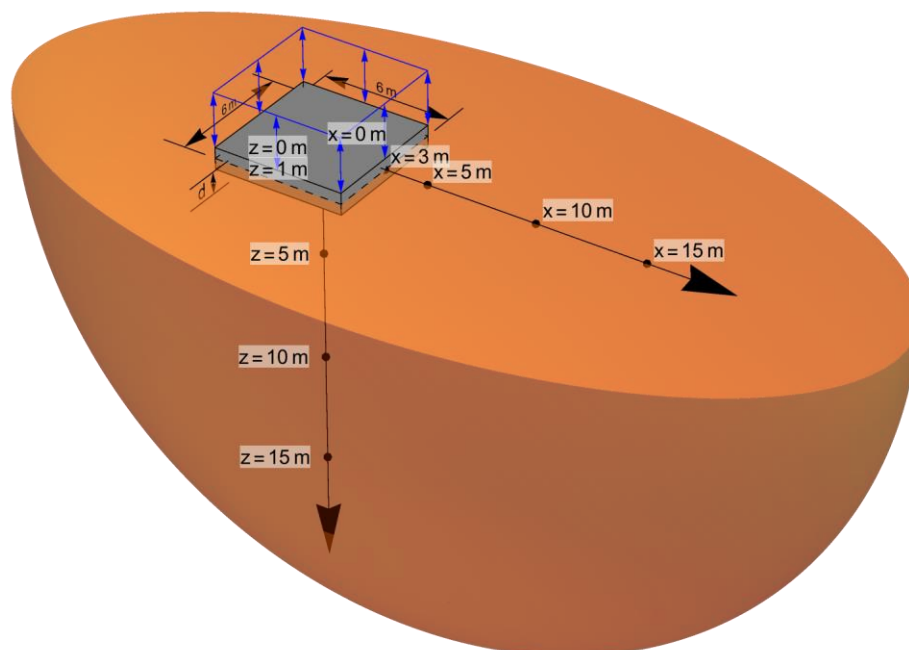


Figure 1. Geometric details of the benchmark example

3.2. Material properties

The foundation is composed of concrete and is supposed to support a fictitious machine, yielding a uniform static soil pressure due to foundation and machine of $\sigma_v = 47.1 \text{ kN/m}^2$. At a thickness of $d = 1.0 \text{ m}$ and dimensions $A = 6 \text{ m} \times 6 \text{ m}$ the aggregated mass is $m = \sigma_v \cdot 36 \text{ m}^2/g = 173 \text{ t}$. For the analyses the total mass is converted into an equivalent mass density of the foundation $\rho_{F,eq} = 4811.4 \text{ kg/m}^3$. Young's modulus of the foundation was chosen as $E = 30\,000 \text{ MN/m}^2$ and Poisson's ratio as $\nu = 0.15$.

For the soil a Young's modulus $E = 224 \text{ MN/m}^2$, Poisson's ratio $\nu = 0.40$, and mass density $\rho_B = 18 \text{ kN/m}^3/g = 1835 \text{ kg/m}^3$ was selected, which is equivalent to a shear modulus $G_d = 80 \text{ MN/m}^2$ and shear wave velocity $c_s = 209 \text{ m/s}$. Damping was prescribed as $\zeta = 0.02$ hysteretic damping in the sense of a complex valued shear modulus $G_{dc} = G_d(1 + 2i\xi)$.

In linear elastic time domain analyses this hysteretic damping cannot be applied straightforwardly. For the case at hand, with a small number of sinusoidal loads, the method of choice is to determine a suitable set of Rayleigh damping parameters from the known or estimated natural frequencies to approximate a hysteretic damping. The actual choice of the Rayleigh damping parameters is of limited significance because the radiation of waves into the half space creates much more dissipation of energy than the material damping.

3.3. Loads and boundary conditions

A uniform sinusoidal load of infinite duration and load amplitude of 36 kN was selected, to be applied at frequencies $f = 4$ Hz, 8 Hz, 16 Hz, 32 Hz, 48 Hz, and 64 Hz.

The boundary condition to approximate the infinite half-space was mostly selected as the well-known Lysmer-Kuhlemeyer transmitting boundary (LKTb) [1][2]. For time domain analyses this is usually the only choice available in commercial software. Other methods applied were Perfectly Matched Layers (PML) [3], as well as Thin Layer Method [4] and Boundary Elements [5], but for the sake of brevity we restrict our discussion to LKTb and PML.

