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H. C. Hege K. Polthier (eds.)

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of the

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Preface

This workshop serves as a forum for experts on mathematical concepts in computer graphics, and for scientists employing visualization as a tool in mathematical research. It is the second event in a series of workshops, bringing together mathematicians and experts from the visualization community.

With increasing complexity of the entitities to be visualized, computer graphics gains by building its concepts on a firm mathematical basis. From the data visualization perspective there is a driving force to employ mathematical concepts due to the fact that the data to be visualized usually embody some mathematical structure. Furthermore, the various mappings to be performed to depict structural properties of data in images are stated most clearly in mathematical terms, and they are performed most efficiently by modern algorithmic techniques.

In an abstract sense, the central goals of scientific visualization therefore are to develop useful representations of mathematical objects and to create efficient methods for mapping important structural aspects of the objects to images. Mathematical disciplines will benefit by this evolution, and can promote it.

The focus of the workshop is on new mathematical concepts and mathematical applications in visualization. This includes also non-mathematical aspects, e.g. perceptual issues, interaction techniques, application specific representations, and last-not-least, software aspects are considered.

The following topics are covered in the workshop:

- applications in differential geometry and partial differential equations
- algorithmic aspects of adaptive and hierarchical techniques in space and time
- time control of animated objects and corresponding algorithms
- algorithmic representation of objects for display, storage and exchange
- new visualization techniques for mathematical structures
- integration of visualization and numerical computation.

To our joy, the announcement of the workshop met with a lively response as this collection of abstracts shows. We wish a good course of the workshop with many fruitful discussions over the fences between the different scientific disciplines.

We would like to thank all authors and speakers for their contributions. The work of the international program committee is very much appreciated. It made possible to ensure a high quality of the papers. The organizers also want to thank all the numerous people involved in the preparation and realization of this workshop, as well as all supporting organizations.

Berlin, August 1997

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Contents

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Auditory Morse Analysis of Triangulated Manifolds

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Visualization of high-dimensional or large geometric data sets is inherently difficult, so we experiment with the use of audio to display the shape and connectivity of these data sets. Sonification is used as both an addition to and a substitution for the visual display. We describe a new algorithm called *wave traversal* that provides a necessary intermediate step to sonification of the data; it produces an ordered sequence of subsets, called *waves*, that allows us to map the data to time. In this paper we focus in detail on the mathematics of wave traversal, in particular, how wave traversal can be used as a discrete Morse function.

The Virtual Album of Fluid Motion – Basic Concepts and Background

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Visualization of high-dimensional or large geometric data sets is inherently difficult, so we experiment with the use of audio to display the shape and connectivity of these data sets. Sonification is used as both an addition to and a substitution for the visual display. We describe a new algorithm called *wave traversal* that provides a necessary intermediate step to sonification of the data; it produces an ordered sequence of subsets, called *waves*, that allows us to map the data to time. In this paper we focus in detail on the mathematics of wave traversal, in particular, how wave traversal can be used as a discrete Morse function.

Tetrahedra Based Volume Visualization

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Volume Visualization techniques have advanced considerably since the first international symposium held on this topic eight years ago. This paper briefly reviews the techniques proposed for the visualization of irregular (or scattered) volume datasets. In particular, methods which adopt simplicial decompositions of E^3 space are considered, and this choice is justified both in terms of modeling and visualization. Simplicial complexes are powerful and robust geometric structures, and a number of efficient visualization algorithms have been proposed. We show that simplicial cells (or simply tetrahedral cells since our target is 3D space) may be conceived as being the unifying kernel primitive for the visualization of not-regular meshes.

Support of Explicit Time and Event Flows in the Object-Oriented Visualization Toolkit MAM/VRS

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We present an object-oriented architecture of a visualization toolkit which integrates geometric modeling and behavioral modeling. It is based on sharing graphics objects between geometrical scene descriptions and descriptions for the flow of time and events. We discuss the properties graphics objects should possess in such a system so that they can be used by different rendering toolkits and can be linked to time- and event-dependent processes. We introduce a new methodology for the symmetric modeling of geometry and behavior based on geometry graphs and behavior graphs.

