

A Bayesian Framework for Simulation-based Digital Twins of Bridges

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Abstract

Simulation-based digital twins have emerged as a powerful tool for evaluating the mechanical response of bridges. As virtual representations of physical systems, digital twins can provide a wealth of information that complements traditional inspection and monitoring data. By incorporating virtual sensors and predictive maintenance strategies, they have the potential to improve our understanding of the behavior and performance of bridges over time. However, as bridges age and undergo regular loading and extreme events, their structural characteristics change, often differing from the predictions of their initial design. Digital twins must be continuously adapted to reflect these changes. In this article, we present a Bayesian framework for updating simulation-based digital twins in the context of bridges. Our approach integrates information from measurements to account for inaccuracies in the simulation model and quantify uncertainties. Through its implementation and assessment, this work demonstrates the potential for digital twins to provide a reliable and up-to-date representation of bridge behavior, helping to inform decision-making for maintenance and management.

Keywords

Digital Twins, Bayesian Inference, Bridge Monitoring, Uncertainty Quantification

1 Introduction

In recent years, the concept of digital twins has gained significant attention in a wide range of industries. The term was first introduced by Dr. Michael Grieves in 2002 [1] in the context of Product Lifecycle Management (PLM) as a digital representation of a physical object or system that is continuously updated with real-time data. Since then, this concept has been applied extensively in many different fields, such as the control of manufacturing processes, the monitorization of real-time data, or as a support for augmented reality (AR) and Internet of Things (IoT) implementations. In the field of engineering, digital twins have become a valuable tool for the assessment and management of physical systems, and bridges are no exception.

Despite their rapid adoption, there is still much to be accomplished regarding the homogenization and standardization of digital twins [2]. Under the definition of digital twin are included very distinct types of digital representations of physical objects, which are worthy of being treated independently. As a general classification based on the source of the information to be treated, we can differentiate geometry-, data- and simulation-based digital twins.

Geometry-based digital twins are usually based on 3D models or Building Information Models (BIM) that provide a detailed representation of the physical geometry of a bridge. The difference with classical systems representations lies in the use of information from the real object. Some examples applied to bridge engineering are the use of scanned point-cloud data to generate the geometry of the model [3, 4], the incorporation of BIM data into an AR model for monitoring the construction and operation of the bridge [5], or checking the clearance for widening projects [6].

Data-driven digital twins use real-time data from various sources, such as sensors, to create a digital representation of the physical system. They are typically focused on the operation, monitoring, and maintenance of the bridge and usually lack information about the system itself. Possible applications are the use of Key Performance Indicators (KPI) to assess the state of the bridge in real-time [7, 8] or as a basis for Structural Health Monitoring (SHM) analysis [9, 10].

Simulation-based digital twins can be understood as a combination of both, where real sensor data is used to en-

rich information from a geometrical model based on physical laws. This paper focuses on their potential to provide valuable information about the state of the bridge and their predictive capabilities.

1.1 Motivation

Standalone simulations have a long tradition in the field of civil engineering, particularly in the design and analysis of bridges. The use of finite element models (FEM) has become a standard tool for engineers to predict the behaviour of bridges under different loads and environmental conditions. In recent years, advances in computational capabilities have made it possible to use more complex models with greater detail, providing engineers with a better understanding of the performance of bridges. Furthermore, a plethora of commercial tools is available for the design and analysis of structures.

Simulation-based digital twins apply this accumulated knowledge from simulation in civil engineering and use it to develop connected models enriched by data from their physical counterparts which can be used for predictions and informed decision-making. Contrary to purely data-driven digital twins, they can provide information where data sources (i.e., sensors) are not available. Additionally, they include robust physical laws that govern the potential behaviour of the bridge, in opposition to statistical ones from data-driven DT. However, their lack of adoption can be traced to several challenges, including the need for information to build the physical model such as material parameters or modelling assumptions, the high computational cost and time required to run the simulations, and the difficulties associated with implementing the technology on already-standing structures.

Bridges are a clear example of such a structure that would benefit from the implementation of simulation-based digital twins. As bridges age, their structural characteristics change, and the predictions made by simulations based on their initial design may no longer be accurate. To address this issue, it is necessary to continuously update the digital twin with real-time data, so that it remains accurate and representative of the physical system. A promising approach is the implementation of a Bayesian framework for updating the model based on sensor data.

