maThTune

Tuning guitars and reading music in major thirds *

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1 Introduction and background

The preference for major thirds tuning, short maThTune, is born out of a mathematicians penchant for regularity and an amateur guitar players longing for simplicity. Also, there is always the desire for doing things a little differently paired with the hope that one may achieve a pleasant variation of familiar guitar sounds. Major thirds tuning was apparently first proposed in 1964 by the American jazz guitarist Ralph Patt*, who later used it extensively in his professional activities. The association with modern jazz and the 'atonal composition system' of Arnold Schönberg combined with the lack of electronic tuners may have prevented the concept from spreading to rock and pop musicians.

To avoid the reduction of the open range on a conventional 6 string instrument, Ralph Patt suggested the use of 7 and 8 string guitars, a proposition that may overstretch both budget and hand of most hobby musicians. A similar approach is being persued by Ole Kirkeby† who maintains a very nice web site on what he calls the *M3 guitar* system. From a mathematical point of view adding a seventh string destroys the regular **mod 3** pairings. There have also been suggestions to tune guitars uniformly in fourths and even fifths like violines and other bow string instruments. Rather more popular have proven various *open string tunings*, where

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fifths are introduced selectively to allow the voicing of standard chords with minimal effort.

Obviously, standard tuning has a few idiosyncrasies, which greatly influence the way guitarists play and limits them to certain keys, or force the use of a capo. A prime example is the almost compulsive twitching of the fourth finger to temporarily turn the major D chord into a Dsus4. In principle one can achieve the same variation on the E and A chord patterns, but that is not nearly as convenient and therefore rarely done by novice guitarists and would-be smash-hit composers. Of course maThTune as proposed here has its own idiosyncrasies as we shall see. However, these are much more uniform with respect to the key played in, so that transpositon up or down the chromatic scale becomes a minor problem.

An inevitable downside of the key invariance is that nothing much can be done without the use of barr chords, the holding down of neighboring strings with one finger, or the damping of at least two strings. In other words, in no key can one use open strings to play the three basic chords for acompaniement by putting down just three fingers on a single string at a time. Standard tuning allows this in A, D, and G, at least if one ignores or dampens the lower E string for the D chord. By prohibiting this basic chording technique maThTune represents a bigger challenge to the absolute beginner and is not well suited for guitars with high action, i.e. a large distance between fretboard and strings. But it does make things a lot easier and flexible once the initial hurdle is overcome. Accompaniement without open strings is rather more convenient since pure major and minor chords have shapes that span only two frets. Adding seconds, fourths, sevenths or ninths can be done quite comfortable with shapes involving just three consecutive frets.

From my point of view an important side-benefit of major thirds tuning is its combination with a non-standard chromatic staff notation. One reason that guitarists are reputedly hopeless at reading music[‡] is that the mapping of the notes from a classical staff notation onto the fret board is a very irregular process indeed. Of course the standard notation is by itself not very convenient, for example if one wishes to transpose between keys. This observation has lead to numerous *chromatic staff* proposals, which attempt a more intuitive representation of all 11 semi-tone positions and their oactaves on a sheet of paper. The *Music Notation Modernization Association* has collected such proposals for 25 years, and compiled a list of seventeen *Desirable Criteria for Alternative Music Notation Systems*[§].

Major thirds tuning of six strings suggests a corresponding musical notation based on six lines with three semi-tone positions between any two of them. The resulting

[‡]http://www.berklee.edu/bt/172/lesson.html

[§]musicnotation.org/musicnotations/criteria.html

notation system is a only a minor variation of existing proposals and satisfies all of the criteria except for two. It uses one more than the recommended five lines and it is somewhat geared towards instruments tuned in major thirds. On the other hand it can be said that the classical staff is most suitable for instruments tuned in fifths. I personally found it reasonably easy to play the violin and viola from standard sheet music, but it always took a lot of time to figure out the fingering on the guitar. It is now quite easy to generate notes in chromatic clefs for any piece that has been encoded in a format suitable for the engraving system Lilypond. That GNU library is freely available, and Kevin Dalley[¶] has provided patches to implement alternative notations painlessly to the user.

This paper is organized as follows. In the following Section 2 we discuss the new tuning and its relation to the temperate scale. In Section 3 we discuss the fingerings of scales and chords. The final Section 4 describes the notation system that fits nicely with the proposed tuning. The Appendix lists more fingerings for various chords. I am endebted to Merlin Rhys-Jones and Dr. Michael Wilms for commenting on an earlier draft of this article.

2 Major third tuning patterns

Standard guitar tuning employs three fourth intervals spanning **5** semi-tones each, and one major third interval between the G and the B string spanning **4** semi-tones. The sum total is $24 = 4 \cdot 5 + 4 = 2 \cdot 12$ semi-tones covering the two octaves between the lowest and the highest strings tuned in E. Instead it is suggested here to use five major third intervals with a total spread of $5 \cdot 4 = 20$ semi-tones. For example one could use the tonal sequences

$$\mathbf{E},\ \mathbf{G\sharp},\ \mathbf{C},\ \mathbf{E},\ \mathbf{G\sharp},\ \mathbf{C} \qquad \text{or} \qquad \mathbf{G},\ \mathbf{B},\ \mathbf{D\sharp},\ \mathbf{G},\ \mathbf{B},\ \mathbf{D\sharp}\,.$$

As one can see the first and fourth, second and fifth, and third and sixth string form pairs of notes that are exactly one octave apart. As a consequence chord shapes are very regular and can be shifted not only *horizontally* along the fret board but also *vertically* across it. In this vertical shift the chord shapes are preserved except for minor adjustments on the lowest and highest string. Throughout this proposal arabic numbers will represent semi-tone intervals or notes relative to the root, with the exception of the usual chord acronyms, e.g. Dm7 used in the appendix of chord table. Hence **3** denotes a minor third, **4** a major third, **5** a fourth, **6** a diminished fifth, **7** a normal fifth and so on.

[¶]www.kelphead.org/chromatic

It is debatable how the tonal range that now falls one major third short of two octaves should be selected. Starting from the low E one has the advantage that all notes usually expected for the guitar in the music literature are playable, assuming the fret board is long enough so that one can still reach very high notes on the sixth string, which is now a C rather than an E. This E based maThTune may be appropriate for single guitar accompaniement, where one plays predominantly chords. In the context of a band, on the other hand, the lower E-G range is already covered by the bass guitar and it may be important that the guitar can solo up to the very high E on a 24 fret guitar.

Currently, I compromise by starting formally with the G but lowering everything by a semi-tone as is frequently done in rock music anyway. One advantage of this effectively F♯ based major thirds tuning is, that one can get by just using a regular set of strings. The lowest and the highest string are only tuned one full tone up or down, compared to their normal designation. Of course, more appropriate would be to select thicker strings at the high end and thinner strings at the low end, depending on where one starts. Starting at least formally from G one also has the advantage that the fourth anf fifth string remain at their standard G and B level so that one can still read the usual tab notation quite easily. Only one fret needs to be added mentally on the sixth string and one subtracted for fingerings on the third string. Of course, further down it is also necessary to subtract three frets on the first and two frets on the second string, but these are played rather rarely ouside chords anyway. Incidentally, starting with G means that the tonal range of the guitar still covers the whole range of the Violin beginning with G and reaching at least up E" and possibly up to G" and beyond.