Expressive Visualization in the Natural Sciences – Recent Work from the Bielefeld Visualization Laboratory

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At the Bielefeld Visualization Laboratory, our aim is to develop conceptual tools for elucidating, visualizing and exploring structural features concealed in complex numerical (or symbolic) data sets. To this end, various specific mathematical theories (regarding e.g. the reconstruction of phylogenetic trees from sequence data or the encoding of molecular geometry) and simulation techniques (regarding e.g. the dynamics of excitable media) have been developed and combined with appropriately chosen visualization procedures (e.g. techniques for visualizing the stable and the instable manifold in fluid dynamics data sets). Our visualizing procedures include fast interactive "real-time" rendering techniques for large-scale geometry and image data and are based on (but not limited to) the industry standard interface to current computer graphics hardware, OpenGL. To further enhance the expressiveness of real-time rendered images we have integrated stylistic means from sketch drawing to pseudo-realism.

In the lecture, we will address the above topics, and we will use them to demonstrate the workings of our visualization procedures.

Computation of Parametrised Surfaces of Prescribed Mean Curvature with Given Boundary

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A finite element method is presented which allows the numerical computation of possibly unstable two dimensional surfaces with prescribed mean curvature H and prescribed boundary Γ, where the boundary conditions are of Plateau type. The algorithm produces a conformal parametrisation of the solution. We give an error estimate in the $H^1(D)$-norm for small solutions which uses former results obtained for the case $H = 0$ together with error estimates for numerical solutions of the nonlinear elliptic system

$$\Delta u = 2H\ u_{x_1} \times u_{x_2}$$

for the parametrisation $u(x) = (u_1(x), u_2(x), u_3(x))$ of the surface.

A Geometrical Puppetshow

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In every age the structure of the technology to which geometry is applied also shapes the geometry – at least its pedagogical formulation. We are familiar with Klein's Erlangen Program and the co-evolution of engineering and geometry in the late 19th century. In the late 20th century the geometrical demands (and opportunities) of wholly immersive virtual environments (e.g. the CAVE) and parallel distributed computing play an analogous role. We explore a metaphor where the graphical constraints of hardware and programming libraries are the puppets in the hands of geometrical puppeteers. We perform live some student projects which illustrate our theme.

An OpenGL Based Visualization Tool Box for Numerical and Statistical Applications

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We give an overview of the concepts of the tool box *gltools* for visualization of numerical and statistical data. This tool box is based on OpenGL and can be used for portable 3D online graphics with easy-to-implement rendering possibilities. This tool box shall not replace large, high quality rendering packages, like AVS, GRAPE or OORANGE, but supplement them. It is intended to do quick and dirty rendering during everyday work in code development, debugging and rapid prototyping. It easily can be integrated into existing applications. Flexibility, portability and modularity as design goals had been put above image quality and sometimes speed. At the same time, the modular structure of the package allows to incorporate more advanced rendering techniques, if this becomes necessary.

Efficient Volume-Generation During the Simulation of NC-Milling

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This paper presents an efficient and robust algorithm for the geometric determination of swept volumes during the simulation of numerical controlled (NC-) milling. The boundary Ψ of the volume swept by a cutter Φ is represented polygonally by using instantaneous helical motions to exactly determine the line of contact between Φ and Ψ.

Applying concepts of differential geometry allows a better and more efficient approximation of tool paths. Tool paths are explicitly calculated when a design surface Γ is to be milled along prescribed curves.

We also describe how to quickly determine a polygonized representation of the truncated material during the milling process by means of "Γ-buffering". This polygon-oriented algorithm is suitable for Boolean subtractions and error assessment.