3.4. Numerical aspects

3.4.1. Computational domain

The problem set requested the steady state solution for a small number of sinusoidal loads. Furthermore, a linear elastic soil medium had to be used, and the material damping of the half-space was given in terms of hysteretic damping. All this strongly instigates the use of a frequency domain solution.

Though, due to the fact that material response in geotechnical design is intrinsically non-linear, most FE software packages with a focus on geotechnical engineering only provide a time domain solution, and hence time domain analyses have been conducted by several participants.

3.4.2. Model dimensions

The mesh size strongly depends on the applied radiation boundary technique. For the sake of brevity we restrict our discussion to Lysmer-Kuhlemeyer transmitting boundaries and Perfectly Matched Layers.

Because LKTb are perfect absorbers in a 1D setting only, partial wave reflections occur at the model boundaries in 2D or 3D models and will pollute the solution if the mesh dimensions were not extended far beyond the region of interest. According to [1] the model shall be extended by at least 1.5 Rayleigh wave lengths beyond the region of interest.

A PML model consists of the region of interest surrounded by absorbing layers that absorb almost perfectly propagating waves of all non-tangential incident angles and all non-zero frequencies. PML can be viewed either as a mean to stretch their local coordinates, or as an artificial material with very particular damping properties. Though PML, unlike LKTb, are not prone to reflections of waves at the interface between the regular FEM region and the PML layer, perfect absorption of waves would occur only in a non-discrete infinitely long PML. Hence the dimensions of the PML layer must be chosen such that any reflections inside the PML will fully attenuate before travelling back into the regular FEM region of the model. Suggestions from several references concerning the extent of the regular domain of a FEM mesh comprising a PML domain for wave propagation into a half-space, as well as the dimensions of the PML itself have been summarized in [8]. With only one exception these references provide mesh dimensions and PML dimensions with respect to the length of a foundation only, with no hints which frequencies were used. Hence the dimensions in terms of wavelengths cannot be deducted from those references. The results obtained in [8] lead to the conclusion that the PML can be attached very close to the region of interest, and that the PML length does not need to extend beyond one wavelength.

3.4.3. Mesh density, time integration method and time step size

The complexity of solving the equation of motion (2) increases to the 3rd power of the number of degrees of freedom N_{DOF} and linearly with the number of timesteps $N_{\Delta t}$, that is the complexity is $\mathcal{O}(N_{\Delta t} \cdot N_{\text{DOF}}^3)$. In particular for solving 3D models, it is desirable to reduce N_{DOF} as well as $N_{\Delta t}$ to a minimum.

In wave propagation problems, like the reference problem herein, the distance between nodes must be chosen such that the wave of shortest wavelength can be captured sufficiently accurate by the piecewise polygon spanned by the nodes. From [9] and [10] 8-12 nodes per wavelength are recommended from numerical experiments of soil-structure interaction under seismic excitation, from [12] even 20 nodes per wavelength. In [11] it is recommended to perform a stepwise mesh refinement such that if the

last mesh refinement does not yield an increase in accuracy above some threshold, e. g. 10 %, then production analyses may be performed with the second to the last mesh density.

An additional requirement, common to all types of FEM analyses, is that the mesh density must be such that any stress and deformation gradient will be captured sufficiently accurate as well. In the case at hand strong stress and deformations gradients will occur at the foundation edges below the foundation and next to the foundation, respectively. Hence this requirement is a local requirement only, while the wavelength condition must be fulfilled globally.

The choice of shortest node-to-node distance is limited by computational resources only, except that for time domain analyses the mesh density must not fall below a limit set by the time step size, the choice of which in turn depends on the choice of the time integration method, see below for more details. Hence, a local mesh refinement must always be accompanied by an appropriate reduction of the time step size. There is no lower limit to the mesh density in frequency domain analyses.

After discretizing the spatial domain the solution of the semi-discrete equation of motion (2) requires the discretization of time into small time steps Δt and the application of an appropriate time integration algorithm to solve equation (2). Any time step integration method must fulfil the generic conditions

- accuracy,
- stability, and
- convergence.