As mentioned before, Ralph Pitt and Ole Kirkeby as well as other proponents of major thirds have considered it important to start on E and to recover the standard range by adding a seventh string tuned again as an E on the top. In the sixties the former had them custom made, some even with 8 strings, but since the nineties there have been some commercial 7 string guitars available. I have shied away from the expense and the further demands on my left hand dexterity. On a 6 string I can manage to get my thumb onto the lowest string and reach the second lowest with the middle or ring one fret higher. That produces a nice fourth, which can form the basis of a triad or support some melody lines played with the remaining three fingers. I don't think that would be possible on a 7-string. It is also interesting that neither Ralph Pitt nor Ole Kirkeby list any barr chords, presumably because they are not used very much in Jazz. Naturally, it would be much harder to play barr chords on a 7- or even 8-string, and a rather densely packed chord of 7 notes within two octaves might sound a bit mushy anyhow.

One rarely discussed aspect is the fact that major thirds tuning is really committed to the temperate scale, more so than the standard tuning. Moreover, playing the open strings in maThTune involves the augmented fifth, or diminished sixth, i.e. exactly two thirds of an octave. No standard eight note scale, whether major, minor or otherwise rooted contains three notes with an equal spacing of one major third. An exception is the 'harmonic minor' scale obtained by just lowering the third note of the major scale by a semi-tone, while keeping the sixth and the seventh unchanged.

All six strings in the standard tuning belong of course to the E-minor and equivalently the G-major scale. The usually recommended procedure is to tune from the lowest E in three fourths upwards to the G, then jump two octaves to the highest E and finally go down one fourth to the B. Alternatively, one may use harmonics, but in either case the result is likely to be very close to a subset of the *natural* E-minor scale. Many bonfire accompanist then rarely stray from the favorite major chords G, C, D, and their parallel minors Em, Am, and Bm, played in the nut position of course. I suspect that the basic chords that are close to G major in the cycle of fifth have shaped the public notion of what a guitar should sound like. The rather dissonmant sound obtained by striking the augmented major chord **4+4+4** formed by the open strings in major thirds certainly does not make it an easy sell to the wider public.

To actually tune the strings in temperate major thirds one has at least the three following options:

By fret: tune from the bottom by clamping each lower string at the fourth fret and tuning the next string to have identical pitch.

By ear: tune in natural, or harmonic major thirds by ear.

Electronic: resort to an electronic tuning device.

We will strongly recommend the last option, even though it might at first look like an easy, amateurish way out. Electronic tuners are by now quite small, affordable and accurate.

The first option is not highly regarded even for standard tuning. It relies on the frets being exactly in the right positions, the strings being in good condition and the player pressing down straight without any bending. I grabbed my son's Ibanez acoustic, which he bought second hand years ago and uses for sing along. Tuning that guitar by fret yielded results that were bad enough to turn anybody off major thirds tuning for good. Specifically, the intervals between first and fourth, second and fifth, third and sixth string turned out audibly larger than the desired octave. The problem was probably that neck and bridge had yielded to the tension a bit

over the years and came a little closer. Consequently, the total length of the strings between nut and bridge was too short and all fret positions were effectively a little too high. Nevertheless, the guitar could be tuned and played reasonably well in the standard pattern. Major third tuning by ear produced a sound that was clearly off even to the untrained ear. Again I am worrying about the lower end of musical and technical sophistication, where the advice: 'have your musical hardware regularly tuned by a professional' is not always well taken.

The second option, tuning by ear, is not feasible either, even on a guitar that is perfectly set up. Natural major thirds sound nearly as harmonic as fifths and fourths, so one can here deviations from them quite well. Also, one may generate the fourth harmonic of the lower string by touching it lightly above the fourth fret and the third harmonic of the upper string by touching it above the fifth fret. Both these harmonics will have exactly the same frequency when the strings are exactly one natural fourth apart. The trouble is that three such natural major thirds fall significantly short of an octave so that the tuning result is necessarily abysmal. Possibly one could learn how to count the beat between the corresponding harmonics in order to get just the right small pitch difference between them. I am a little sceptical whether this can be done by anybody but a highly trained pofessional.

To see why tuning by ear works in the standard tuning but nor for major thirds let us look briefly at the theory of the 12- tone scale.

semis	1	2	3	4	5	6	7	8	9	10	11	12
ratios	16/15	9/8	6/5	5/4	4/3	7/5	3/2	8/5	5/3	9/5	15/8	2/1

We will compare the frequency ratios $q = m/n <= 2$ of the *natural* intervals that sound nice to the human ear with those of the corresponding notes in the equally tempered scale. Actually there is no strict agreement about the natural frequencies, but we'll go with the ones that have the smallest possible numerator, irrespective of whether that is a power of two and thus closely related to the overtones. The denominator n determines by what factor the smallest common period of the root and the other note is larger than that of the root note itself. In other words, after m times the period of the root vibration the combined oscillation repeats itself exactly. Presumably a comparatively short common period signals harmony to the ear for whatever physiological reasons. The simplest ratios, which we will call natural are listed in the table above.

A well known property is that the ratios q for the second($\mathbf{3}$), minor third($\mathbf{4}$), major third($\mathbf{5}$), and fourth($\mathbf{7}$) are consistent with their complementary interval within the whole octave where one gets $2/q$ for the minor seventh($\mathbf{10}$) down to the fifth($\mathbf{7}$). The

semi-tone(**1**) is also in this way reciprocal to the major seventh(**11**). The tritone or deminished fifth is given as $q = 7/5$, which is of course not complementary to itself as $2/q = 10/7 \neq 7/5$. The only solution to the quation $2/q = q$ is the algebraic number $\sqrt{2}$, which everybody knows to be irrational. In any case the triton remains somewhat enigmatic. It is traditionally considered a very dissonant interval but now frequently occurs as transitionary *blues note*, of which there are several. Going up a harmonic second from the major third one obtains the ratio 45/32, which is a much better approximation to $\sqrt{2}$ than the more nicely sounding frequency ratio 7/5.

For the equally tempered scale the frequency ratio for the interval k semi-tones above the root is the $k-$ the power of the twelfth root of 2, i.e.

$$q \;=\; 2^{k/12} \;=\; (\sqrt[12]{2})^k \;\approx\; 1.059463^k$$

The logarithm base two $\log_2(q)$ of the frequency ratio q starts with 0 for the root note and then grows exactly by 1/12 with each semi-tone until the value $1 = \log_2(2)$ for the octave. Hence it makes sense to compare $log_2(q)$ for the harmonic frequencies to the corresponding multiple of 1/12 as we have done in the plot on page 8.

