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Estimation of the Kitagawa-Takahashi diagram by cyclic R curve analysis

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Abstract

The Kitagawa-Takahashi (KT) diagram is a proven concept for describing the fatigue limit in presence of a defect or crack. It can be determined empirically with great experimental effort. It can also be estimated by means of the El Haddad relationship if the endurance limit and the long fatigue crack propagation threshold are available in reasonable accuracy. A third option is the determination using the cyclic R-curve, which describes the dependency of the fatigue crack propagation threshold on the crack growth at the short crack propagation stage. This can be experimentally determined using a closure-free initial pre-crack. It can then be applied to the determination of crack arrest for a given applied load and a given defect or crack size. Compared to the other two methods mentioned above, this option has considerable advantages: It can be applied to any component and any stress ratio. It allows the treatment of multiple cracks and provides estimations of the S-N curve in the finite life regime as well as at the endurance limit. Compared to the empirical determination of the KT diagram, the experimental effort is significantly lower and compared to the El Haddad approach it avoids problems such as the use of non-conservative long fatigue crack propagation thresholds (when the conventional load reduction method is applied to materials prone to corrosion) and the mathematical predetermination of the curve shape. The work introduces the method and provides a critical discussion as well as quantitative comparison between the different methods.

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1. Introduction

The basic safety of a component loaded in fatigue is provided by the S-N curve concept, with the fatigue limit being the lower bound stress range or amplitude below which no failure occurs at all (endurance limit) or below which no failure occurs up to a certain number of loading cycles (in materials without an endurance limit). It is important to understand that the endurance limit is not specified by the avoidance of crack initiation but by crack arrest (see Miller (1993) and Murakami (2002)). In any case, cracks are initiated at defects. These can be material defects or geometrical defects. Material defects are non-metallic inclusions, pores, defective microstructures or pre-existent microcracks. Geometrical defects are micro-notches, e.g. due to pronounced surface roughness, to indentations or scratches. They can be formed during fabrication but can also arise in operation or by faulty maintenance. Typical defects which originate in service are corrosion pits and spallings caused by contact damage. No detailed discussion on this topic will be provided here, see, however, the overview by the authors in Zerbst et al. (2019a,b,c).

If the crack-like defects are small enough, the cracks emanating from them grow over a certain distance and then arrest at certain structural features, preferably grain boundaries. The largest of these **microstructurally short** cracks, which just like that arrests, controls the fatigue strength of the material. This is to be distinguished from the fatigue strength of the component which is lower, and for which designed notches play an additional role.

Nomenclature

a	crack depth
a_i	initial crack depth (for the cyclic R-curve analysis)
a_0	El Haddad parameter
d_{\pm}	transition from region I to region II in the KT diagram, crack depth at crack arrest
K	stress intensity factor
K_{\max}	maximum K in the loading cycle
K_{\min}	minimum K in the loading cycle
K_{op}	K above which the crack is open in the loading cycle
R	load ratio ($= \sigma_{\min}/\sigma_{\max} = K_{\min}/K_{\max}$)
Y	boundary correction factor
ΔK	stress intensity factor range ($= K_{\max} - K_{\min}$)
ΔK_{eff}	effective ΔK (crack closure corrected)
ΔK_p	plasticity corrected K , formally derived from the cyclic J -integral
ΔK_{th}	threshold against fatigue crack propagation
$\Delta K_{\text{th,eff}}$	intrinsic threshold (without crack closure)
$\Delta K_{\text{th,LC}}$	ΔK_{th} for long cracks
$\Delta K_{\text{th,op}}$	component of ΔK_{th} due to crack closure
$\Delta\sigma$	stress range
$\Delta\sigma_e$	endurance limit
$\Delta\sigma_{\text{th}}$	threshold stress range

As long as all micro-cracks arrest, the size of the defect, i.e. the initial crack size has no effect on the fatigue limit. However, this is no longer the case if the defect size is larger than the arrest crack size would be. Under such conditions a crack is no longer considered a microstructurally short crack but a mechanically/physically short one. Mechanically short means that the crack size is in the order of the plastic zone size and consequently fracture mechanics can be applied to calculate the crack driving and resistance force. Nevertheless, small-scale yielding conditions are not met at this stage, so that the crack driving force must be expressed in terms of elastic-plastic fracture mechanics parameters. Physically short means that extrinsic crack-tip shielding mechanisms such as crack closure (see e.g. Suresh (2003) and Tanaka et al. (2003)) does not exist at the beginning but gradually builds up with crack propagation. Crack arrest occurs when the crack driving force ΔK falls below the fatigue crack propagation threshold ΔK_{th} . In notches this

causes what is sometimes designated as “anomalous crack growth”. First, driven by the stress concentration at the notch root, the crack propagates rather fast. Then it slows down due to the combined effect of the decreasing stresses in wall thickness direction and the build-up of the crack closure phenomenon. Finally, the crack arrests or speeds up again, with crack arrest occurring in particular at very sharp notches.