Detailed discussions of this subject can be found, e. g., in [6] and [7] and references therein, hence we restrict our discussion to aspects relevant to the reference problem and the solutions provided by the participants. Stability and convergence are necessary, but not sufficient conditions for accuracy. Many time integration methods have been suggested in the literature since the late 1950s, but only a very limited number of methods are implemented in current FEM software. Procedures used by the participants are the Newmark method, the HHT method, and flavors of the Central Difference method, see [6] and [7] and references therein. The differences between methods are the assumptions made for the distribution of accelerations and velocities within one or two consecutive time steps, the weights applied to them during quadrature over time, as well as how the equilibrium condition expressed by the equation of motion (2) is incorporated into the procedure.

Accuracy means that the time step size must be chosen such that the smallest wave period within the load function is captured sufficiently accurate. From the literature, e. g. [6], $\Delta t \leq 0.01 T_{n,\min}$ is recommended as a very safe but in many cases an overly conservative choice, with $T_{n,\min}$ being the smallest natural period of the model. In [13] it is recommended that if stability requirements do not dictate otherwise, $\Delta t/T_g \leq 0.1$ already yields an error below 10 %, and $\Delta t/T_g \leq 0.04$ yields a negligible error, with T_g the smallest period contained in the forcing function.

Stability means that the solution will not grow above all bounds when advancing in time. It is expressed as a condition for the ratio of time step size to smallest natural period $\tau = \Delta t/T_{n,\min}$ of the model. Some time integration methods do not impose any limit on τ , while others do. The former are called “unconditionally stable”, while the latter are called “conditionally stable”. Often a distinction is also made between “implicit” and “explicit” methods, but the criteria are not univocal in the literature (compare e. g. [6] and [7]).

An important property of time integration methods is the numerical (also: “algorithmic”) damping. As with physical damping, the application of a numerical time integration method may lead to an amplitude reduction and to a period elongation. While by physical damping both phenomena occur concurrently and at a fixed relationship, by numerical damping they occur independently of each other and not necessarily concurrently. Clearly, physical and numerical damping interfere with each other, but do not necessarily accumulate. For example, in [16] it is stated that the Newmark trapezoid method cancels out all physical damping for large ratios of $\Delta t/T_{n,\min}$ if applied to the equation of motion of a damped SDOF oscillator. Only a small number of papers from the literature deal with the impact of physical damping on the damping properties of time integration methods, see e. g. [13] and [14].

As already mentioned in section 3.4.2 the time step size is limited for a given mesh density, though not vice versa. The limit is imposed by the CFL condition [17]. Essentially it states that for a stable computation the time step size must not exceed the time a wave needs to travel over the distance between two grid nodes. The CFL condition is strictly applicable to finite difference meshes. To the knowledge of the authors there is no paper which proves the applicability to FEM methods. From the literature, e. g. [6], it is nevertheless recommended to comply with the CFL condition at least when applying explicit time integration methods.

The requirements for mesh density and time step can be summarized as follows:

$$\Delta h_K \leq \frac{c}{10 f_{\max}}, \quad \Delta t \leq \min \left\{ \frac{T_{n,\min}}{10}; \frac{\Delta h_E}{c}; \frac{T_{p,\min}}{10} \right\} \quad (3)$$

with Δh_K the smallest distance between nodes, c the wave velocity (usually shear wave), f_{\max} the highest frequency in the forcing function, $T_{n,\min}$ the lowest natural period of the model, Δh_E the smallest element edge length, and $T_{p,\min} = (f_{\max})^{-1}$. The first limit in equation (3) does not apply for unconditionally stable time integration methods.

4. Reference solution for model verification

The solution of a SDOF oscillator with the stiffness and damping coefficients taken from impedance tables [15] and the mass taken as the mass of the foundation according to section 3.2, assuming that the mass is a rigid body, is used as the reference solution for model verification. A harmonic load with amplitude 36 kN is used as excitation. From the transfer function between excitation at the mass and vertical displacement of the mass' centre the values according to table 1 have been obtained.

Table 1. Reference solution

f in Hz	4	8	16	32	48	64
u in μm	20.4	21.0	13.7	4.14	2.00	1.20

5. Submitted solutions

As can be seen from table 2 the solutions submitted by the participants cover a wide range of commercially available FEM software and the full range of potential features. All 2D solutions are axisymmetric models effectively approximating the square foundation by a circular foundation of equal area.

