

20th European Conference on Fracture (ECF20)

A model for the determination of the fatigue life in technical alloys containing large and small defects

M. Madia^{a,*}, U. Zerbst^a, H.Th. Beier^b^a*BAM Federal Institute for Materials Research and Testing, Unter den Eichen 87, D-12205 Berlin, Germany*^b*Technische Universität Darmstadt, Petersenstraße 12, D- 64287 Darmstadt, Germany*

Abstract

The determination of the fatigue life in technical alloys containing large and small defects must rely on a propagation model which accounts for short and long crack growth. Recently an analytical model which incorporates propagation in the short crack regime and plastic correction for the crack driving force has been presented by two of the authors.

This work is intended to show further validation of the model, taking into account data sets for different materials with different testing conditions.

Despite the assumptions about missing parameters, the value of which had to be taken from the literature, the predictions showed a fairly good approximation of the fatigue lives. A possible interpretation of the results in terms of multiple crack initiation and propagation at higher loads is proposed.

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Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering

Keywords: Fatigue strength; Crack propagation; Short cracks.

1. Introduction

The overall lifetime of a cyclically loaded structure can be subdivided into three stages: i) crack initiation; ii) small and long fatigue crack propagation; iii) final fracture. In the small crack growth stage both micro-structurally

* Corresponding author. Tel.: +49 30 8104-4166; fax: +49 30 8104-1539.

E-mail address: mauro.madia@bam.de

Nomenclature

| | |
|----------------------------|--|
| a | crack length |
| a_i | initial crack length |
| f | plastic correction function |
| k | exponent in the McEvily's law |
| m | exponent in the Paris' law |
| C | constant in the Paris' law |
| E | modulus of elasticity |
| R | stress ratio |
| R_m | ultimate tensile strength |
| U | crack closure function |
| U_{LC} | value of the crack closure for long cracks |
| ϕ | boundary correction function |
| σ_f | flow stress |
| σ_{\max} | maximum applied stress |
| σ_Y | yield strength |
| ΔK | stress intensity factor range |
| ΔK_{eff} | effective stress intensity factor range |
| ΔK_{pl} | plasticity-corrected stress intensity factor range |
| ΔK_{th} | threshold stress intensity factor range |
| $\Delta K_{th,eff}$ | effective threshold stress intensity factor range |
| ΔL_r | cyclic ligament yielding parameter |
| $\Delta \varepsilon_{ref}$ | reference strain range |
| $\Delta \sigma_{ref}$ | reference stress range |

and mechanically small cracks must be considered, see Polak (2003). Micro-structurally small means a crack size in the order of micro-structural features (such as the grain size), whereas mechanically small refers to the order of mechanical quantities, such as the plastic zone size or a notch stress field.

In the case of engineering materials with large second phase particles the initiation stage is rather small and the overall lifetime is usually controlled by the extension of small cracks which are, if the initial defects are large enough, mechanically small cracks.

In Zerbst et al. (2011) two of the present authors proposed a model for fracture mechanics based determination of the fatigue strength and life, based on the assumption of a negligible short crack initiation stage which allowed them to base the analysis on a pre-existing defect treated as initial crack. The model is briefly presented in this paper and further validation is provided using literature data. Some limitations emerged and a possible extension of the propagation model is proposed based on experimental evidences in order to improve the predictions.

2. Description of the proposed model

The crack propagation model is based on a modification of the Paris' equation in order to take into account the following effects:

- Calculation of the crack growth rates in the short and long crack regime
- Plastic correction to the crack driving force
- Plasticity-induced crack closure

The resulting crack growth rates are calculated as

$$\frac{da}{dN} = C \cdot \Delta K_{eff}^m \quad (1)$$

in which the effective stress intensity factor range is given by

$$\Delta K_{eff} = U(a) \cdot \Delta K_{pl} = U(a) \cdot \Delta K \cdot [f(\Delta L_r)]^{-1}. \quad (2)$$

