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Vibration-based Model Updating and Identification of Multiple Axial Forces in Truss Structures

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Abstract

Safety assessment of existing iron and steel truss structures requires the determination of the axial forces and corresponding stresses in truss structural members. The results of the axial force determination can be integrated as part of a structural health monitoring scheme for existing trusses. In this work, a methodology is proposed to identify multiple axial forces in members of a truss structure based on the modal parameters. Vibration test allows the identification of the natural frequencies and mode shapes, globally of the truss structure as well as locally of the individual bars. The method calibrates the numerical model of the truss structure using a genetic algorithm and strategic validation criteria. The validation criteria are based on the identified natural frequencies and global mode shapes of the truss structure as well as information of the axial forces in the individual bars of the truss, which are estimated from the natural frequencies and five amplitudes of the corresponding local mode shapes of the single bars based on an analytical-based algorithm. The calibration allows the identification of the axial forces in all bars of the truss structure. For mode pairing strategy, a technique makes use of the enhanced modal assurance criteria with the calculation of the modal strain energies. Moreover, the modal strain energies are also used to select the relevant local mode shape of the individual bars. The feasibility and accuracy of the proposed methodology is verified by laboratory experiments on several truss structures. In situ tests on existing trusses are intended. The results from one of the laboratory tested structures, i.e. a two-bar system, are included in this paper.

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1. Introduction

Civil engineering structures composing of axially loaded members such as roof trusses, truss girders, steel offshore or aerospace structures, due to their inherent degradation, service loads and accidental actions, necessitate maintenance overtime. To assess the safety and design structural health monitoring schemes for existing truss-type structures, it is important to identify the axial forces and corresponding stresses which the members are subjected to.

In this work, iron and steel trusses supporting the roof of buildings are of particular interest. A number of those structures are still in use today, especially in historic and heritage monuments. Examples of iron roof trusses are those of the State Hermitage Museum in Saint Petersburg. From cultural heritage point of view, they are of enormous significance due to a great variety of constructive prototypes from the early days of the European iron construction [1, 2]. Precise determination of the stresses plays a crucial role for the preservation of historic trusses. The structural assessment measures require non-destructiveness, minimum intervention and practical applicability.

For truss structures, the member axial forces can be estimated by methods such as method of joints, method of sections or finite element method [3], provided information about parameters such as external loads, support conditions and member end connections are known. However, accurate information about those parameters is difficult to obtain in practice. Inverse method has therefore been applied, in which the unknown input parameters, in particular the axial load, are determined based on the output parameters such as modal parameters of natural frequencies and mode shapes.

The nondestructive inverse identification of the axial forces in axially loaded structural members has been studied by a number of researchers. The identification techniques can be classified according to static, static-dynamic or purely dynamic, as well as depending on the types of applications on different civil engineering structures, for instance columns, cables, tie-rods or space frames. Considering axially loaded members as part of a structure such as trusses, methods making use of the finite element formulations coupled with model updating techniques have been described in [4-6], which concern the determination of the axial forces in all structural members. However, these methods are based on sensitivity or gradient-based searching algorithms, whose success is significantly dependent on the assumptions of the initial values of the target parameters. In addition, the joint connections of investigated structures are ideally assumed as pinned or rigid, which in reality do not exist. Other approaches are analytically-based methods for single beams with unknown boundary conditions that have the generality to be applied to beams as part of a structure [7-9]. Nevertheless, these methods are sensitive to small errors on the input parameters and subjective to the mode selection. In [9], the identified force can be averaged from a selected range in the frequency domain approach, but the selection of this range is not generalized. In general, the methods of single beams do not apply in an obvious way to assemblies of members in trusses and frameworks [5]. In cases where the structural responses under new load scenarios are to be predicted, a global model of the structure is required.

The present work proposes a procedure to enable identification of multiple loads in members of truss structures and give information about the stiffness of the joint connections in order to acquire a global model of the structure. The focus of the work are planar truss-type structures consisting of slender members with the possibility to extend to space trusses. The identification procedure adopts optimization-based model updating driven by genetic algorithms (GA) [10, 11]. GA have been employed in applications such as model updating of railway bridges [12, 13] or inverse identification of axial force in tie-rods [14]. The proven advantage of GA is that the initial values for the target parameters are insensitive to the identification results. The proposed procedure is first verified using numerical data of a two-bar structure. After that, experimental validation is performed using laboratory test data of the two-bar system.