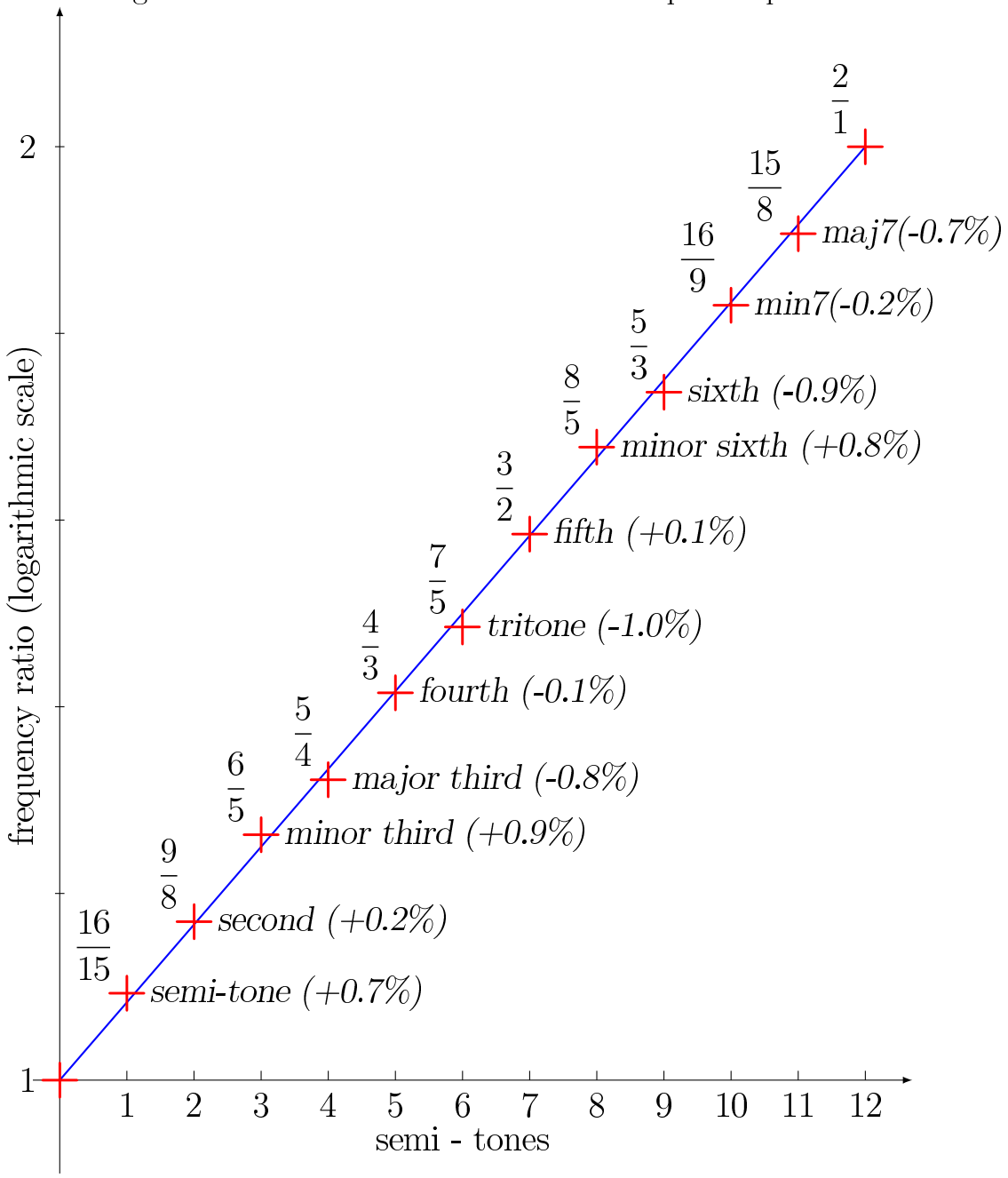
As one can see the natural fifth lies nearly unperceptibly 0.1% above the straight line representing equally tempered tuning. Correspondingly, the fourth whose frequency ratio 4/3 equals twice the reciprocal 2/3 of the fifth lies just 0.1% below the tempered tuning line. This explains why tuning by ear works so well in the standard setup. On the other hand, the natural sixth lies a much more significant 0.9% below the tempered sixth and correspondingly its reciprocal, the minor third quite the same percentage above. Finally the augmented fifth lies 0.8% above the tempered version and its reciprocal note, our beloved major third, the same percentage below the tempered ratio $\sqrt[12]{2}^4 = \sqrt[3]{2}$. Consequently, three consecutive harmonic major thirds generate the frequency ratio

$$\frac{5}{4}\cdot\frac{5}{4}\cdot\frac{5}{4} \;=\; \frac{125}{64} \;=\; 2-\frac{3}{64} \;=\; 2\left[1-\frac{3}{128}\right]$$

which is about $2.4\% = 3 \cdot 0.8\%$ below the desired factor 2 for the octave. This discrepancy is very audible even to the untrained listener so that tuning by ear is really not a feasible proposition.

Finally let us note that all natural notes above the tempered line can be reached by slightly bending the string, which has been recommended specifically for the minor third in at least one guitar book. There is nothing one can do for the other notes, whose tempered version is already to high, except possibly pushing the root note by bending towards them.

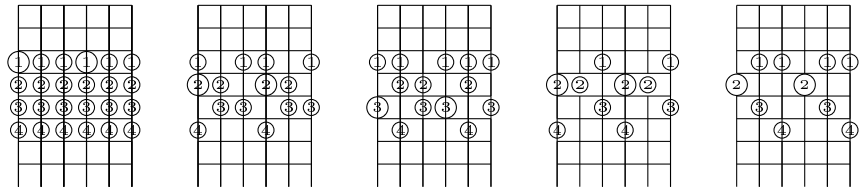
Figure 1: Deviation of harmonic tones from equal tempered scale



3 Fingering patterns

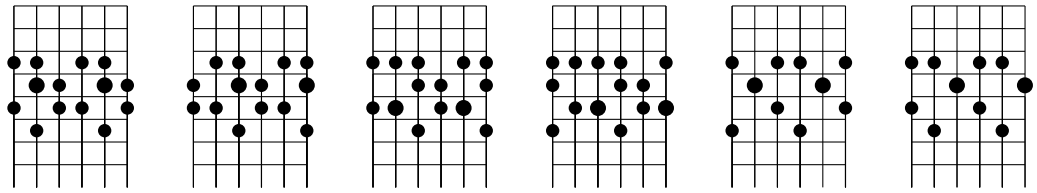
3.1 Scales

First we note that with the hand in any fixed position along the fret board, one can play a full chromatic scale covering 23 semi-tones without stretching the first or fourth finger as is neccessary in standard tuning. That should make playing melodies and solos simpler for novices and faster for experienced guitar players.



chromatic scale major scale minor scale major pentat. minor pentat.

