

Efficient identification of random fields coupling Bayesian inference and PGD reduced order model for damage localization

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One of the main challenges regarding our civil infrastructure is the efficient operation over their complete design lifetime while complying with standards and safety regulations. Thus, costs for maintenance or replacements must be optimized while still ensuring specified safety levels. This requires an accurate estimate of the current state as well as a prognosis for the remaining useful life. Currently, this is often done by regular manual or visual inspections within constant intervals. However, the critical sections are often not directly accessible or impossible to be instrumented at all. Model-based approaches can be used where a digital twin of the structure is set up. For these approaches, a key challenge is the calibration and validation of the numerical model based on uncertain measurement data.

The aim of this contribution is to increase the efficiency of model updating by using the advantage of model reduction (Proper Generalized Decomposition, PGD) and applying the derived method for efficient model identification of a random stiffness field of a real bridge.

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1 Introduction

Model updating approaches, which are inverse optimization processes, are used to calibrate and validate a numerical model based on uncertain measurement data. This can be done with deterministic as well as Bayesian approaches [1]. In the latter, a joint probability distribution of all model parameters is determined. The calibration is computationally very expensive due to a huge number of model computations. A way to improve the efficiency is to apply reduced order modeling. The idea is to find the lowest dimensional system, which is able to capture the dominant behavior of the structure. In the last decade, the PGD method [2] has been developed for a variety of applications, among these also model updating techniques [3]. The key procedure is to generate a simple parametric solution given by a number of smaller-dimensional PGD modes representing all solutions in a fixed parameter space. After a more elaborate computation of such a numerical abacus, the PGD reduced model can be evaluated very efficiently making PGD a powerful method for the inverse model calibration of a digital twin.

At the Federal Institute for Materials Research and Testing (BAM), a pre-stressed two-span concrete bridge (fig. 1) was build for testing various measurement systems for monitoring purposes. By reducing the pre-stress, a damage of the structure could be introduced due to cracking under tension that is “healing” by adding the pre-stressing forces. Using deflection measurements along the complete length of the bridge, a global stiffness reduction for decreasing pre-stressing forces was observed (fig. 2).

The goal of this paper is to identify the stiffness reduction, using an efficient model calibration method based on a PGD reduced model combined with a random field approximation for the Young’s modulus representing the stiffness distribution.

2 PGD model updating procedure

The inverse model calibration problem is reformulated as a Bayesian inference problem [2,4], where the posterior distribution $P(\theta|\mathbf{y}) \propto P(\mathbf{y}|\theta)P(\theta)$ of the model parameters θ with given measurement data \mathbf{y} is computed as a product of the likelihood $P(\mathbf{y}|\theta)$ and the prior information $P(\theta)$. The likelihood function is expressed as a probability distribution (multivariate normal distribution) of the model error $\mathbf{k} = \mathbf{y} - \mathbf{g}(\theta)$, the difference between the measurement data and the model response $\mathbf{g}(\theta)$. It includes a noise term related to both real measurement noise and model bias.



Fig. 1: BAM two-span prestressed testing bridge **a** and its prestressing device **b**.

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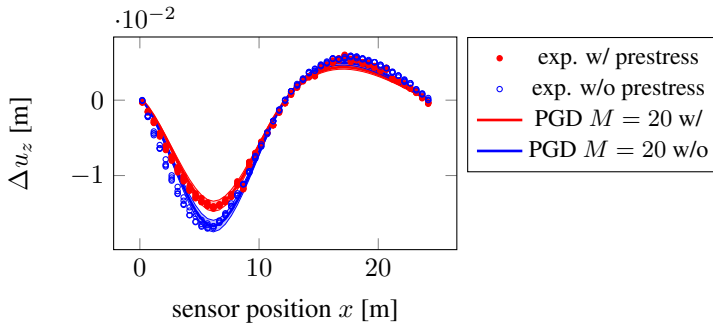


Fig. 2: PGD model against measurement data.