2. Proposed Methodology

Structures composing of axially loaded members, in particular trusses, are a constitution of individual straight members connecting at joints. The behavior of the constituent members governs the behavior of the structure as a whole. Global analysis is referred in this work as analysis of the whole structure, while local analysis is associated with analysis of single members. Considering an example of a two-member structure as shown in Fig. 1 (a). At joints, rotational springs are introduced to account for the unknown joint stiffness. Although the joints are often assumed as either pinned or rigid for design purposes, all truss and frame structures are semi-rigid in nature. The effect of the joint stiffness on the behavior of a structure will be addressed in the later part of the paper. When a single member of the structure is analyzed, a beam model as shown in Fig. 1 (b) is assumed, with translational and rotational springs.

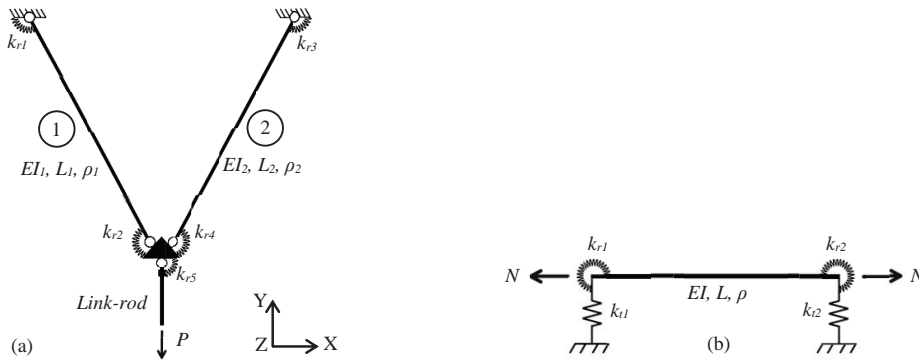


Fig. 1. Structural system for global and local analyses: (a) two-bar system; (b) single bar as part of a structure.

2.1. Local and global analyses including stress stiffening

It is known that an axial force acting in a structural member causes change in its stiffness and thus affect the vibration behavior of the structure. This phenomenon is called stress stiffening and has been discussed for instance in [4, 15]. For a simply supported Euler-Bernoulli beam subjected to a time-invariant constant axial force throughout its length, a closed-form frequency expression including stress stiffening exists [16]. For other boundary conditions of the beam including the axial-force effects, only approximate solutions can be studied, for example in [17, 18]. Similarly, when rotational inertia and shear deformation effects are considered based on Timoshenko beam theory, closed-form solutions do not exist. In such cases for single beams and for complex structures, the finite element method provides a powerful mean for the global analysis of a structure under stress stiffening. In the finite element method, a new element called geometric stiffness is included to account for the presence of the axial-force effects [4].

For the inverse estimation of the axial force in members as part of a structure, analytical-based algorithms were proposed using local vibration measurements [7-9]. The analytical-based method in [9] is based on Timoshenko beam theory and requires the installment of five or more sensors on a member. The identified axial force is obtained from each of the frequencies and five amplitudes of the corresponding mode shapes of the member, given the characteristics of the member are accurately known. As mentioned, the application of the methods for single beams is not straightforward on assemblies of members in trusses and frameworks. However, integration of the information acquired from the methods for single beams is a beneficial strategy. In the present work, the modal characteristics approach described in [9] is applied as part of the proposed methodology for the identification of the axial forces in multiple-bar structures.

