

Numerical Analysis for an Electro-Magneto-Mechanical Phenomenon with High-Order Accurate Methods

Tobias Gleim^{1,2,*} and Hefeng Chen²

¹ Safety of Transport Containers, Federal Institute for Materials Research and Testing, Unter den Eichen 44-46, 12203 Berlin, Germany

² Chair of Mechanics and Dynamics, University of Kassel, Mönchebergstr. 7, 34125 Kassel, Germany

This paper establishes an axisymmetric model for a levitation device. Therein, the Maxwell equations are combined with the balance of linear momentum. Different possible formulations to describe the MAXWELL equations are presented and compared and discussed in the example. A high order finite element discretization using GALERKIN's method in space and the generalized NEUMANN- α method in time are developed for the electro-magneto-mechanical approach. Several studies on spatial and temporal discretization with respect to convergence will be investigated. In addition, the boundary influences and the domain size with respect to the levitation device are also examined.

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1 Motivation

Electromagnetic phenomena occur in all electrically operated devices. The electric and the magnetic fields are unconsciously generated in a coupled fashion or even consciously produced. There are a large number of electronic devices that perform mechanical work using electrical energy or, conversely, convert mechanical work into electrical energy. The motivation of this article lies in the activation of SMA fibers that are heated or directly influenced by electromagnetic fields and thereby change their shape, cf. [1]. For fundamental research, it is of crucial importance that benchmark problems are as simple as possible and tailor-made for observing effects to characterize certain influences. The experiment of the TEAM Workshop group named Problem 28 [2] will be used as a predestined and well-known example. The example is characterized by the simplicity of the geometry, see Figure 1 left.

To solve complex problems like electro-magneto-mechanical phenomena, high-order accurate methods in space and time are required, which are investigated on the problem of the levitation device.

2 Weak Form of the Electro-Magneto-Mechanical Field Equations

Electromagnetic phenomena are generally described by MAXWELL's equations. They are a set of partial differential equations in space and time. These equations deal with electric and magnetic effects including their interaction. Since the electric and magnetic fields vary in time, reciprocal actions are induced:

$$\nabla \cdot \mathbf{D} = \rho_R, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mathbf{J} + \dot{\mathbf{D}}, \quad \nabla \times \mathbf{E} = -\dot{\mathbf{B}}. \quad (1)$$

The studies in this paper follow the strategy of a potential formulation and details about the derivation and mathematical transformations can be seen in [3].

For a numerical computation of a levitation device, a description of the movement of an elastic body in the electromagnetic fields is required. Similar to the electromagnetic phenomena governed by MAXWELL's equations, the movement and the elastic behavior of a body are described by the balance of linear momentum:

$$\rho \ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b}, \quad (2)$$

To couple the electromagnetic equations with the mechanical equations, another equation is needed. In the presence of an electric and a magnetic field, the force that acts on a moving electrical charged particle can be described by LORENTZ force law, which takes the following form in the case of a continuous charge distribution:

$$\mathbf{F}^{\text{Lor}} = \rho_c \mathbf{E} + \mathbf{J} \times \mathbf{B}, \quad (3)$$

To enable the application of the finite element method, the Maxwell equations and the balance of linear momentum and their NEUMANN boundary conditions and the LORENTZ force must be formulated weakly through a multiplication with test functions and integration over the domains Ω , respectively.

* Corresponding author: e-mail gleim@uni-kassel.de



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3 Solution Procedure

To solve this multifield problem (1) - (3) a spatial discretization is realized using the finite element method. Therein, the primary variables, its time derivatives and the test functions are approximated by LAGRANGE shape functions. For arbitrary test functions, the linear semidiscrete balance equation of the multifield problem is determined by evaluating the integrals of the spatially discretized terms with the GAUSS integration and leads to ordinary differential equations. This multifield problem of second order in time can be solved for example with a generalized NEWMARK- α method, cf. [3, 4].

