



2nd International Conference on Structural Integrity, ICSI 2017, 4-7 September 2017, Funchal, Madeira, Portugal

Approximation of the crack driving force for cracks at notches under static and cyclic loading

M.Madia^{a*}, D. Tchoffo Ngoula^b, U. Zerbst^a, H. Th. Beier^b

^aBundesanstalt für Materialforschung und -prüfung (BAM), Division 9.1, Unter den Eichen 87, D-12205 Berlin, Germany

^bTechnische Universität Darmstadt, Materials Mechanics Group, Franziska-Braun-Str. 3, D-64287 Darmstadt, Germany

Abstract

The work deals with the efficient calculation of the elastic-plastic crack driving force (J -integral for monotonic loading and ΔJ -integral under cyclic loading) for short cracks at notches as essential parameter for the reliable static and fatigue assessment of notched structures. The J - or ΔJ -integral is calculated based on analytical solutions for stress intensity factors, estimated by means of well-known weight function solutions in the case of cracks under power-law stress distributions. A plasticity-correction function is applied to the stress intensity factors to obtain the final expression of the crack driving force. The comparison between analytical solutions and finite element calculations in case of cracks at the weld toe in welded joints shows good agreement.

© 2017 The Authors. Published by Elsevier B.V.

Peer-review under responsibility of the Scientific Committee of ICSI 2017

Keywords: Structural integrity; fracture mechanics; crack driving force; notches

1. Introduction

The most widely spread standards and guidelines for the structural assessment of structures containing cracks, such as EPRI, R6, BS7910 or SINTAP/FITNET, rely on the so-called EPRI estimation scheme (Kumar et al., 1981) or on the reference stress method introduced by Ainsworth (1984) for the evaluation of the crack driving force, see

* Corresponding author. Tel.: +49 30 8104-4166.

E-mail address: mauro.madia@bam.de

Nomenclature

a	crack depth
C_i	material parameter in the Chaboche model
ds	infinitesimal arc length along the integration path Γ
E	elastic modulus
k_t	elastic stress concentration factor
K	stress intensity factor for mode I
K'	parameter in the Ramberg-Osgood equation
$f(L_r)$	plasticity correction function
J	J -integral
L_r	ligament yielding parameter
$m(\xi, a)$	weight function
M_{ij}	coefficients in the expressions of the weight functions
n'	exponent in the Ramberg-Osgood equation
n_j	normal outward vectors
R	stress ratio
T_i	components of the traction vector
u_i	components of the displacement vector
W	strain energy density
ΔJ	cyclic J -integral
ΔW	cyclic counterpart of the strain energy density
ε_{ij}	components of the strain tensor
ε_p	equivalent plastic strain
γ_i	material parameter in the Chaboche model
Γ	integration contour
$\sigma(\xi)$	through-thickness stress distribution
σ_{app}	applied remote stress
σ_{ij}	components of the stress tensor
σ_y	yield stress
σ_0	reference yield stress
ξ	in-depth local coordinate at the notch
χ	stress gradient at the notch

Zerbst et al. (2000, 2007). Note that the reference stress method, compared to EPRI, has a wider application range in that it is not restricted to configurations for which the finite element based h -functions are available. In the present context, it is important that it is not restricted to semi-circular surface cracks, but also allows the analysis of semi-elliptical cracks.

Approximate formulas are also used in fracture mechanics-based methods for the estimation of the fatigue life of components, but with a substantial difference: In monotonic fracture assessment procedures, elastic-plastic fracture mechanics is applied to allow for ductile tearing and plastic collapse, whereas in fatigue assessment most of the calculation schemes apply simplified approaches, which are based on linear-elastic fracture mechanics. Nevertheless, it is well-known that the small-scale yielding condition is not always applicable even in fatigue, for instance in case of the propagation of mechanically/physically short cracks, for which the extension of the plastic region is in the same order of magnitude as the crack length. A further important application of this kind is given by cracks growing at notches. Note, however, that proposals for elastic-plastic fatigue crack growth evaluation are available in the literature, see McClung et al. (1999).