In Eq.(2) the function $U(a)$ characterizes the development of the plasticity-induced crack closure from the short to the long crack regime. The model accounts for the gradual build-up of closure, which means that the crack is considered by hypothesis fully open at the beginning of the propagation, i.e. $U(a_i)=1$, and the closure evolves up to the saturated value $U=U_{LC}$ according to

$$U(a) = [1 - e^{-k(a-a_i)}] \cdot (U_{LC} - 1) + 1. \quad (3)$$

The idea behind Eq.(3) is to mirror the development of threshold $\Delta K_{th}(a)$, as described in McEvily et al. (2003), into the evolution of the plasticity-induced crack closure.

The value of the closure function for long cracks U_{LC} which appears in Eq.(3) is calculated according to Newman (1984). Nevertheless, the original model formulation has been modified, according to McClung (1994), by substituting the ratio σ_{max}/σ_f with

$$\frac{K_{max}}{K_f} = \frac{\phi \cdot \sigma_{max} \cdot \sqrt{\pi \cdot a}}{\sigma_f \cdot \sqrt{\pi \cdot a}} = \phi \cdot \frac{\sigma_{max}}{\sigma_f} \quad (4)$$

in order to account for other geometries and loading conditions.

Eq.(2) encloses another important feature of the model, namely the function

$$f(\Delta L_r) = \left(\frac{E \cdot \Delta \varepsilon_{ref}}{\Delta \sigma_{ref}} + \frac{1}{2} \cdot \frac{\Delta \sigma_{ref}}{E \cdot \Delta \varepsilon_{ref}} \cdot \Delta L_r^2 \right)^{-\frac{1}{2}} \quad (5)$$

which allows for the plastic correction of the cyclic stress intensity factor ΔK , being

$$\Delta L_r = \frac{\Delta \sigma_{ref}}{2 \cdot \sigma_Y} \quad (6)$$

the ligament yielding parameter, defined in a similar fashion as in R6 (2009), but modified in the present model in order to account for cyclic loading.

3. Validation

The model is used in the following to calculate the fatigue lives for worked examples which have been found in the literature. In case of missing parameters, reasonable assumptions have been made.

3.1. Tension loaded plates of Al2024-T3

The experimental data have been collected from Newman et al. (1999), Grover et al. (1953) and Wanhill (1988). The basic mechanical properties of this alloy are: $\sigma_Y = 360$ MPa, $R_m = 490$ MPa, modulus of elasticity $E = 72$

GPa. S-N curves were obtained for dog-bone specimens under uniaxial tensile load at $R=0$ and $R=-1$ (see Fig. 1). Long fatigue crack propagation curves ($da/dN - \Delta K$) were generated at NASA and fatigue threshold values of $\Delta K_{th} = 3.0 \text{ MPa}\sqrt{\text{m}}$ for $R=0$ and $\Delta K_{th} = 5.5 \text{ MPa}\sqrt{\text{m}}$ for $R=-1$ have been considered. The intrinsic threshold was estimated to be $\Delta K_{th,eff} = 0.75 \text{ MPa}\sqrt{\text{m}}$. The exponent k in Eq.(3) has been set equal to 65.2 mm^{-1} in accordance to previous results presented in Zerbst et al. (2011), as no value has been found in the literature for the Al 2024-T3.

The initial crack was assumed to be semi-circular with a depth of $20 \mu\text{m}$ according to Newman et al. (1999), who obtained this value by a trial-and-error procedure using his strip yield model.

Since the S-N specimens did show no stress concentration ($K_t = 1$), the stress intensity factor and reference load solutions already applied to the validation example in Zerbst et al. (2011) have been used. The simulations have been stopped when the crack reached a final depth of 80% the plate thickness.

Despite of the necessary assumptions to apply the model to the literature data, the results have been quite satisfactory for both R ratios, as it is shown in Fig. 1. The only discrepancy appears at higher loads ($\sigma_a > 300 \text{ MPa}$), where the predictions diverge from the test values, which are clearly characterized by shorter lives.