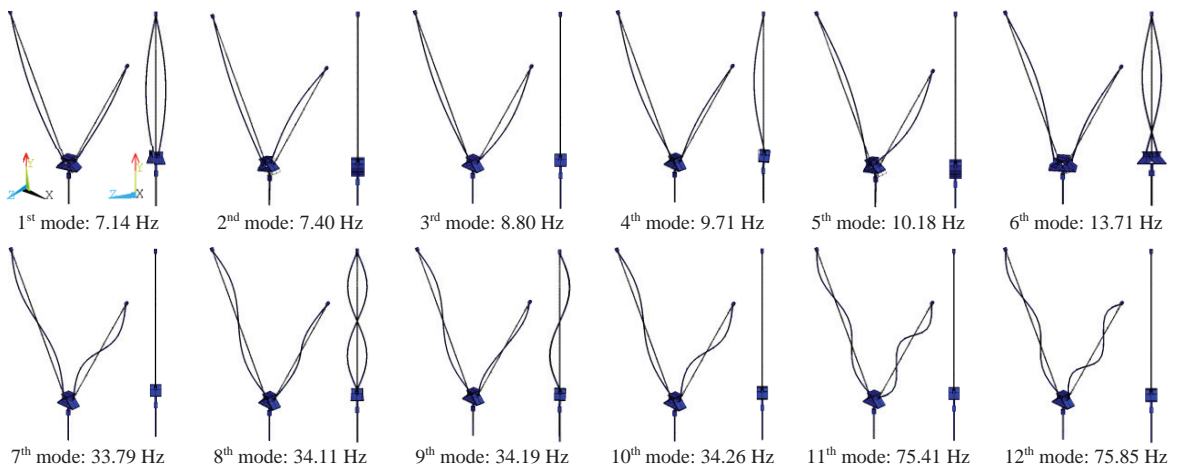


Fig. 2. Numerical natural frequencies and mode shapes of first twelve modes of pinned system at zero applied force in perspective and side view.

2.2. Numerical model with semi-rigid connections

For truss structures, the common assumption of pinned joints simplifies greatly hand calculations to analyze the forces in the structure. However, assumption of pinned or rigid joints can have pronounced effects on the structural dynamic behavior. To illustrate the importance of numerical modelling with semi-rigid connections for a multiple-bar structure, the effect of joint rigidity on the static and dynamic responses of the two-bar system in Fig. 1 (a) is examined.

The two-bar structure was modeled by the finite element method based on Timoshenko beam theory. The two members are assumed to have the same geometrical and material properties. Each member has a length of 1510 mm and solid circular cross-section of $\phi 9.5$ mm. The material properties for the system are assumed as the modulus of elasticity of 205000 N/mm², mass density of 7850 kg/m³ and the Poisson’s ratio of 0.30. Three-dimensional linear elastic rotational springs with varying stiffness were introduced to five connection points at beginning and end of the two members and the joint to the link-rod.

Four target systems were considered, i.e. pinned vs. rigid systems and zero- vs. high-load systems. For the pinned system, the rotational spring stiffness are assumed to be infinitesimal; whereas for the rigid system, they tend to infinity. The applied axial load takes two values of zero and 30 kN. The load of 30 kN for the high-load system is equivalent to an axial force and normal stress of approximately 17.14 kN and 242 N/mm² in the two members. As the inclusion of geometric stiffness is often linked with non-linear geometrical analysis, linear static analysis with stress stiffening as well as geometric non-linear analysis, i.e. stress stiffening and large deformations, have been performed. However, because the system experiences minor out of plane bending or deformations, the analysis taking into account large deformations provides highly similar results to the linear static analysis and thus will not be presented.

From the static analysis, the assumption of pinned or rigid joints have little effects on the stresses of the members, as the bending stress is nearly negligible compared to the normal stress. For the dynamic analysis, the first twelve modes of the pinned system are presented in Fig. 2. As expected, the global modes of a structure are governed by the modal configurations of its members. Fig. 3 shows the results of the natural frequencies of the pinned and rigid systems taking into account stress stiffening. It can be seen that the assumptions of joint rigidity have significant effects on the dynamic behavior, in particular the natural frequencies, of the system. Evidently, the numerical model should consider the unknown joint stiffness to correctly approximate the structural dynamics. For practical purposes, constraint parameters in the form of stiffness indexes or percentages of semi-rigid connection have been introduced to assess the degree of joint flexibility [19]. The fixity factor representing the semi-rigid connection as percentage is expressed as

$$\gamma_i = \frac{k_{ri}L}{3EI + k_{ri}L} \tag{1}$$

where k_{ri} is the rotational spring stiffness at i end of a member, EI is the flexural stiffness and L is the length of the member. The factor γ_i takes values from zero to one, with zero indicating pinned and one specifying rigid condition.