KEYWORDS: computer aided manufacturing, NC-milling, NC-verification, sweeps, solid modeling, Γ-Buffer.

Surface Reconstruction Based on Optimization Techniques

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Being a substantial part in the process of "reverse engineering", surface reconstruction has obtained a revival in recent years. Several interesting new approaches have been developed, allowing reconstruction of a surface (starting from a set of discrete points) of higher accuracy, better quality and more efficiency than traditional methods.

First we will briefly review some of the most interesting methods. Then we focus on optimization methods, whereby additional degrees of freedom (control points) are introduced and used for optimizing the shape of the surface.

We concentrate on approaches that use B-splines for the representation of the resulting surface. This fact is of importance if the method aims at integration in a commercial CAD/CAM-system.

Constant Mean Curvature Surfaces with Cylindrical Ends

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R. Schoen has asked whether the sphere and the cylinder are the only complete (almost) embedded constant mean curvature surfaces with finite absolute total curvature. We propose an infinite family of such surfaces. The existence of examples of this kind is supported by results of computer experiments we carried out using an algorithm developed by Oberknapp and Polthier.

Mesh Optimization and Multilevel Finite Element Approximations

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Mesh reduction techniques are used for accelerating the visualization process for large datasets. Typical examples are scalar or vector valued functions defined on complex 2 or 3 dimensional meshes. Grosso et al. presented a method for mesh optimization based on finite elements approximations with the L_2 norm and adaptive local mesh refinement. Starting with a very coarse triangulation of the functional domain a hierarchy of highly non-uniform tetrahedral (or triangular in 2D) meshes is generated adaptively by local refinement. This process is driven by controlling the local error of the piecewise linear finite element approximation of the function on each mesh element. In this paper we extend the algorithm to the Sobolev space H^1, where the error norm allows for gradient information to be included. This improves the convergence of the algorithm in regions, where the function has high frequency oscillations. In order to analyze the properties of the optimized meshes we consider iso-surfaces of volume data.

Morse Theory for Implicit Surface Modeling

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Morse theory describes the relationship between a function's critical points and the homotopy type of the function's domain. The theorems of Morse theory were developed specifically for functions on a manifold. This work adapts these theorems for use with parameterized families of implicit surfaces in computer graphics. The result is a theoretical basis for the determination of the global topology of an implicit surface, and supports the interactive modeling of implicit surfaces by direct manipulation of a topologically-correct triangulated representation.

Fast LIC with Higer Order Filter Kernels

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Line integral convolution (LIC) has become a well-known and popular method for visualizing vector fields. The method works by convolving a random input texture along the integral curves of a vector field. In order to accelerate image synthesis significantly, an efficient algorithm has been proposed that exploits pixel coherence in field line direction. Originally, this so-called 'fast LIC algorithm' was restricted to simple box-type filter kernels.

In this paper we describe a generalization of this algorithm for higher-order filter kernels. Expanding the filter kernels in terms of truncated power functions allows us to exploit certain convolution theorems. In the discrete case, the convolution integral is expressed as a linear combination of repeated sums. Compared to the original algorithm the additional expense for using higher-order filter kernels, e.g. of B-spline type, is very low. Such filter kernels produce smoother results than a simple box filter. This is evident from visual investigation as well as analysis of pixel correlations. Thus, our method represents a useful extension of the fast LIC algorithm for the creation of high-quality LIC images.