1.2 Bayesian framework for model updating

Bayesian inference is a well-known statistical framework that allows for the quantification of uncertainty in the parameters of a model. It involves updating prior beliefs about the parameters with new data to obtain a posterior distribution, which encapsulates the updated knowledge about the parameters. This ability to quantify the uncertainty present in the model results is key in the context of digital twins, as it allows the user to make informed decisions knowing the certainty of the provided results.

Bayesian inference has been commonly used to incorporate information from measurements into simulation models. In the field of civil engineering, it is generally applied in the development of predictive maintenance strategies [11] or model updating strategies [12]. The latter ones are the focus of this study, as they allow the estimation of parameters based on information from sensors and introduce

a measurement of the uncertainty of the predictions.

Furthermore, the Bayesian framework allows to introduce a mismatch term to measure the discrepancy between the model and the reality [13], which can provide further insight into the adequacy of the digital twin. This approach has yet to be applied to model parameter estimation.

1.3 Structure and objectives

The objective of this article is threefold: the identification of requirements and challenges for simulation-based digital twins of bridges, the implementation of a Bayesian framework in that context, and the validation of such a workflow in a demonstrator example.

Therefore, in the following section, an analysis of the requirements, challenges, and potential applications of simulation-based digital twins for bridges is presented. Next, the Bayesian framework is formally introduced. Finally, its application to a simple demonstrator based on the Nibelungenbrücke of Worms (Germany) is presented.

2 Simulation-based digital twins

A more precise definition of simulation-based digital twins is provided by Lu et al. in [14]: "A digital twin (DT) refers to a digital replica of physical assets, processes, and systems. DTs integrate artificial intelligence, machine learning, and data analytics to create living digital simulation models that are able to learn and update from multiple sources as well as represent and predict the current and future conditions of physical counterparts." Based on that, a set of requirements, potential applications and challenges for their implementation can be analysed.

2.1 Requirements

The main features that must be present in a digital twin for bridges, identified by Cong et al. in [9], can be summarized as follows:

- It must be a digital replica of the physical bridge.
- The DT must be rich in data.
- It must be connected to the physical bridge, allowing for its update and monitoring in near-real time.
- It spans the whole life cycle of the bridge.
- The data is stored in a common environment.
- It can be used as a visualization tool.
- It can be used as a simulation tool.
- It can learn from new measurements.

It can be appreciated that both the fidelity, interconnectivity and accessibility of the data and the digital twin play a key role in its deployment. Therefore, it is key that the implemented framework allows for such capabilities.

Additionally, one of the main goals of the digital twin is to provide useful information to the end users that they can use for making improved decisions. A "black-box" approach for the digital twin would hinder the insights provided by the models, as well as hide its potential shortcomings. Measurements of the fitness of the current model must be provided together with the predictions, allowing an assessment of its quality.

Finally, the results provided by the digital twin must be reproducible and traceable. Otherwise, obtaining robust predictions is not possible, reducing the applicability of the model to real-case scenarios. This is especially noteworthy in the case of bridges, as accounting for extreme events and factoring in operational risks requires extensive reliable information.

2.2 Challenges

Based on the given requirements, several challenges to the building of a simulation-based digital twin can be identified. First, simulations are based on physics models that include a set of simplifications and assumptions. This limits the applications to such a set of assumptions and introduces potential errors and model biases. A methodology that limits these biases must be considered. When possible, they should be quantified as well and given to the user as a quantification of the model fitness.

Additionally, the physical systems usually filter the errors coming from faulty sensor measurements. Anomalies and artificially extreme values coming from the measured data may produce unexpected outcomes from the simulation as well, which should be corrected when acquiring and pre-processing it.

To properly generate predictions, it is necessary to provide the simulation with sufficient data coming from measurements to calibrate it. However, in many cases it is not possible to place sensors in suitable locations at the bridge, either due to technical or economic reasons. The simulation part of the digital twin can be an answer to this shortcoming, but a compromise must be available between the amount of available data to calibrate the simulation model and the target values to be predicted by such a model.