When the crack size reaches the order of the long crack, no arrest occurs any longer, provided the cyclic crack driving force exceeds the long fatigue crack propagation threshold $\Delta K_{th,LC}$. The successive stages of fatigue crack propagation are illustrated in Fig. 1.

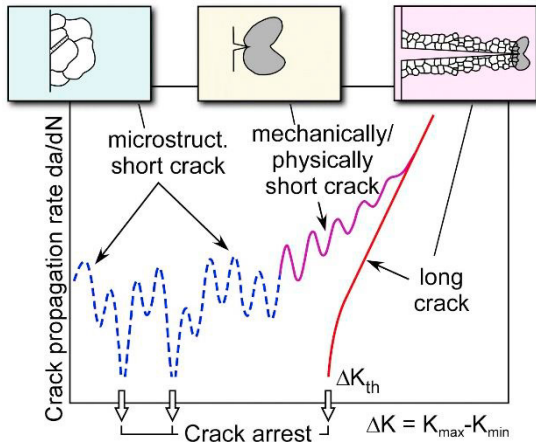


Fig. 1. Successive stages of fatigue crack propagation, schematic view.

2. The Kitagawa-Takahashi diagram

The Kitagawa-Takahashi (KT) diagram, Kitagawa and Takahashi (1976), is a double-logarithmic plot of the fatigue limit (in general terms designated as $\Delta\sigma_{th}$ with “th” standing for “threshold”) versus crack depth a (Fig.2). It covers all the stages of fatigue crack propagation shown in Fig. 1.

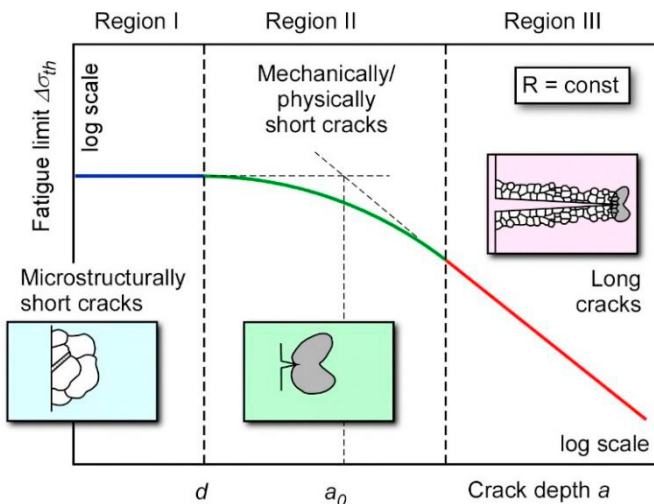


Fig. 2. Kitagawa-Takahashi diagram, schematic view.

Region I is the range of the microstructurally short crack. It is terminated when the defect/crack size is larger than $d_{d_{\pm}}$. Up to this point all cracks are arrested. For steels and aluminum alloys $d_{d_{\pm}}$ is in the order of $10 \div 25 \mu\text{m}$, Zerbst et al. (2019a,b,c). Its magnitude depends on the strength of the material. As an example, Zerbst et al. (2018), using fracture mechanics simulation, have obtained an arrest crack depth of $19 \pm 2 \mu\text{m}$ for steel S355NL (ultimate tensile strength: 555 MPa) and of $11 \pm 1 \mu\text{m}$ for steel S960QL (ultimate tensile strength: 1057 MPa).

Crack arrest takes also place in Region II but now due to the gradual build-up of the crack closure phenomenon. Eventually, in Region III the crack is arrested as long as the cyclic crack driving force ΔK does not exceed the long fatigue crack propagation threshold $\Delta K_{\text{th,LC}}$. The KT curve is controlled by $\Delta K_{\text{th,LC}}$ and shows a slope of $-1/2$ in double-logarithmic scale. Note, that the diagram is different for different stress ratios $R = \sigma_{\text{min}}/\sigma_{\text{max}}$.

