

Acoustic-structure interaction in the Scaled Boundary Finite Element Method for prismatic geometries

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Abstract. Due to the short wavelength compared to the dimensions of the structure, the simulation of ultrasonic waves is still a challenging task. A numerical method well suited for this purpose is the semi-analytical Scaled Boundary Finite Element Method (SBFEM). When applying this method, only the boundary of a computational domain is discretized using finite elements, while the interior is described by an analytical ansatz. Hence, the number of degrees of freedom is reduced significantly compared to the classical Finite Element Method (FEM). In recent years, a particular formulation of the SBFEM for the simulation of ultrasonic guided waves was developed. The method constitutes an efficient algorithm for prismatic structures of arbitrary length, such as plates, pipes, or beams. Wave propagation phenomena in such structures can be modeled for isotropic and anisotropic inhomogeneous waveguides. Even though the method is an efficient tool for the simulation of guided waves in solid media, a reliable model for the simulation of acoustic wave propagation in fluids as well as acoustic-structure interaction in terms of SBFEM is still missing. In principle, the fluid can be described by a displacement-based formulation and thus be implemented in existing SBFEM algorithms for solid bodies. However, due to the discretization with classical finite elements, spurious modes occur, which cannot be separated from the physical modes straightforwardly. The spurious modes can be suppressed using a penalty parameter. Although very accurate results were achieved for some problems, this procedure has been proven unreliable for certain cases. For this reason, we propose a different approach in this contribution. We employ a pressure model to simulate the acoustic behavior of fluids. The implementation of the pressure model results in a higher effort due to the necessity of incorporating coupling terms, but it presents a stable alternative without spurious modes. The accuracy of the method is demonstrated in comparison with analytical solutions and results obtained using the FEM.

Introduction

The goal of non-destructive testing (NDT) is to detect flaws, such as cracks or pores, inside a component without destroying it. Besides other physical phenomena ultrasound can be used in NDT for this purpose. So-called guided waves are hereby of special interest for NDT as they travel long distances. Guided waves are found in structures with a thickness in range of the wavelength e.g. plates or pipes, so-called waveguides. However, the behavior of guided waves is very complex as they exhibit a strong dispersion. The possibilities to analytically describe the complex dispersion behavior is limited. An efficient simulation tool for this task is therefore essential to further develop NDT methods. Besides other methods the classical Finite Element Method (FEM) can be used to investigate the wave behavior. However, it results in general in large numerical systems, due to fine mesh needed at ultrasonic frequencies. An efficient alternative presents the Scaled Boundary Finite Element Method (SBFEM). In the SBFEM, a semi-analytical method, only the boundary of the computational domain is discretized with finite elements. The physical behavior inside the domain is described analytically. A special formulation for guided waves in prismatic geometries was developed by Gravenkamp and Krome [1, 3, 4]. In [3] it was shown that it presents an efficient alternative to the FEM to compute guided waves.

The method was shown to be an efficient tool for the simulation of guided waves in solids. However, a stable method for the simulation of acoustic behavior in fluids is still missing. In a first try a displacement-based ansatz was used to describe the fluid. This ansatz results in spurious modes, widely reported in literature. By applying a penalty parameter the spurious modes can be suppressed. Despite good results for 2D and axisymmetric geometries [6], for the case of 3D geometries the implemented method did not show results without spurious modes.

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We, therefore, in this work implement the Helmholtz equation to describe the acoustic fluid into the SBFEM.

1 Theoretical framework

The SBFEM relies on a coordinate transformation. In general the cartesian coordinates are mapped onto the so-called SBFEM coordinates, η and ξ . η is the local coordinate of the finite elements on the boundary. ξ is the scaling direction pointing from so-called “scaling center” to the boundary. In the case of prismatic geometries the scaling center is placed at $x - \infty$. It follows that ξ is equivalent to the cartesian x coordinate. In 2D the local coordinate η is parallel to the y coordinate, respectively in 3D the η lies in the y - z -plane .

1.1 Acoustic-structure interaction

How to handle dynamic behavior in solids can be found in [1, 3]. It is therefore not repeated here. The acoustic behavior of a fluid is described by the Helmholtz equation

$$K\Delta p - \rho_f \ddot{p} = 0, \quad (1)$$

where K is the bulk modulus and ρ_f the density of the fluid. By applying the coordinate transformation and performing some computation steps (equivalent to the computation steps for solid found in [1]), the SBFEM equation is obtained.

$$\mathbf{E}_{0p} \mathbf{p}_{n,xx}(x) - \mathbf{E}_{2p} \mathbf{p}_n(x) - \mathbf{M}_{0p} \ddot{\mathbf{p}}_n = 0 \quad (2)$$

\mathbf{E}_{0p} , \mathbf{E}_{2p} and \mathbf{M}_{0p} are coefficient matrices integrated over the finite elements and \mathbf{p}_n is a vector containing the integrated nodal pressure values. A solution algorithm for the dispersion behavior can be found in [1].