The results of the final solutions of all participants in terms of the displacements of the foundation's center are displayed in figure 2. It is apparent that the error at each frequency is less than 5 % of the reference solution, which is considered as an excellent agreement. The misfit between numerical solutions and reference solutions increases with excitation frequency, because for the reference solution the foundation was assumed to be rigid, which the more ceases to be a realistic assumption the higher the excitation frequency. Furthermore, it is apparent that the peak amplitude, that is the damped natural frequency of the foundation, is near 8 Hz.

The path which led to the set of results displayed in figure 2 was not straightforward for the entirety of the participants. Some contradicting results were submitted at first hand, which instigated several sensitivity studies. The results of some of these studies are summarized in section 6.

Table 2. Overview of submitted solutions

ID	Software	Spatial order	Time-/frequency domain	Radiation BC	Material damping type	Element shape half-space	Shape function order
1	Plaxis3D	3D	Time	LKTB	Rayleigh	Tet	2
2	Plaxis2D	2D	Time	LKTB	Rayleigh	Tri	4
3	SASSI	3D	Freq	TLM	Hysteretic	Hex	1
4	Tochnog	2D	Time	LKTB	Rayleigh	Quad	1
5	Numgeo	2D	Time	LKTB	Rayleigh	Quad	2
6	Ansys	3D	Freq	PML	Hysteretic	Tet	2
7	Ansys	2D	Freq	LKTB	Hysteretic	Tri	2
8	Ansys/SSI [5]	3D	Freq	BEM	Hysteretic	Matrix	—
9	Abaqus	3D	Freq	LKTB	Rayleigh	Hex	1

6. Results of sensitivity studies

6.1. Computational domain

The distribution of the surface deformation of solutions listed in table 2 are displayed in figure 3, and deformations below the foundations centre in figure 4. At 4 Hz all solutions are very close, except at the edge of the foundation. At 64 Hz a clear distinction between 2D and 3D solutions becomes apparent at the soil surface. The reason is that the axisymmetric solutions effectively use a circular foundation of radius 3.40 m which is larger than the half-width 3 m of the square foundation.

In the high frequency range, a circular foundation is not a conservative approximation of a square foundation of equal area. While the deformations of the foundation itself are overestimated by axisymmetric solutions, the deformations of the free field are underestimated by a factor of 2.