Efficient Visualization of Data on Sparse Grids

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Sparse grids are nowadays frequently used in numerical simulation. With their help the number of unknown in discrete PDE or approximation problems can drastically be decreased. On sparse grids in d space dimensions $O(N\log(N)^{d-1})$ nodes are required to achieve nearly the same approximation quality for sufficiently smooth data as on a standard finite difference grid with N^d nodes. This allows numerical methods with small error tolerances on a corresponding very fine mesh width. Mapping the complete sparse grid data on an N^d standard grid in order to analyse them visually would burst a currently available work station storage for large $O(N\log(N)^{d-1})$ numbers of nodal values provided by numerical code on a sparse grid. We present an efficient, procedural approach to sparse grids for the visual post processing. A procedural interface addresses data on grid cells only temporarily, only if actually requested by the visualization methods. The cell access procedures extract local numerical data directly from the users sparse grid data base. No additional memory is needed. The procedural approach is combined with a multiresolution strategy based on a recursive traversal of the grid hierarchy. It includes hierarchical searching for features such as isosurfaces and a locally adaptive stopping on coarser grids in areas of sufficient smoothness. This data smoothness corresponds to a user prescribed error tolerance. Examples underline the benefits of the presented approach.

Discrete Rotational CMC Surfaces and the Elliptic Billiard

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It is a well known fact, that the meridian curve of rotational constant mean curvature (cmc) surfaces (which determines the surface completely) can be obtained as the trace of one focal point of an ellipse or hyperbola when rolling it on a straight line. In this paper it will be shown, that discrete rotational cmc surfaces can be obtained in a similar way. In fact the meridian polygons are closely related to the elliptic standard billiard: The discrete analouge of the ellipse or hyperbola will be the trace of a billiard in a continuous ellipse.

A Survey of Parallel Coordinates

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Visualization provides *insight through images* [1], and can be considered as a collection of application specific mappings: *Problem Domain* $\longrightarrow$ *Visual Range.* For the visualization of multivariate problems a multidimensional system of *Parallel coordinates* is studied which provides a one-to-one mapping between subsets of N-space and subsets of 2-space. The result is a systematic and rigorous way of doing and *seeing* analytic and synthetic N-dimensional geometry. Lines in N-space are represented by N-1 indexed points. In fact all p-flats (planes of dimension p in N-space) are represented by indexed points where the number of indices depends on p and N. The representations are generalized to enable the visualization of polytopes and certain kinds of hypersurfaces as well as recognition of convexity. Several algorithms for constructing and *displaying* intersections, proximities and points interior/exterior/or on a hypersurface have been obtained. The methodology has been applied to Visual Data Mining, Process Control, Medicine, Finance, Retailing, Collision Avoidance Algorithms for Air Traffic Control, Optimization and others.

[1] B. H. McCormick, T. A. Defanti, and M. D. Brown. *Visualization in Scientific Computing.* Computer Graphics 21-6, ACM SIGGRAPH, New York, 1987.

Computational Steering and Interactive Visualization

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In this seminar I will present the design, implementation and application of SCIRun, a scientific programming environment that allows the interactive construction, debugging and steering of large-scale scientific computations. Using this "computational workbench," a scientist can design and modify simulations interactively via a visual programming environment. With SCIRun scientists can automatically change parameters associated with a particular numerical method, boundary conditions or even the geometric structure of the computational mesh. As opposed to the typical "off-line" simulation mode - in which the scientist manually sets input parameters, computes results, visualizes the results via a separate visualization package, then starts again at the beginning - SCIRun "closes the loop" and allows interactive steering of the design, computation, and visualization phases of the simulation.

A Meta Scheme for Iterative Refinement of Meshes

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Subdivision is a powerful method for smooth visualization of coarse meshes. An initial mesh of arbitrary topology is subsequently subdivided until the surface appears smooth and visually pleasing. Because of the simplicity of the algorithms and their data structure they have achieved much attention. Most schemes exhibit undesirable artifacts in the case of irregular topology. A more general description of subdivision algorithms is required to be able to avoid artifacts by changing parameters and also the topology of subdivision. We develop a partition of the algorithms to sub-algorithms and combinatoric mappings. An algebraic notation enables the description of calculations on the mesh and steering the algorithm by a configuration file. Opposite to index based notation of tensor-product surfaces the algebraic notation is even applicable for irregular and non-tensor-product surfaces. Variations of these sub-algorithms and mappings lead to known and new subdivision schemes. Further, the unified refinement scheme makes the efficient depth-first subdivision available to all these subdivision schemes.