The work presents a simplified calculation scheme based on the reference stress method, according to which the crack driving force for short cracks at notches can be calculated for both static and cyclic loading. Analytical solutions have been generated for different cracks at the weld toe of various welded joints and compared to finite element

solutions. In the finite element analyses, the material behavior has been described by a Ramberg-Osgood relationship in case of static loading, whereas the Chaboche material model has been employed in the simulations of cyclic loading. Results show that the approximation formulas provide values of J and ΔJ very close to the finite element results even in case of mechanically short cracks under cyclic loading, which allows to use this methodology for reliable and efficient calculation of the fatigue life of notched components.

2. Calculation of the crack driving force

The crack driving force has been calculated using two different strategies: (i) analytical formulas and (ii) finite element simulations.

2.1. Analytical approach

The calculation of the elastic-plastic crack driving force is based on the R6 calculation scheme (R6, 2013). In case of monotonic loading the following equation is used:

$$J = J_e \cdot [f(L_r)]^{-2}, \quad (1)$$

where J_e is the elastic component of the J -integral, $f(L_r)$ is the plasticity correction function and L_r is the ligament yielding parameter. For a detailed overview of the method the reader may refer to Zerbst et al. (2007). Here it is important to remark that a different determination of L_r has been proposed in Madia et al. (2014):

$$L_r = \sigma_{app}/\sigma_0, \quad (2)$$

in which the reference yield stress σ_0 has been introduced in place of the limit load. The advantage of the proposed formulation has an optimized accuracy, which is particularly important in fatigue crack propagation analysis.

The correct determination of the J -integral according to Eq.(1) needs the information of the elastic component of the J -integral (J_e) and a formulation of the plastic correction function $f(L_r)$.

Notched structures are characterized by non-linear through-thickness stress profiles, in many engineering cases with high values of the elastic stress concentration factor k_t and steep gradients χ . One example for this is given by stress profiles at the weld toe in welded structures. Therefore, it is important to use a method for the determination of the elastic crack driving force for a potential crack at the notch, which can be employed in case of an arbitrary stress distribution. Methods based on weight functions have the advantage to be very versatile and applicable nearly to any problem, even those involving complex stress fields such as notch fields with steep gradients, residual stress profiles or thermal stresses. For mode I problems, the general expression of the stress intensity factor is

$$K = \int_0^a [\sigma(\xi) \cdot m(\xi, a)] d\xi, \quad (3)$$

where $\sigma(\xi)$ is the through-thickness stress distribution normal to the crack plane and $m(\xi, a)$ is the weight function.

Different expressions of the weight functions are available in the literature according to the considered geometrical configuration. The present work relies on the functions developed for semi-elliptical cracks in a finite plate under non-linear stress profiles for the determination of the crack driving force for surface cracks at weld toes. The expressions of the weight functions for the deepest (A) and the surface point (C) are

$$m_A(\xi, a) = \frac{2}{\sqrt{2\pi a(1-\frac{\xi}{a})}} \cdot \left[1 + M_{1A} \cdot \left(1 - \frac{\xi}{a}\right)^{\frac{1}{2}} + M_{2A} \cdot \left(1 - \frac{\xi}{a}\right) + M_{3A} \cdot \left(1 - \frac{\xi}{a}\right)^{\frac{3}{2}} \right] \quad (4)$$

and

$$m_c(\xi, a) = \frac{2}{\sqrt{\pi}\xi} \cdot \left[1 + M_{1c} \cdot \left(\frac{\xi}{a}\right)^{\frac{1}{2}} + M_{2c} \cdot \left(\frac{\xi}{a}\right) + M_{3c} \cdot \left(\frac{\xi}{a}\right)^{\frac{3}{2}} \right] \tag{5}$$

The coefficients M_{ij} have been calculated according to Wang et al. (1995) and an efficient integration scheme is adopted to avoid singularity problems (Moftakhar et al., 1992).

There exist various options for the definition of the plasticity correction function. If the stress-strain behavior of the material is known (approximated by a Ramberg-Osgood in the present work), $f(L_r)$ can be determined by the highest analytical analysis level of R6 (2013)

$$f(L_r) = \left(\frac{E \cdot \varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2} \cdot \frac{\sigma_{ref}}{E \cdot \varepsilon_{ref}} \cdot L_r^2 \right)^{-\frac{1}{2}}, \tag{6}$$

where ε_{ref} and σ_{ref} are the reference strain and reference stress respectively, and E is the elastic modulus. The meaning of ε_{ref} and σ_{ref} is depicted in Fig. 1(a).