There are two options for the experimental and theoretical determination of the Kitagawa-Takahashi diagram.

(a) Empirical determination using artificial notches:

Specimen sets are prepared with narrow (crack-like) artificial notches. Within each set, the notch size is identical. They are used for the determination of the fatigue strength. The result is one point of the KT diagram. This is straightforward. Nevertheless, this technique might be problematic when the notch effect is not adequately considered in the determination of the threshold stress. Furthermore, as mentioned above, the question whether or not a crack arrests at a given load depends on the notch sharpness. A further issue is related to the extensive experimental campaign needed due to the large number of specimens to be tested at the fatigue limit and for different notch sizes.

(b) The El Haddad approach:

El Haddad et al. (1979) have proposed a description of the KT curve by means of a continuous curve.

$$\Delta\sigma_{\text{th}}(a)/\Delta\sigma_e = \sqrt{a_0/(a + a_0)} \quad (1)$$

In Eqn. (1), $\Delta\sigma_e$ is the endurance limit and a_0 is the so-called El Haddad parameter provided by the intersection between the straight curve sections of regions I and III. When $\Delta\sigma_e$ and the long crack threshold $\Delta K_{\text{th,LC}}$ are known, it can be obtained by

$$a_0 = 1/\pi \cdot [\Delta K_{\text{th,LC}}/(Y \cdot \Delta\sigma_e)]^2, \quad (2)$$

Y being the boundary correction factor. Since this option is less extensive than (a) it is the most applied. However, it also has problems. The most important one is that the determination of $\Delta K_{\text{th,LC}}$, although subject of testing standards, provides different values dependent on the method of experimental determination, at least for materials which are prone to corrosion, Zerbst et al. (2016). Another problem is related to the values found in the literature, some of which are even summarized in compendia. Frequently (not always) they are given for $Y = 1$ which refers to the through crack in an infinite plate. The usual case is, however, the semi-elliptical crack in a plate. For this $Y = 0.728$, Tanaka and Akiniwa (2003). Finally, the shape of the curve is mathematically pre-determined by Eqn. (1), even if the material would display a different behavior.

3. Cyclic R curve analysis

A cyclic R-curve describes the dependence of the fatigue crack propagation threshold on the crack extension Δa in the physically short fatigue crack propagation range, as schematically illustrated in Fig. 3. In principle, the threshold value can be divided into two parts

$$\Delta K_{\text{th}} = \Delta K_{\text{th,eff}} + \Delta K_{\text{th,op}}(\Delta a) \quad (3)$$

While $\Delta K_{th,eff}$ is an intrinsic, closure-free value which depends on the elastic constants and the lattice type only, $\Delta K_{th,op}$ (“op” stands for “opening”) represents the extrinsic part depending on the material, the R ratio and the crack extension Δa , until the crack reaches the long crack propagation stage.

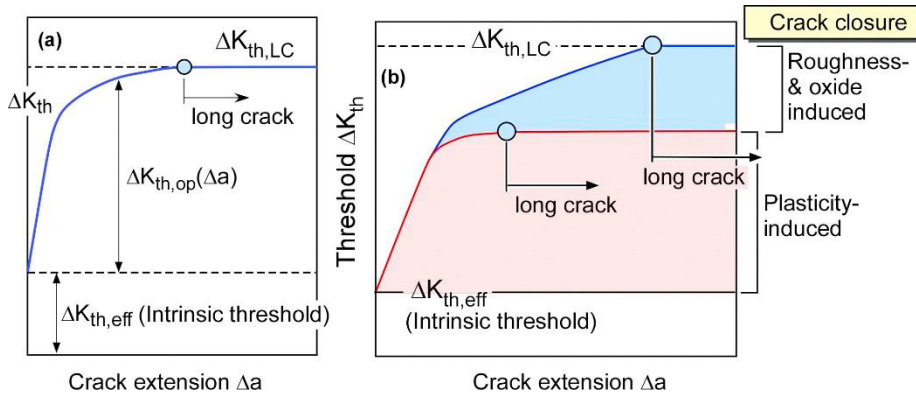


Fig. 3: Cyclic R-curve. (a) Principle; (b) Contributions of the different crack closure phenomena.