Mathematics and Visualization for Problems of Fluid Dynamics and Elasticity

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Computational fluid dynamics and elasticity have become important techniques in applied mathematics and engineering. The treatment of complicated problems demands the ability of treating complex geometries with a large number of grid points. The recent development of super computers and algorithms has enabled the simulation of many realistic problems. The effective development of software and interpretation of the results require a powerful visualization which is to produce dynamic images of the numerical results which are close to the physical reality.

I would like to show some examples here of our work for which the visualization is a necessary tool for developing the software, to analyse the produced numerical data and to present the results. For the following problems, in particular, I shall describe the problem which has to be solved and shall show the visualization of the results.

- Transport phenomena in density driven flow through porous media.
- Two phase flow through porous media.
- Compressible flow through a two stroke engine.
- High speed flow and chemical reaction.
- Parallelization.
- Deformation of an elastic cylindrical shell under pointwise loads.

Zonotope Dynamics in Numerical Quality Control

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Mathematical rigorous error bounds for the numerical approximation of dynamical systems have long been hindered by the wrapping effect. We present a new method which constructs high order zonotope (special polytopes) enclosures for the orbits of discrete dynamical systems. The wrapping effect can made arbitrarily small by controlling the order of the zonotopes. The method induces in the space of zonotopes a dynamical system of amazing geometrical complexity. We emphasize the visualization of the zonotopes to better understand the involved dynamics.

Four Dimensional Stereoscopy Using a Pair of Two Dimensional Retinas

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We discuss an issue of mathematical visualization as well as of cognitive science: how might one be trained to see objects such as surfaces in 4-dimensional space using a pair of 2-dimensional retinas? To aid the understanding, a cartoon of ordinary stereo vision in 3-space is presented, and the effect of repositioning the retinas is considered. We explore the configuration spaces of such stereo-retina-pairs in both $\mathbb{R}^3$ and $\mathbb{R}^4$, and experiment with what we might see of some familiar 3- and 4-dimensional objects were our eyes so positioned.

Visualizing Poincaré Maps Together With the Underlying Flow

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We present a set of advanced techniques for the visualization of 2D Poincaré maps. Since 2D Poincaré maps are a mathematical abstraction of periodic or quasi-periodic 3D flows, we propose to embed the 2D visualization with standard 3D techniques to improve the understanding of the Poincaré maps. Methods to enhance the representation of the relation $x \leftrightarrow P(x)$, e.g., the use of spot noise, are presented as well as techniques to visualize the repeated application of P, e.g., the approximation of P as a warp function. It is shown that animation can be very useful to further improve the visualization. For example, the animation of the construction of Poincaré map P is a very intuitive visualization. During the paper we present a set of examples which demonstrate the usefulness of our techniques.

Accuracy in 3D Particle Tracing

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This paper presents a novel way of identifying and illustrating the accuracy in the particle tracing method for flow visualization. We make use of explicit Runge-Kutta methods for particle tracing in steady velocity fields, and describe three approaches to estimate the accuracy of the calculated path. These approaches are: re-integration (with smaller tolerance or in a backward direction), global error estimators and residuals in the velocity field. Visualization paradigms are also presented to convey data accuracy information and these ideas are implemented in an Open Inventor / IRIS Explorer environment.

Visualization in Topology: Projective Plane and Projective Space

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In this talk we present visualization of two topologically non-trivial objects: projective plane and projective space. Various properties and representations of these objects are visualized. General topological constructions and concepts are visually demonstrated, such as immersion and embedding of the manifolds, their non-orientability and non-trivial connectivity and their fundamental groups. The actual problems arising in visualization of topologically non-trivial objects and their representation in virtual environment are discussed, concerned with fast rendering of self-crossing transparent surfaces and smooth mapping of colors and textures onto topologically complex surfaces. Two video films are prepared for demonstration.