Above we have displayed the chromatic, major, minor, and the corresponding pentatonic scales using the package of Kasper Peeters gchords[‖]. The root note has been placed on the second finger in major and the third finger in minor in order to minimize the use of the fourth finger. As one can see the patterns exactly repeat themselves on the lower and higher triplet of strings because these are exactly one octave apart. The minor pentatonic pattern over three chords looks especially simple. It can be viewed as a little hook with the minor triad as its basis and the fourth and seventh convienently reachable by the third and fourth finger.



major scale major scale minor scale minor scale minor pentat. minor pentat.

As shown above one can easily shift the scales vertically one string up or down. For the 6-string guitar the highest string aquires the pattern from the lowest and vice-versa. Thus we have the second advantage, namely that the fingering of any chord or tune can be moved vertically across strings. For example repeating a chord or

[‖]www.damtp.cam.ac.uk/user/kp229/gchords

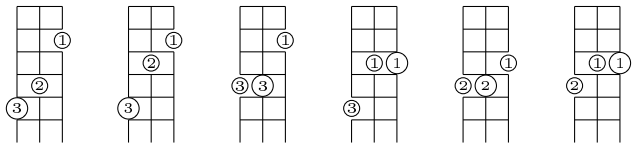
tune a fourth=**5** semi-tones higher simply requires repeating exactly the same finger action one fret and one string higher. As suggested by Ole Kirkeby the orientation along the key board is greatly simplified if one replaces the usual board markings by symbols repeating themselves every four frets.

Finally let us note that maThTune makes it a little easier to simultaneously play two melodies that go along at the same rhythm but at a varying distance of thirds and fourths. I have been told that especially with thirds that does not sound good on a distorted guitar, but as yet I do not quite understand that supposed effect.

3.2 Base triads

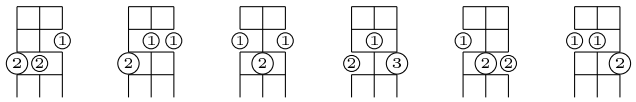
A major or minor triad and any one of its 2 inversions consists of two successive intervals of semi-tone sizes **3**, **4**, or **5**. We may denote these six triads acordingly as **4**+**3**, **3**+**4**, **5**+**3**, **3**+**5**, **5**+**4**, and **4**+**5**. In standard tuning only the last two inverted triads with a total spread of 9 can be played conveniently with two fingers on the lower four strings. The reason is that these strings are 5 semi-tones apart and thus spread a little too far for convenient fingerings of minor thirds.

Base triads on strings tuned in fourths



4+3 **3+4** **5+3** **3+5** **5+4** **4+5**

Base triads on strings tuned in major thirds

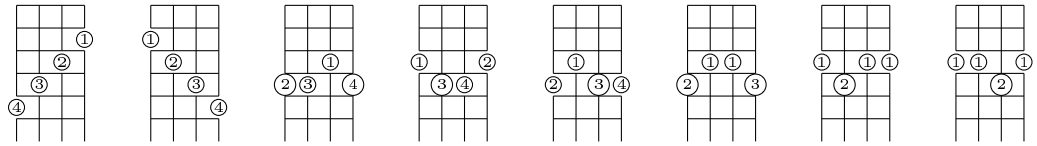


4+ 3 **3+ 4** **5+3** **3+5** **5+ 4** **4+ 5**

In maThTune only the triad **3**+**5** requires the use of three fingers and the other five can be done by placing the index and ring finger just one fret apart. As the first, index finger may serve as a bar the remaining third and fourth finger can cover the full chromatic scale on the remaining two or three strings without undue stretching. Hence the two small fingers can be employed to play melodies or to form more elaborate chords. People with spider hands like Jimmy Hendrix could even free up a third finger by playing the base triad with a thumb and either the index or the middle finger.

3.3 4-, 5-, 6-note chords

It is easy to augment the triad by a forth note, for example by replicating the root note an octave higher or adding a minor or major sept. There are 27 such quad chords formed as a succession of three intervals of semi-tone size 3,4,or 5 (i.e. minor and major thirds or fourths in the standard terminology). Their total range varies between $9 = 3 \cdot 3$ for the diminished minor chord **3**+**3**+**3** and $15 = 3 \cdot 5$ for the extended sus7 chord **5**+**5**+**5**. Since $9 = 3 \cdot 4 - 3$ and $15 = 3 \cdot 4 + 3$ even these two rather bizarre chords and certainly the more regular ones like **4**+**3**+**5**, **4**+**3**+**4**, and **3**+**4**+**3** can be played without any finger stretching on four successive strings. In contrast the four lower chords in standard tuning span 15 semi-tones so that the minimal spread of any quad chord played without stretching is 15-3=12, which excludes already minor and major sept chords. Consequently, their standard fingering always jumps certain intermediate notes of the quad chord. For example, the popular standard C7 shape omitts the fifth note G alltogether but repeats the root C on top of the seventh B once more. Hence we may denote this quad chord more precisely as **4**+**6**+**2**. It can be played in maThTune by damping the middle string in the chord **4**+**3**+**3**+**2** played on five strings.

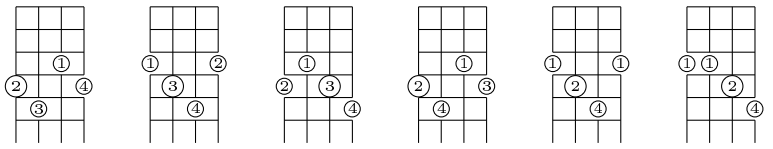


3+**3**+**3** **5**+**5**+**5** **4**+**3**+**5** **5**+**4**+**3** **3**+**5**+**4** **3**+**4**+**5** **5**+**3**+**4** **4**+**5**+**3**

In standard tuning the tones of the four lower strings are 5 semi-tones apart, whereas the average interval in a repeated triad like E,G♯,B,E,G♯,B spans only 4 semi-tones. Hence it is impossible to play them as a consecutive chord in standard tuning. Rather one has to omitt certain tones playing instead for example E,B,E,G♯,B,E in the standard E major chord shape. This particular, scattered triad contains the root note E thrice, the fifth B twice, and the major third G♯only once. In other words it is quite close to a power chord consisting solely of the root and the fifth, which gives it a clear and definitely major sound. In contrast the tightly packed chord E,G♯,B,E,G♯,B contains each tone of the simple triad E,G♯,B exactly twice. Consequently, it includes the minor third between G♯and B twice, which makes it sound rather soft, more like a minor C♯chord with a suspended root. On the other hand the inverted major chord B,E,G♯can be repeated to yield B,E,G♯,B,E,G♯without losing its clearly major like character. The second inversion G♯,B,E in includes also a minor third and thus

sounds in its repeated form G♯,B,E,G♯,B,E more like the soft minor g♯6b chord. To avoid these effects one may damp out the last or first string and thus preserve only one minor third within the chord. Fortunately, these reductions to a five tone chord are suggested by the finger positions anyway and thus come rather natural.