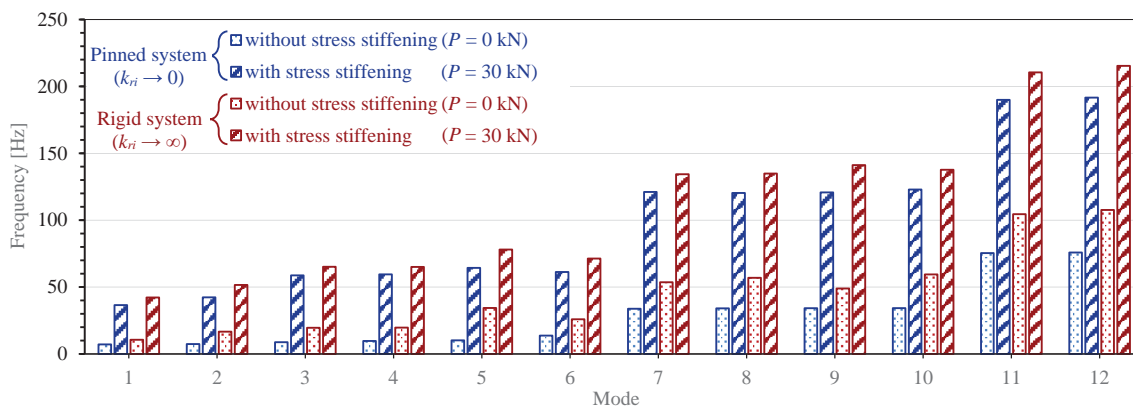


Fig. 3. Numerical natural frequencies of first twelve modes of pinned and rigid two-bar system without and with stress stiffening.

2.3. Identification procedure

The inverse identification of the axial forces and the corresponding stresses for a truss-type structure is performed by the finite element model updating using an optimization scheme driven by genetic algorithms (GA). Descriptions of the GA can be found for instance in [10]. Several recognized advantages of GA are such as non-independence of initial starting value, capacity to handle large number of parameters and find global minimum. For the inverse identification of the axial force in single beams, a procedure based on GA has been applied successfully [14]. For truss structures compared to single beams, the identification process requires a comprehensive objective function. Based on the results of the vibration measurements, the calibration between the numerical model and experimental test data is carried out, in which the updating parameters are the axial load and the rotational spring stiffness of joints. An objective function which includes terms related to the residuals of the frequencies and mode shapes as well as additional information of the member axial forces in selected members is minimized. Thus the objective function f_{obj} is defined as

$$f_{obj} = \sum_{i=1}^{nmodes} \frac{|f_i^{num} - f_i^{exp}|}{f_i^{exp}} + \sum_{i=1}^{nmodes} |\text{MAC}(\phi_i^{exp}, \phi_i^{num}) - 1| + \sum_{j=1}^{nbars} \frac{|N_j^{num} - N_j^a|}{N_j^a} \tag{2}$$

where f_i^{exp} , f_i^{num} , ϕ_i^{exp} and ϕ_i^{num} are the experimentally identified and numerically calculated natural frequencies and mode shapes of mode i ; N_j^{num} is the numerically calculated member axial force by static load equilibrium and N_j^a is the axial force identified by an analytically-based algorithm based on [9] in selected member j ; $nmodes$ is the total number of modes and $nbars$ is the total number of selected members. The modal assurance criteria (MAC) is given by

$$\text{MAC}_{ij} = \frac{(\hat{\phi}_i^T \hat{\phi}_j)^2}{(\hat{\phi}_i^T \hat{\phi}_i)(\hat{\phi}_j^T \hat{\phi}_j)} \tag{3}$$

where $\hat{\phi}_j$ is the numerically derived vector containing the coordinates from numerical mode j corresponding to the experimental degrees-of-freedom and $\hat{\phi}_i$ is the experimentally obtained vector containing the experimental information of mode i . Concerning the mode pairing, based on the results of the modal analysis of the numerical model, the two-bar system possesses closely spaced in-plane as well as out of plane modes. Therefore, the energy-based modal assurance criterion (EMAC) [20] is applied for the mode pairing criteria in addition to the MAC

$$\text{EMAC}_{ijk} = \prod_{jk} \text{MAC}_{ij} \tag{4}$$

where \prod_{jk} is the relative modal strain energy of a certain cluster k of numerical degrees-of-freedom related to a numerical mode j . For each mode, the relative modal strain energy for selected cluster of in-plane degrees-of-freedom and the EMAC are calculated. The assignment of modes is given similarly to the MAC, where the numerical mode with the largest value is assigned to the respective experimental mode.