KEYWORDS: scientific visualization, topology, geometry, time critical rendering, animation, applications of virtual environment.

Mathematics and Computer Aided Radiology

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Computer aided radiology is a rapidly emerging new field which will serve as an area of very particular new mathematical and scientific challenges. As a guiding example from our work at MeVis in Bremen (Center for Medical Diagnostic Systems and Visualization) I will discuss the long standing problem of determining the 3D Couinaud-Segments of a patients liver based on CT or MRI crossectional clinical data. The solution of this problem represents the typical complexity of medical imaging problems: image processing, 3D visualization, 3D rendering, 3D interaction, modelling, and validation. Applications to segment-oriented cancer surgery and split-liver transplantation will also be discussed.

Discrete Geodesics and Numerics on Surfaces

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Geodesic curves are a fundamental concept in mathematics to generalize the idea of a straight line to curves on arbitrary smooth surfaces and to smooth manifolds. We define the notion of discrete geodesic curvature of curves on polygonal surfaces, and characterize discrete geodesics by having vanishing geodesic curvature. In contrast to the definition of geodesics as shortest curves, our discrete geodesics are straightest curves and are not necessarily shortest.

An immediate consequence of the concept of straightest curves is the possiblity to parallel translate a vector along a curve on a polygonal geometry, especially across edges and vertices. This allows us e.g. to extend higher order integration methods to the integration of vector fields on polygonal surfaces.

The method of using geodesics as straightest curves allows us to give an intrinsic definition of a number of operations on polygonal surfaces without taking the ambient space into consideration, thereby avoiding e.g. ambiguous projection operations.

Special Relativity in Virtual Reality

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The appearance of fast moving objects can be calculated according to the Theory of Special Relativity. In addition to the Lorentz contraction the effects of finite light speed and aberration play an important role. These phenomena were first discovered and described correctly in 1959 by Penrose and Terrel. Concerning the visualization of the phenomena there already exist systems with relativistic ray tracing and polygon rendering. Investigating these approaches in detail we found a reformulation of the problem which allows the treatment of acceleration in real-time. Therefore, user interaction could be integrated and a virtual reality for special relativity was possible.

Exploring Low Dimensional Objects in High Dimensional Spaces

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We discuss general principles and also software implementation for visualizing low dimensional objects in high dimensional spaces. By high dimensional space we mean euclidian space of dimension greater than four. The low dimensional objects are modeled, mathematically, by simplicial complexes of dimension 4 or less. A particular software visualization project, named Hew, is discussed. An example of an embedded three dimensional projective space is featured in the figures.

Efficient Visualization of Large–Scale Data on Hierarchical Meshes

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Modern numerical methods are capable to resolve fine structures in solutions of partial differential equations. Thereby they produce large amounts of data. The user wants to explore them interactively by applying visualization tools in order to understand the simulated physical process.

Foundations of a multiresolution concept will be discussed. We consider data on a large class of hierarchical and nested grids with data functions in various types of discrete function spaces. This especially includes hierarchical grids consisting of elements which are tensor products of simplices in two and three dimensions generated by recursive subdivision applying an arbitrary but finite set of refinement rules. Finite–volume and hp–finite–element function spaces are taking into account. Data could be timedependent and adaptive in space and time.

We will especially emphasize on

- the economical storing of large scale data for postprocessing purposes,
- adaptive and hierarchical visualization (especially isosurface extraction),
- different types of error monitoring (data and geometry error),
- local magnifying lenses,
- the efficient searching and vector field intergration on nested grid,
- and resolving h and p refinement in the visualization.

A Scheme for Edge-based Adaptive Tetrahedron Subdivision

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A new scheme for adaptive refinement of tetrahedra based on an edge criterion is presented. This scheme guarantees consistent subdivision without cracks without the necessity for checks between neighboring tetrahedra of a mesh, allowing a recursive algorithm with low space requirements and parallel implementation.