2.3 Applications

A digital twin based on the framework developed here would be suitable for numerous applications. First, it would provide physical predictions at points where measurements are not available. This can be used for monitoring the bridge's behaviour. Moreover, the analysis of the predictions of the digital model can influence an efficient sensor placement, saving costs and materials. By applying a Bayesian framework, quantification of the uncertainty should be available for any provided measurement.

As previously mentioned, one prominent use case is the fitting of digital twins to standing structures subjected to aging processes. Fitting the simulation with data from measurements allows the implementation of such cases, which can be used to assess life-cycle indicators, establish maintenance schedules, and identify regions needing inspection due to potential damage.

3 The Bayesian inference framework

The proposed Bayesian framework can be divided into three main steps: the definition of the model, the fitting of the parameters and the generation of predictions based on the updated model. This framework has been extensively applied in general problems from a variety of fields [15]. Here we will adapt it for its incorporation in a digital twin of a bridge.

3.1 Model definition

The model is defined as a Finite Element (FE) discretization of the geometry of the bridge, which will be analyzed under a set of physical laws governed by equations (generally PDEs). The output of this model is represented by \mathbf{u} (in this case, the displacement field of the bridge under its own weight), which depends on a set of latent parameters $\boldsymbol{\theta}$ and on a set of known parameters $\boldsymbol{\lambda}$ assumed to be real and fixed. Therefore, we can generally express \mathbf{u} as a function of the parameters and the sources as $\mathbf{u}(\boldsymbol{\theta}, \boldsymbol{\lambda})$. This formulation is independent of the model and can be adapted to an arbitrary parametrization or discretization.

Additionally, we define a set of sensors, that can be real or virtual measurement. The set of input values supplied to \mathbf{u} will be expressed as \mathbf{X} and the measured outputs as \mathbf{Y} . At the same time, $\mathbf{u}(\mathbf{x})=\mathbf{y}$, where \mathbf{x} is in the set of possible inputs and \mathbf{y} is in the set of possible outputs generated by \mathbf{u} . In the case of the \mathbf{u} as displacement field, \mathbf{x} can be a set of material parameters and \mathbf{y} the measured displacements.

In a Bayesian framework, it is necessary to include the available information on the system through *prior* probability distributions. These prior distributions will be denoted by $\pi_{pr}(\boldsymbol{\theta})$. The objective will be inferring the *posterior* probability distributions $\pi_{pos}(\boldsymbol{\theta})$ of the parameters. These parameters can have a predetermined correlation structure or be considered independent, depending on the problem and its applications.

Information on the error model and the model bias can be included as well, allowing a better-informed use of the fitted model. The measurement noise \mathbf{e} is typically known beforehand and can be applied additively or multiplicatively to the response model. In this project, we will define:

$$\mathbf{y} = \mathbf{u}(\boldsymbol{\theta}, \boldsymbol{\lambda}) + \mathbf{e} \quad (1)$$

where $\mathbf{e} \sim N(0, \sigma^2 \mathbf{I})$. This noise term supposes independence between different sensors with the same uncertainty modelled as white noise, as is usual the case. This structure is generally provided during the measurement phase after processing the raw sensor signals. If the model bias is to be included as in [13], it suffices with adding a term \mathbf{d} that represents that bias and whose parameters are to be inferred. It is often modelled as a Gaussian Process (GP) governed by its mean and correlation structure. The inclusion of such a bias requires further considerations on the chosen inference procedure that are out of the scope of this work, therefore it will not be included.

3.2 Fitting of the parameters

A Bayesian inference approach is implemented to solve the inverse problem by fitting the model parameters to a set of initial data measurements. In this approach, Bayes' theorem is used to obtain the posterior distributions of the parameters. The theorem reads:

$$p(\mathbf{u}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{u})p(\mathbf{u})}{p(\mathbf{y})} \quad (2)$$

Where $p(\mathbf{u}|\mathbf{y})$ is the posterior distribution of \mathbf{u} given the

measurements \mathbf{y} , $p(\mathbf{y}|\mathbf{u})$ is the likelihood of the model given a set of measurements, $p(\mathbf{u})$ is the prior distribution of the model and $p(\mathbf{y})$ is the marginal likelihood of the measurements.