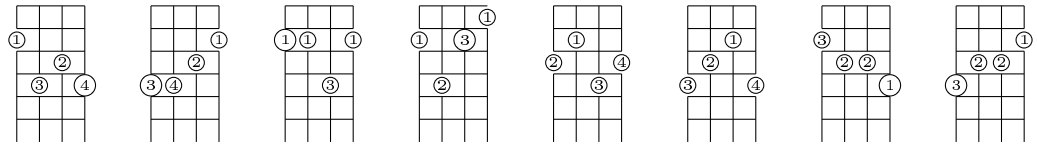
Sus4 Chords



5+2+5 **5+5+2** **3+5+5** **5+2+5** **5+5+2** **4+5+5**

The chords **4+3+5**, **5+4+3**, and **3+5+4** are the three inversions of the major triad appended with the octave on the lowest note. The corresponding minor chords are **3+4+5**, **5+3+4**, and **4+5+3**, which are also depicted above. Obvious and simple variations are adding the fourth by extending the first interval from the root and thus suspending the third note. In major one obtains the chords **5+2+5**, **5+5+2**, and **3+5+5**. In the last variant the third is not fully suspended since is still present on the lowest string. In minor one obtains the chords **5+2+5**, **5+5+2**, and **4+5+5**, where the third survives again in the last variant. The first are the same as in the major case but their fingerings will be different if one wishes to conveniently switch back and forth to the basic chord. Similarly, one can easily obtain a sept chord by suspending the **5** or dropping the octave. Thus we obtain the following four note variants, of which only the last two are minor chords.

Dominant 7 Chords

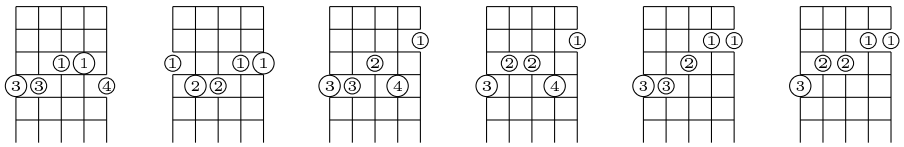


6+3+3 **4+3+3** **4+6+2** **6+2+4** **3+6+3** **3+3+6** **5+4+5** **3+4+3**

Here we have three variants for the sept chord of the basic inversions because either the root note or its octave can be dropped to the minor seventh or the latter may suspend the fifth as also done in the familiar C7 shape. On the other hand dropping a singly represented root in a minor chord turns it into a plain major chord so these two variants have not been included. Even though they include seconds the sus4 and 7 chords listed encompass only three fret levels each, so they can be fingered

without too much difficulty. The same is still true when we consider 9 chords, which can be obtained by either dropping the third or suspending the root itself. In either case it is better to play five strings, which is also true for major seventh chords.

Major 7 and others



4+3+4+5 **5+4+3+4** **4+3+5+2** **3+4+5+2** **4+3+3+4** **3+4+3+4**

The first two chords **4+3+4+5** and **5+4+3+4** are major with the root or its octace dropped to a major seventh. The third chord **4+3+5+2** is a complete 9 chord as it is played for example in *Every breath you take* by the Police. However here there is no stretching of fingers and the 9 can be played concurrently with the octave. If the latter is also dropped by one full tone one obtains the fourth chord **4+3+3+4** whose minor version **3+4+3+4** comes last. These last two 7/9 chords are really my favorites, as the fourth finger can hammer onto the fifth and sixth string two frets above the first finger to reestablish the *octave* or raise the ninth to a major third. When the root is on the second string they can also be underpinned with another fifth using the thumb, which yields the 6-note chords **5+4+3+3+4** and **5+4+3+3+4**. Of course most other chords can be extended to 6-note versions too, which yields the nonexaustive listing in the Appendix.

Transitions between minor and major chords are very simple. The same is true for the corresponding major seven variants and one can always hammer on the tonic from half a note below when playing a barr code. This use of the major seventh as an upbeat is a cheap but quite pleasent effect. Just like on the E-shape in the standard tuning the fourth suspending the third and the sixth suspending the fifth can be played with the fourth finger, which can hammer-on or pull-off. However, to free the fourth finger when playing major, one has to hold down the neighboring tonic and fourth with the third finger, which may be difficult on a wide finger board. For me reaching the dominant seventh in this way is already quite a stretch. Hence I can only switch easily between major or minor chords and their major seven variants by using the latter as a base and suspending the major seven by the octave with the fourth finger. Except in the first position, i.e. for C and E, the classical blues change fifth-sixth-seventh(dominant) and back is not nearly as easy to do as in the E or A shape for standard tuning. Rather, modifying a full chord one can only move conveniently on top or below the fixed tonic and major third interval.

4 Music notation in major thirds

As we mentioned in the introduction, reading music in the classical notation seems particularly hard for guitarist. It is never really easy, but the fife line staff is steeped in tradition and a cherished tool of the trade for classical musicians. For example, transposing or transscribing a melody into another key is a task that really tells the professionals form amateurs, and can hardly be done mentally on the fly.

There is a whole community of people who have been debating and proposing alternative notations, mostly based on chromatic staffs. There the vertical position of a note symbol uniquely determines its pitch, at least after some reference level, typically that of c' has been established. There is no need for flat or sharp symbols; neither as part of a *key signature* at the beginning of a line nor later on for raising or lowering individual notes within a measure. If defined at all, the key of a piece is usually recognizable to the reader, but it may also be annotated somewhere near the top by the engraving system.

The chromatic notation systems go apparently back to Schönberg and are surveyed on the site**. They differ mainly in their placement of staff and ledger lines. Some require also changing the shape of note heads and the representation of note duration, but there does not seem to be any real need for that. That claim is supported by the observation that literature for rock and pop guitar usually specifies rhythm by the classical system of flags but encodes pitch in numbers from 0-24 representing fret positions. These widely used *tabulature* notations constitute a rather curious mixture of analog and digital information, whose overall appearence is not very intuitive. In contrast our proposal, like all chromatic staffs, represents pitch fully analog as vertical elevation. Additional information for example regarding fingering may be annotated in a digital fashion without unduely cluttering the sheet.