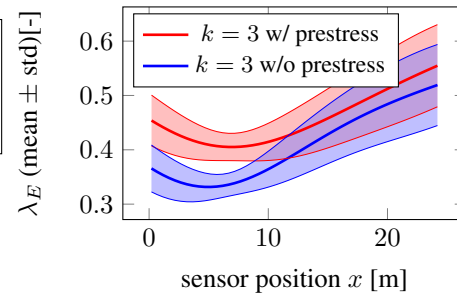


Fig. 3: Identified Young's-modulus factor.

Using a PGD model [2, 5] as model response, the model error is given in an explicit form

$$\mathbf{k} = \mathbf{y} - \sum_{i=1}^M \mathbf{F}_1^i(\mathbf{x}_s) \prod_{l=1}^d F_{1+l}^i(\theta_l) \quad (1)$$

by means of the PGD modes F each depending only on a single model parameter θ_l . The first PGD mode represents the dependency on the physical space given by the location of the sensors $\mathbf{x} = \mathbf{x}_s$. The PGD modes are computed based on the progressive PGD solver [5] in an iterative manner. Afterwards, the PGD based model error (1) and its derivative w.r.t. the parameters required for the variational inference can be evaluated in real time for each parameter set within the predefined parameter space. Of course, the accuracy of the model calibration is strongly related to the accuracy of the numerical model $\mathbf{g}(\boldsymbol{\theta})$ depending on the discretization in each parameter as well as on the number of PGD mode sets M . A goal-oriented adaptive PGD refinement process to control the model quality is introduced.

3 Numerical results

In this section, the identification of the stiffness distribution, by means of the given measurement sets with and without prestress of our testing bridge at BAM, is shown. A PGD model of the three-dimensional bridge with a linear elastic material was set up. The Young's modulus was modeled as a Gaussian random field $E = E_0 \cdot (\mu + \sum_{i=1}^K \bar{\Phi}_i(\mathbf{x})\xi_i)$ with an exponential squared Matern covariance function [6] (parameters: correlation length: 8 m, coefficient of variation: 0.05, $E_0 = 38.5 \cdot 10^6$ kN/m² and Poisson's ratio $\nu = 0.2$). In the model calibration procedure, the mean μ , the random parameters ξ_i as well as the noise precision are identified. For that reason, the PGD model is a function of the PGD coordinates $[\mathbf{x}, \mu, \xi_1, \xi_2, \dots]$. Figure 2 shows a very good agreement of the measurement data with the PGD model response after calibration. In fig. 3, the identified distribution of the Young's modulus along the bridge length is shown. Based on the identified random field with three random parameters, the derived model can express the measured stiffness reduction very well and localize the actual damage and stiffness reduction caused by cracks in midspan.

4 Conclusion

An efficient model calibration method for digital twins is presented by embedding a PGD reduced model in a variational Bayesian inference schema. Based on that method, it is possible to efficiently calibrate a digital twin of a testing bridge by different measurement data. Furthermore, a stiffness reduction caused by a damage of the real structure could be identified in the numerical model.

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References

- [1] D. M. Blei, A. Kucukelbir, and J. D. McAuliffe, *J. Am. Stat. Assoc.* **112**, 859-877 (2017).
- [2] F. Chinesta, E. Cueto, E. Abisset-Chavanne, J. L. Duval and F. El Khaldi, *Arch. Comp. Method. E.* **27**, 105-134 (2020).
- [3] P. B. Rubio, F. Louf, and L. Chamoin, *Comput. Mech.* **62**, 1485-1509 (2018).
- [4] M. A. Chappell, A. R. Groves, B. Whitcher and M. W. Woolrich, *IEEE Transactions on Signal Processing* **57**, 223-236 (2009).
- [5] A. Falcò, N. Montès, F. Chinesta, L. Hilario and M. Mora, *Journal of Computational and Applied Mathematics* **330**, 1093-1107 (2018).
- [6] C. E. Rasmussen and C. K. I. Williams, *Gaussian Process for Machine Learning* The MIT Press, 2006.