The application of the reference stress method to cyclic loading has been proposed by Zerbst et al. (2012), who applied the method to the determination of the stress-life curve of specimens made of Al5380-H321, containing large second phase particles. The method has been developed further and the cyclic J -integral is determined by

$$\Delta J = \Delta J_e \cdot [f(\Delta L_r)]^{-2}, \tag{7}$$

$$f(\Delta L_r) = \left(\frac{E \cdot \Delta \varepsilon_{ref}}{\Delta \sigma_{ref}} + \frac{1}{2} \cdot \frac{\Delta \sigma_{ref}}{E \cdot \Delta \varepsilon_{ref}} \cdot \Delta L_r^2 \right)^{-\frac{1}{2}}, \tag{8}$$

and

$$\Delta L_r = \frac{\Delta \sigma_{app}}{2 \cdot \sigma_0}. \tag{9}$$

It is easy to recognize that the expressions of Eqs. (7) to (9) are formally equivalent to the expressions derived in case of monotonic loading, in that the loading variables are replaced by their variation (range). Note that it has been assumed that the Masing’s hypothesis is verified for the materials under investigation, so that either branch of the cyclically-stabilized stress-strain material response is geometrically similar to the Ramberg-Osgood material law by a scale factor of two. The meaning of this statement is depicted in Fig. 1(b).

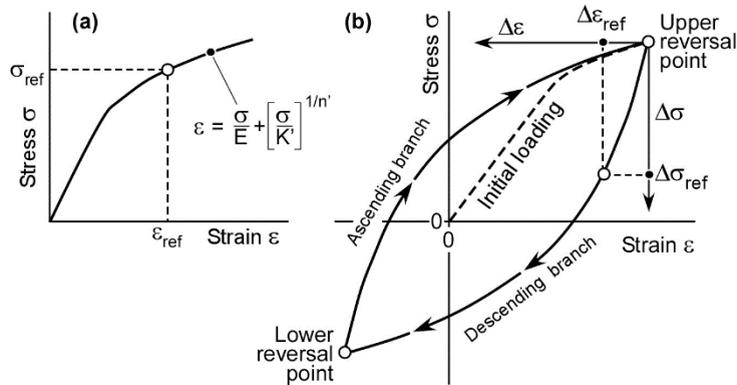


Fig. 1. Determination of the reference stress and strain in case of (a) monotonic and (b) cyclic loading.

2.2. Numerical approach

The calculation of the J -integral for monotonic loading by means of finite elements is widely applied and implemented in many finite element commercial codes, therefore no detailed discussion is needed here. In contrast, the numerical definition of the cyclic J -integral is not trivial and some specific requirements have to be met for its calculation. The general expression is given by

$$\Delta J = \int_{\Gamma} \left(\Delta W dy - \Delta T_i \frac{\partial \Delta u_i}{\partial x} \right) ds, \quad (10)$$

where

$$\Delta W = \int_0^{\Delta \varepsilon_{ij}} \Delta \sigma_{ij} d\Delta \varepsilon_{ij}. \quad (11)$$

Note that the operator Δ presented in Eqs. (10) and (11) has not a univocal meaning. When used in conjunction with the stress tensor, strain tensor, traction vector and displacement vector components, it denotes the changes of these quantities. These changes must be evaluated referring to a reference state, which serves thus as new origin for the calculation of the increments of the variables. For instance, considering the stress field, the load reversal points are natural reference states, see Fig.1(b). However, this definition of Δ does not apply to the J -integral and to the strain energy density W . Therefore ΔJ and ΔW shall be treated as functions, which depend on the variation of their arguments, as defined in Eqs. (10) and (11).

The central issue in the numerical calculation of the cyclic J -integral lies, therefore, in the numerical integration of Eqs. (10) and (11), which can be done by postprocessing the values of stresses, strains and displacements derived by finite element calculations. Once the integration path is defined and the local normal outward vectors n_j calculated, the integration can be performed from one load reversal point to the other, as far as the ascending and descending branches of the cyclic stress-strain response are identical and no crack closure effects are considered. The trapezoidal rule is employed for the numerical determination of cyclic strain energy density of Eq.(11):

$$\Delta W \approx \sum_{L=L_{lower}}^{L_{upper}-1} \left[\frac{\Delta \sigma_{ij}^{L+1} + \Delta \sigma_{ij}^L}{2} \cdot (\Delta \varepsilon_{ij}^{L+1} - \Delta \varepsilon_{ij}^L) \right], \quad (12)$$

where L is the load increment counter, L_{lower} and L_{upper} denote the lower and upper load reversal point respectively, which are used as limits of the integration. The variation of the traction vector components are evaluated at every load increment as follows:

$$\Delta T_i^L = \Delta \sigma_{ij}^L \cdot n_j \quad (13)$$

Finally, the partial derivative of the displacement vector components $\partial \Delta u_i / \partial x$ is approximated by difference quotients.