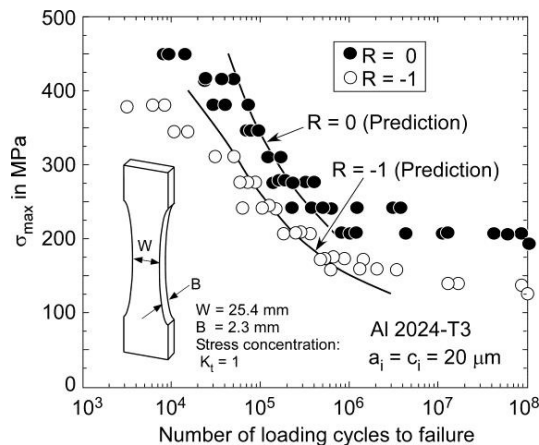


Fig. 1. Experimental and predicted S-N data for tensile plates made of aluminium Al2024-T3.

3.2. Tension loaded round bars made of ductile cast iron EN-GJS-400-18-LT

S-N curves on tensile loaded round bars of EN-GJS-400-18-LT ductile cast iron cut from a rejected wind turbine hub casting were published in Shirani et al. (2011). The basic mechanical properties of the material comprised: $\sigma_Y = 230 \text{ MPa}$, $R_m = 400 \text{ MPa}$ and modulus of elasticity $E = 167 \text{ GPa}$. Fatigue threshold values were estimated as $\Delta K_{th} = 9.5 \text{ MPa}\sqrt{\text{m}}$ for $R=0$ and $\Delta K_{th} = 14.3 \text{ MPa}\sqrt{\text{m}}$ for $R=-1$ in Hübner et al. (2007) and Zambrano (2011). The intrinsic threshold was estimated to be $\Delta K_{th,eff} = 3.5 \text{ MPa}\sqrt{\text{m}}$. The exponent k in Eq.(3) has been set equal to 1.5 mm^{-1} according to the results published in Clement et al. (1984) and Journet et al. (1989) (see also Fig. 2). This value has been determined for $R \approx 0$ (actually 0.1) and, in this paper, an estimate of the $\Delta K_{th}(a)$ function had to be provided for the tests at $R=-1$. Since the authors expect a moderate effect (at least when taking into account the experimental scatter) they used the same k as above in conjunction with the long crack threshold for $R=-1$. Note that the crack closure function is characterised by a rather low exponent, which means that the crack closure needs a crack extension of almost 2 mm in order to be fully developed, i.e., to reach the threshold for long cracks.

A major problem of this case study is the nature and size of the pre-existent defect and the initial crack. This is because besides the graphite nodules which, according to Hübner et al (2007), have usually a diameter in the range 20 to $200 \mu\text{m}$ depending on the cooling rates, also micro-shrinkage pores up to few millimetres act as preferential sites for crack initiation as shown in Shirani et al. (2012), see also Fig. 3.

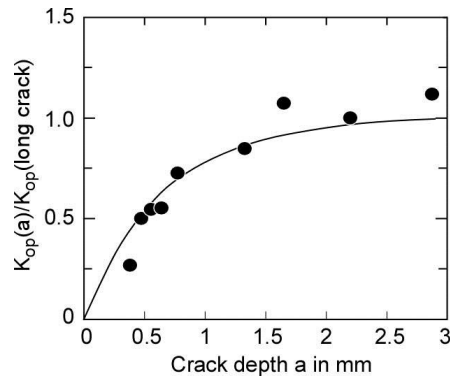


Fig. 2. Crack opening function as provided in Clement et al. (1984) and Journet et al. (1989). In those papers, the authors realised the small crack growth experiment by machining away the wake of an originally long crack and subsequent stress relief heat treatment.