4 Axisymmetric Model of a Levitation Device

The influence of time integration and its time step size and the spatial discretization as well as the polynomial degree on the oscillation will be investigated and compared with the aid of the experiment result of the Team Workshop Problem 28 [2]. Figure 1 illustrates the magnetic as well as the electric field at times $t = 0.005$ s and $t = 0.01$ s. The contour plot of the

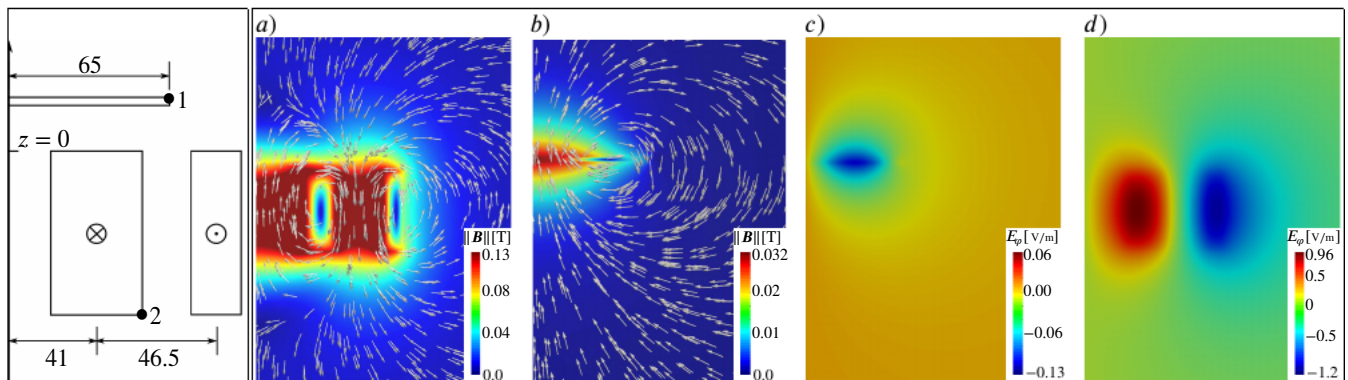


Fig. 1: Left hand side: Axisymmetric example with two excitation coils and conductor, Right hand side: Numerical solutions of the magnetic (a) and (b)) and the electric (c) and (d)) field at time $t = 0.005$ s (a) and (c)) and $t = 0.01$ s (b) and (d)).

magnetic field in both subfigures shows the norm of the vector-valued quantity while the arrows of uniform length demonstrate the direction of action of the magnetic field. The magnetic field clearly depicts a clockwise rotating effect and pushes the conductor upwards. Figure 2 demonstrates the levitation height of the conductor for a linear (left hand side) and quadratic

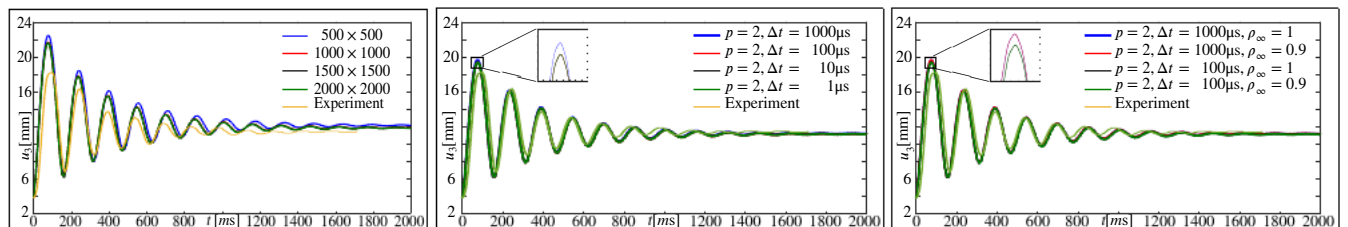


Fig. 2: Left hand side: Displacement of the plate at point 1 for the polynomial degrees $p = 1$ for different domain sizes in millimeters with $\Delta t = 1 \mu s$ and $\rho_\infty = 1$, Middle: Displacement of the plate at point 1: for different time step sizes with $\rho_\infty = 1$ and right hand side for different time step sizes and different spectral radii each for the domain size 500×500 .

(middle and right hand side) polynomial, different time step sizes (middle) and the parameter ρ_∞ (right hand side) of the generalized NEWMARK- α method. In the left subfigure, it can be observed that a small domain indicates a too height solution and decreases with a larger domain towards the experimental data. A higher spatial polynomial can improve the solution and the time step size has a significant influence on the numerical solution, while the spectral radius for the generalized NEWMARK- α method has a minor effect. A detailed study for different formulations, spatial domain, polynomial degree and time integration parameters can be seen in [3].

5 Outlook

The Benchmark Problem 28 [2] helps to verify complex electro-magneto-mechanical phenomena. For this fundamental study, a simple implementation without any additional numerical techniques was depicted. It could be demonstrated that high-order accurate methods in space and time are needed to improve the quality towards the experimental data. Further investigations are needed with respect to more complex geometries, which requires then additional techniques, cf. [5].

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