3. Examples of application of the approaches

The analytical and the numerical approaches have been compared extensively on selected types of welded joints. Here the results are presented for semi-circular cracks at the weld toe of Double-V butt-welds and cruciform joints made of steel S355NL under cyclic loading.

3.1. Material data

The analytical approach needs the characterization of the cyclically-stabilized stress-strain response of the material. The experimental data of low cycle fatigue tests have been used to determine this for the S355NL steel in terms of the Ramberg-Osgood equation:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}, \tag{14}$$

with $K' = 1115 \text{ MPa}$, $n' = 0.1961$ and $E = 210000 \text{ MPa}$. In addition to this, the numerical model needs the cyclic elastic-plastic material behaviour, which has been implemented in the form the Chaboche model:

$$\sigma = \sigma_y + \sum_{i=1}^5 \left[\frac{C_i}{\gamma_i} \cdot (1 - \exp(-\gamma_i \cdot \varepsilon_p)) \right], \tag{15}$$

in which C_i and γ_i are material parameters, σ_y is the yield strength and ε_p is the equivalent plastic strain. The values of the parameters are reported in Table 1.

Table 1. Material parameters for the Chaboche model in Eq.(15) for the steel S355NL.

i	1	2	3	4	5
C_i [MPa]	305034	98322	90427	26762.3	7607
γ_i	12318.92	3654.02	1204.24	230.50	21.07
σ_y [MPa]	152.17				

3.2. Comparison of the results

Several semi-circular cracks have been simulated also in the finite element analysis by using the node release technique. The value of the crack depth was in the range from 0.05 mm to 0.5 mm, i.e. well within the mechanically short crack range for the steel S355NL (Madia et al., 2016).

The first level of comparison has been performed with linear elastic material behavior in order to evaluate the precision of the approximation of the weight function method with respect to the finite element calculations. The results are depicted in Fig. 2, where a very good agreement can be noted between analytical and numerical approach.

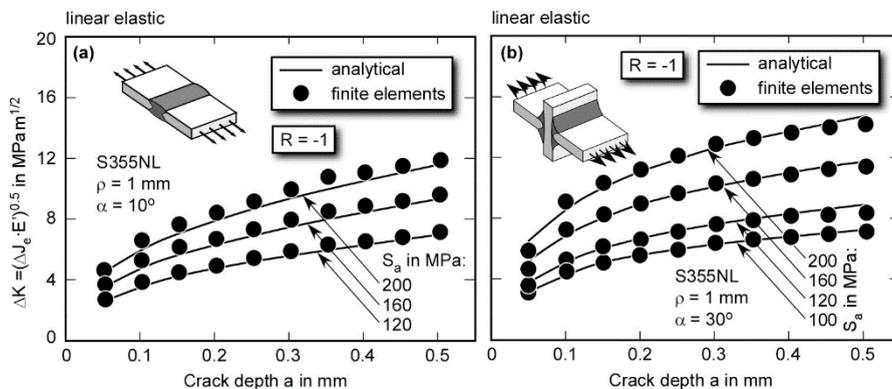


Fig. 2. Comparison between the analytical and numerical approaches on the basis of the linear elastic stress intensity factor: (a) Double-V butt-weld; (b) cruciform joint. The calculations have been performed for the material S355NL and for different levels of applied remote stress.

The calculations have been repeated for the same examples, but using elastic-plastic material behavior. The results reported in Fig. 3 show that the approximation of the cyclic elastic-plastic behavior by means of the plasticity correction based on the reference stress method yields values of the cyclic crack driving force very close to those obtained with the more complex finite element calculations. Note that the results are given in terms of the plasticity-corrected stress intensity factor range ΔK_p , which is formally derived from ΔJ according to the following equation:

$$\Delta K_p = \sqrt{E' \cdot \Delta J}, \quad (16)$$

where $E' = E$ for plane stress and $E' = E/(1 - \nu^2)$ for plane strain.