On Space Partitions for Fast Isosurface Extraction

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We consider the problem of fast isosurface extraction from scalar data sampled on a regular or irregular grid. Such data may stem from empirical measurements, computer simulations such as for fluid flow dynamics, or from mathematical models, e.g., implicitly defined surfaces. The known methods used for this task can be grouped in two classes: (i) hierarchical geometrical space decomposition, and (ii) working with range intervals in value space. The first category applies to volume data on structured grids where octrees have been proven useful. We provide a framework for the evaluation of hierarchical space partitions for isosurface extraction, based on probablity measures for the isovalue. This motivates a new solution of the isosurface extraction problem using generalized binary space partitions. It provides a speed up relative to the octree-based approach at the cost of increased preprocessing. We also give a survey of the other methods and relate them in terms of speed up and costs in memory space, based on our simulations for some test data sets. Finally, we show, how some of the faster methods can be used in connection with rendering of shaded points (an OpenGL feature) for a real-time display of isosurfaces where the isovalue is under interactive user control.

Clifford Algebra in Vector Field Visualization

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The visualization of vector fields is still based on piecewise linear approximation. This is fast and good enough in large areas but has drawbacks if the non-linear behavior of a field has local topological implications like close simple critical points or higher order singularities. This article introduces the concept of Clifford algebra into the visualization of vector fields to deal with these difficulties. It derives a close relationship between the description of some polynomial 2D vector fields in Clifford algebra and their topology, especially the index and the position of critical points. This is used to develop an algorithm for vector field visualization without the problems of conventional methods.

Fast Marching Methods for Propagating Interfaces

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Fast Marching Methods are computational techniques for tracking the evolution of fronts. In the case of some particular evolution laws, interfaces problems can be converted into Eikonal equation and static Hamiltonians. By relying on a blend of upwind schemes, finite difference approximations, viscosity solutions of Hamilton-Jacobi equation, and fast sort algorithms, Fast Marching techniques provide optimal solutions to a variety of problems. For example, in a computational domain of N total points, they compute the complete solution in O(N log N) steps, accurately accounting for topological change and sudden changes in propagation speed. We will explain and demonstrate these techniques applied to problems in robotic navigation, medical imaging, seismic travel times, and couple them to accurate level set techniques to build optimally fast shape and surface reconstruction techniques.

Hierarchical Techniques for Global Illumination Computations – Recent Trends and Developments –

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Since the beginning of computer graphics, one of the primary goals has been to create convincingly realistic images of three-dimensional environments that would be impossible to distinguish from photographs of the real scene. The goal to create photo-realistic images has lead to the development of completely new software techniques for dealing with the inherent geometric and optical complexity of real world scenes. This paper gives an overview of advanced algorithms for photo-realistic rendering and in particular discusses hierarchical techniques for global illumination computations.

Two-Dimensional Image Rotation

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We study the problem of approximating a rotation of the plane, $\alpha : R^2 \longrightarrow R^2$ $\alpha(x,y) = (x\cos\theta + y\sin\theta, y\cos\theta - x\sin\theta)$, by a bijection $\beta : Z^2 \longrightarrow Z^2$. We show by an explicit construction that one may choose β so that $\sup_{z\in Z^2} |\alpha(z) - \beta(z)| \leq \frac{1}{\sqrt{2}}\frac{1+r}{\sqrt{1+r^2}}$, where $r = \tan(\theta/2)$. The scheme is based on those invented and patented by the second author in 1994.

Multiresolution Surface Modeling

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The visualization of large models of real-world objects, like cars, trains, airplanes, etc. is a major challenge in the context of virtual reality and scientific visualization. In most scenarios several such models have to be visualized and animated simultaneously. Examples are the optimization of the cabin of a train or the optimization of a driver's position in a car. In such a setting not only the particular train or car has to be visualized but also other cars, pedestrians, buildings, etc. Furthermore, due to new acquisition techniques and more elaborated modeling and animation techniques the size of the models is still growing. Despite of the performance of modern graphics hardware the visualization of such scenarios cannot be realized in real-time without special techniques to reduce the number of triangles that are rendered.