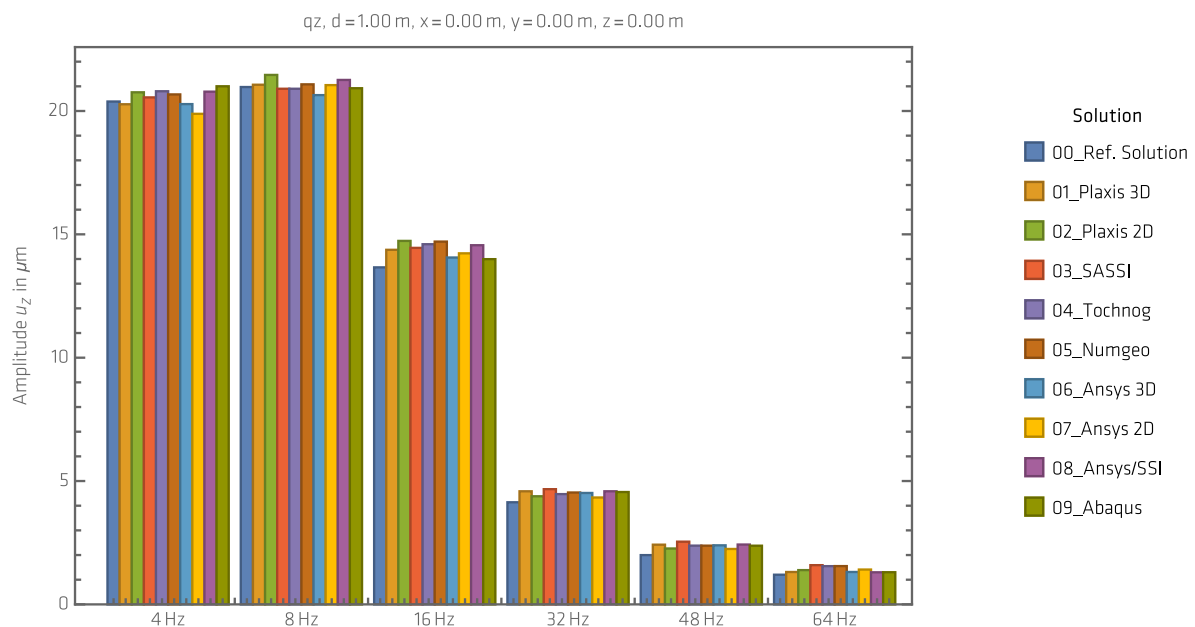


Figure 2. Vertical amplitude of foundation center of all solutions from table 2

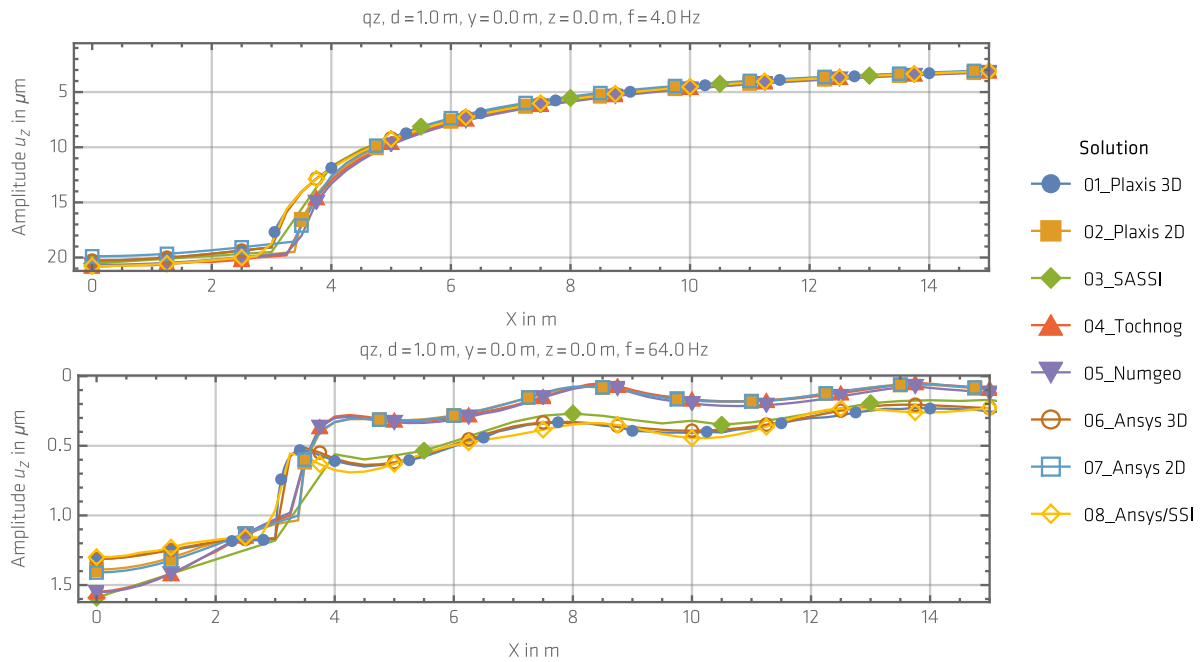


Figure 3. Vertical displacement amplitude at the soil surface, frequencies 4 Hz (top) and 64 Hz (bottom).