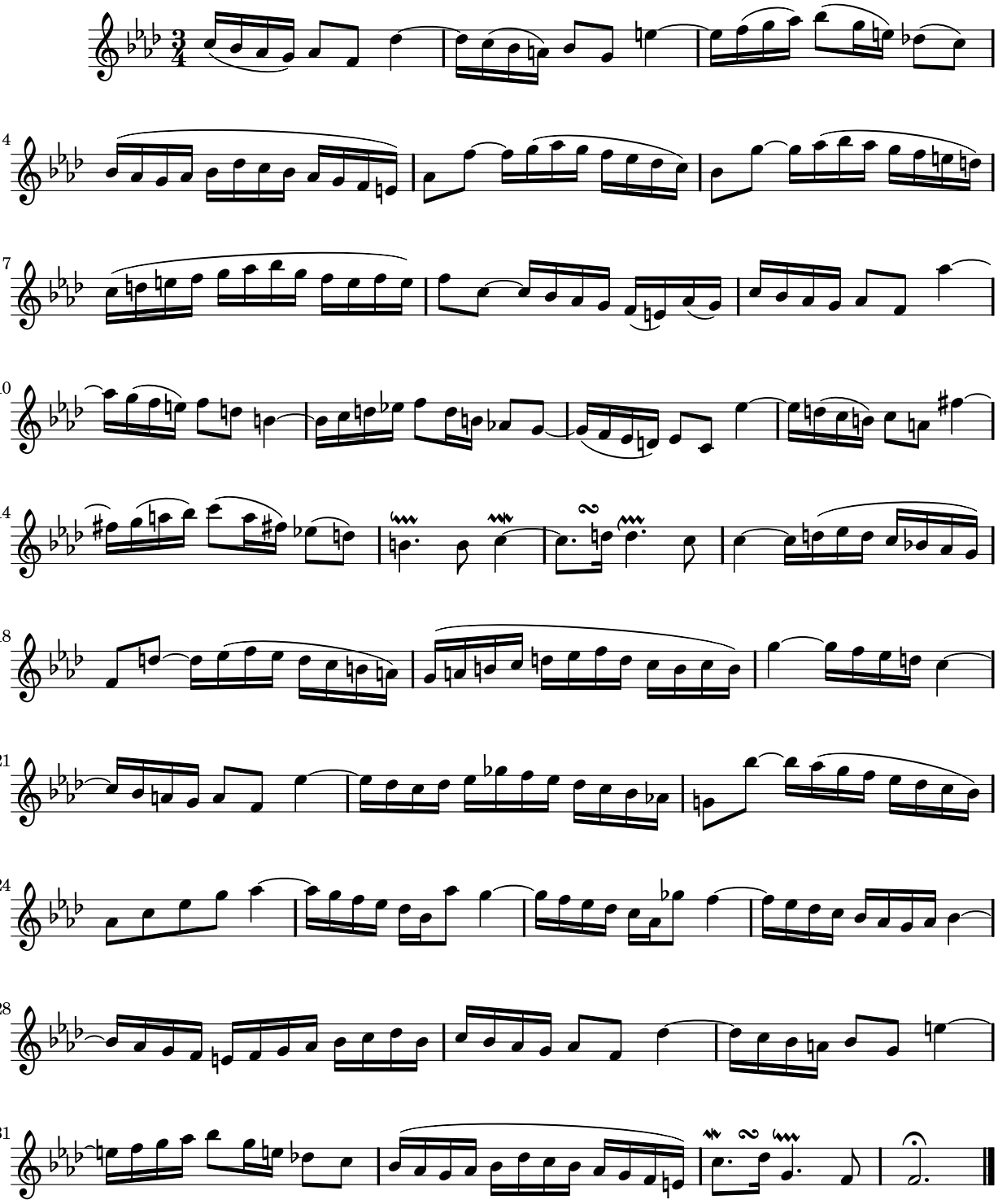
Disregarding the upper limit of five lines recommended in the criteria of the music notation association we propose to use six, representing exactly the pitches of a six string guitar tuned in major thirds. In the remainder we will assume that these levels are the notes G, B, D♯, G, B, D♯, but of course a simultaneous shift up or down could easily be accomodated. In any case the G-clef will have to indicate the position of the G'. Because of the 1-1 association with the strings it would make for us little sense to leave any one of the lines off. One might consider drawing a pair of them, e.g. the two B's just a bit fatter to simplify orientation. Although, again tabulature readers seem to have no trouble visually plucking information from six vertically spaced but otherwise identical lines. We discuss the proposal on the example of Bach's Invention 9 provided by Kevin Dalley.

**musicnotation.org/musicnotations/index.html

Invention 9

J. S. Bach (1685-1750)

BWV 780



Music engraving by LilyPond 2.12.3—www.lilypond.org

First we see the classical notation of the peice in F minor requiring four ♭ in the signature and involving quite a few accidental ♭ and ♯ in the body. On the next page 16 we see a preliminary version of the maThTune notation. Between any two lines we need to visually distinguish three semitone positions, just above the lower line, right in the middle, and just below the upper one. It looks as though one should be able to do that without any extra annotations. That should also apply for the ledger lines above and below the six staff lines. Unfortunately I have not yet managed to eliminate the in-between ledger lines that are not major thirds but rather an odd number of full seconds apart from the staff lines. The even multiples should show up whenever the notehead is on, just above, or just below its position. Whenever a ledger is activated in this way all the other ledgers between it and the staff lines should also be displayed, as is customary in standard notation. The treble clef's center indicates the position of middle G. I have experimented drawing the two G lines a bit thicker, but that makes the visiual impression just more busy.

Overall, the display appears already quite satisfactory and not too different to what one is used to in the standard notation. The growth in vertical depth compared to the classical sheet shown on the preceding page is about 50%. That seems reasonable because we have one more line and the interval between lines is now always 4 semitones, whereas for the standard notation it varies between 3 and 4. Since the lowest string is never played in this particular piece we could achieve a compression by simply leaving the lowest staff lines off, and possibly adding one for the high B at the top. To make this custom made staff identifiable one could either rely on the G clef in front of each line or emphasize the corresponding staff line by making it noticeably thicker than the others. Even within a piece one could use such a variable system of up to six staff lines always including the central G level as sort of an achor. That would give a total range of 11 major thirds and thus almost three octaves without any additional ledgers.