We can differentiate four steps to solve the inverse problem in a Bayesian framework [15]:

1. Define the prior distributions of the parameters.
2. Define the data generating process.
3. Define the likelihood function of the model.
4. Explore the posterior density from Bayes' theorem.

The prior distributions are chosen from experience, as already mentioned. The likelihood model includes information on the variability of the measurements and the error and differs from case to case. Obtaining the posterior distribution is usually not possible analytically and involves the evaluation of integrals over high-dimensional parameters. Therefore, iterative sampling approaches are commonly used to estimate the posterior distribution from samples of the priors. The approach to be used is the application of Markov-Chain Monte-Carlo sampling (MCMC). This method generates a chain of samples of θ that eventually converges to a stable distribution that represents the posterior distribution of the parameters $\pi_{\text{pos}}(\theta)$.

3.3 Prediction on the updated model

Once the model parameters are fitted, they can be used to perform predictions on the model. One of the advantages of using the Bayesian framework is that the parameters are fitted in a posterior probability distribution, which can be used to quantify their uncertainty and evaluate their dispersion.

An important application of these fitted posterior distributions is the generation of so-called posterior predictive distributions. Using the parameters that have been fitted from real measurements, the forward model is evaluated to generate predictions that were not available from the data. An example would be generating measurements at points where no data is available. These predictions will generally have a probability distribution that allows for uncertainty quantification on them.

Alternatively, the predictions from the fitted model can be used to calculate the required KPI with information on its variability. This is an invaluable tool to assess the behaviour of the fitted model integrated into the digital twin.

4 Use case: the Nibelungenbrücke

Located in the city of Worms (Rheinlad-Palatinate), the Nibelungenbrücke (see **Figure 1**) connects it across the river Rhine with the state of Hesse [16]. The history and conditions of the bridge make it the perfect candidate for a digital twin approach. A close monitoring of its behaviour is key for the continuation of its service life, and accurate predictions of the state of the bridge can be used to take decisions on the safety of its operation. Additionally, the bridge's properties have been subjected to large modifications due to ageing processes from its use and the rehabilitation process. This renders the original model assumptions outdated; hence a Bayesian framework can be useful

to provide information on the new parameters and behaviour of the bridge.



Figure 1 Nibelungenbrücke, Worms. Source: Bob Cortright, Bridge Ink

4.1 Model descriptions

The model will represent a simplified version the first complete span on the western shore of the bridge. It is a box-girder bridge built using the balanced cantilever method. The measurements of the simplified model are taken from the available datasheets [17]. A scheme of the cross-section is observed in **Figure 2**.

The geometry is modelled as a 3D purely elastic solid. As prestressed concrete is the main material, it is chosen a Young's modulus E of 40 GPa, a Poisson's ratio ν of 0.2 and a density ρ of 2350 kg/m³ as the real material parameters. This simple model suffices for demonstrative purposes of the Bayesian framework.

This set-up is going to be tested under the self-weight of the bridge. In this case, artificial measurements of the displacements in vertical and transversal directions with respect to the deck are collected at 27 virtual sensors positioned across the bridge (see **Figure 3**). No noise has been added to such measurements, and only one data point has been collected at each location. The geometry and mesh have been generated using the package Gmsh [18], the finite element problem and solution were implemented using FEniCSx [19].

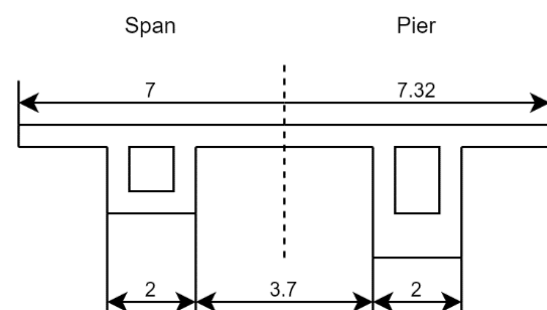


Figure 2 Scheme of the model's cross-section, in meters. Source: BAM

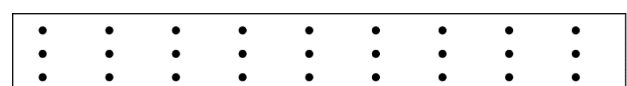


Figure 3 Sensor positions on the deck. Source: BAM

4.2 Inference procedure

The objective of the inference procedure is to estimate the real density, Young's modulus E and Poisson's ratio ν of the bridge. To do that, the prior distributions indicated in **Table 1** are introduced in the model. Despite not including an error term σ in the measurements, an additive Gaussian error model is added to the procedure, expecting it to become zero.

