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Model based sensitivity analysis in the determination of viscoelastic material properties using transmission measurements through circular waveguides

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Abstract

Several ultrasonic approaches for material determination are formulated in terms of an (nonlinear) inverse problem, e.g. immersion technique (Castaings et al. (2000)) or plate-waveguide techniques (Marzani et al. (2012)). In this contribution we focus on cylindrical waveguides for ultrasonic material determination and especially on the sensitivity of recorded transmission signals to the material properties. We utilize composite scaled sensitivities to determine the information content that can be achieved by the setup to certain parameters and discuss the limitations of the approach.

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1. Introduction

The knowledge about material properties is the basis of engineering from statics to dynamics. Whilst material characteristics in quasi-statics can be measured using standardized test (ISO 899, ISO 527, ISO 6721, etc.), the identification of material characteristics in a high dynamic range, i.e. ultrasonic frequencies, is generally more complex and has not been standardized yet.

In this contribution we focus on material property determination in cylindrical samples, i.e. extruded thermoplastics. Extruded rods can be processed using shape cutting technology and are therefore often preferred, if injection molding is not suitable due to geometric limitations or economic considerations, e.g. small series or custom products. Rautenberg (2012) first presented an approach to reconstruct material properties from polymeric cylindrical samples. Here, measurements are performed in transmission between the parallel faces of a hollow cylindrical sample, see Fig. 1. The received signal then contains information about the sample's material and geometric properties, whereas the geometric properties can be measured separately. Using a model of the guided wave problem, the material properties can be reconstructed in the sense of a nonlinear inverse problem.

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1.1. The nonlinear inverse Problem

An inverse problem is defined as the evaluation of causes based on the monitoring/measurement of observations. Hence, given a set of observations $\mathbf{y} \in \mathcal{Y}^N$ one intends to find the causes $\mathbf{p} \in \mathcal{P}^M$ that lead to these observations. For that purpose, a forward model is given with

$$\mathbf{y} = f(\mathbf{p}) \quad \text{with} \quad f : \mathcal{D}(f) \subset \mathcal{P}^M \rightarrow \mathcal{Y}^N \quad (1)$$

As there might be no suitable inverse of this model, one intends to solve the problem by minimizing the Euclidean norm of modeled observations as given by the forward model and measured observations. A solution of this problem can be achieved by variation of the modeled causes with

$$\mathbf{p}^{\text{ML}} = \operatorname{argmin}_{\mathbf{p}} \{\varepsilon(\mathbf{p}, \mathbf{y})\} \in \mathcal{D}(f) \quad \text{with} \quad \varepsilon(\mathbf{p}, \mathbf{y}) = \|f(\mathbf{p}) - \mathbf{y}\|_2^2 \quad (2)$$

with \mathbf{p}^{ML} being the maximum-likelihood estimate of \mathbf{p} . There exist three conditions for the well-posedness (in the sense of Hadamard) of the inverse problem, see Kabanikhin (2012). (1) The existence condition: For any \mathbf{y} there exists a solution described by $f(\mathbf{p})$. (2) The uniqueness condition: The solution of the problem is unique, i.e. $f(\mathbf{p})$ is bijective. (3) Stability condition: The solution is a continuous function of \mathbf{y} .

In this contribution we focus on two aspects of the nonlinear inverse problem. First, we discuss the sensitivity of the observations, here N discrete samples of the recorded signal $\mathbf{y} = [y(1), y(2), \dots, y(N)]^T$, to the causes, here material parameters as defined in the next section. Obviously, only those causes can be reconstructed that have a measurable influence on the observations. Second, in the context of nonlinear optimization it is advantageous to have a somehow balanced sensitivity distribution. We therefore discuss the effect of observation transformation on the information content in the sense of sensitivities.

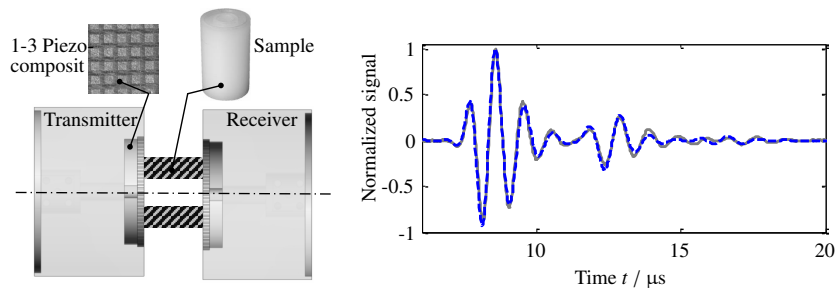


Fig. 1. Sketch of transmission experiment (left) and comparison of simulated (dashed) and measured (solid) signal (right).

2. Forward modeling of transmission experiment

2.1. Material model

For hexagonal materials, the elastic compliance matrix $\mathbf{S} = \mathbf{C}^{-1}$ depends on five linear independent elastic constants, i.e. $E_T, E_L, \nu_T, \nu_L, G_L \in \mathbb{R}^+$.

$$\mathbf{S} = \begin{bmatrix} 1/E_T & -\nu_T/E_T & -\nu_L/E_L & 0 & 0 & 0 \\ & 1/E_T & -\nu_L/E_L & 0 & 0 & 0 \\ & & 1/E_L & 0 & 0 & 0 \\ & \text{sym.} & & 1/G_L & 0 & 0 \\ & & & & 1/G_L & 0 \\ & & & & & 1/G_T \end{bmatrix} \quad \text{with} \quad G_T = \frac{E_T}{2(1 + \nu_T)} \quad (3)$$