An option for its determination is shown in Fig. 4. First, a pre-crack is generated in compression pre-cracking starting from a very sharp notch (the authors use razor blades for sharpening). It is essential that the crack generated by means of this procedure is closure-free. Subsequently, the crack is loaded with a certain value of ΔK at a constant R ratio. If $\Delta K < \Delta K_{th,eff}$, no propagation occurs. Slightly above $\Delta K_{th,eff}$, the crack grows, but then arrests again because of the build-up of the crack closure phenomenon. By further increasing ΔK , the crack continues to grow up to the next arrest event, etc. When the long crack threshold $\Delta K_{th,LC}$ is reached, no crack arrest occurs any longer. The cyclic R-curve is obtained by interconnecting the crack arrest points.

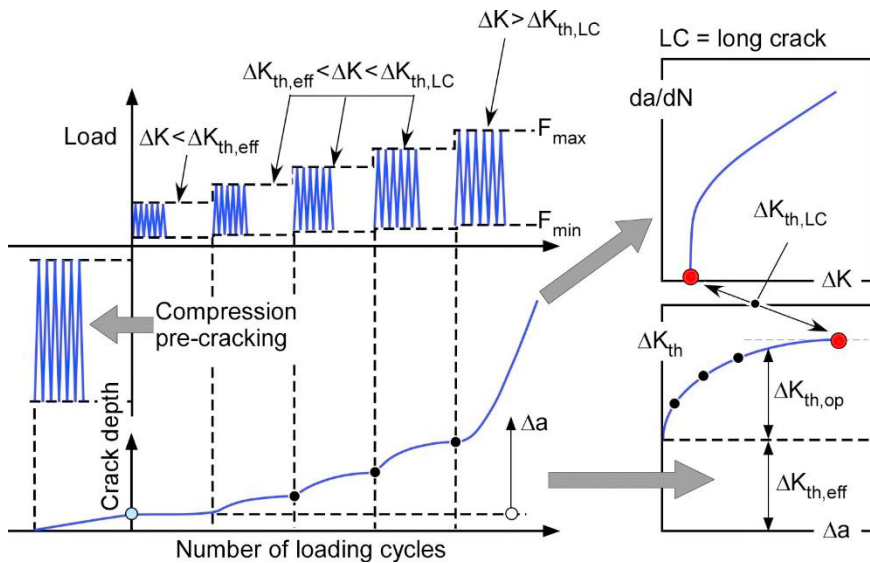


Fig. 4: Determination of a cyclic R-curve according to Tabernig et al. (2002)

The cyclic R curve can be used for the prediction of crack arrest in components. The principle is illustrated in Fig. 5 using the example of a butt weld seam. The (blue) $\Delta K_p - a$ curves mark the cyclic crack driving force in the

component for different stress levels ($\Delta\sigma_1 < \Delta\sigma_2 < \Delta\sigma_3$). The index "p" stands for plasticity-corrected (in fact, it is a cyclic J -integral formally converted to ΔK). The transition from crack arrest to crack propagation is defined by the curve that touches the (red) cyclic R-curve tangentially. It is decisive in the present context that in this way KT diagrams can be theoretically generated when the initial crack depth a_i is used as an input variable. The theoretical determination of KT diagrams has some significant advantages over the methods discussed above:

- (a) The KT diagram can be determined for any specimens and components.
- (b) The method can also be applied in case of multiple cracks.
- (c) With minor modifications, KT diagrams can be determined for the finite life branch of the S-N curve (instead of fatigue limit).

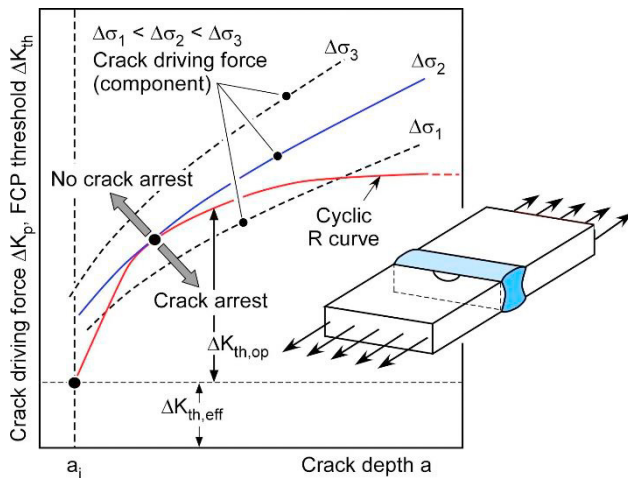


Fig. 5: Principle of a cyclic R curve analysis.