The boundary condition between fluid and solid reads as

$$K \frac{\partial p}{\partial n} = -\rho_f \mathbf{n}^T \ddot{\mathbf{u}} \quad (3)$$

where \mathbf{n} is the normal vector pointing from the fluid domain into the solid domain. For the SBFEM this results in following equation system.

$$\begin{bmatrix} \mathbf{E}_{0s} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{0p} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{n,xx} \\ \mathbf{p}_{n,xx} \end{bmatrix} + \begin{bmatrix} \mathbf{E}_{1s}^T - \mathbf{E}_{1s} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{n,x} \\ \mathbf{p}_{n,x} \end{bmatrix} - \quad (4)$$

$$\begin{bmatrix} \mathbf{E}_{2s} & \mathbf{Q} \\ \mathbf{0} & \mathbf{E}_{2p} \end{bmatrix} \begin{bmatrix} \mathbf{u}_n \\ \mathbf{p}_n \end{bmatrix} - \begin{bmatrix} \mathbf{M}_{0s} & \mathbf{0} \\ \rho_f K \mathbf{Q}^T & \mathbf{M}_{0p} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_n \\ \ddot{\mathbf{p}}_n \end{bmatrix} = \mathbf{0} \quad (5)$$

where \mathbf{E}_{0s} , \mathbf{E}_{1s} , \mathbf{E}_{2s} and \mathbf{M}_{0s} are the coefficient matrices of the solid domain. The coupling matrix \mathbf{Q} is given by

$$\mathbf{Q} = \mathbf{N}^T \mathbf{n} \mathbf{N}_p \quad (6)$$

where \mathbf{N} are the shape functions of the solid element and \mathbf{N}_p the shape functions of the fluid element. By transforming (5) into the frequency domain an eigenvalue problem is obtained, which yields the dispersion behavior of acoustic waves inside the geometry.

2 Examples

2.1 Liquid-layer between two plates - dispersion behavior

To begin with the dispersion curves of two aluminium plates with a thin liquid layer in between are calculated (see Fig. 1a). Both plates have a thickness of $h = 5$ mm, whereas the liquid layer is $d = 1$ mm

thick. The material parameters for aluminium are given by: ($E = 70 \text{ GPa}$, $\nu = 0.33$, $\rho = 2700 \frac{\text{kg}}{\text{m}^3}$). The liquid layer is made from water ($K = 2.1904 \text{ GPa}$ and $\rho_f = 1000 \frac{\text{kg}}{\text{m}^3}$). The phase velocity is shown in Fig. 1b. The results are in perfect agreement with dispersion curves computed with COMSOL Multiphysics[®].

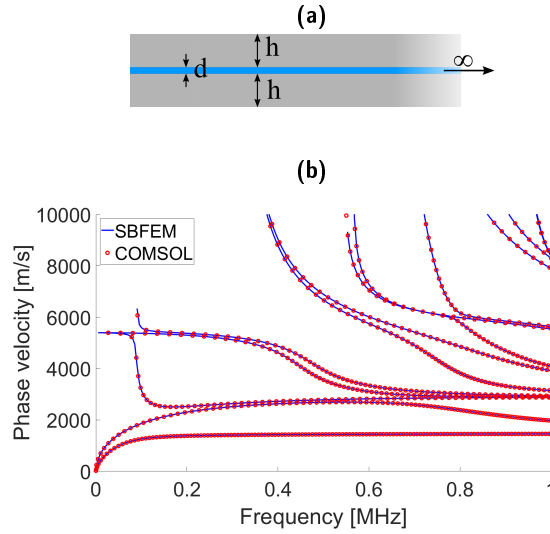
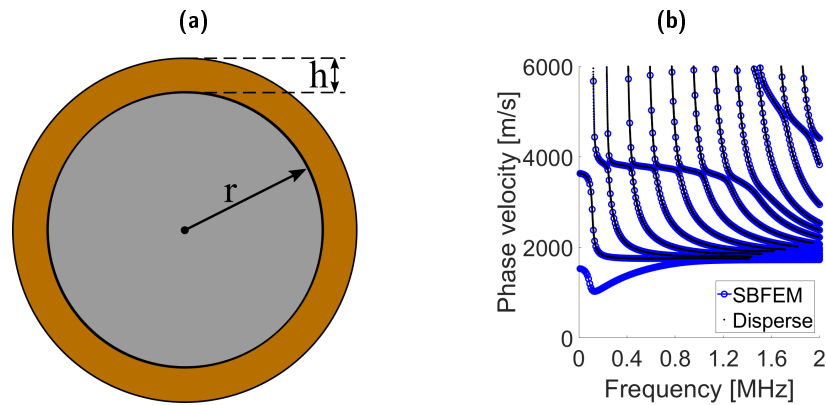


Figure 1. Dispersion behavior of plate with a fluid layer in between. a) simulated geometry b) phase velocity over frequency.

2.2 Liquid-filled pipe

Guided waves can be used very efficiently to investigate or monitor piping systems. For this purpose the wave behavior in such a structure is of interest. In Fig. 2b the dispersion behavior of an brass pipe filled with motor oil ($\rho_f = 870 \frac{\text{kg}}{\text{m}^3}$ and $K = 2.634 \text{ GPa}$) is displayed. The pipe is made of brass ($\rho = 8400 \frac{\text{kg}}{\text{m}^3}$, $\nu = \frac{1}{3}$ and $E = 108.42 \text{ GPa}$). The inner radius r of the pipe is 5 mm and the wall thickness h is 1 mm (see Fig. 2a). Here, an axisymmetric model is used, where in the SBFEM only one radius of the pipes has to be discretized with one-dimensional finite elements [2]. The results of SBFEM are compared to results from DISPERSE [5], which relies on the analytical Global Matrix Method (GMM), and COMSOL Multiphysics[®]. Both, SBFEM and COMSOL Multiphysics[®], yield one mode more than DISPERSE. It is the fluid mode M1 [Woekel.2015].



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