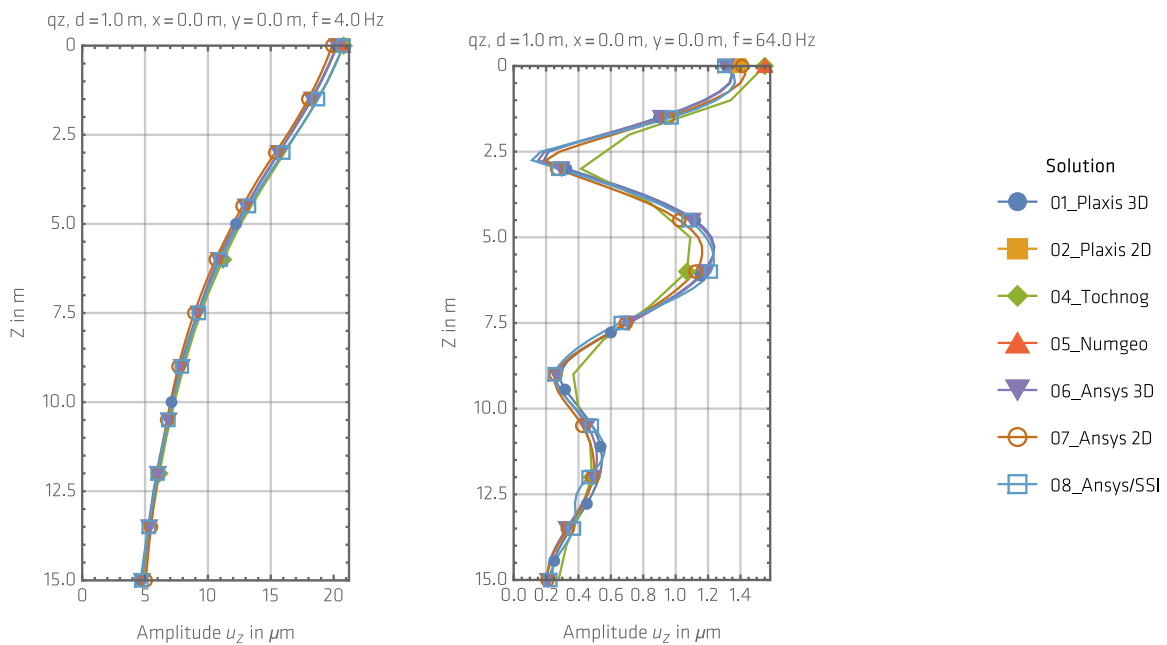


Figure 4. Vertical displacement amplitude below the foundations center, frequencies 4 Hz (left) and 64 Hz (right).

6.2. Model dimensions

Recommendations from the literature for model dimensions for the case of time domain solutions using LKTB are mostly provided in multiples of the foundation dimensions. Some participants followed these

recommendations, but the results scattered significantly between the participants. Therefore, a sensitivity study was performed to establish recommendations for the model dimensions in terms of the buffer length between the region of interest and the LKTB required to obtain reasonably accurate results. A 2D axisymmetric model in frequency domain was built with ANSYS and using LKTB as boundary conditions for this purpose.

The region of interest was taken as the area up to 16 m from the foundation center, which is 1 m beyond the most distant evaluation point. The dimensionless buffer length α was defined as the ratio of the distance from the region of interest to the Rayleigh-wave length, which for simplicity was approximated by the shearwave length. See figure 5 for details.

In total 12 models with buffer length ratio α between 1/8 and 10 were analyzed for all frequencies. The result in terms of the vertical displacement amplitude of the evaluation point closest to the transmitting boundaries is displayed in figure 6. It is apparent that the results turn insensitive to an increment in buffer length if $\alpha \geq 2$. Note that this result is beyond the recommendations of [1] which have been obtained by a very similar model.

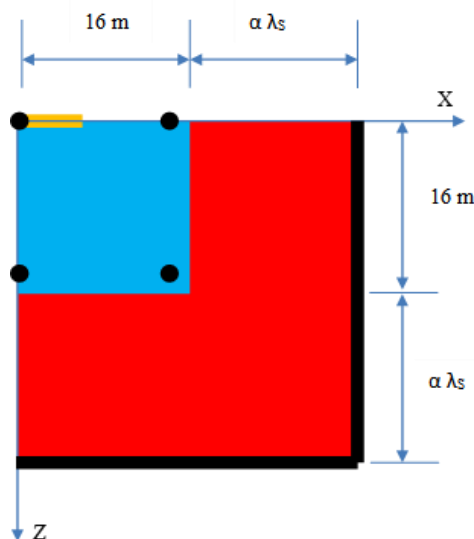


Figure 5. Dimensions of axisymmetric model for sensitivity study on buffer length between region of interest and LKTB. Blue: Region of interest with foundation at the upper left. Red: buffer. Black lines: LKTB. Black dots: evaluation points.