These results are very encouraging and useful in view of the implementation of the analytical approximations for the assessment of the fatigue life of notched components, where the finite element modelling would be not applicable due to computational costs. Note that the authors successfully applied the presented calculation scheme to the fatigue life assessment of welded joints, where even multiple crack propagation and interaction have been simulated (Madia et al., 2017). The procedure gave good approximations of the stress-life curves for different welded joints both in the finite life and in the fatigue limit regime.

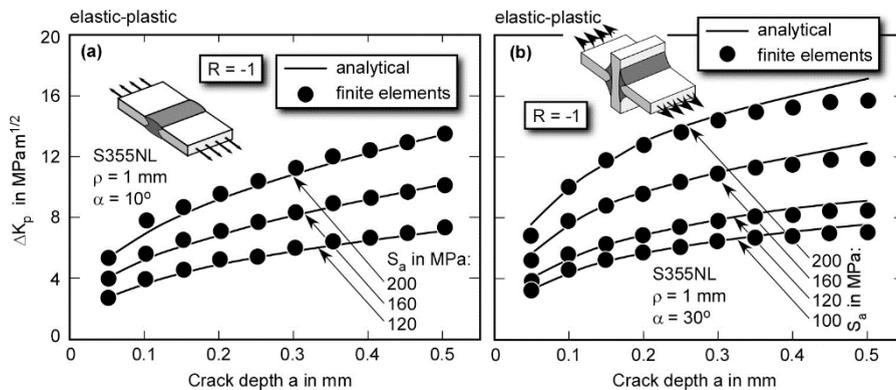


Fig. 3. Comparison between the analytical and numerical approaches on the basis of the plasticity-corrected stress intensity factor: (a) Double-V butt-weld; (b) cruciform joint. The calculations have been performed for the material S355NL and for different levels of the applied remote stress.

References

- Ainsworth, R.A., 1984. The assessment of defects in structures of strain hardening material. *Engng Fract Mech* 19, 633-642.
- Kumar, V., German, M.D., Shih, C.F., 1981. An Engineering Approach for Elastic-Plastic Fracture Analysis. Report NP-1931, Electric Power Research Institute.
- Madia, M., Arafah, D., Zerbst, U., 2014. Reference load solutions for plates with semielliptical surface cracks subjected to biaxial tension loading. *Int J Pres Ves Piping* 119, 19-28.
- Madia, M., Zerbst, U., 2016. Application of the cyclic R-curve method to notch fatigue analysis. *Int J Fatigue* 82, 71-79.
- Madia, M., Zerbst, U., Beier, H.Th., Schork, B., 2017. The IBESS model – Elements, Realisation and Validation. Submitted to *Engng Fract Mech*.
- McClung, R.C., Chell, G.G., Lee, Y.-D., Russell, D.A., Orient, G.E., 1999. Development of a Practical Methodology for Elastic-Plastic and Fully Plastic Fatigue Crack Growth.
- Moftakhar A.A., Glinka G., 1992. Calculation of stress intensity factors by efficient integration of weight functions. *Engng Fract Mech* 43, 749-756.
- R6, Revision 4, 2013. Assessment of the Integrity of Structures Containing Defects. EDF Energy Nuclear Generation Ltd., Gloucester, UK.
- Wang, X., Lambert, S.B., 1995. Stress intensity factors for low aspect ratio semi-elliptical surface cracks in finite-thickness plates subjected to nonuniform stresses. *Engng Fract Mech* 51, 517-532.
- Zerbst, U., Ainsworth, R.A., Schwalbe, K.H., 2000. Basic principles of analytical flaw assessment methods. *Int J Pres Ves Piping* 77, 855-867.

- Zerbst, U., Madia, M., Hellmann, D., 2012. An analytical fracture mechanics model for estimation of S–N curves of metallic alloys containing large second phase particles. *Engng Fract Mech* 82, 115-134.
- Zerbst, U., Schödel, M., Webster, S., Ainsworth, R.A., 2007. *Fitness-for-Service Fracture Assessment of Structures Containing Cracks. A Workbook based on the European SINTAP/FITNET Procedure*, Elsevier.