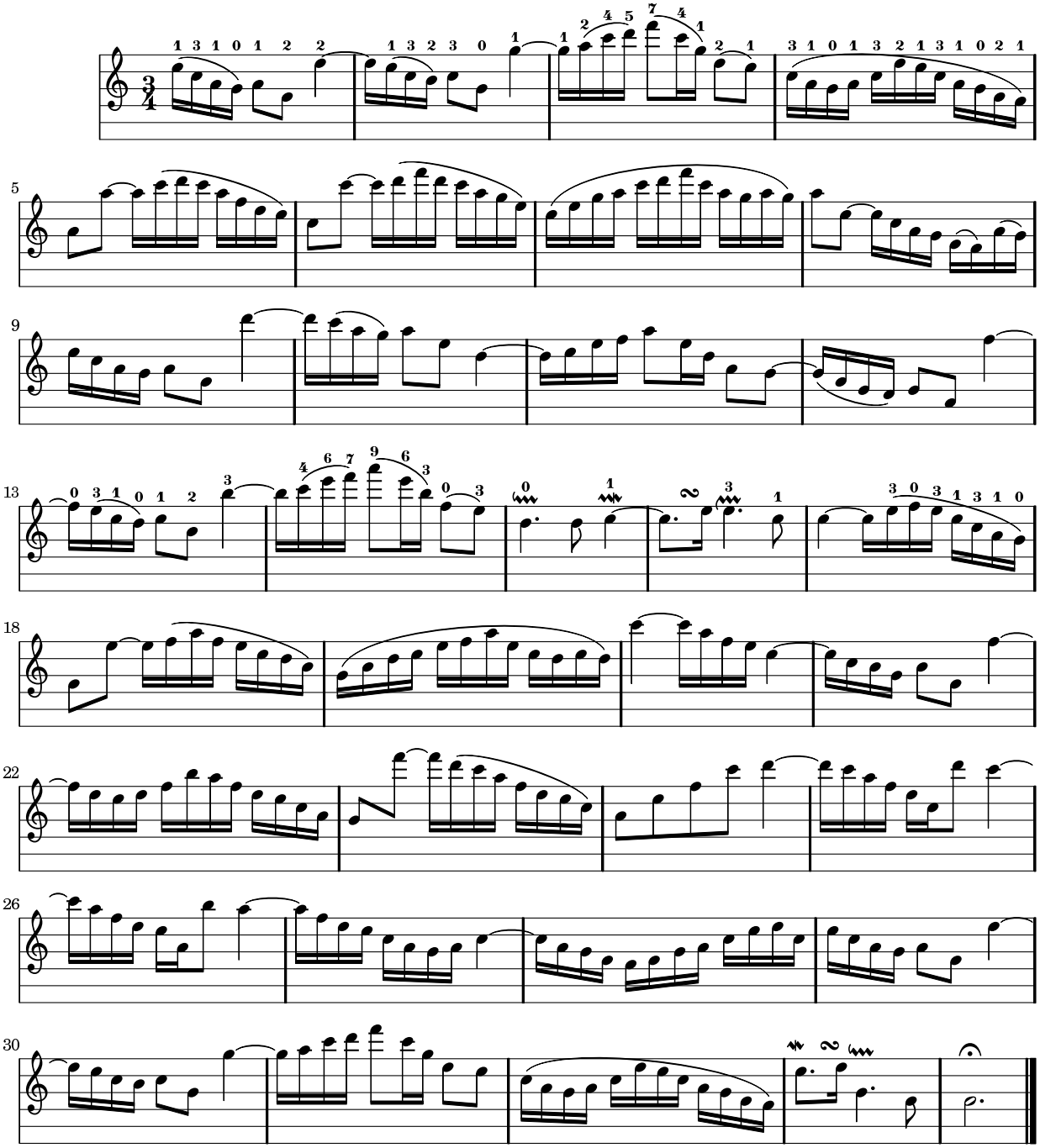
Even the preliminary version is eminently playable for guitarist. With the left hand in first position right at the nut, all notes on lines are open strings, all notes just above are played with the first (index) finger, all notes in between with the second (middle) finger, and all just below a line by the (third) ring finger. In other words the finger number coincides exactly with the fret number. That simple fingering is annotated above the first two measure of the score. Here the fourth (little) finger is not yet needed, it only comes into play in higher hand positions, for example the fifth. There the same assignment as above applies except that the fourth finger now has to cover all notes right on the lines.

Invention 9

maThTune

J. S. Bach (1685-1750)

BWV 780



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Music engraving by LilyPond 2.12.3—www.lilypond.org

Above the third measure I have annotated the fret positions on the highest D# string, which somewhat resembles tabulature notation. Of course fret 7 on the highest string corresponds to fret 11 on the second highest and fret 19 on the third. On the highest string it can be played with the third finger in the fifth hand position, and similarly fret 5 with with the first finger. The first twelve measures of the Invention 9 range from C nearly two octaves up to B♭. That segment can be played throughout in the fifth hand position just discussed with the root of F major resting on the second finger, which we have used in the scales on page 10. Only in measure 14 one hat to go up once to C", which can be done by stretching the forth finger just one semi-tone, or of course moving up to a higher position.

In contrast to tablature the chromatic notation does not prescribe the hand position, but leaves that choice to the player. Of course as we have done selectively on the example one may annotate particular fingerings. This is customary in classical music education and provided for by Lilypond. Of course, one is not restricted to that particular engraving system and also might write the chromatic notation by hand. By the way, at a time when barely anybody writes letters by hand anymore it seems just a tadd quaint that the music notation association stipulates that pencil and paper or black board and chalk must be sufficient tools for writing down music in the new notation. We beleive our proposal fulfills this requirement in principles, although the placement of the note heads between the lines must be quite accurate. A possible variant would be to provide all notes in the middle between two staff lines also with a short ledger line.

5 Summary and Conclusion

The proposal discussed in this paper concerns the tuning of guitars and the transcription of music in a notation system that goes along with it very nicely. Both aspects are based on major thirds, which have already been suggested as uniform tuning interval for guitars by various accomplished musicians. We see the following main advantages

- full chromatic scales without stretching
- vertical transposition of fingerings
- base triads using just two fingers
- 4-, 5-, and 6-note chords without skips

Whether or not one likes the tighter chord structure over the scattered ones enforced by standard tuning seems to be a matter of taste. Of course, in maThTune one can include larger intervals and thus obtain sparser chords by damping certain strings.

Regarding the placing of the slightly reduced total range we prefer G, B, D♯, G, B, ♯ to E, G♯, C, E, G♯, C, or an extension thereof with additional strings. For people playing on the full fret board with cut-off the reduction of the total range of 4 octaves by one major thirdis probably not a serious disadvantage.

We noted that unless the frets and strings of a guitar are in perfect condition maThTune tuning should best be realized using an electronic tuning device. Tuning by ear is definitely out because three harmonic major thirds fall short off an octave by some rather audible 2.4%.

The major challenge of the maThTune proposal is the need to relearn all melody and chord fingerings. The much increased regularity should certainly help in that respect as well as the proposed adaption of a chromatic staff notation to the needs of guitar players. Here we see the following advantages

- clear representation of pitch as elevation without ♯s and ♭s.
- notehead position w.r.t nearest line fixes finger selection on guitar
- rhythmic notation and general appearance remains unchanged

No matter where one is on the fret board one of four assignments between note head position and finger selection applies.