It must be noted that this problem is very ill-posed, as there are many possible combinations of ρ , E and ν such that a given set of loads generates the measured displacements. Therefore, the inferred parameters are not expected to reflect exactly the initial ones. The inference procedure is performed using MCMC for 300 steps with 50 steps of burn-in, which is enough in this case for clear convergence. The inference problem is defined using probeye (<https://github.com/BAMresearch/probeye>), a python package developed to this end. It creates an interface for the methods of the package emcee [20], that runs the FE model for every proposal of the parameters. The program is executed on a virtual machine with 4 CPUs of an AMD EPYC 74F3 processor and 8 Gb of RAM.

Table 1 Material parameters to be inferred

Parameter	Unit	Prior
E	Pa	LogNormal(31,0.2)
ν	-	LogNormal(-1.5,0.2)
ρ	kg/m ³	LogNormal(8.0,0.5)

4.3 Results

The pair plot of the inference procedure can be observed in **Figure 4**. As previously mentioned, the problem is clearly ill-posed, and the true values, represented by the red dot, lays far from the obtained level curves of maximum probability. This is the case despite the posterior distributions having been updated in the inference procedure, as observed in the diagonal plots. New methodologies to reduce this effect must be considered to generate results that allow drawing conclusions on these parameters. The predicted parameters are ρ with mean 1700 kg/m³ and standard deviation 89 kg/m³, E with mean 28.9 GPa and standard deviation 0.71 GPa, ν with 0.22 and standard deviation 3E-3, and noise (σ) with mean 2.6E-5 (which lays on the limit of the feasible do-main) and standard deviation 2.1 E-6. As expected, there is also a clear correlation between E and ν for a given ρ , as seen by the corresponding noise plot displaying a narrow linear band. The noise error was predicted accurately as almost zero. Prior distributions with means closer to the true value would potentially render better results for material parameters, proving the importance of this choice in the inference procedure.

Figure 5 shows the histogram of 100 sampled displacements for the point with coordinates (0.0, 0.0, 50.0) in x and y directions. The obtained values are shown in **Table 2**. We can observe a great agreement with the results, and

the probability distribution provides invaluable information about the uncertainty of the measurement. Generating results on demand at an arbitrarily located virtual sensor enriches the information provided by the digital twin and allows better-informed decision-making.

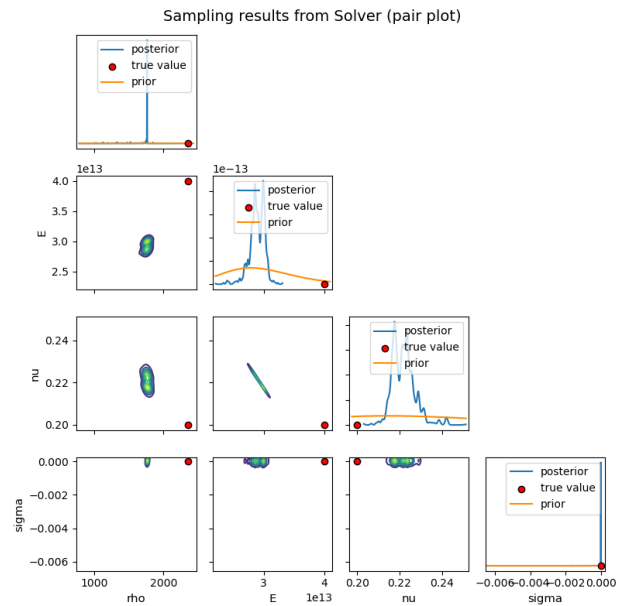


Figure 4 Pair plot of the marginal distributions of inferred parameters. ρ in kg/m³, E in Pa, ν and σ dimensionless. Source: BAM