Introducing viscoelastic effects, we restrict ourselves to a loss tangent that is equal to each entry in \mathbf{C} . This modeling has been found by Hosten et al. (2008) to be a suitable approximation for most polymers and composites. For modeling viscoelastic behavior, we use the fractional Zener model, see e.g. Mainardi (2010). The complex matrix of viscoelasticity then reads

$$\tilde{\mathbf{C}} = \mathbf{C} \frac{1 + (i\omega\tau_\varepsilon)^\beta}{1 + (i\omega\tau_\sigma)^\beta}, \quad (4)$$

with $\tau_\varepsilon, \tau_\sigma, \beta \in \mathbb{R}^+$ denoting retardation time, relaxation time and power parameter (fractional derivative order in time domain), respectively. The causes under consideration in the following study are $\mathbf{p} = [c_3, c_1, \nu_L, \nu_T, G_L, \tau_\varepsilon, \tau_\sigma, \beta]^T$ with $c_3^2 = 1/(S_{33}\rho)$ and $c_1^2 = 1/(S_{11}\rho)$.

2.2. Guided wave modeling

In this contribution, we use a forward model that is optimized with respect to computing costs, but in fact comprises some approximations. The overall structure of the forward model is:

- Compute dispersion curves using Scaled Boundary FEM, see Gravenkamp et al. (2014), using an equivalent real-valued frequency dependent material model, see Bause et al. (2014).
- Utilize modal expansion to compute the (undamped) transfer function of the waveguide given a spatial excitation and receiving model, see Bause et al. (2014).
- Re-introduce material damping, see Bause et al. (2014). This procedure is of course an approximation, essentially regarding the negligence of structural damping.

A comparison between simulation and forward model given the causes \mathbf{p}^* which is defined as working point in the next section can be seen in Fig. 1(right).

3. Sensitivity and information content

When calculating sensitivities, it is often useful for comparability to scale them into a dimensionless form. We follow the definitions given by Hill (1998) and define the dimensionless scaled sensitivity of observation y_j to cause p_i as

$${}_{y_j}dss_{p_i} = \left(\frac{\partial y_j}{\partial p_i} \bigg|_{\mathbf{p}^*} \right) p_i^* . \quad (5)$$

As no analytical derivatives are available, we approximate the derivatives using central difference approximations with $\pm 2\%$ change of the parameter under consideration. To indicate the total amount of information provided by all observations \mathbf{y} with respect to the cause p_i , the root mean square operation is used to define the composite scaled sensitivity, see Hill (1998)

$${}_{y}css_{p_i} = \sqrt{\frac{1}{N} \sum_{j=1}^N ({}_{y_j}dss_{p_i})^2} \quad (6)$$

Examples for the dimensionless scaled sensitivity of the time domain observations \mathbf{y} can be seen in Fig. 2(a,b), where the sensitivity to c_3 and ν_L has been calculated. It is obvious that the information content with respect to c_3 is much higher than to ν_L , which can also be observed when calculating the composite scaled sensitivities as depicted in Fig. 2(c). Transformation of the time domain observations into amplitude spectra information $\mathbf{y} \rightarrow \mathbf{Y}$ and calculating the composite scaled sensitivities, see Fig. 2(d) reveals that a more balanced information content can be achieved by using the spectral information.

4. Conclusions

It has been demonstrated that the choice of the representation of the observations has large influence on the information content provided to each cause and therefore highly influences the success of the solution of the inverse

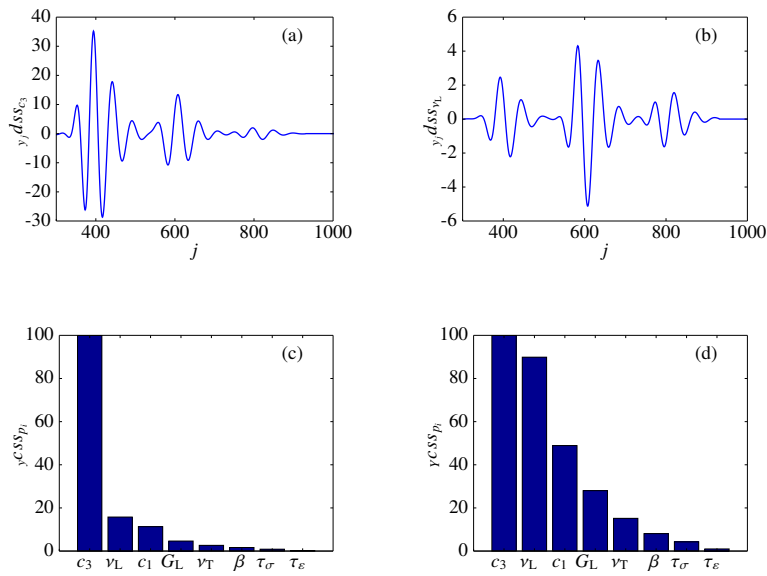


Fig. 2. Dimensionless scaled sensitivities and composite scaled sensitivities.

problem. It is interesting to note that although the transformation is linear and hence the information content of the observations y and Y are equal, the information content with respect to sensitivities changes dramatically. It has further been found that, despite the transformation, there is a much higher sensitivity to the 'elastic' parameters than to the 'viscoelastic' parameters that might be treated by multi-stage optimization, i.e. first optimizing $c_3, c_1, \nu_L, \nu_T, G_L$ and then $\tau_\sigma, \tau_\epsilon, \beta$. In addition, more information on the cross-sensitivities need to be taken into account to address the uniqueness problem. In this context, physical considerations about the symmetry class of extruded polymers, e.g. quasi isotropy, might lead to a more reliable formulation of the problem.

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