4. Comparison of the results

When KT diagrams obtained by the El Haddad approach and by a cyclic R curve analysis are compared, a typical pattern shows up such as illustrated in Fig. 6. An example of a calculation carried out for the steel S355NL is shown in Fig. 7.

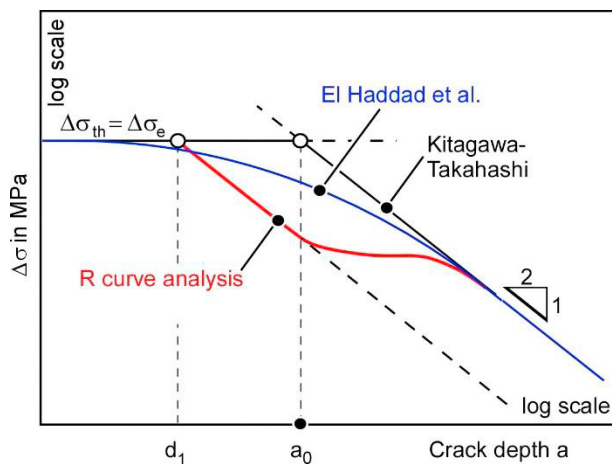


Fig. 6: Comparison of KT diagrams generated by the El Haddad approach and by cyclic R-curve analyses, schematic view.

While the diagrams coincide in regions I and III there is a substantial difference in region II. Here the calculated diagrams obtained using the cyclic R-curve result in significantly lower tolerable stresses. Note that a similar trend has been obtained by Akiniwa et al. (1997) investigating the fatigue behavior of low- and medium-carbon steels, showing a very good agreement with the experimental results. This poses the question, whether the simple El Haddad relationship describes the real crack propagation behavior in the physically/mechanically short crack regime. In fact, Maierhofer et al. (2015) claimed that the El Haddad model is giving non-conservative estimations because it is not considering the built up of crack closure in the short crack regime. Another important issue they mention is the dependence of the threshold stress (non-propagating cracks) on the notch depth.

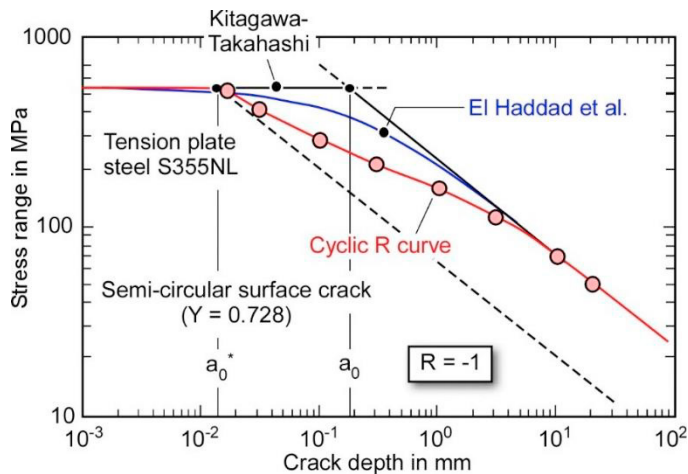


Fig. 7: Comparison of KT diagrams generated by the El Haddad approach and by cyclic R-curve analyses, example (S355NL steel).

5. Summary and outlook

The topic of the paper was the determination of Kitagawa-Takahashi diagrams. Besides the conventional methods (empirical determination using specimens with artificial notches and the El Haddad approach) a theoretical approach based on a cyclic R-curve analysis has been discussed. It turned out that the cyclic R-curve-based approach provided lower permissible stresses in region II of the KT diagram compared to the El Haddad approach. The latter provides a smooth mathematical transition from the microstructurally short to the long crack regime and does not take into account the buildup of the closure effect in the physically/mechanically short crack regime, which can be well captured by the cyclic R-curve analysis. This, however, is the subject of ongoing work by the authors. For the case that the cyclic R-curve approach proves to be the correct one, this would be highly relevant to avoid non-conservative predictions when applying KT curves.

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