Regarding the selection of mode for the member force identification using the analytical-based algorithm [9], because the analytical-based method gives more solutions of the axial force, i.e. one for each frequency, a criterion using the relative modal strain energy is applied. The relative modal strain energy is calculated for individual members of the structure for each mode. The vibration of each member in relation to other members and the dominant vibration of the members in certain modes can be recognized. The mode with the highest relative modal strain energy in the form of the first mode of a single beam is selected. The proposed selection criterion agrees with recommendations given in [8] about the selection of a proper flexural mode shape to increase the accuracy of the identified force.

Table 1. Parameters of target systems for the numerical study of the two-bar structure.

Target system	Axial load	Member axial force	Member axial stress	Rotational spring stiffness					
	P^{true} [kN]	N^{true} [kN]	σ^{true} [N/mm ²]	$k_{r1}^{true} = k_{r3}^{true}$ [kNm/rad]	γ_1^{true} -	$k_{r2}^{true} = k_{r4}^{true}$ [kNm/rad]	γ_2^{true} -	k_{r5}^{true} [kNm/rad]	γ_5^{true} -
1	5.00	2.89	40.78	0	0.000	2	0.925	500	0.996
2	15.00	8.59	121.19	2	0.925	20	0.992	5	0.692
3	25.00	14.29	201.60	20	0.992	200	0.999	50	0.957

3. Numerical verification

Case studies were conducted for the two-bar system simulating the experimental data, assuming the unknowns are the applied axial load at the end of the link-rod and the stiffness of the rotational springs (see Table 1). Three values of the axial load were chosen as 5 kN, 15 kN and 25 kN representing low, intermediate and high tensile stress in the members. The values of the rotational springs were chosen randomly to represent different constraint flexibility. The rotational spring stiffness at symmetric locations are assumed to be identical, since the identification problem in this case is symmetric. For clarity, the parameters used as simulated experimental data in the numerical study are denoted as “true”, whereas the parameters that are denoted as “id” are those identified from the proposed methodology.

Implementing the proposed method, the first five to six modes were used. The axial load was varied simultaneously with the rotational spring stiffness in the range from zero to 30 kN, while the rotational spring stiffness were varied from pinned to rigid conditions. The values of the initial populations were assumed random with no knowledge of the design parameters. Ten independent optimization runs with different initial populations were performed and the mean values from the runs were adopted.

The results of the numerical study for three target systems are given in Table 2 and Table 3. In addition, the identified axial loads and member forces as well as frequencies of the first five modes are presented in Fig. 4. The identified loads match very well to the true ones. Furthermore, the identified loads are highly consistent in all runs. Regarding the modal parameters, the true and identified frequencies are in high agreement; the MAC range from 0.988 to one. For the rotational spring stiffness, overall, the identified fixity factors indicate correctly joint rigidity in relation to hinged or rigid condition. The ratios of the fixity factors between the true and identified stiffness are close to one.

Table 2. Results of the identified axial forces and stresses in the numerical study of two-bar system.

Target system	P^{id} [kN]	Δ [%]	N^{id} [kN]	Δ [%]	σ^{id} [N/mm ²]	Δ [%]
1	4.97	-0.60	2.87	-0.69	40.49	-0.71
2	14.98	-0.13	8.58	-0.12	121.05	-0.12
3	24.96	-0.16	14.26	-0.21	201.18	-0.21

Table 3. Results of the fixity factors representing the identified rotational spring stiffness in the numerical study of two-bar system.