In order to close the gap between the desired and actual hardware performance, simplification algorithms are iteratively applied to obtain a hierarchy of successively coarser approximations to the input object. Such multiresolution hierarchies usually have a number of distinct levels of detail, usually 5 to 10, for a given object. At run time, the perceptual importance of a given object in the scene is used to select its appropriate level of representation from the hierarchy. However, for scientific visualization applications where the goal often is to visualize one or two highly detailed objects at close range, an algorithm is desirable, that allows several different levels of detail to co-exist across different regions of the same object.

Therefore, in this context multiresolution modeling means maintaining objects at different levels of detail or approximations, where the level of detail may be different in distinct areas of the object. With respect to visualization it means to render the model at any given moment with minimal level-of-detail (LOD) sufficient to produce an image of reasonable quality.

This talk gives a brief overview over current techniques for simplifying polygonal surface models in order to accelerate rendering, speed network transmission and conserve memory. The construction of multiresolution models of 3-D shapes and compression techniques for multiresolution models are discussed. A new approach to light sensitive rendering is presented.

Minimax Sphere Eversions and other Optimal Geometry

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We have computed several sphere eversions from an infinite family with rotational symmetry of increasing order. These are driven by minimizing the Willmore bending energy, starting with critical halfway models proposed by Kusner. Although these evolutions never seem to lead to singularities, we will show other cases where the Willmore flow evidently does pinch off handles. We will also discuss other geometric optimization problems, including tight knots and soap bubbles.

An Object-Oriented Interactive System for Scientific Simulations: Design and Applications

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Better insight in complex physical processes requires integration of scientific visualization and numerical simulation in a single interactive framework. This paper presents an object-oriented environment which combines the tasks of numerical simulation, visualization, simulation specification, run-time monitoring and steering. We first review the different existing approaches to the above tasks and outline their relative limitations. Next, we present a model for a framework which attempts to provide a general approach to the tasks of simulation specification and steering in an object-oriented manner. An implementation of the framework is described.

We have built an object-oriented library for finite element computations and integrated it into the simulation system. An example in which our system has been used to solve a practical engineering problem illustrates the combination of object-oriented numerics and interactivity.

Visualization of Complex ODE Solutions

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This paper presents a visualization method for Complex Ordinary Differential Equations (CODEs), based on real extended phase portraits. This model enables classical ODE–theory functions to be visualized, for example flow functions, and the CODE integration results to be interpreted visually. This paper comprises three parts : firstly a presentation of the problem and related existing methods, secondly an introduction to the extended phase portraits, and how they apply to complex equations, and thirdly two applications of this model.

Topology in Grades 6-12: The Intrinsic View of Surfaces

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Students in grades 6-12 (and even younger) easily and happily learn about multiply-connected spaces by playing games in them. The first half of this talk will demonstrate a collection of computer games which present closed surfaces from an intrinsic point of view. The second half will present a simplified proof of the classification of surfaces inspired by the games, and will discuss some design issues which arise when representing higher genus surfaces in software. The talk will conclude with an explanation of how satellite data expected in 2001 may tell us the topology of the universe.

"uG" – A Software Toolbox for Numerical Simulation

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Numerical simulation is an important tool for science as well as for industry. The realiability of the results, however, strongly depends on the numerical techniques used. To obtain reliable results often advanced numerical techiques are needed. This makes tools desirable providing advanced numerical techniques to the standard user. The "uG" software provides such a toolbox for the simulation of processes modeled by partial differential equations. In the lecture we discuss the structure of the toolbox, numerical methods and strategies, and show a wide range of applications.