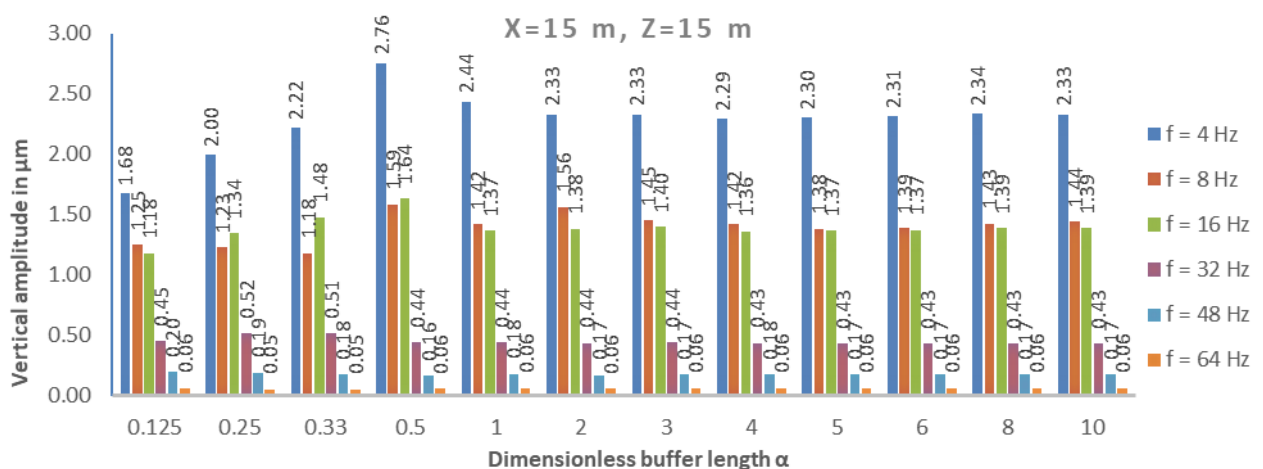


Figure 6. Impact of buffer length on the displacement amplitude at the evaluation point closest to the transmitting boundaries

6.3. Mesh density

Because of the large computational effort required for 3D analyses the number of DOFs should be kept at the lowest reasonable. The requirement becomes obvious considering the model size and element count shown in figure 7. Note that quadratic elements were used such that the total number of nodes is twice the number of element corner nodes. Therefore, the requirement of a mesh refinement near the foundation edge, an area with strong stress and deformation gradients, was investigated.

A 2D frequency model built with ANSYS was employed for that purpose. Because elements with midside nodes were used, the basic requirement is to discretize the model with five elements per shear wavelength λ_s . The mesh density was halved in the close vicinity to the foundation, and further halved at the soil-foundation interface. This mesh follows the recommendations according to section 3.4.3. It is displayed in figure 8 on the left. The mesh setting was then globally refined by dividing all elements by two in two consecutive steps. From table 3 it can be observed that the error with respect to the displacement of the foundation center was approximately 10 % of the reference solution at 4 Hz. The global refinement steps significantly improved the results.

The impact if only a local refinement is performed is shown in figure 8 center and right. While the element size at large remains as large as possible, the automatic mesh refinement is restricted to the close vicinity of the foundation such that the strong gradients there can be fully captured. Since the element size near the foundation of models F05-E20 and F05-E05-5 is approximately equal, the depth of the refinement apparently is the ruling improvement.

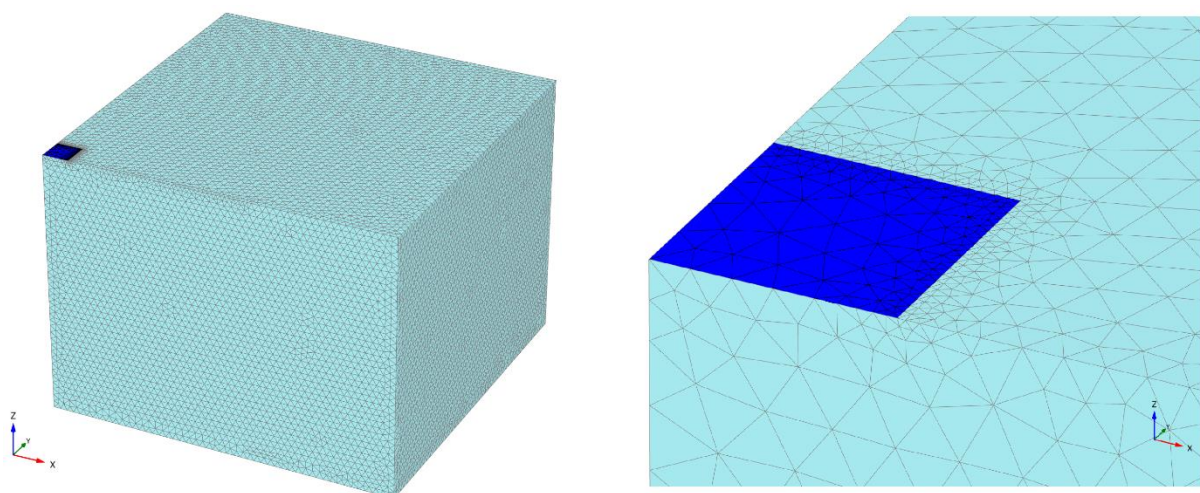


Figure 7. Example of an appropriately refined mesh near the foundation edge. Plaxis3D model at 16 Hz, largest/smallest element size ca. 80/10 cm.