Target system	$\gamma_1^{id} = \gamma_3^{id}$ -	$\frac{\gamma_1^{id}}{\gamma_1^{true}}$	$\gamma_2^{id} = \gamma_4^{id}$ -	$\frac{\gamma_2^{id}}{\gamma_2^{true}}$	γ_5^{id} -	$\frac{\gamma_5^{id}}{\gamma_5^{true}}$
1	0.000	-	0.967	1.045	0.994	0.998
2	0.928	1.003	0.998	1.006	0.809	1.169
3	0.998	1.006	0.999	1.000	0.975	1.019

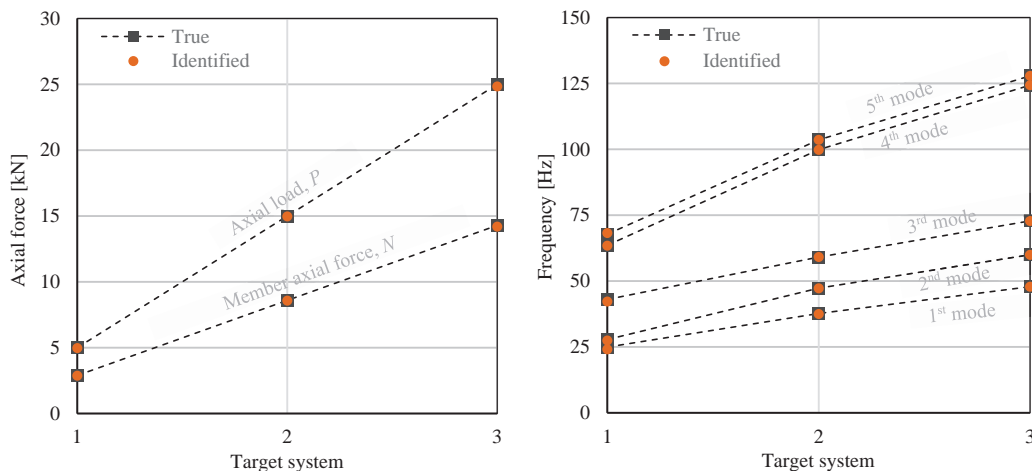


Fig. 4. Results of the numerical study of two-bar system: (a) Axial forces; (b) Natural frequencies of the first five modes.

4. Experimental validation

The laboratory experiments for two-bar structure with the geometric and material characteristics like the system in the numerical study were carried out. The elastic modulus of the steel bars was determined by tensile test as 205000 N/mm². Tension loads were applied by the hydraulic jack at the end of the link-rod in vertical pulling downward direction. Five load steps were performed in an increasing manner from approximately 7.05 kN to 26.45 kN. The force by the hydraulic jack was measured by a load cell. In addition, two strain gauges were used to measure the strain in the members; one was used for each member. As the assembled laboratory system is not absolutely symmetric, the axial forces and corresponding stresses in the two members are not absolutely the same. The applied loads result in an axial stress of about 58 N/mm² to 221 N/mm² measured by the strain gauge in the second member.

For the dynamic tests, twelve accelerometers were installed in total; six accelerometers on each member. Measurements were carried out with hammer excitations on the two members of the system at a sampling frequency of 2048 Hz. The modal parameters were extracted from the measured responses using a covariance-driven stochastic subspace identification technique [21]. From the identified modal parameters of the two-bar system, the first five to six modes corresponding to the in-plane mode of vibration were used in the identification process. Similar to the numerical study, the updating parameters are the applied axial load and the rotational spring stiffness assuming the same at symmetrical positions.

The results of the experimentally measured and identified loads and member axial forces for the two-bar system together with the identified rotational spring stiffness are given in Table 4. The identified joint stiffness indicate that the two-bar system does not exhibit high flexibility as the system is loaded and stressed. This illustrates that the assumption of pinned joints for the system could result in erroneous results of the dynamic responses. Fig. 5 shows the experimentally measured and identified forces as well as natural frequencies of the first five modes. The obtained axial forces match reasonably well between the experimental measured and numerical identified values for all load states. Moreover, for the identified modes, the natural frequencies agree well with the experimental results. The obtained MAC values range from 0.936 to 0.988, indicating a high consistency of the paired mode shapes.

Table 4. Results of the identified axial forces and rotational spring stiffness in laboratory experiments of the two-bar system.