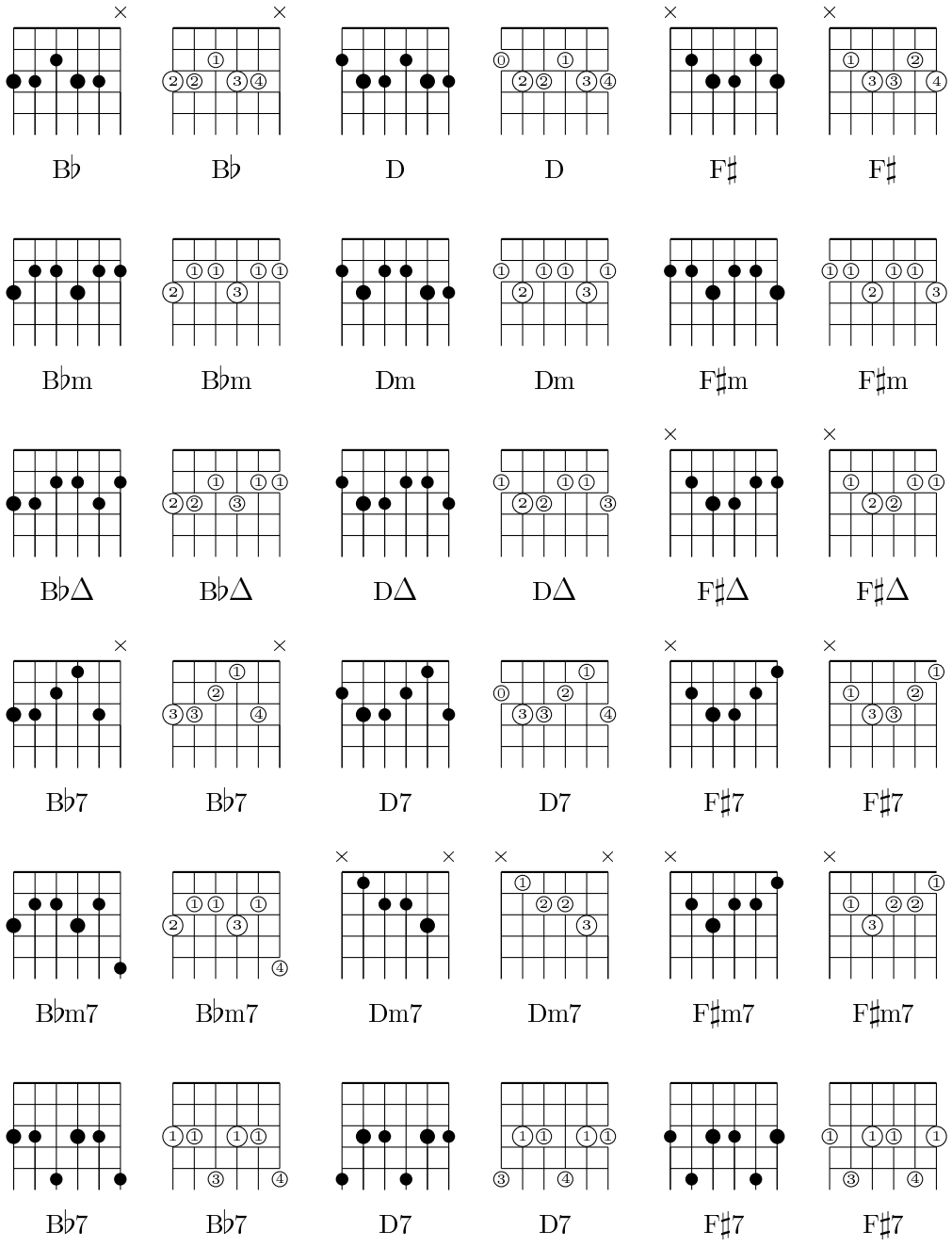
While some details remain to be worked out maThTune notation can be generated within LilyPond using Kevin Dalleys patches and adding the line

```
(cons "maThTune" (vector '(2 6 10) '() -5 7 12 ly:pitch-semitones #()))
```

to his common.ly file. All pieces to be engraved must be provided in the LilyPond format, which can be generated automatically from some other formats. There is a growing data base of music in the Mutopia project[††] .

[††]http://www.mutopiaproject.org/piece-list.html

6 Appendix


B♭
B♭
D
D
F♯
F♯
B♭m
B♭m
Dm
Dm
F♯m
F♯m
B♭Δ
B♭Δ
DΔ
DΔ
F♯Δ
F♯Δ
B♭7
B♭7
D7
D7
F♯7
F♯7
B♭m7
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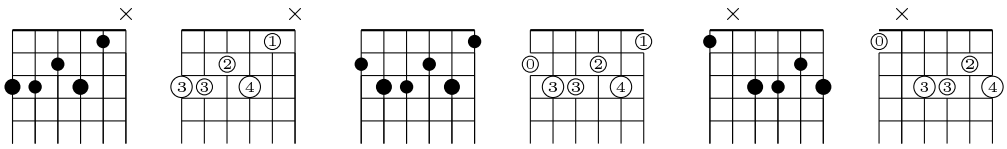
D7
F♯7
F♯7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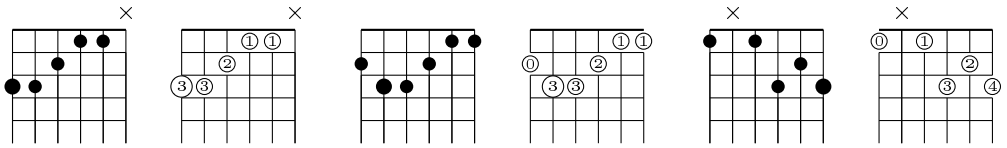
B♭add9
Dadd9
Dadd9
F♯add9
F♯add9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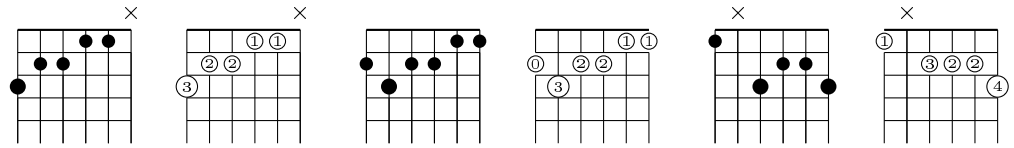
B♭9
D9
D9
F♯9
F♯9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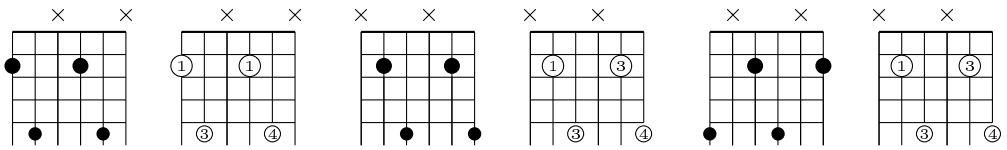
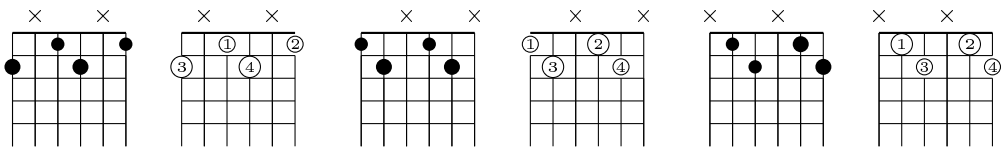
B♭m9
Dm9
Dm9
F♯m9
F♯m9


A5
A5
C♯5
C♯5
F5
F5


A5
A5
C♯5
C♯5
F5
F5