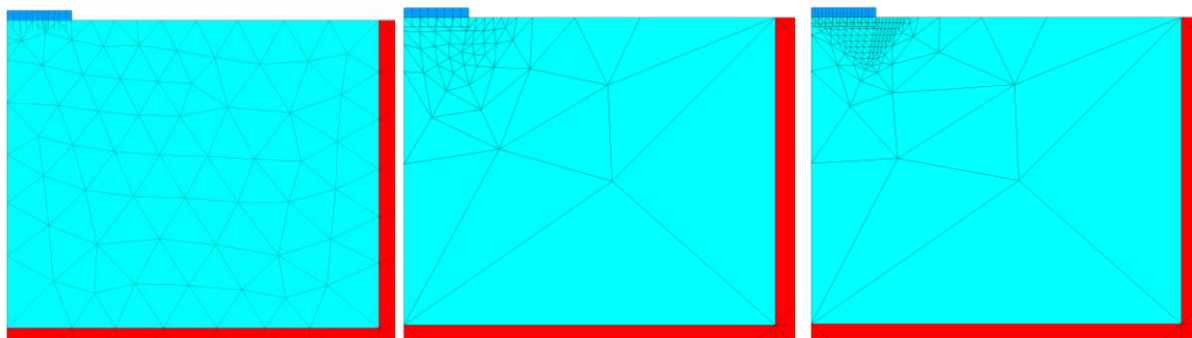


Figure 8: Meshes of the region of interest of models after table 3 (F05-E20, F05-E05-3, F05-E05-5, from left to right) for excitation frequency 4 Hz.

Table 3. Impact of global and local mesh refinements

Model designation	Global element size	Element size near foundation edge	Element size below foundation	$ u_z $ at foundation center in μm	
				4 Hz	64 Hz
Global mesh refinement					
F05-E05	$\lambda_S/5$	$\lambda_S/10$	$\lambda_S/20$	17.97	1.41
F05-E10	$\lambda_S/10$	$\lambda_S/20$	$\lambda_S/40$	19.48	1.41
F05-E20	$\lambda_S/20$	$\lambda_S/40$	$\lambda_S/80$	19.95	1.41
Local mesh refinement					
F05-E05-3 ^(a)	$\lambda_S/5$	$\lambda_S/5$	$\lambda_S/5$	19.85	1.41
F05-E05-5 ^(a)	$\lambda_S/5$	$\lambda_S/5$	$\lambda_S/5$	20.00	1.41
Reference solution after section 4				20.38	1.20

(a) last digit is the number of recursive element divisions

7. Summary and outlook

Some results of a study to compile recommendations for numerical analyses in soil dynamics are presented. The focus is on human induced vibrations in the elastic regime. Because most such problems are intrinsically three-dimensional, the computational costs in terms of mesh size, mesh density and time step size (if applicable) must be minimized. A machine foundation on homogeneous half-space subject to a sinusoidal load is used as a reference example. Some results of sensitivity studies are presented. The most important findings so far are as follows:

- Widely applied recommendations for model size in time domain analyses using Lysmer-Kuhlemeyer transmitting boundaries may be unconservative. The distance between the region of interest and model boundaries shall be at least two Rayleighwave lengths, while previously only 1.5 Rayleighwave lengths were recommended in the literature.
- Approximations of foundation geometry may lead to unconservative results as well. In the case at hand the approximation of a square foundation by a circular foundation of equal area leads to free field vibration amplitudes only half of the corresponding amplitudes than if a square foundation is applied. On the other hand, the vibrations of the foundation itself are overestimated by a circular foundation.
- A local mesh refinement in areas of strong stress or deformations gradients is strictly required.

Further studies will focus on the interaction of numerical and physical damping in connection with the non-proportional damping intrinsic to transmitting boundaries for time domain analyses.

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