P^{exp} [kN]	P^{id} [kN]	Δ [%]	σ_2^{exp} [N/mm ²]	N_2^{exp} [kN]	N_2^{id} [kN]	Δ [%]	$k_{r1}^{id} = k_{r3}^{id}$ [kNm/rad]	γ_1^{id} -	$k_{r2}^{id} = k_{r4}^{id}$ [kNm/rad]	γ_2^{id} -	k_{r5}^{id} [kNm/rad]	γ_5^{id} -
7.05	6.07	-13.90	58.03	4.11	3.49	-15.09	2.56	0.940	38.71	0.996	1.61	0.419
12.46	11.53	-7.46	102.00	7.23	6.61	-8.58	4.90	0.968	12.47	0.987	157.43	0.986
16.74	15.88	-5.14	136.60	9.68	9.09	-6.10	17.68	0.991	168.53	0.999	154.53	0.986
22.25	21.48	-3.46	177.80	12.60	12.27	-2.62	7.66	0.979	181.51	0.999	118.64	0.982
26.45	26.40	-0.19	220.50	15.63	15.09	-3.45	24.44	0.993	199.08	0.999	498.09	0.996

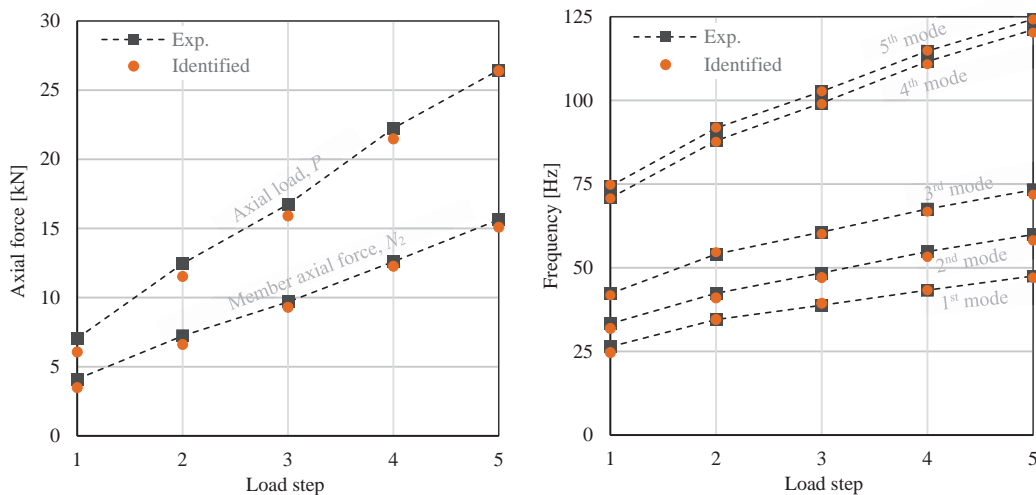


Fig. 5. Results of the laboratory experiments of the two-bar system: (a) Axial forces; (b) Natural frequencies of the first five modes.

5. Conclusions

A methodology is presented in this paper to identify the multiple axial forces and corresponding stress states in members of truss and truss-like structures. The numerical finite element model of the structure is calibrated based on the modal parameters using a genetic algorithm. The validation criterion is based on the identified natural frequencies and global mode shapes of the structure. Moreover, additional information of the axial forces in selected individual members of the structure based on an analytical-based algorithm are utilized. For the vibration test, a minimum number of five sensors per selected member is required by the application of the analytical-based algorithm.

The proposed method has been validated using simulated and experimental data from a two-member system. The axial loads in the structure have been correctly identified. In addition, information about the joint constraints in relation to pinned or rigid condition have been obtained, which are useful for the global modeling of the structure such as to predict the structural responses under different loading scenarios or design structural health monitoring schemes.

The results of the two-bar system in this paper are based on one of the laboratory experiment campaigns as part of a doctoral research work at Brandenburg University of Technology Cottbus–Senftenberg in cooperation with Bauhaus University Weimar and Federal Institute for Materials Research and Testing (BAM). Further results of different systems are to be subsequently presented as well as in the next step, in situ tests on existing truss structures are prepared.

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