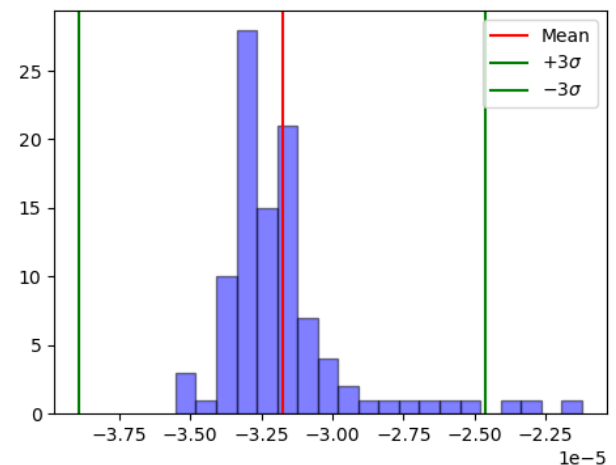


Figure 5 Histogram for posterior predictive at $y=50$ m. Displacement for Y (vertical), in meters. Source: BAM

Table 2 Posterior predictive results

Parameter	Displacement X [m]	Displacement Y [m]
True	7.200E-9	-3.136E-5
Mean	7.867E-9	-3.148E-5
Standard deviation	6.550E-10	2.372E-6

4.4 Potential technical challenges

Once the main inference model is implemented and defined, it must be integrated into the digital twin. This involves numerous potential technical challenges that must be addressed. First, it must be observed that the model that is updated in the inference procedure will never perfectly reflect the real system. This is an inherent effect of the modelling choices, and it must be considered when deciding on the monitoring conditions. Increasing the number of parameters or adding a greater number of modelling assumptions could lead to a better-fitted model, but at the expense of an increase in complexity and cost, and a potential risk of overfitting and loss in prediction power. Additionally, the more complex a system, the more difficult is to choose the correct assumptions and data that reflect the current state of the system. This includes the choice of noise, correlation and bias structures or the decision of which results to consider.

Additional issues arise from the technical implementation of the digital twin. Simulation-based DTs require a continuous flow of data supplied for refitting purposes while being accessible to the users for predictive calls. The location of the databanks, the digital services, and the interfaces with each other and with the users must be decided beforehand, considering the requirements of the implementation. Online access to the DT can be advantageous but may be discouraged for some uses due to security and reliability concerns, or directly unavailable. A monitoring/updating schedule with must be set for an efficient use of resources. The same monitoring concept must be carefully designed, deciding which data is to be supplied to the DT, with which temporal windows and under which conditions. This information influences greatly the modelling assumptions previously mentioned.

Finally, in the context of Structural Health Monitoring, it must be decided which values are to be provided to the end user. The predictions from the fitted simulation-based DT can be used for estimations of the remaining service life of an ageing bridge, the calculations of Key Performance Indicators of the structural state, or even for further structural optimization in the case of systems with active components. Each type of end user will have different requirements to be satisfied, which must be considered when designing the DT. Nevertheless, every part involved in the process benefits from well-documented and referenced elements. This translates to appropriate and comprehensive data descriptions within a clear and general meta-data framework, and tested, maintained, and documented software implementations with transparent code.

5 Conclusion

In this study, the viability of a Bayesian framework for a simulation-based digital twin of bridges was analysed. It was observed that the advantage of providing richer and more informative data from the simulation and of allowing for uncertainty quantification outweighs the potential challenges of lacking data or ill-posed problems. Furthermore, the framework is proved in a demonstrative example of a hypothetical digital twin of the Nibelungenbrücke of

Worms. In this simplified example, it was possible to generate reliable predictions out of a very limited set of measurements, as well as measurements of their uncertainty.

The implementation of such DT is a very complex problem that involves many different parts. This generates a plethora of potential technical challenges to be resolved, ranging from the modelling assumptions to the transference of data to the end user. The early identification of such challenges is found to be key for a swift implementation of simulation-based DTs. In future work, the application of the framework to a more realistic example with real measurements will be pursued. Additional analysis on the fitness of the model, the error and correlation models and the model bias must be carried out. Nevertheless, the current analysis serves as a first step in the implementation of the Bayesian framework in a functional digital twin of a bridge.

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