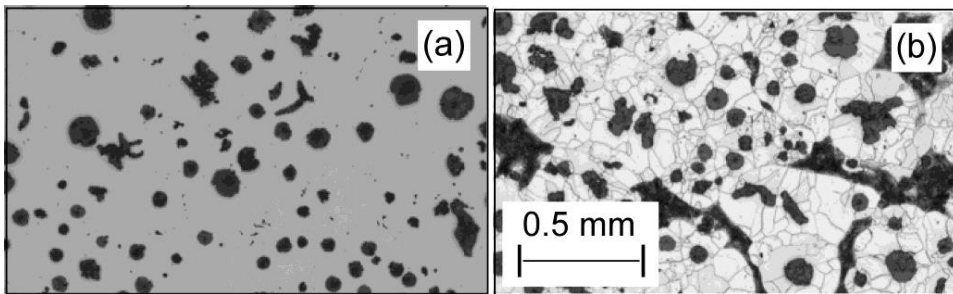


Fig. 3. Optical micrographs in Shirani et al. (2012): (a) polished; (b) polished and etched surfaces. The dark areas mark graphite nodules (a) and graphite nodules plus pearlite zones (a) and (b).

The present study uses an assumption on the initial crack dimension and location in the specimens since no more precise information has been available. For the analysis an initial circular defect of 0.2 mm diameter has been placed in the centre of a round specimen. This dimension has been set according to Shirani et al. (2011), who reported that the relevant defects expected in the material where microshrinkages of a size smaller than 0.2 mm. For determining the stress intensity factor an equation according to Isida et al. (1987) has been adopted.

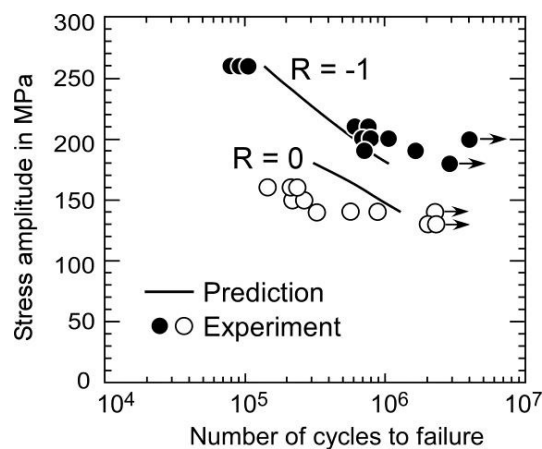


Fig. 4. Experimental and predicted S-N data for EN-GJS-400-18-LT ductile cast iron.

The experimental S-N curves and the results of the analytical simulations are depicted in Fig. 4 for both fatigue tests at $R = 0$ and $R = -1$. Due to large variation in the content of natural defects and pores the S-N curves show substantial scatter bands as they are typical for this kind of material. As can be seen, despite the rather crude assumption which the authors applied, the model worked fairly good in the case of $R = -1$, except for the highest load level where the prediction seems to become slightly non-conservative. A similar trend has been observed already in the previous case study. In contrast, a large non-conservative deviation between experiment and prediction characterised the data for $R = 0$.

4. Concluding remarks

Although based on limited input information sampled from different sources and modelling assumptions, the model has proved to yield fairly satisfactory predictions of the S-N behaviour for different materials and loading conditions. There are, however, exceptions to that rule. In particular, it seems that the model works properly for lower loads, whereas it provides non-conservative results in case of higher load amplitudes.

The main problem is that the model has been developed for alloys containing large second phase particles, in which the overall life is governed by the propagation of the largest defect. But the occurrence and size of smaller (and sometimes perhaps even larger) initial cracks is not just a material property, but also depends on the loading level. This is an experimental evidence in case of weldments which was demonstrated in Otegui et al. (1989), where the authors found that the crack initiation sites increased with increasing load amplitudes. In turn this means that at higher load amplitudes, where more than one defect can be activated, multiple crack propagation must be employed, which would shift the prediction towards shorter lives. In the opinion of the authors, this is the main reason why the present model yields non-conservative results for higher load amplitudes, therefore the model needs an extension to deal with multiple crack initiation and propagation.

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